

The Quality Adjusted Price Index in the Pure Characteristics Demand Model

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Abstract

This paper computes a quality adjusted price index for the personal computer CPU from 1996 to 2000. The index is based on the pure characteristics demand model. I first compute the quality adjusted price index for the whole market, and show that it is very comparable with the hedonic price index but more sensitive to changes in product quality. Two types of the hedonic index are considered. One is the dummy variable index and the other is the formulation in Pakes (2003). When I group consumers by their willingness to pay for attribute improvement, the index shows consumer groups are differently affected by their product choices.

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1 INTRODUCTION

This paper computes a price index in the computer processing unit (CPU) market using the pure characteristics demand model (PCM hereafter) developed by Berry and Pakes (2007). The index is constructed using the compensating variation from an estimated utility function. The index constructed in this way is called the quality adjusted price index or the cost of living index (Trajtenberg 1990; Nevo 2003.)

The quality adjusted price index has been computed for the CT scanner (Trajtenberg 1990), the automobile (Pakes, Berry, and Levinsohn 1993), and the ready-to-eat cereal (Nevo 2003). However, all of these studies use the (random coefficient) logit demand model, which has the idiosyncratic logit error term in the utility function (Berry, Levinsohn, and Pakes 1995).

In the PCM the compensating variation captures surplus changes solely related to product characteristics, and consumer heterogeneity and product choice are fully described by random coefficients on product characteristics.

I also compute the hedonic price index for comparison. I consider two types of the hedonic index. One is the dummy variable index that is based on coefficients on the time dummy variables in the hedonic regression (Griliches 1971.) The other is the formulation in Pakes (2003) (the Pakes index hereafter) that provides the upper bound on the quality adjusted price index. Trajtenberg (1990) argues that the quality adjusted price index captures changes in product quality more accurately in any cases, while the use of the hedonic price index is justified for the case where quality changes result from process innovation.

Exploiting the advantage of having the estimated utility function, I group consumers according to their willingness to pay for attribute improvement and compute the index for consumer subgroups. Consumers are heterogeneous in two aspects. First, they differ in their willingness to pay for quality improvement, mainly measured by the processing speed. Second, they differ in their valuations on capacity of extra data storage inside the processor (the level 2 cache.)

The group index captures welfare changes consumers experience from particular products that they

purchase. For example, in a market with vertically differentiated products, consumers endowed with low values of the random coefficient buy high quality products. When new products of improved quality are introduced, they experience different price changes compared to consumers who buy low quality products (i.e., those with high values of the random coefficient.)

Using product-level data for the personal computer CPU from 1993 to 2000, I first compute the quality adjusted price index for the whole market in the one random coefficient PCM. The biggest decline occurs in the second and the third quarters of 1998 with 34 % and 32.8 % declines respectively. These are periods when Intel, a dominant manufacturer of the CPU, slashed price to recover its decreasing market share. The smallest decline occurs in the first quarter of 1999 and in the third quarter of 2000 with 1.4 % and 6.7 % declines respectively. From the second quarter of 1999 to the second quarter of 2000, the index steadily decreases by 20 ~ 28% thanks to a rapid increase in the processing speed and stable introductory prices for new products.

In the two random coefficient PCM the market index shows that consumers gain much more than in the one random coefficient model when new products are introduced at the low end of the market. For example, in the first quarter of 1999 a half of new products were introduced at the low end of the market. While the index in the one random coefficient model declines by 1.4 %, it declines by 41.2 % in the two random coefficient model. This is because the one random coefficient model averages out welfare gains from these new products.

The comparison with the hedonic price index shows that the two indices are very comparable, but also shows that the quality adjusted price index is more sensitive to changes in product quality. Differences are prominent when new products are introduced, and when prices of existing products fall without new products being introduced.

The group index shows that price changes are significantly different across groups. For example, in the second and the third quarter of 1998, periods with the biggest index declines, consumers who bought new

products experienced more than a 40 % decline, while consumers who bought the lowest quality products experienced less than a 10 % decline. The group index also shows that the larger price decline in the two random coefficient PCM in the first quarter of 1999 is driven by consumers at the low end of the market.

2 THE MODEL

2.1 Consumer Demand

Suppose I observe $t = 1, \dots, T$ markets, and in each market there are N_t consumers. Given market t , the indirect utility of consumer i from purchasing product j is

$$\begin{aligned} v_{ij}^t &= \delta_{ijt} - \alpha_i p_{jt} = \mathbf{x}_{jt} \boldsymbol{\beta}_i - \alpha_i p_{jt} + \xi_t + \Delta \xi_{jt}, \quad \text{for } 1 \leq j \leq J \\ v_{i0}^t &= \delta_{0t} \end{aligned}$$

where \mathbf{x}_{jt} is a vector of observable characteristics of product j , $\boldsymbol{\beta}_i$ represents a marginal utility that consumer i derives from product characteristics, p_{jt} is the price of product j , and α_i is the individual-specific price coefficient. ξ_t is the mean utility of unobservable characteristics at period t , $\Delta \xi_{jt}$ is a detrended unobservable characteristic with $E(\Delta \xi_{jt}) = 0$, and δ_{0t} represents the utility of choosing the outside option.

This demand model is called the pure characteristics demand model, since it does not have the additive idiosyncratic taste component (Berry and Pakes 2007.) The simplest version of PCM is so called the vertical model in which there is only one random coefficient. In the vertical model consumers agree on product quality but differ in their willingness to pay for quality improvement (Bresnahan 1987; Shaked and Sutton 1982; Berry 1994.) The consumer consensus on the ranking imposes a substitution pattern that only products in the adjacent "neighborhood" are substitutes for each other. However, more random coefficients can be added to product attribute variables. With multiple random coefficients, the consensus on the product

ranking does not hold any more, and a substitution pattern becomes more flexible.

As in all discrete choice models, the utility from the outside option and the mean utility of unobserved characteristics cannot be separately identified without further assumptions. If the value of the outside option is assumed to be zero, *i.e.*, $\delta_{0t} = 0$, for all periods, time dummy variables identify changes in the mean value of unobservable characteristics over time. On the other hand, if the mean value of unobservable characteristics is assumed to be zero, *i.e.*, $\xi_t = 0$, for all periods, then the same time dummy variables identify changes in the utility from the outside option.

