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A LINEAR DYNAMIC PROGRAMMING APPROACH TO IRRIGATION SYSTEM MANAGEMENT WITH DEPLETING GROUNDWATER*

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A model for measuring the economic benefits of irrigation system development over a depleting aquifer is presented, along with related methodology for detailed long-range farm planning. The paper considers management issues, such as distribution system configuration, drilling policy, area developed for irrigation, and crop production.

A Linear Dynamic Programming (LDP) method is developed and applied to derive optimal temporal investments in the use of stock resources and long-term cropping plans. First, parametric linear programming (PLP) is used to maximize periodic profits subject to specified values of state variables related to annual water use and irrigation system capacity. The PLP results are then used in a dynamic programming model to determine the optimal allocation of water and irrigation resources over time. The impact of aquifer depletion on the profitability of furrow and pivot irrigation systems is illustrated for a typical farm situation in the Texas High Plains. Results indicate that the economic benefits of modern water and energy efficient irrigation systems may come from the expansion of current irrigation intensity rather than from an extended period of irrigation when water is initially scarce relative to land. Several conditions are identified where economic depletion may occur before the point of physical exhaustion is reached.

(GOVERNMENT SERVICES/WATER; AGRICULTURE; DYNAMIC PROGRAMMING APPLICATIONS)

1. Introduction

The impact of a continuously declining water table and an escalating energy price has intensified the development and adoption of more water and energy efficient irrigation systems throughout the High Plains area of the United States. In the semi-arid irrigated Texas High Plains, farmers are using groundwater from the Ogallala Aquifer at the rate of 7 million acre feet per year. This rate varies from 2 to 6 percent of the total extractable quantity of groundwater (Wyatt et al. 1977). The use of groundwater for irrigation far exceeds the use of this water for municipal and industrial purposes. The Ogallala Aquifer in the Texas High Plains is hydrologically isolated from adjacent areas with virtually no recharge from the surface (Grubb 1979). For these reasons, water that is being withdrawn from the aquifer is not replaced quickly by natural recharge and is in effect being mined. Farmers' reactions to the continuous aquifer decline have included such actions as the drilling of additional wells to maintain their current pumpage volume, switching to more expensive water efficient irrigation systems, changing the crop mix, and reducing the amount of irrigation for each crop. That the efficient use of remaining groundwater reserves is of concern to private farmers, the business community, and public officials is evident by the recent 6 million dollar Six State Ogallala Aquifer Area Study (High Plains Associates 1982). That study focused on the probable consequences of water management alternatives for the High Plains area.

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groundwater. Most of the studies by Burt (1964a, b, 1966, 1967), Brown and Deacon (1972), Biere (1972), and Remson and Gorelick (1980) use dynamic programming or control theory to derive decision rules for the aggregate temporal use of ground water. Fogel et al. (1976) employed inventory theory concepts for intraseasonal irrigation management assuming unlimited water supply. Recently, Bras and Cordova (1981) provided a single-crop irrigation model which accounts for the dynamics of the soil moisture content. Minimal treatment, however, is given to the simultaneous determination of the detailed annual farm production plans and the long-term planning of the irrigation systems on site. As a result, these studies tend to be more theoretical than applied.

There are more applied studies which deal with the temporal utilization of groundwater in the Southern High Plains. The studies by Hughes and Harmon (1969), Lacewell and Grubb (1970), Young and Coomer (1980), Hardin and Lacewell (1980) utilize a recursive linear programming (RLP) technique. The RLP technique is a reasonable predictor of producer behavior in unmanaged common property situations. An annual model can be made large enough to develop cropping plans in which attention can be paid to such factors as irrigation scheduling, varying soil types, labor constraints, etc., without great computational expense. However, the recursive format does not provide information as to the optimal temporal pattern of investment and the use of stock resources. This information is vital for the local decision makers since permanent capital structures in the agribusiness sector may become worthless due to aquifer depletion.

Most of the crops in the Southern High Plains are irrigated with groundwater and in the real planning situation, the determination of crops depends on the availability of underground water, the area developed for irrigation, and the technology of the irrigation system itself. On the other hand, water requirements, acquisition of irrigation capacity, and the expected annual water level decline are dictated by the choice of crops to be grown (Rose 1973). The planning method, therefore, should simultaneously analyze all of these interdependent aspects of the problem and provide detailed optimal solutions for real size problems.

