Material Flow, 3 (1986) 69–82
Elsevier Science Publishers B.V., Amsterdam – Printed in The Netherlands

ANALYSIS OF RECIRCULATION IN ROBOTIC SYSTEMS USING FEEDBACK MODELS

R.G. Wilhelm

Cincinnati Milacron, Cincinnati, Ohio (U.S.A.)

S.Y. Nof

School of Industrial Engineering, Purdue University, W. Lafayette, IN (U.S.A.)

and A. Seidmann

Department of Industrial Engineering, Tel Aviv University (Israel)

ABSTRACT

Task recirculation is defined for production systems and several models for predicting its effects are shown. The random product feedback flow model is compared with simulation results for a single-cell model and simulation results are also presented for a two-cell recirculation model. Applications of recirculation in programmable assembly systems are illustrated, and the recirculation models are used to predict performance for example systems.

1. INTRODUCTION

With the advent of capable sensing systems and flexible production equipment, it is now quite possible to perform several operations on a workpiece at a single production station. Ideally, a part will require one operation but, if errors arise, remedial steps can be taken to repair or rework the part while it is still at the workstation.

These additional activities at the station can be characterized by additional arcs and nodes on a network diagram, as depicted in

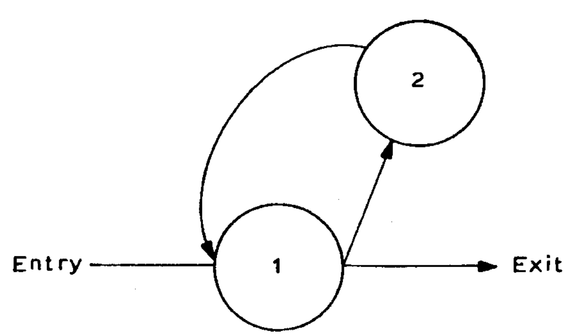


Fig. 1. Recirculation in a production system.

0167-1936/86/$03.50

Fig. 1. As the diagram shows, a workpiece enters the system and operation 1 is performed. Upon completion, the workpiece either exits the system or is reworked (operation 2). Once the rework operation is complete, operation 1 is repeated and, if successful, the workpiece leaves the system. The direction of arcs in the network model suggests recirculation—the workpiece cycles through operations at a workstation until the task is completed successfully. It is assumed that, if a problem cannot be fixed, components will be replaced as part of the rework so that a good product will eventually emerge.

This strategy of successive operations, or recirculation, is already being used in several robotic assembly applications. One example involves the assembly of computer circuit cards. Integrated-circuit modules with pin connectors are inserted by an industrial robot into a printed circuit board. Sometimes the pins of a module may be bent and prevent proper insertion. The robot is equipped with sensors that can detect an improper insertion and, when this is sensed, the robot takes the module in question to a straightening device and aligns the connector pins. Once this has been done, the insertion is attempted again. This example maps directly to the recirculation shown in Fig. 1.

Recirculation relies heavily on in-process inspection, both of workpieces and processes. Clearly, this approach adds manufacturing costs, but additional benefits come with this cost. Expected errors in parts and production can be handled routinely during the regular production process without system shutdowns or operator intervention. The percentage of acceptable parts produced by a system with recirculation can approach 100% since, within limits, only good parts leave the system. Also, costs associated with faulty processing are reduced since additional tasks are only done after one task has been completed successfully.

This problem has been studied by Dooley [1] for an integrated-circuit chip assembly system, where the probability of recirculation was very high. His approach was to use a simulation of the material flow to predict equipment requirements for the system. The simulation accounted for over 50 causes of recirculation and included several different paths that a product might take. Most notably, because of the size of the problem, this approach modeled the movement of defects in the system rather than the movement of parts. Newhart and O'Leary [3] have also looked at recirculation in a production system for integrated circuits. A simulation model was used to evaluate the application of different scheduling rules.

Seidmann and Nof [6] have approached recirculation in a more general way with a random-product feedback flow model (RPFF) which can predict system performance for a unitary manufacturing cell with feedback. Such a cell operates an automated group of workstations which are tended by a material-handling robot. It produces one single product at a time. The unitary design is applied in light assembly lines with relatively short cycle times, e.g.: insertion of solid state chips into PC (printed circuit) boards.