Pakes et al. (1993) propose another identifying assumption that on average unobservable characteristics of continuing products do not change over time, where continuing products are defined as products that exist in both periods of $t - 1$ and t . Then, a change in the utility from the outside option can be identified from the mean difference between $(\delta_{jt} - \delta_{0t})$ and $(\delta_{jt-1} - \delta_{0t-1})$ of those continuing products (see Song (2007) for details.)

2.2 Construction of the Quality Adjusted Price Index

The quality adjusted price index is computed using the compensating variation. For the discrete choice model, the compensating variation (CV) for consumer i in period t is defined as

$$CV_{it} = \frac{u_i^t - u_i^{t-1}}{\alpha_i},$$

where $u_i^t = \max_j (v_{ij}^t)$ (Small and Rosen 1981.) One should note that, since income effect is assumed away, there is no difference between the equivalence variation and the compensated variation.

Summing CV_{it} over all consumers, I obtain

$$CV_t = \sum_{i=1}^{N_t} \left(\frac{u_i^t - u_i^{t-1}}{\alpha_i} \right).$$

Assuming that changes in prices from period $t-1$ to t take the form $p_{jt} = (1 - \mu_t)\bar{p}_{t-1} + \Delta p_{j(t-1)}$, that is, the distribution of prices moves leftward by a factor of $(1 - \mu_t)$ but the variance remains the same, Trajtenberg (1990) shows that a quality adjusted price index can be computed as

$$(1 - \mu_t) = \frac{PI_t}{PI_{t-1}},$$

where $\mu_t = \overline{CV}_t / (\overline{CV}_t + \bar{p}_t)$ and $PI_0 = 100$. $\overline{CV}_t = CV_t/N_t$ and \bar{p}_t is the average price of products in period t . $\frac{1}{(1-\mu_t)}\bar{p}_t$ can be interpreted as the reservation price that would make the consumer indifferent between the set of improved products in period t and the older set in $t-1$. In other words, if the products in t were "offered at an average price of $\frac{1}{(1-\mu_t)}\bar{p}_t + \varepsilon$ (for any small $\varepsilon > 0$), the consumer would prefer to have the older set instead" (Trajtenberg 1990, p. 33).

Exploiting the structure of the utility function, I go one step further and construct the quality adjusted price indexes for consumer subgroups. Consumers are grouped according to their values of random coefficients. Consumers with different values of random coefficients make different consumption choices in each period. For example, if products are vertically differentiated so that the utility function with one random coefficient represents consumer preferences, *i.e.*,

$$\begin{aligned} v_{ij}^t &= \delta_{jt} - \alpha_i p_{jt} = \mathbf{x}_{jt}\boldsymbol{\beta} - \alpha_i p_{jt} + \xi_t + \Delta\xi_{jt}, \quad \text{for } 1 \leq j \leq J \\ v_{i0}^t &= \delta_{0t}, \end{aligned}$$

a consumer with a low value of α_i always buys a higher quality, or equivalently a more expensive, product than a consumer with a high value of α_i . As a result, these two types of consumers experience different welfare changes over time, and the price index for each group reflects this difference.

In the case of the one random coefficient model, consumers can be divided into ten groups if the ten percentiles of α_i are chosen, five groups if the twenty percentiles of α_i are chosen, and so on. If the

utility function has two random coefficients, consumers will be grouped along with two dimensions of their preferences, and divisions on each dimension are determined by values of each random coefficient.

Given a consumer group I_g , CV_{it} is summed over consumers belonging to I_g to obtain

$$CV_{gt} = CV_t(i \in I_g) = \sum_{i=1}^{n_g} \left(\frac{u_i^t - u_i^{t-1}}{\alpha_i} \mid i \in I_g \right),$$

and the quality adjusted price index for I_g is computed as

$$(1 - \mu_{gt}) = \frac{PI_{gt}}{PI_{gt-1}}, \quad (1)$$

where $\mu_{gt} = \overline{CV}_{gt} / (\overline{CV}_{gt} + \bar{p}_{gt})$ with $\overline{CV}_{gt} = CV_{gt} / n_g$. \bar{p}_{gt} is the average price of products that consumers in this group purchase in period t .

It is not the unique feature of the PCM that consumers are grouped by values of random coefficients. In the random coefficient logit demand model (BLP), one can also construct the group index. However, the implication of this index is different due to the idiosyncratic logit error term. Consider BLP with α_i as the only random coefficient. There are two sources of consumer heterogeneity. One is α_i and the other is the idiosyncratic taste term (ε_{ijt}) that is independent across consumers, products and time.

There are two ways of grouping consumers in this case. The first way is to group consumers by the value of α_i and then draw *i.i.d.* random variables from $G(\varepsilon)$ to independently assign them to each consumer every period. However, this can be problematic as consumers in the same group can make totally different purchase decisions over time due to *i.i.d.* ε_{ijt} . A consumer who buys the most expensive product in period t may buy the least expensive one in period $t + 1$, and one may question implications of this group index.

The second, maybe a more reasonable, way is to group consumers by the value of α_i and compute the expected welfare changes. However, all consumers have a non-zero probability of buying a given product. Therefore, impact of product entry/exit is not as significantly different across groups as in the PCM, so it is

much less meaningful to construct the group index in the model with an idiosyncratic error term.

2.3 Comparison with Hedonic Price Index

I consider two types of the hedonic price index in comparison to the quality adjusted index. One is the dummy variable hedonic index and the other is the formulation in Pakes (2003). The dummy variable index refers to the index built from the time dummy variables in the hedonic regression. It is hard to link this index to the quality adjusted index due to its lack of a theoretical basis. In perfect competition coefficients in the hedonic regression represent the marginal cost of changing product characteristics, but in oligopolistic markets they also include markup. This makes it hard to predict their signs. Pakes (2003) discusses this problem in details. Nevertheless, it is often used in markets where product characteristics significantly change over time. I present two specifications, one with price as the dependent variable and the other with the log of price as the dependent variable.