2. Linear Dynamic Programming Formulation

2.1. *Overview*

The purpose of this paper is to describe an efficient computer-aided planning method using linear programming and dynamic programming to determine the temporal pattern of investment in irrigation systems and the resulting use of groundwater reserves which will maximize expected net present value of future returns. A linear dynamic programming (LDP) method is used to sweep out a series of profit maximizing temporal resource plans dependent upon the type of distribution system, the irrigation pumping capacity, the land area developed for irrigation, and the water supply-pumping lift situation. The LDP method combines the techniques of parametric linear programming (PLP) and discrete dynamic programming (DP).

The LDP method is based on a variation of previous studies by Nemhauser (1964) and Nazareth (1980) who show that DP can be used in the decomposition of block diagonal LP problems as an alternative decomposition LP routine. The polyperiod linear programming (PLP) model of the type formulated by Harmon et al. (1976) is amenable to solution by the Decomposition Principle of Dantzig and Wolfe (Nemhauser 1964). That is, the problem can be partitioned into $N + 1$ subsets such that N subsets (or stages) form mutually independent systems. The $(N + 1)$st subset contains equations or constraints which form a common linkage among the variables in the above N subsets. The constraints in the $(N + 1)$st subset will be considered as

State or Control variables. The LDP method is designed to handle agricultural problems with the use of standard LP software products and simple programmable DP routines.

The procedure presented here uses the dynamic programming (DP) framework for those decisions which have carry-over effects on the states from year to year, e.g. decisions about lowering the water table by pumping, number of wells drilled, and amount of irrigation equipment procured. It then applies standard linear programming for single year optimization of resource use (land, water, labor, etc.) and crop choice subject to the constraints upon water availability and irrigation capacity which are determined by the multiyear DP framework. These LP decisions (e.g.: crop mix) are assumed to have no carry-over effects from year to year. The one-year profits from the LP model serve as inputs for optimization in the multiyear DP model.

The procedure presented here consists of two computational phases. Phase 1 uses LP to generate detailed optimal one-year farm plans, intratemporal allocation of water and net returns for specified aquifer state and irrigation decisions. These are the one-year profits which are used as inputs to the multi-year DP. Phase 2 is the DP that deduces the optimal allocation of water and irrigation resources over time and computes the resulting overall multiperiod benefits of the plan.

2.2. *Phase* 1

Several PLP models are used to determine the maximum one-year returns achievable by a farmer with a given aquifer level, a given irrigation system, a known number of wells, and a specific water use policy. The LP notation is summarized in Table 1. Several types of subscripts are used here: (1) year, or stage, denoted n, where $n = 1, 2, \ldots, N$ and N is the number of years in the planning horizon; (2) area developed for irrigation in quarter sections, denoted s, where $s = 1, 2, \ldots, S$ and S is less than or equal to the number of quarter sections in the farm; (3) wells, denoted w, where $w = 1, 2, \ldots, W$ with W being the upper bound on the number of wells in the area; (4) annual level of water pumped, denoted q, where $q = 1, 2, \ldots, Q$ and Q is the number of potential levels of aggregate water use in each year; (5) activities, denoted j, where $j = 1, 2, \ldots, J$ and J is the number of potential crops and related activities in the region; (6) resources, denoted i, where $i = 1, 2, \ldots, I$ and I is the number of relevant restrictive resources such as labor, tractors, capital and land, and (7) months of the year, denoted t, where $t = 1, 2, \ldots, 12$.

The model variables corresponding to the production activities are Y_{jn} and the net unit price of the jth activity in year n is given by c_{jn}. The amount of the ith resource available for use in year n is defined by b_{in} and a_{ijn} is the amount of the ith resource required by a unit level of the jth production activity in year n. Aquifer depth and yield determine well discharge rate and the cost of pumping. Let V_n, $(V_n(X_n, W_n, S_n))$, denote the current farm state in terms of the remaining saturated thickness X_n, the number of operating irrigation wells W_n, and the area developed for irrigation S_n, in year n.

At the beginning of every year, management can change the operational mode of the irrigation system by drilling additional wells, by restaging existing wells, by changing the size of the distribution system, as a whole, or by changing time and/or amount of water applied to each acre. D_n denotes the set of permissible decisions which are appropriate in year n.

The one-year intratemporal model consists of four groups of constraints: (1) resource class, (2) well supply, (3) annual water pumpage, and (4) distribution capacity.