The RPFF model requires only that the mean and variance of process times be provided so any process-time distribution may be used. Using this model, the mean and variance of the total number of recycles and the total time that a part spends in the system can be calculated. The time spent at each station can also be calculated. From these quantities, system performance can be evaluated.

Building on the RPFF model, methods are now being developed to evaluate production systems with product recirculation. Several simulation models have been used to investigate the behavior of single-cell systems with recirculation. Results from the simulation models have been compared with those from the RPFF model. A simulation model has also been used to consider multiple-cell systems with recirculation. In the subsequent sections, these models are described and results are presented. From these interim results, examples show where these recirculation models can be applied to production systems.

2. THE RANDOM PRODUCT FEEDBACK FLOW MODEL

The random product feedback flow model, as developed by Seidmann and Nof [6], uses probabilistic analysis to calculate performance measures for a single manufacturing cell with feedback for rework, as shown in Fig. 2.

In this system, a product enters and is processed at the main work station M1. Once processing is complete, the product will either leave the system or proceed to the rework station R1, according to a branching probability p. When a product branches to the rework station, some rework is done and then the product reenters the main work station. It is assumed that the single-passage times through each station are independent random variables. A probability distribution function

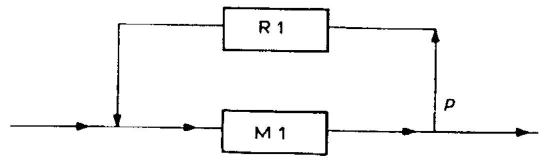


Fig. 2. Single cell RPFF model.

for each of these times can either be theoretically developed or it can be based on a statistical analysis of industrial data.

By analyzing the conditional probability that a part exits the cell after a specific number of passes, one can calculate the mean and variance for the time that a part spends in the system and also the areal times, or the total time that a part spends in each of the stations. Since no specific distribution has to be assumed, this method can be used for general applications. A thorough explanation of the model is provided in [6]. The equations for calculating performance measures are summarized in Table 1.

The RPFF model has been extended to accommodate batches of products in a cell. Since single passage times are assumed to be independent, the mean time and variance for

TABLE 1

RPFF equations

Measure	Mean value	Variance value
Number of visits in area M1	$\mu_k = 1/q$	$\sigma_k^2 = p/q^2$
Total time in Area M1	$\mu_{\overline{MT}} = \mu_{M1} q^{-1}$	$\sigma_{\overline{MT}}^2 = q^{-1}(pq^{-1}\mu_{M1}^2 + \sigma_{M1}^2)$
Total time Area R1	$\mu_{\overline{RT}} = \mu_{R1} pq^{-1}$	$\sigma_{\overline{RT}}^2 = q^{-1}(pq^{-1}\mu_{R1}^2 + p\sigma_{R1}^2)$
Total time in cell	$\mu_W = (\mu_{M1} + p\mu_{R1})q^{-1}$	$\sigma_W^2 = (\mu_{M1} + \mu_{R1})^2 pq^{-2} + (\sigma_{M1}^2 + p\sigma_{R1}^2)q^{-1}$
Covariance between Total areal times	$\mathrm{Cov}(\overline{M1}, \overline{R1}) = pq^{-2}\mu_{R1}\mu_{M1}$	

Nomenclature: p = rework probability; q = exit probability ($q = 1 - p$); μ_{M1} = mean single passage time in M1; μ_{R1} = mean single passage time in R1; σ_{M1}^2 = variance of single passage time in M1; σ_{R1}^2 = variance of single passage time in R1.

INPUT

DATA SET	MAIN MEAN	MAIN VARIANCE	REWORK MEAN	REWORK VARIANCE	REWORK PROB.	BATCH SIZE
1	.20	.04	.20	.04	.200	1.
2	.80	.64	.80	.64	.200	1.
3	1.00	.20	1.00	.20	.200	1.
4	1.00	.80	1.00	.80	.200	1.
5	.60	.12	.60	.12	.200	1.
6	2.40	1.92	2.40	1.92	.200	1.