The Pakes index, on the other hand, provides an upper bound on the quality adjusted price index. Its main idea is to adjust income such that consumers can buy the same bundle available in a base period at prices of a comparing period. This income adjustment is an upper bound of true compensating variation. When product quality improves without significant increases in price, money is taken away until the consumer is moved back to a consumption bundle in the base period, and the amount taken is less than to move her to the base period utility level. So the Pakes index is supposed to be no higher than the quality adjusted index in an absolute term in this case.

Unfortunately I cannot construct the exact Pakes index with my data set. To construct this index the price hyperplane should be estimated in the hedonic space for each period, but I do not have enough observations to do this. So I impose a constraining assumption that all periods share the same constant and error terms. This makes the estimated hyperplane less flexible than that of the Pakes index, but still allows me to apply it to my data set. I call this index the constrained Pakes index.

In particular, I first run the hedonic regression with processor speed and processor speed interacted with the time dummy variables. The constant term and dummy variable for smaller level 2 cache are included but not interacted with the time dummy variables. The price hyperplane for period t is estimated by $h_t(c_t) = X_t \hat{\beta}_t$. When the log of price is used as the dependent variable, $h_t(c_t) = \exp(X_t \hat{\beta}_t) \exp(0.5 \hat{\sigma}^2)$ where $\hat{\sigma}^2$ is a consistent estimate of the variance of the error term. The average compensating variation for period t is calculated as

$$\sum_{j \in c_t} (h_{j,t+1}(c_t) - h_{jt}(c_t)) \times \frac{q_{jt}}{\sum_{j \in c_t} q_{jt}}$$

where $h_{t+1}(c_t) = X_t \hat{\beta}_{t+1}$ and q_{jt} is the quantity of product j sold in period t . $h_{t+1}(c_t)$ denotes the price hyperplane for a consumption bundle of period t at prices of period $t + 1$.

3 THE PRICE INDEX IN THE CPU MARKET

3.1 Estimates of the Demand System

The data used in estimation consists of price, quantity sold, and characteristics of products of Intel and AMD from the second quarter of 1993, the first period in which Intel introduced Pentium processors, to the third quarter of 2000. Main characteristics are the processing speed in mega hertz (MHz) and the capacity of the level 2 cache which is extra memory storage inside the processor. The level 2 cache enables the processor to speed up computation by reducing a communicating time between the processor and the main memory chip outside the processor. Usually Intel and AMD produce two sizes of the level 2 cache, and there is a significant price difference between the two types. The appendix describes the data used in demand estimation.

I use the second quarter of 1996 as the base period in reporting all price indices. This is mainly to eliminate the role of 386 and 486 processors in comparing the quality adjusted index with the hedonic index. In that period Intel stopped producing 386 and 486 processors, and since then their market share remained under 5% until they disappeared in the third quarter of 1997. These processors are treated as the outside

option in demand estimation due to no data on price. The quality adjusted price index is sensitive to this treatment as the utility of buying a product is in a relative term to that of choosing the outside option. The hedonic index is less sensitive as it only considers prices of products sold.

Table 1 shows market trends during the sample periods. First of all, product quality, measured by the processor speed, improved significantly. The first column shows that the (quantity weighted) average speed increased from 137 MHz to 701 MHz in less than five years. This is more than a 500 % increase. The maximum speed rapidly increased in the late 1990s. The average lifetime of the CPU was about eleven quarters before 1998, and shortened to five quarters afterwards with higher rates of product introduction and obsolescence.

Secondly, despite the drastic change in product quality, the price distribution in each period did not change significantly. The third and fourth columns in Table 1 shows that the (quantity-weighted) average price has declined slightly. The price of new products has been stable. The first Pentium Pro and the first Pentium II processors, introduced in 1995 and 1997 respectively, were marketed at higher than \$1,000.00, but the subsequent new products were never priced at higher than \$1,000.00.

Lastly, the last two columns of Table 1 shows that the market expanded. More consumers bought the CPU as its quality improved and price went down. There are a few periods where the total quantity went down compared to the previous period, but the market expanded by about 260 % in less than five years. This suggests that consumers prefer new products increasingly over time.

Considering all these trends, one may conclude that consumers are better off with improved and cheaper CPUs. However, it is not trivial to measure how much they are better off with product quality changing drastically over time.

Table 2 shows estimates of the PCM. The estimation procedure is explained in details in Song (2007, 2008). Observable characteristics include the processor speed (*Speed*), the processor speed squared (*Speed*²), and the dummy variable for a smaller capacity of the level 2 cache (*No_Cache*) in the first specification,

and the log of the processor speed ($\log(\textit{Speed})$) and the dummy variable for a smaller capacity of the level 2 cache in the second specification. The dummy variables for quarters are included in all specifications, but are not reported in the table.

In the one random coefficient model, the random coefficient is put on the price variable, and is assumed to be distributed log normal with the location parameter fixed at 0, *i.e.*, $\log(\alpha) \sim N(0, \theta_1)$. This model imposes that consumers agree on the ranking of product quality but differ in their willingness to pay for quality improvement. In the two random coefficient model, another random coefficient is put on the dummy variable for a smaller level 2 cache, and is assumed to be distributed normal, *i.e.*, $\beta_{is} \sim N(\beta_s, \sigma)$. In this model consumers evaluate product quality in two aspects. One is whether a processor has a larger level 2 cache, and the other is the overall quality in terms of all other characteristics, mainly the processing speed.

Each model is estimated with two specifications. The first specification uses \textit{Speed} and $(\textit{Speed})^2$, and it shows that product quality is a concave function of the processor speed. Both \textit{Speed} and $(\textit{Speed})^2$ are significant at the 5% significance level. In the second specification, $\log(\textit{Speed})$ is used instead of \textit{Speed} . By using the log, I restrict product quality to increase monotonically with the speed. $\log(\textit{Speed})$ is significant at the 5% level, and also shows that a higher processor speed is less appreciated over time.

In the two random coefficient model the average consumer is willing to pay \$200 ~ \$400 for putting the level-2 cache into the processor, while in the one random coefficient model the average consumer is willing to pay \$80 ~ \$110. The willingness to pay in the former model is more consistent with price differences between products of different sizes of the level-2 cache.