Resource Class. The resource class constraints specify the well-known resource

TABLE 1

Notation Summary for Intratemporal Model

Subscripts:	
Year (stage)	$n = 1, 2, \ldots, N$
Area	$s = 1, 2, \ldots, S$
Wells	$w = 1, 2, \ldots, W$
Activities	$j = 1, 2, \ldots, J$
Resources	$i = 1, 2, \ldots, I$
Months	$t = 1, 2, \ldots, 12$
Water	$q = 1, 2, \ldots, Q$
Variables (Output From Model)	
$Y_{jn} =$	production level of commodity j in year n.
$\theta_{swq} =$	optimal return with w wells irrigating s quarter sections with q units of water.
Parameters (Input To Model)	
$a_{ijn} =$	amount of the ith resource required by the jth activity in year n.
$a'_{jt} =$	monthly well capacity required by one unit of the jth production process in month t.
$a''_j =$	distribution capacity required by the jth production process.
$c_{jn} =$	net unit price of the jth production process in year n.
Standards (Input to the Model)	
$b_{in} =$	available amount of the ith resource in year n.
$M(W, X) =$	maximum aggregate water supply of w wells from an aquifer with x feet of water saturated sand.
$A(s) =$	maximum area, measured in quarter sections, which can be irrigated by season with a system of size s.

allocation problem in farm management (Beneke and Winterboer 1973, Rose 1973):

$$\sum_j a_{ijn} Y_{jn} \leqslant b_{in} \qquad (i = 1, \ldots, I;\ n = 1, \ldots N). \tag{1}$$

Well Supply. The supply constraints ensure that there is a sufficient supply of water to meet the demand faced by the irrigation schedule of the selected activities. $M(w, x)$ denotes the monthly supply of irrigation water available from w wells drilled into an aquifer with x remaining feet of saturated thickness and a'_{jt} is the amount of monthly well capacity required by one unit of the jth production process in month t. The well supply constraints for each month and year in the planning horizon are:

$$\sum_j a'_{jt} Y_{jn} \leqslant M(W_n, X_n) \qquad (t = 1, 2, \ldots, 12;\ n = 1, 2, \ldots, N). \tag{2}$$

Annual Water Pumpage. The production activities during each year cannot consume more water in the irrigation system than is available due to the pumping decisions, and this allows the possibility of running the irrigation system at less than its full capacity in one year if it is profitable to allocate the water to another year. The model considers Q discrete levels of pumpage each year. The constraint on water pumpage for year n is:

$$\sum_{t}\sum_{j} a'_{jt} Y_{jn} \leqslant AP(q_n) \qquad (n = 1, 2, \ldots, N) \tag{3}$$

where q_n is the pumpage level chosen for year n and $AP(q_n)$ is the associated water volume in acre-feet.

Irrigation Distribution Capacity. The capacity of the water distribution system has to meet the seasonal irrigation demands of the crops. The parameter a''_j refers to the amount of distribution capacity required by the jth production process and the amount of area which can be irrigated by season with a system of size s is $A(s)$. For example, a single quarter section center pivot system might be towed between fields for winter and summer use. Therefore:

$$\sum_{j} a''_j Y_{jn} \leqslant A(s) \qquad (n = 1, \ldots, N; s = 1, \ldots, S). \tag{4}$$

Given specific values for s, w, and q, the objective function to be maximized for year n is the *LP*:

$$\theta^{(n)}_{swq} = \underset{Y_{jn} > 0}{\mathrm{Max}} \left[\sum_{j} c_{jn} Y_{jn} \right]_{(1 < s < S, 1 < w < W, 1 < q < Q)} \tag{5}$$

subject to constraints (1)–(4) and $(s, w, q) \in d_n$.

The set of optimal, feasible, current returns in year n, state v_n, $(v_n = (x_n, w_n, s_n))$ and the decision d_n is given by:

$$R_n(V_n) = \left[\theta^{(n)}_{111}, \theta^{(n)}_{112}, \ldots, \theta^{(n)}_{swq}\right]. \tag{6}$$

The output from this phase is a series of tables $(R_n(V_n D_n), n = 1, 2, \ldots, N)$ which provides the one-year net farm income obtainable for each possible state, stage and value of the decision variables. In this study, the remaining saturated thickness at year t, which is a continuous stratum, is approximated through the coarse grid approach (Nemhauser 1964). For example, if the water table is allowed to take on some 200 possible values, then each increment represents 0.5% of the initial supply.