OUTPUT
======

Total Cell Values

DATA SET	MEAN TIME IN CELL	VARIANCE	COVARIANCE (MAIN & REWORK)	C.OF VAR (PASSES)	C.OF VAR (TIME)
1	.30	.11	.01	.4472	1.1055
2	1.20	1.76	.20	.4472	1.1055
3	1.50	1.55	.31	.4472	.8300
4	1.50	2.45	.31	.4472	1.0435
5	.90	.63	.11	.4472	.8819
6	3.60	10.08	1.80	.4472	.8819

Areal Values

DATA SET	BATCH SIZE	MAIN MEAN	MAIN VARIANCE	REWORK MEAN	REWORK VARIANCE
1	1.	.25	.06	.05	.02
2	1.	1.00	1.00	.20	.36
3	1.	1.25	.56	.25	.36
4	1.	1.25	1.31	.25	.51
5	1.	.75	.26	.15	.14
6	1.	3.00	4.20	.60	2.28

OUTPUT
======

Performance Measures

DATA SET	PERCENTAGE REWORK TIME	PRODUCTION RATE	EFFICIENCY
1	16.67	3.33	.833
2	16.67	.83	.833
3	16.67	.67	.833
4	16.67	.67	.833
5	16.67	1.11	.833
6	16.67	.28	.833

Fig. 3. Typical output for RPFF analysis.

a batch will simply be the sum of the mean and variance for each part in the batch.

A FORTRAN program has been written to calculate performance measures for the batch RPFF model. The program operates via a series of subroutines to accept input, calculate measures, and print output and plots. The subroutines are linked with common statements allowing additions and deletions for particular applications. For example, a cost equation has been added for some analyses. Sample input and output are shown in Fig. 3.

3. SIMULATION OF A SINGLE CELL SYSTEM

The single-cell model has been simulated using the SLAM [4] simulation language. The system was modeled with a discrete-event simulation.

An example of the simulated system is shown in Fig. 2; parts flow through the system and branch to rework or finish, according to a given branching probability. At all times, only one part is in the system. Once it finishes service, another part enters.

Statistics were collected for the mean and standard deviation of time in each station and the total time in the system. The simulations were stopped by using a sequential stopping rule that employed batch means analysis to estimate the confidence interval for the estimated total time in the system. Once the half-width of the confidence interval was less than 5% of the estimated mean, the simulation was stopped [2,5].

4. RECIRCULATION IN A TWO-CELL SYSTEM

A two-cell system with recirculation has been modeled as illustrated in Fig. 4. Parts enter at station M1 and are passed to rework at station R1 with a probability of p_1. If a part is not reworked after completing service at station M1, it passes to station M2 for the next task. After completing service at M2, it may also require rework and it will pass to station R1 with a probability of p_2. In this model, it is assumed that all rework is performed in a common rework station. After completing rework, the part will always begin service first at station M1. Experience with several programmable assembly operations with rework led to this approach. It is likely that other systems will require different assumptions about the rework paths and stations.

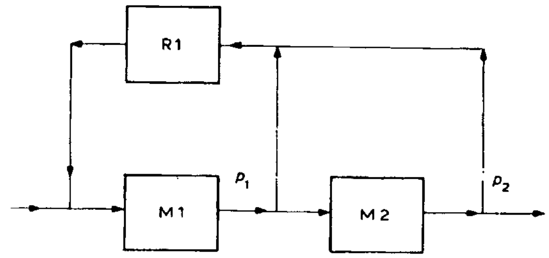
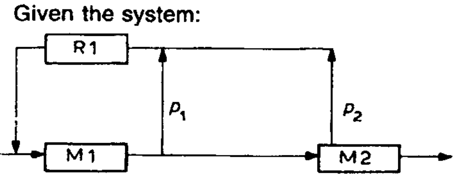


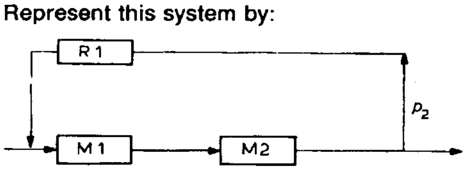
Fig. 4. Two cell system with recirculation.