The variance of β_{is} is significant, meaning that consumers endowed with the same α_i value the level-2 cache differently. The one random coefficient model does not reflect consumer heterogeneity on the level-2 cache by forcing products to be differentiated on the single dimension. Adding another random coefficient enriches the model by allowing consumers to be heterogeneous in multi-dimensions. This difference between

the two models will be more clearly demonstrated in the quality adjusted price index for different consumer groups.

As mentioned earlier, the coefficients on time dummy variables capture a mixture of changes in the mean utility of unobservable characteristics and in the value of the outside option. One cannot separate these two without assumptions. I use three different assumptions and compare results.

3.2 The Price Index for the Whole Market

I first compute the quality adjusted price index using the one random coefficient PCM. I use the first specification in Table 2. Table 3 lists price changes that the average consumer experiences ($-\mu_t$) under different assumptions on the outside option and unobservable characteristics. The average consumer is the one whose α_i is equal to the mean. The price index (PI_t) is also reported with the second quarter of 1996 as the base period. $-\mu_t$ multiplied by 100 gives a percentage change in price from period $t - 1$ to period t and

$$PI_t = (1 - \mu_t) \times PI_{t-1}.$$

Following Nevo (2003), I first assume that the value of the outside option does not change over time (the columns labeled u_0^t *Fixed*) and then assume that the mean value of unobservable characteristics (ξ_t) does not change over time (u_0^t *Change*). Under the former assumption the time dummy variables identify changes in the mean value of unobservable characteristics. Under the latter they identify changes in the value of the outside option.

For each assumption on the time effect I assume that unobservable characteristics change over time ($\Delta\xi_{jt}$ *Change*) and that they do not ($\Delta\xi_{jt}$ *Fixed*). In the former assumption all residuals are interpreted as unobservable characteristics. The latter allows for measurement error in estimating product quality with a linear function of characteristics. In this case $\Delta\xi_{jt}$ is fixed at the first period value for each product. Note that the motivation for the assumptions on $\Delta\xi_{jt}$ is not the same as in Nevo (2003). The well known problem

of red bus and blue bus is not relevant to the PCM as it does not have an idiosyncratic taste term.

Table 3 shows that the index increases in about a half of time periods when the value of the outside option is fixed, and at the end of the sample period consumers become worse off. The assumption on $\Delta\xi_{jt}$ makes a significant difference so that change in the compensating variation is much smaller when $\Delta\xi_{jt}$ is fixed. However, the index still increases by more than two folds at the end of the sample period. The increased index is not consistent with the observed market trend. A larger number of consumers bought more improved products at lower prices in later periods. This contradiction suggests that the outside option improved over time. Consider a period from the third quarter of 1999 to the second quarter of 2000. This is a period where the (nominal) average price went up. The first two columns in the table show that consumers continued to be worse off during this period. This implies that quality improvement did not offset price increase. However, if the outside option actually improved, quality improvement is underestimated.

On the other hand, the index continuously declines when u_0^t changes over time, and it goes down below two at the end of the sample period. This is more consistent with the observed market trend. The index tends to go down further when $\Delta\xi_{jt}$ is fixed. However, the assumption on $\Delta\xi_{jt}$ does not make a significant difference as much as when u_0^t is fixed.

In the columns labeled u_0^t *Identified*, the value of the outside option is identified using the assumption that on average product quality of the continuing products does not change over time. Song (2007) uses this assumption and shows that the estimated outside option increases and the mean of unobservable characteristics decreases in the late 1990s. The index is similar to when ξ_t is fixed (u_0^t *Change*), but declines less rapidly. That is probably because the improvement in the outside option is exaggerated when ξ_t is fixed.

Table 4 lists the quality adjusted price index in the two random coefficient PCM. The value of the outside option is identified using the same assumption as for u_0^t *Identified* in Table 3. The two indices are significantly different for periods after the third quarter of 1998. The index declines faster with two random coefficients so that it goes down to 0.9 at the end of the sample period. Most notably, in the first quarter

of 1999 the index declines by 41.2 % in Table 4, while it declines by 1.4 % in Table 3. This is the first period in which Celeron processors captured considerable market shares. Although Celeron processors were first introduced in the third quarter of 1998, their market shares were tiny and only two Celeron processors were introduced. Moreover, one of two Celeron processors disappeared from the market in the last quarter of 1998. However, about four out of ten Intel processors are Celeron processors from the first quarter of 1999. The benefit consumers receive from these new products does not appear when products are ranked on a single dimension, but becomes significant with another random coefficient.

Table 5 reports the hedonic price index and compares it with the quality adjusted price index. The first two columns list the dummy variable hedonic index, and the third and fourth columns the constrained Pakes index. Each hedonic index has two specifications, one with price as the dependent variable and the other with the log of price. The dummy variable index uses processor speed, processor speed squared, the dummy variable for lower level 2 cache and the time dummy variables as regressors. For the constrained Pakes index the processor speed and the processor speed squared variables are interacted with the time dummy variables. The last two columns list the quality adjusted price index for the one and the two random coefficient PCM.

The most significant difference between the dummy variable index and the Pakes index is their sensitivity to the econometric specification. The mean square difference between the two dummy variable indices is 0.038, while it is only 0.001 between the two Pakes indices. This suggests that the Pakes index is less prone to the specification error than the hedonic index. This is also consistent with Pakes (2003) who shows that his results are robust to various econometric specifications.

The comparison also shows that the second specification of the dummy variable index (the second column) is more similar to any of the Pakes indices than the first specification. Their squared mean difference is only 0.004, while it is about 2.3 with the first one. It also shows that the second specification of the dummy variable index tends to produce a higher number than the Pakes index in an absolute term. It is true for all

periods from the first quarter of 1998 to the end of the sample period.

It is hard to make analytical comparison between the dummy variable index and the Pakes index due to the former index's lack of theoretical foundation, but my results give stronger support to the Pakes index. The dummy variable index produces dramatically different numbers depending on the econometric specification, and it is not clear which specification should be chosen.

In the last two columns of the table I list the quality adjusted price index with the value of the outside option identified. As explained earlier the Pakes index is supposed to be no higher than the quality adjusted index in an absolute term if both the utility function and the price hyperplane are precisely estimated.