2.3. *Phase* 2

The optimal temporal allocation of the state resources that will maximize the net present value of future returns is determined in Phase 2 by DP. Decisions are made at the beginning of each stage, and it is assumed that the resulting benefits, as well as the appropriate changes in the states, are known with certainty. As before, the stages are equally spaced along the planning horizon and they are numbered in chronological order. Hence, V_1 and V_n denote the initial and final states, respectively.

The profitability of an existing irrigation system in year n with a given saturated thickness X_n depends on the current number of wells W_n and the current type and size of the irrigation system S_n. The common options open to the farmer include the conversion to dryland crop production, the restaging of existing wells (if indicated by the state of the aquifer), and the drilling of additional wells. The options also include the replacement of all or part of the existing distribution system (depending on the age of the system) or the expansion of the distribution system. Within the capacity of an irrigation system, the farmer can choose to use varying quantities of groundwater. The anticipated return from choosing an irrigation system capacity in year n with distribution size s^* and well capacity w^* is equal to the current return from that system less current conversion costs plus the maximum discounted net future returns given that a system of capacity (s, w) is used in year n. The rational producer under certainty would

TABLE 2

Notation Summary for Intertemporal Model

Subscripts	
Year (Stage) $n =$	$1, 2, \ldots, N$.
Variables (Output From Model)	
$d^* =$	optimal temporal decisions.
$R_n(V_n) =$	maximum net present value at stage n of returns from stages n to N.
Parameters (Inputs To Models)	
$Z =$	$1/(1 + r)$ where r is the periodic rate of discount.
$D_n =$	set of permissible decision for stage n.
$R_n(V_n, D_n) =$	array of current returns at stage n for a producer operating in state V_n using all feasible decisions in set D_n.
$V_n =$	set of time varying state variables $= (X_n . W_n, S_n)$.
Functionals (Model Structure Elements)	
$f_{n+1}(t(V_n, d_n)) =$	net present value of returns in future stages with decision d_n and state V_n.
$t_n(V_n, d_n) =$	single-valued transformation function providing the value of V_{n+1} given V_n and d_n.

make those conversions which maximize the net present value of his profit stream. Since for a given farm state at any point in time, the optimal decisions for the future are independent of the past policy; and since the returns are additive, the problem is amenable to DP maximization according to Mitten's (1964) Composition Principles.

Recall that D_n is an admissible space of irrigation planning variables among which w is the number of wells used, s is the size of the irrigation system and q is the pumpage level. Let D_n be the specific decisions which can be made at the beginning of stage n and define by d_n the specific decisions taken at the beginning of stage n ($d_n \in D_n$). Table 2 summarizes the DP notation. The single value transformation function indicating the state of the system at the begining of stage ($n + 1$) given that the system is currently in state V_n stage n when decision d_n is taken is $t_n(V_n, D_n)$.

Returns in future time periods are discounted to a present value in stage n. The parameter z is equal to $1/(1 + r)$ where r is the periodic rate of discount for one stage and $f_{n+1}(t_n(v_n, d_n))$ is the NPV of returns in future stages (i.e., $n + 1, n + 2, \ldots, N$) if the farm is currently in stage n, state V_n and decision d_n is taken. The objective is to determine $R_n(V_n)$, the maximum net present value at stage n of returns from stages n to N when an optimal decision rule is followed. The appropriate recurrence relations associated with the backward analysis of the net present value for a finite planning horizon of this irrigation planning problem is:

$$R_n(V_n) = \operatorname*{Max}_{d_n}\left[R_n(V_n, d_n) + zf_{n+1}(t_n(V_n, d_n))\right], \quad d_n \in D_n; \quad n = 1, 2, \ldots N - 1, \tag{7}$$

and the boundary condition is:

$$R_N(V_N) = \operatorname*{Max}_{d_N}\left[R_N(V_N, d_N)\right], \quad d_N \in D_N. \tag{8}$$

The answer is a set $d^* = (d_1, d_2, \ldots, d_N)$ of optimal farm management and irrigation development decisions.