The RPFF model may be used to calculate performance measures for this system since it can be represented as shown in Fig. 5. The two-cell system has also been simulated using the discrete-event portion of SLAM. The simulation operates nearly the same way as

Given the system:



With μ_{M1}, μ_{M2}, μ_{R1}, σ^2_{M1}, σ^2_{M2}, σ^2_{R1},
Represent this system by:



where: $q_1 = 1 - p_1$ and $\mu_{\widetilde{M1}} = (\mu_{M1} + p_1\mu_{R1})q_1^{-1}$
with: $\sigma^2_{\widetilde{M1}} = (\mu_{M1} + \mu_{R1})^2 p_1 q_1^{-2} + (\sigma^2_{M1} + p_1\sigma^2_{R1})q_1^{-1}$
Since the distributions are independent, the means and variances are additive.
Therefore:

$$\mu_W = (\mu_{\widetilde{M1}} + \mu_{M2} + p_2\mu_{R1})q_2^{-1}$$

and

$$\sigma^2_W = (\mu_{\widetilde{M1}} + \mu_{M2} + \mu_{R1})^2 p_2 q_2^{-2} + (\sigma^2_{\widetilde{M1}} + \sigma^2_{M2} + p_2\sigma^2_{R1})q_2^{-1}$$

where $q_2 = 1 - p_2$

Fig. 5. RPFF solution for two cell system.

the single-station simulation. Only one part is allowed into the system at a time, and statistics are collected for the time that a part spends in each area of the system. The simulations were stopped according to the procedure described earlier.

TABLE 2

Input for RPFF and simulation analysis of single-cell model

Distribution	Case	Mean	Variance	Batch	Rework probability
Exponential	1	0.20	0.04	2	0.2
	2	0.40	0.16	2	0.2
	3	0.80	0.64	2	0.2
	4	1.00	1.00	2	0.2
	5	1.20	1.44	2	0.2
	6	2.00	4.00	2	0.2
Normal	1	1	0.20	10	0.2
	2	1	0.40	10	0.2
	3	1	0.80	10	0.2
	4	1	1.00	10	0.2
	5	1	1.20	10	0.2
	6	1	2.00	10	0.2
Uniform	1	0.60	0.12	5	0.2
	2	1.20	0.48	5	0.2
	3	2.40	1.92	5	0.2
	4	3.00	3.00	5	0.2
	5	3.60	4.32	5	0.2
	6	6.00	12.00	5	0.2

TABLE 3

Input for simulation analysis of two-cell model

Distribution	Case	Mean	Variance	Batch	Rework probability
Exponential	1	0.20	0.04	1	0.2
	2	0.40	0.16	1	0.2
	3	0.80	0.64	1	0.2
	4	1.00	1.00	1	0.2
	5	1.20	1.44	1	0.2
	6	2.00	4.00	1	0.2
Normal	1	1	0.20	1	0.2
	2	1	0.40	1	0.2
	3	1	0.80	1	0.2
	4	1	1.00	1	0.2
	5	1	1.20	1	0.2
	6	1	2.00	1	0.2
Uniform	1	0.60	0.12	1	0.2
	2	1.20	0.48	1	0.2
	3	2.40	1.92	1	0.2
	4	3.00	3.00	1	0.2
	5	3.60	4.32	1	0.2
	6	6.00	12.00	1	0.2

5. MODEL INPUTS

For the RPFF model and the two simulation models, six parameter sets were used for each of three distributions. Inputs for each model are shown in Tables 2 and 3. For ease of comparison, the means and variances for each distribution were adjusted so that the ratios of mean to variance were the same across the three distributions. For example, for the first parameter set in each distribution, the variance is 0.2 times the mean. For simplicity, all the rework tasks were simulated with the same parameter sets and distributions as for the main tasks.

In the two-cell simulation, the parameter sets were identical for the two main tasks and rework.