The table shows that the quality adjusted index is almost always higher in an absolute term. At least one of the quality adjusted indices is higher than both of the Pakes indices in eleven quarters out of seventeen. In eight quarters out of these eleven quarters both of the quality adjusted indices are higher than both of the Pakes indices. In two quarters, the first quarter of 1997 and the fourth quarter of 1997, one of the quality adjusted indices is higher than one of the Pakes indices. Only in four quarters, both of the Pakes indices are higher than both of the quality adjusted indices.

A quarter-by-quarter comparison suggests that the quality adjusted index gives more weight to quality changes, and this may be the main reason why the Pakes index is higher in some quarters. For example, in the third quarter of 1997 the quality adjusted index goes down by about 20 %, while the Pakes index goes down by about 30 %. This is a period right after the introduction of the first Pentium II processor. This processor was introduced at almost \$2,000 and then its price dropped to below \$1,000 in the next period. Comparing these two periods, the Pakes index declines much more in the third quarter of 1997, while the quality adjusted index declines more in the second quarter of 1997.

The fourth quarter of 1998 and the third quarter of 2000 are another examples. These are periods when no new product is introduced so there are no significant quality changes. In the first period the quality adjusted index goes down by about 10 %, while the Pakes index declines by about 24 %. In the second

period the former index goes down by about 15 %, while the latter declines by about 20 %.

Pakes (2003) points out that one of the disadvantages of the Pakes index is that it is not necessarily close to the least upper bound of the compensating variation. My results show that its ratio to the quality adjusted index is about 0.70 on average, conditional on the former being lower than the latter. It is no lower than 0.61 and is as high as 0.75, depending on the specification. The period in which the two indices are the most different is the fourth quarter of 1996. The ratio is as low as 0.05 and no higher than 0.22. However, this period is clearly an outlier. Excluding this, the average ratio is no lower than 0.65 and as high as 0.82. The period with the two indices being the closest is the second quarter of 2000. The ratio is as high as 0.99 and no lower than 0.85.

3.3 The Price Index for Groups

In Table 6 I report the quality adjusted price index for five consumer groups. They are grouped according to their values of α_i , where $\log(\alpha_i) \sim N(0, 0.89)$, so that I_j includes consumers whose α_i are in the j^{th} quintile of the distribution. Consumers in I_1 include those who buy the highest quality products and consumers in I_5 include those who buy the lowest quality products and who do not buy any product. The value of the outside option is identified by the aforementioned way.

The table shows that each group experiences a different degree of price changes. For example, in the second and the third quarter of 1998 when the index for the whole market (the market index) falls by more than 30 %, consumers in I_1 (the high end) experience more than a 40 % drop, while those in I_5 (the low end) experience less than a 10 % drop. This suggests that a large decline in the market index was caused by new product introduction at the high end of the market.

The table confirms that the introduction of 300 MHz Pentium II processor in the second quarter of 1997 benefits consumers considerably despite its high price. The index falls by 36 % for I_1 in this period. However, in the third quarter of 1997 the index falls by only 16.3 % for the same group despite an almost

50 % price decline. Considering the fact that no new products were introduced in this period, this suggests that consumers benefit more from the quality improvement than from the price decline.

The table also confirms that the high end market is responsible for a modest decline in the market index in the fourth quarter of 1998, while the low end market is responsible for a modest decline in the first quarter of 1999. In the former period consumers in I_1 experience an increase in the index, while consumers in I_5 experience a 36.3 % decline. In the latter period consumers in I_5 experience an increase, while consumers in I_1 experience a 22.9 % decline.

In the third quarter of 2000 both the high and the low ends are responsible for a modest decline in the market index. Consumers in I_1 experience a 10.8 % decline and those in I_5 experience a 0.1 % decline. A steady decline of the market index from the second quarter of 1999 to the second quarter of 2000 is evenly contributed by all sectors of the market.

One should note that the presence of ξ_t in the utility function may cause a problem in computing the group index. ξ_t measures the mean value of unobservable characteristics of all products in period t , and its value changes by both product entry/exit and changes in unobservable characteristics of continuing products. The following example shows how product entry/exit may distort the group index in presence of ξ_t .

Suppose, as an extreme case, unobservable characteristics of individual products do not change over time once they are introduced in the market. Suppose further that no new products are introduced at the high end of the market in period t and some products at the low end with relatively better unobservable characteristics exit the market at the end of period $t - 1$. For a group of consumers at the high end, the quality adjusted price index should merely depend on price changes. If this group buys the same products that they bought in period $t - 1$ and prices does not change, the quality adjusted price index should not change either. However, since ξ_t goes down due to the product exit at the end of period $t - 1$, it will unjustly lower these consumers' surplus, and so result in an increase in the price index.

The fourth quarter of 1998 is a good example. According to Table 6, consumers at the high end experienced an increase in the index. This is a period where no new products were introduced and six products exited the market. Since unobservable characteristics of the continuing products are assumed not to change over time, an increase in the index means that nominal prices of existing products went up. However, prices of the high end products actually decreased. A real cause of the index increase may come from a decrease in ξ_t , which may be irrelevant of the high end products.

To address this issue I compute the index after subtracting the estimated ξ_t from product quality. Although most of periods are not considerably affected, the exclusion of ξ_t lowers the index for I_1 when no new products are introduced. In the fourth quarter of 1998 the index for I_1 changes from a 44.7 % increase to a 34.7 % increase and the index for I_2 changes from a 2.1 % decline to a 5.4 % decline. In the third quarter of 2000, another period without new products, the index for I_1 changes from a 10.8 % decline to a 18.7 % decline.

Another interesting period to compare is the first quarter of 1999. This is a period with six new product entries and one product exit, and the index for I_1 changes from a 22.9 % decline to a 37 % decline without ξ_t . The index for other groups also declines significantly by excluding ξ_t . This suggests that unobservable characteristics of new products in this period are relatively worse than those of existing products. In this case, the index for consumers at the high end is distorted by dropping ξ_t . Nevertheless, it is hard to predict how the exclusion of ξ_t will affect the group index generally since product entry and exit take place at the same time for most of periods.