A closed form solution may be derived under the following conditions: (1) $R_n(V_n, d_n)$ is concave in d_n, (2) D_n is a convex set; and (3) t_n is linear in V_n and d_n (Thomas 1976). Since the irrigation system model restricts D_n to be an integer, a closed form solution is unachievable. The maximization is equivalent to an integer program and the recursive equations are solved numerically. The computational work involved in obtaining a solution can be reduced by approximating continuous states, such as the aquifer level, with a tractable number of intervals and by exploiting the physical constraints on the system states. For example, irrigation ceases when the state of the underground water level comes within 25′ of the bottom of the aquifer. Note that this two-phase methodology resembles the classical way in which irrigation-system related decisions are made (Hedges 1963, Johl 1980). That is, first one figures the best annual resource allocation plan and then interacts it with the longer-term water management decisions.

3. Application of the Model

An example application is provided below. The application is concerned with the evaluation of investments in alternative irrigation distribution systems in the Texas High Plains. It deals with a 640-acre tract with 630 tillable acres of land in the medium textured soil area of the region. At the beginning of the planning horizon, the operator is assumed to want to compare the profitability of furrow (FUR) and low pressure center pivot (LPCP) systems. With the FUR system, water would be delivered onto 1 to 4 quarter sections through underground pipe. Under the LPCP system, the farmer would install from 1 to 4 quarter section pivots. Either system can be maintained by replacing the equipment every 15 years over a 45-year planning horizon. The operator will drill additional wells as the water table declines so as to maintain irrigation capacity for the farm, if such action is profitable. The LPCP system is designed to operate on 35 psi of pressure at the pivot head and have a 66% application efficiency. The FUR system requires 10 psi at the well head and has a 50% application efficiency. Due to technical limitations, the "corners" will be unirrigated with the LPCP system; therefore, the maximum area irrigated is 528 acres.

Crop	Corn	Cotton				Wheat		PVTS	Wells	RHS
	X_{01}	X_{31}	X_{32} . . .	X_{40}	X_{41}	X_{85}	X_{96} . . .	X_{98}	X_{99}	
Net Rev. $	68 . . .	93	103 . . .	101	125 . . .	40	42 . . .	0	0	
Restraints										
Land	1.0 . . .	1.0	1.0 . . .	1.0	1.0 . . .	1.0	1.0 . . .		<	630
T. Irr in.	20 . . .	0	6 . . .	10	10 . . .	0	20 . . .		<	7560
AI Jan–Feb							3.6		− 449 <	0
AI Mar–Apr	5.4		5.4	5.4	5.4		7.2		− 449 <	0
AI May	3.6 . . .		. . .		. . .				− 225 <	0
AI June I	. . .		. . .		. . .				− 112 <	0
AI June II	3.6								− 112 <	0
AI July I	3.6								− 112 <	0
AI July II									− 112 <	0
AI Aug I				3.6					− 112 <	0
AI Aug II	. . .				3.6 . . .				− 112 <	0
AI Sep–Dec	. . .				. . .		9.0		− 899 <	0
AC Summer	1.0 . . .		1.0	1.0	1.0 . . .			− 132	<	0
AC Winter							1.0	− 132	<	0
Max AC	1.0		1.0	1.0	1.0		1.0		<	528
Pivots								1	<	3
Wells									1 <	4

Abbreviations used: AI = Acre Inches, AC = Acres Irrigated, June I = 1 – 15, PVTS = Number of Pivot Systems Used

FIGURE 1. Structure of Annual Linear Programming Tableau Used to Maximize Short-Term Profits from the Study Farm with Three Pivot Systems, Four 200 Gallon per Minute Wells, and a Maximum Annual Water Use of 630 Acre Feet.

In aquifers such as the Ogallala in West Texas, the specific water yield is almost equal to the storage coefficient, which is defined as the volume of water that an underground formation will drain under the force of gravity. The area studied overlays 200 feet of saturated thickness, and based on previous studies (Wyatt et al. 1977), a coefficient of storage of 15 percent is selected. The approximated volume of water in storage, in acre-feet per surface acre, is obtained by multiplying the saturated thickness by 0.15. Natural recharge from infiltration of precipitation is estimated to be about one inch per year. The relationship between the annual pumpage volume and the average drawdown of the saturated thickness determines the stage transformation function for the state of the aquifer. Following empirical approximations derived from regional hydrologic studies (Wyatt et al. 1977) the new state of the aquifer is given by:

$$X_n = X_{n+1} - gP_{n-1} + NR, \tag{9}$$

where X_n is the aquifer state at n, P_n is the depletion due to the annual pumpage, g is a constant depending on the storage and recirculation coefficients, and NR is the average contribution of the natural recharge. Similar approaches are used in other studies (e.g.: Bredehoeft and Young 1970, Hardin and Lacewell 1980, Young and Coomer 1980).