One should note that the index for I_1 in the fourth quarter of 1998 still increases even with ξ_t excluded. There are two plausible explanations. One is that unobservable characteristics of continuing products change over time. In other words, unobservable characteristics of a new product introduced in the previous period became so much worse in this period that consumers who bought this product became worse off despite a price decline. The other explanation is that the assumption that product quality linearly depends

on product characteristics may not be appropriate to fully capture relationship between characteristics and product quality.

Table 7 reports the quality adjusted price index for consumer groups in the two random coefficient PCM for some selected periods. Consumers differ in their willingness to pay for quality improvement mainly measured by the processing speed (α_i), and differ in their valuations on capacity of the level 2 cache (β_{is}). As a result, those who have the same value of α_i may purchase different products because of different values of β_{is} .

In the table consumers are divided into fifteen groups according to their values of α_i and β_{is} . Groups in the upper rows prefer a larger capacity of the level 2 cache and groups in the left columns prefer products of higher overall quality. The table starts with the second quarter of 1998, one period before the introduction of Celeron processors, so the index in this period is the same across rows. In the following period the index in the third row is much lower than those in the first two rows, showing that consumers who do not care much about capacity of the level 2 cache become much better by Celeron processors.

In the fourth quarter of 1998 when Intel withdrew one Celeron processors (out of two) from the market, some groups in the third row became worse off. However, with a new line of Celeron processors introduced in the following quarter all groups in the third row experience a considerable decline in the index. This explains the difference between Table 3 and Table 4 for the first quarter of 1999. The table also shows that a relatively modest decrease in the market index for the third quarter of 2000 is related to consumers who purchase Pentium processors.

4 CONCLUSIONS

Using the pure characteristics demand model I compute the quality adjusted price index for the CPU market. The index goes down by more than 25 % when quality improves without a price increase. It goes down by less than 10 % when no new product is introduced. It is comparable with the hedonic price index, but more

sensitive to quality change.

I group consumers according to values of the random coefficients and compute the group index. In the one random coefficient PCM consumers only differ in their willingness to pay for quality improvement. In the two random coefficient model consumers also differ in their willingness to pay for extra memory storage inside the processor. The group index shows how each group is affected by product entry and exit.

One caveat is that the index is based on a static demand model. This means that all consumers enter the market every period and buy at most one product. If they do not buy at all, it means that they decide not to own any product. This implies that the utility of products lasts for only one period. However, consumers purchase durable goods intermittently. Some consumers already own products and decide whether to replace them with new ones.

A more proper price index should incorporate the durable good feature. If a consumer purchases a product for replacement, the index should be based on a utility difference between buying a new product and using her "old" product. A dynamic demand model that explicitly models a replacement decision is necessary to construct an index for durable goods.

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References

- [1] Berry, S. (1994), "Estimating Discrete-Choice Models of Product Differentiation," *Rand Journal of Economics*, 25, 242-262.
- [2] Berry, S., Levinsohn, J., and Pakes, A. (1995), "Automobile Prices in Market Equilibrium," *Econometrica*, 63, 841-890.
- [3] Berry, S., and Pakes, A. (2007), "The Pure Characteristic Demand Model," *International Economic Review*, 48, 1193-1225.
- [4] Bresnahan, F. (1987), "Competition and Collusion in the American Auto Industry: The 1995 Price War," *Journal of Industrial Economics*, 35, 457-482.
- [5] Griliches, Z. (ed.) (1971), *Price Indexes and Quality Change: Studies in New Methods of Measurement*, Cambridge, MA: The Harvard University Press.
- [6] Nevo, A. (2003), "New Products, Quality Changes and Welfare Measures Computed from Estimated Demand Systems," *The Review of Economics and Statistics*, 85(2), 266-275.
- [7] Pakes, A., Berry, S., and Levinsohn, J. (1993), "Applications and Limitations of Some Recent Advances in Empirical Industrial Organization: Price Indexes and the Analysis of Environmental Change," *American Economic Review, Papers and Proceedings*, 83, 240-246.
- [8] Pakes (2003), "A Reconsideration of Hedonic Price Indices with an Application to PC's," *American Economic Review*, 93 (5), 1578-1596.
- [9] Shaked, A., and Sutton, J. (1982), "Relaxing Price Competition Through Product Differentiation," *Review of Economic Studies*, 49, 3-13.
- [10] Small, K. A., and Rosen, H. S. (1981), "Applied Welfare Analysis with Discrete Choice Models," *Econometrica*, 49, 105-130.
- [11] Song, M. (2007), "Measuring Consumer Welfare in the CPU Market: An Application of the Pure Characteristics Demand Model," *RAND Journal of Economics*, 38, 429-446.
- [12] Song, M. (2008), "Estimating the Pure Characteristics Demand Model: A Computational Note," unpublished manuscript, University of Rochester, Rochester, NY.
- [13] Trajtenberg, M. (1990), *Economic Analysis of Product Innovation: The Case of CT Scanners*, Cambridge, MA: The Harvard University Press.

Table 1: Market Trend

Time	Processor Speed		Price		Quantity Sold	
	Avg. Speed ^a	Change	Avg. Price ^b	Change	Total ^c	Change
96Q2	137.0	100.0	327.3	100.0	16.2	100.0
96Q3	149.7	109.3	310.9	95.0	19.8	122.1
96Q4	157.1	114.7	334.9	102.3	23.0	142.1
97Q1	164.5	120.0	326.3	99.7	23.2	143.5
97Q2	189.1	138.0	400.7	122.4	20.9	129.1
97Q3	202.0	147.4	295.2	90.2	23.0	142.0
97Q4	213.6	155.9	278.3	85.0	23.1	142.9
98Q1	241.1	175.9	306.2	93.6	20.4	125.8
98Q2	293.3	214.1	373.6	114.1	20.5	126.4
98Q3	356.5	260.3	374.1	114.3	26.1	161.2
98Q4	372.5	271.9	287.1	87.7	30.7	189.5
99Q1	382.8	279.4	264.0	80.7	26.0	160.5
99Q2	427.8	312.2	255.8	78.2	23.7	146.6
99Q3	467.7	341.4	285.3	87.2	30.4	187.7
99Q4	526.1	384.0	297.2	90.8	37.7	232.8
00Q1	591.3	431.6	306.3	93.6	46.3	285.8
00Q2	645.1	470.8	286.9	87.6	50.1	309.1
00Q3	701.3	511.9	263.4	80.5	42.0	259.2

^aThe quantity weighted average in mega hertz.

^bThe quantity weighted average in U.S. dollars.

^cIn million units.

Table 2: Estimates of the Pure Characteristics Demand Model

Variable	One random coefficient model		Two random coefficient model	
	Specification 1	Specification 2	Specification 1	Specification 2
Constant	-1.08* (0.23)	-0.73* (0.08)	-1.11* (0.06)	-0.70* (0.08)
Speed	0.41* (0.09)		0.42* (0.02)	
Speed ²	-0.02* (0.01)		-0.02* (0.00)	
log(Speed)		0.60* (0.04)		0.62* (0.04)
No_Cache ^a				
Mean (β_s)	-0.13* (0.03)	-0.13* (0.01)	-0.32* (0.02)	-0.44* (0.04)
Standard	0	0	0.34* (0.04)	0.55* (0.03)
Deviation (σ)				
Price ^b				
θ	0.89* (0.09)	0.54* (0.02)	0.84* (0.01)	0.50* (0.00)

Note: Number of observations is 321. Dummy variables for quarters are included in all specifications. Standard errors are reported in parenthesis.

^aNo_Cache is a dummy variable for processors with smaller capacity of the level 2 cache.

^bThe price coefficient, α , is distributed log normal with mean set to zero; i.e., $\log(\alpha) \sim N(0, \theta)$

*significant at the 5% level.

Table 3: The Quality Adjusted Price Index in the One Random Coefficient PCM.

Time	u_0^t Fixed				u_0^t Change				u_0^t Identified ^c	
	$\Delta\xi_{jt}$ Change		$\Delta\xi_{jt}$ Fixed		$\Delta\xi_{jt}$ Change		$\Delta\xi_{jt}$ Fixed		$-\mu_t$	Index
	$-\mu_t^a$	Index ^b	$-\mu_t$	Index	$-\mu_t$	Index	$-\mu_t$	Index		
96Q3	-0.093	90.7	-0.142	85.8	-0.105	89.5	-0.151	84.9	-0.143	85.7
96Q4	-0.255	67.6	-0.2495	64.5	-0.039	86.0	-0.038	81.6	-0.082	78.7
97Q1	0.195	80.7	0.247	80.4	-0.167	71.6	-0.134	70.7	-0.142	67.5
97Q2	-0.258	59.9	-0.306	55.8	-0.230	55.2	-0.286	50.4	-0.256	50.2
97Q3	0.475	88.4	0.105	61.7	-0.163	46.2	-0.298	35.4	-0.197	40.3
97Q4	0.281	113.2	0.229	75.8	-0.145	39.5	-0.167	29.5	-0.146	34.4
98Q1	0.141	129.2	1.109	159.9	-0.367	25.0	-0.165	24.7	-0.290	24.5
98Q2	-0.331	86.4	-0.378	99.5	-0.338	16.6	-0.380	15.3	-0.340	16.1
98Q3	-0.291	61.3	-0.255	74.1	-0.361	10.6	-0.338	10.1	-0.328	10.8
98Q4	7.553	524.0	1.313	171.5	-0.136	9.1	-0.315	6.9	-0.083	9.9
99Q1	-0.412	308.1	-0.241	130.2	-0.153	7.7	0.257	8.7	-0.014	9.8
99Q2	-0.117	272.1	-0.168	108.3	-0.261	5.7	-0.308	6.0	-0.217	7.7
99Q3	0.059	288.1	0.053	114.1	-0.285	4.1	-0.281	4.3	-0.255	5.7
99Q4	0.533	441.7	0.431	163.2	-0.341	2.7	-0.351	2.8	-0.283	4.1
00Q1	0.988	878.1	0.682	274.5	-0.275	2.0	-0.321	1.9	-0.287	2.9
00Q2	0.233	1082.7	0.241	340.6	-0.205	1.6	-0.215	1.5	-0.214	2.3
00Q3	-0.164	905.1	-0.163	285.0	-0.131	1.4	-0.107	1.3	-0.067	2.1

Note: The reported indexes are based on the first column in Table 2.

^a $-\mu_t$ multiplied by 100 gives a percentage change in price from period $t - 1$ to period t .

^b $PI_t = (1 - \mu_t) PI_{t-1}$. 96Q2 = 100.

^cThe value of the outside option is identified with the assumption that on average $\Delta\xi_{jt}$ of the continuing products does not change.

Table 4: The Quality Adjusted Price Index in the Two Random Coefficient PCM.

<i>Time</i>	Two Random Coefficients PCM		
	\overline{CV}_t^a	$-\mu_t^b$	index ^c
96Q3	49.6	-0.138	86.2
96Q4	24.2	-0.067	80.4
97Q1	53.5	-0.141	69.1
97Q2	124.5	-0.237	52.7
97Q3	77.1	-0.207	41.8
97Q4	45.0	-0.139	36.0
98Q1	121.7	-0.284	25.8
98Q2	177.3	-0.322	17.5
98Q3	234.6	-0.385	10.7
98Q4	52.5	-0.155	9.1
99Q1	184.8	-0.412	5.3
99Q2	118.0	-0.316	3.7
99Q3	117.5	-0.292	2.6
99Q4	46.2	-0.135	2.2
00Q1	157.4	-0.340	1.5
00Q2	95.7	-0.250	1.1
00Q3	53.0	-0.168	0.9

Note: The reported indexes are based on the third column in Table 2. The value of the outside option is identified with the assumption that on average $\Delta\xi_{jt}$ of the continuing products does not change.

^aIn U.S. dollars

^b $-\mu_t$ multiplied by 100 gives a percentage change in price from period $t - 1$ to period t .