A simplified form of the single period (annual) linear programming model used to illustrate the LDP method is shown in Figure 1. The model contains one class of land and seasonal constraints relating to the irrigation requirements of various crops. The natural gas and crop prices shown in Table 2 were assumed to remain constant. Accordingly, a 5% real rate of discount is used.

4. Results and Conclusions

The beginning situation (state) for a particular producer is characterized by the initial water supply, the number of existing wells, and the size and type of the distribution system. Because farmers pump from a common water supply, the results are valid to the extent that all farmers make similar decisions. For brevity of this case analysis, we have not attempted to introduce risk or tax credits, or to consider the implications of energy prices rising relative to crop prices. Crop prices used in the analysis are given in Table 3. Table 4 presents cost of drilling irrigation wells and

TABLE 3

Prices for Crops and Natural Gas Used in the Analysis

Crop	Unit	Price
Cotton Lint	lb	$ .66
Corn	bu	3.10
Grain Sorghum	cwt	4.80
Wheat	bu	3.80
Natural Gas	MCF	2.75

TABLE 4

Cost of Drilling Irrigation Wells and Replacing Pump Bowls Used in the Analysis

Remaining Feet of Saturated Thickness	Capacity of Well (GPM)	New Well (Complete)	Replacement of	
			Bowls	Engine
200–100	1,000	$19,700	$2,680	$4,800
100–75	800	19,600	2,670	2,185
75–50	600	15,515	2,710	1,450
50–25	200	12,575	2,790	1,225

TABLE 5

Cost of Establishing a Low Pressure Central Pivot or a Furrow Distribution System

Low Pressure Center Pivot		Furrow Distribution System	
Central Pivot System	$34,000	Underground Pipe	$4,225
Underground Pipe	3,666	Risers	1,125
Less: Discounted Salvage		Gated Pipe	1,100
Value	− 2,182	Hydrants	300
Total	$35,484	Total	$6,750

replacing pump bowls, and Table 5 presents the costs of establishing the irrigation systems.

The PLP phase is accomplished by using standard MPS-IBM software. The results were tabulated and entered into a DP routine which has been programmed in FORTRAN IV according to §2. Technical programming details are beyond the scope of this paper; however, auxiliary disk storage, 2 word integer arrays and implicit search techniques are used to reduce core memory requirements. Phase 2 of the case discussed here called for a memory partition of 264K and took about 160 seconds of CPU time on the National Advanced Systems AS/6 computer to reach an optimal solution. A single solution generates an optimal trajectory for each of 10,000 possible starting states. However, only the optimal trajectories from a few selected states are presented here. The DP computer program is available to readers on request to the author.

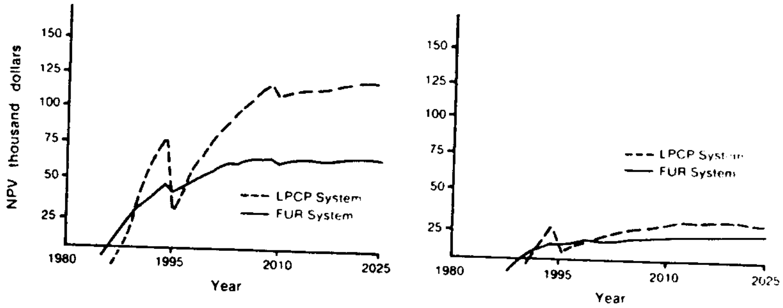
The estimated NPV of returns to irrigation from the LPCP or FUR systems for a farmer with no initial irrigation development or wells, but who has 200, 150, 100 or 50 feet of water saturated sand are shown in columns 1 and 2 of Table 6. The farmer with 100 feet (or more) of saturated sand who used the LPCP system would, under the assumption of the study, find that system to be more profitable. However, a comparison of the cumulative discounted profits from the respective systems (Figure 2) indicates that it would require approximately 20 years before the discounted profits from the LPCP system permanently surpass those from the FUR system. The sharp declines in the level of cumulative discounted profits in Figure 2 reflect expenses for the replacement of the distribution and/or the restaging of the irrigation wells at various points throughout the planning horizon.

The optimal time and amount of capital expenditure for a producer developing an irrigated farm for either a furrow or center pivot system is summarized in Table 7. When there is 200 feet of saturated sand per surface acre it is optimal for the producer with the LPCP system to drill 3 wells and irrigate 528 acres, while under the FUR system it is most profitable to drill 4 wells and irrigate 630 acres. However, when the initial saturated thickness is 150 feet or less, it is most profitable to operate 3 wells with the LPCP system but only 2 wells with the FUR system. This occurred because the net marginal cost per acre inch of water applied with the LPCP system is less than with the

TABLE 6

Comparison of Net Discounted Returns from a Furrow and from a Low Pressure Central Pivot System for Alternative Initial Resource Situations

	Total NPV for Initially Undeveloped Farm		Gain From Conversion on Initially Irrigated Farm	
Saturated Thickness (Feet in 1980)	LPCP	FUR	FUR to LPCP	LPCP to FUR
200	$1,242,150	$1,180,225	$35,363	− $204,299
150	1,161,438	1,128,590	32,779	− 201,715
100	1,071,472	1,058,268	− 32,811	− 136,131



Initial Saturated Thickness = 150

Initial Saturated Thickness 100

FIGURE 2. Comparison of Cumulative Net Discounted Profits from a Low Pressure Central Pivot and a Furrow Distribution System When the Initial Depth of Water Saturated Sand is 150 and 100 Feet.

FUR system. Hence, it is profitable to apply more groundwater per irrigated acre with the LPCP system than with the FUR system. Total number of acre feet used is approximately the same with either system because more acres were irrigated (630) with the FUR system than with the LPCP system (528) (Table 7). As the supply of ground water declined, it is no longer profitable to drill additional wells to maintain the total pumping capacity of the farm. Rather, the pumps in the existing wells are restaged. However, when the producer with the FUR system had fewer than 50 feet of saturated sand remaining, it is optimal to restage only enough wells to complete the preplant irrigation within the alloted 2 month time period. The proportion of the acreage irrigated a second time within the season is limited by the well capacity.

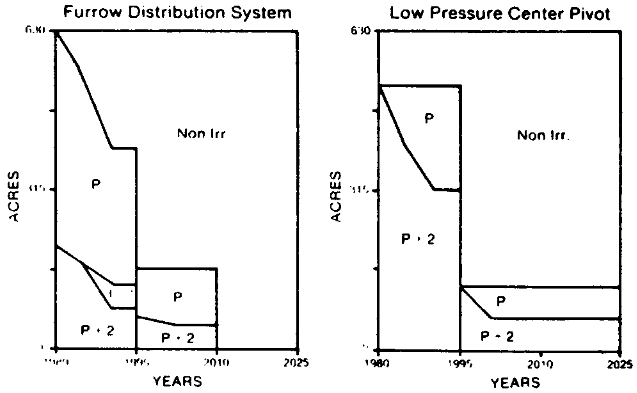
If the producer has 150 feet of water-saturated sand at the beginning of the planning horizon, it would be most profitable to irrigate all four quarters during the first 30 years (Table 7) and irrigate only one quarter section during the last 15 years. When either the FUR or LPCP system is used, the LPCP system provides a greater annual stream of returns over variable cost than the FUR system but has substantially greater capital costs (Table 7).

TABLE 7

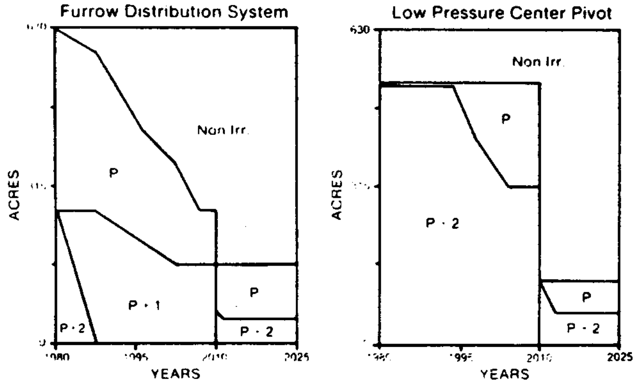
Optimal Capital Expenditures Required to Maximize Long-Term Discounted Profits from Furrow and Center Pivot Irrigation Systems when the Initial Saturated Thickness was 150 and 100 Feet