^c $PI_t = (1 - \mu_t) PI_{t-1}$. 96Q2 = 100

Table 5: Comparison with the Hedonic Price Index

	Dummy Variable Index ^a		Constrained Pakes Index ^b		Quality Adjusted Price Index ^c	
	p on X	$\log(p)$ on X	p on X	$\log(p)$ on X	one r.c.	two r.c.
96Q3	-0.232	-0.193	-0.163	-0.171	-0.143	-0.138
96Q4	-0.031	0.009	-0.004	-0.015	-0.082	-0.067
97Q1	-0.141	-0.175	-0.136	-0.152	-0.142	-0.141
97Q2	0.057	-0.157	-0.210	-0.151	-0.256	-0.237
97Q3	-0.445	-0.359	-0.306	-0.323	-0.197	-0.207
97Q4	-0.088	-0.123	-0.143	-0.157	-0.146	-0.139
98Q1	-0.240	-0.331	-0.282	-0.233	-0.290	-0.284
98Q2	-0.151	-0.312	-0.204	-0.230	-0.340	-0.322
98Q3	-0.142	-0.325	-0.215	-0.218	-0.328	-0.385
98Q4	-0.160	-0.337	-0.239	-0.235	-0.083	-0.155
99Q1	-0.083	-0.307	-0.183	-0.198	-0.014	-0.412
99Q2	-0.101	-0.310	-0.238	-0.195	-0.217	-0.316
99Q3	-0.056	-0.246	-0.192	-0.166	-0.255	-0.292
99Q4	-0.081	-0.344	-0.220	-0.240	-0.283	-0.135
00Q1	-0.063	-0.247	-0.198	-0.204	-0.287	-0.340
00Q2	-0.059	-0.291	-0.214	-0.212	-0.214	-0.250
00Q3	-0.051	-0.247	-0.200	-0.202	-0.067	-0.168

^aThe dummy variable index is calculated as $PI_t = (\gamma_t - \gamma_{t-1}) / \gamma_{t-1}$ when the dependent variable is price and $PI_t = \exp(\gamma_t - \gamma_{t-1}) - 1$ when the dependent variable is the log of price, where γ_t is a coefficient on the dummy variable for period t . p denotes price and X denotes product characteristics which include the constant term, speed, speed squared, and the dummy variable for smaller capacity of the level-2 cache. The time dummy variables are included in both specifications.

^bThe constraining assumption is that all periods share the same constant and error terms. The index is constructed by $-\mu_t = \widehat{CV}_t / (\widehat{CV}_t + \bar{p}_t)$ where \widehat{CV}_t is the upper bound of the average compensating variation. See the text for how \widehat{CV}_t is estimated. p denotes price and X denotes product characteristics which include the constant term, speed interacted with the time dummy variables, speed squared interacted with the time dummy variables, and the dummy variable for smaller capacity of the level-2 cache.

^cThe value of the outside option is identified with the assumption that on average $\Delta\xi_{jt}$ of the continuing products does not change.

Table 6: The Quality Adjusted Price Index for Groups.

<i>Time</i>	$-\mu_{gt}^a$				
	$i \in I_1$	$i \in I_2$	$i \in I_3$	$i \in I_4$	$i \in I_5$
96Q3	-0.246	-0.163	-0.159	-0.158	-0.004
96Q4	-0.165	-0.081	-0.024	0.037	0.059
97Q1	-0.211	-0.102	-0.100	-0.186	-0.185
97Q2	-0.360	-0.184	-0.157	-0.093	-0.112
97Q3	-0.163	-0.266	-0.344	-0.409	-0.350
97Q4	-0.095	-0.124	-0.149	-0.176	-0.197
98Q1	-0.333	-0.163	-0.220	-0.284	-0.278
98Q2	-0.457	-0.311	-0.194	-0.138	-0.049
98Q3	-0.401	-0.316	-0.309	-0.197	-0.094
98Q4	0.447	-0.021	-0.189	-0.255	-0.363
99Q1	-0.229	-0.203	-0.220	-0.194	0.037
99Q2	-0.285	-0.235	-0.262	-0.234	-0.190
99Q3	-0.314	-0.181	-0.160	-0.157	-0.200
99Q4	-0.318	-0.238	-0.203	-0.261	-0.328
00Q1	-0.250	-0.237	-0.257	-0.211	-0.348
00Q2	-0.191	-0.246	-0.285	-0.234	-0.271
00Q3	-0.108	-0.190	-0.152	-0.161	-0.001

Note: The reported indexes are based on the first column in Table 2. Consumers are grouped such that I_j includes consumers whose α_i are in the j^{th} quintile of the distribution. The value of the outside option is identified with the assumption that on average $\Delta\xi_{jt}$ of the continuing products does not change.

^a $-\mu_{gt}$ multiplied by 100 gives a percentage change in price for group g from period $t - 1$ to period t .

Table 7: The Group Index in the Two Random Coefficient PCM.

		$-\mu_{gt}^a$				
		I_{11}	I_{12}	I_{13}	I_{14}	I_{15}
98Q2	I_{21}	-0.488	-0.321	-0.198	-0.150	-0.054
	I_{22}	-0.488	-0.321	-0.198	-0.150	-0.054
	I_{23}	-0.488	-0.321	-0.198	-0.150	-0.054
98Q3	I_{21}	-0.415	-0.333	-0.324	-0.210	-0.078
	I_{22}	-0.415	-0.333	-0.324	-0.210	-0.078
	I_{23}	-0.707	-0.738	-0.728	-0.693	-0.695
98Q4	I_{21}	0.409	-0.001	-0.162	-0.231	-0.249
	I_{22}	0.409	-0.001	-0.162	-0.231	-0.249
	I_{23}	0.039	0.322	0.061	0.090	-0.280
99Q1	I_{21}	-0.221	-0.150	-0.157	-0.139	0.127
	I_{22}	-0.221	-0.150	-0.157	-0.139	0.175
	I_{23}	-0.671	-0.578	-0.501	-0.267	-0.235
00Q3	I_{21}	-0.011	-0.093	-0.048	-0.013	0.402
	I_{22}	-0.011	-0.093	-0.048	-0.141	-0.247
	I_{23}	-0.472	0.043	-0.238	-0.705	-0.510

Note: The reported indexes are based on the third column in Table 2. Groups in the upper rows prefer a larger capacity of the level 2 cache and groups in the left columns prefer products of higher overall quality. The value of the outside option is identified with the assumption that on average $\Delta\xi_{jt}$ of the continuing products does not change.

^a $-\mu_{gt} \times 100$ gives a percentage change in price for group g from $t - 1$ to t .