Furrow System			Low Pressure Center Pivot		
Year	Action	Cost	Year	Action	Cost
Initial Saturated Thickness = 150 Feet					
1980	Drill 2–1,000 gpm wells	39,400	1980	Drill 3–1,000 gpm wells	59,100
1980	Underground pipe 4 qts*	27,000	1980	Buy 4 LPCP systems	141,936
1991	Convert 2 wells to 800 gpm	5,340	1993	Convert 3 wells to 800 gpm	8,010
1995	Underground pipe 4 qts*	27,000	1995	Buy 4 LPCP systems	141,936
1998	Convert 2 wells to 600 gpm	5,420	2001	Convert 3 wells to 600 gpm	8,130
2010	Underground pipe 1 qts*	6,750	2010	Buy 1 LPCP system	35,484
2010	Convert 2 wells to 200 gpm	5,580	2012	Convert 3 wells to 200 gpm	5,580
Initial Staturated Thickness = 100 Feet					
1980	Drill 2–800 gpm wells	39,200	1980	Drill 3 – 800 gpm wells	58,800
1980	Underground pipe 4 qts*	27,000	1980	Buy 4 LPCP systems	141,936
1987	Convert 2 wells to 600 gpm	5,420	1987	Convert 3 wells to 600 gpm	8,130
1995	Underground pipe 1 qts*	6,750	1995	Buy 1 LPCP system	35,484
2000	Convert 2 wells to 200 gpm	5,580	2000	Convert 2 wells to 200 gpm	5,580
2010	Cease irrigation	—	2009	Cease irrigation	—

*qts = 160 acres



Initial Saturated Thickness 100 Feet



Initial Saturated Thickness 150 Feet

Figure 3. Comparison of Long-Term Cotton Production with the Number of Preplant and Post Plant Irrigations between a Low Pressure Center Pivot and a Furrow Distribution System for a Farm with 630 Croppable Acres and 100 or 150 Feet of Water Saturated Sand.

The cropping patterns for an initial saturated thickness of 150 and 100 feet for both the LPCP and FUR systems are shown in Figure 3. Alternative crops, such as wheat, corn and grain sorghum, were found to be less profitable than cotton under the assumptions of the study. Figure 3 shows that over time the amount of irrigation water used each year decreases for both systems. The general cropping pattern, changing from a high level of irrigation on cotton to a moderate level of irrigation on cotton, and in the last years to dryland cotton, reflects the declining water use over time. The rate of income accumulation decreases over time for both systems, as shown in Figure 2, resulting from the declining use of irrigation water as it becomes more expensive over time. The decline in the water level increases the pumping lift which reduces the profit per unit of water. When the net profit per unit of water reaches zero, irrigation is terminated. Thus, the optimal irrigation management policy shows that, in this case study, economic depletion will occur before the point of physical exhaustion is reached. The foregoing irrigation trend, along with the anticipated economic depletion at certain regions, has an important bearing on the current public debate regarding the need for legislative limitations on the annual pumpage rates by farmers in the Southwest.

In Figure 3, the number of acres of cotton in each time period which would most profitably receive a preplant irrigation is designated by the symbol *P*. The optimal number of post plant irrigation applications in each case follows the "+" symbol. The number of acres irrigated and the number of irrigations per acre decline over time as the groundwater supply diminishes. It is of interest that under the assumptions of the study, the producer using the system with the higher application efficiency (the LPCP system) would use the "saved" water to increase the number of irrigations in the near term rather than to increase the number of years in which irrigation is possible. It seems that benefits from more water-energy efficient irrigation systems may come from the expansion of current irrigation rather than from an extended period of irrigation when water is initially scarce relative to land (Figure 3).

This study demonstrates that it is feasible to simultaneously determine optimal, discrete capital expenditure patterns for both intratemporal and intertemporal groundwater use. The annual LP models used by the LDP procedure provide the "shadow prices" (simplex multipliers) which can be used for evaluation of resources and managerial identification of major constraints on farm income.

The LDP procedure is currently being used to evaluate alternative irrigation systems in conjunction with water conservation practices under various future price scenarios. Possible extensions to the model include analysis of stochastic weather effects and their interaction with the optimal irrigation policy (Bras and Cordova 1981). Further research may incorporate the effects of more detailed groundwater equations of hydraulic management of the aquifer (Maddock 1972, 1973 and Gorelick 1983); additional investigation should also regard such issues as the impact of peak load electrical power pricing and federal tax incentives to foster the adoption of water conservation technologies.[1]

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