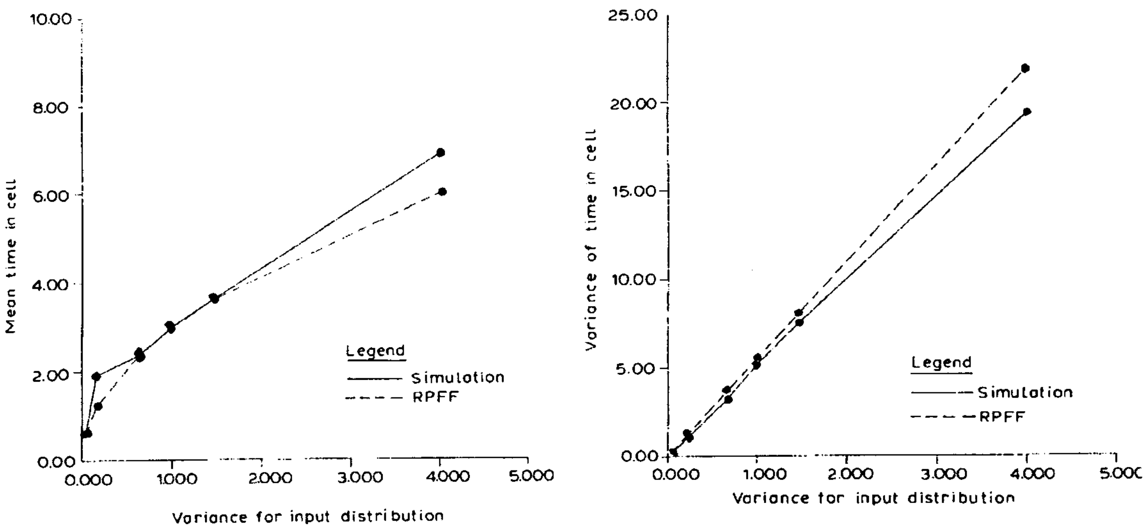


Fig. 6a. Single cell simulation and RPFF output for exponential distribution (batch = 2).

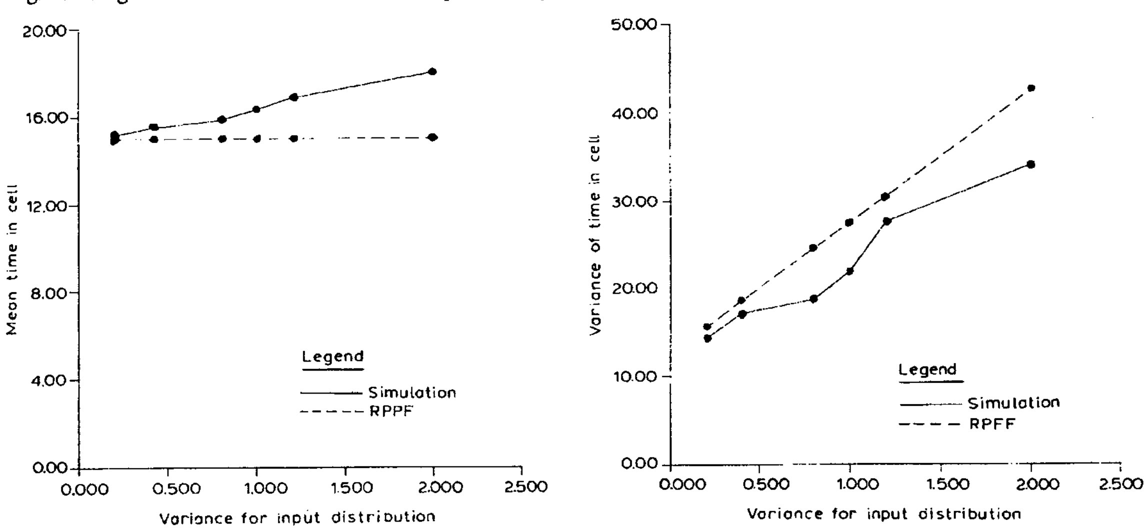


Fig. 6b. Single cell simulation and RPFF output for normal distribution (batch = 10).

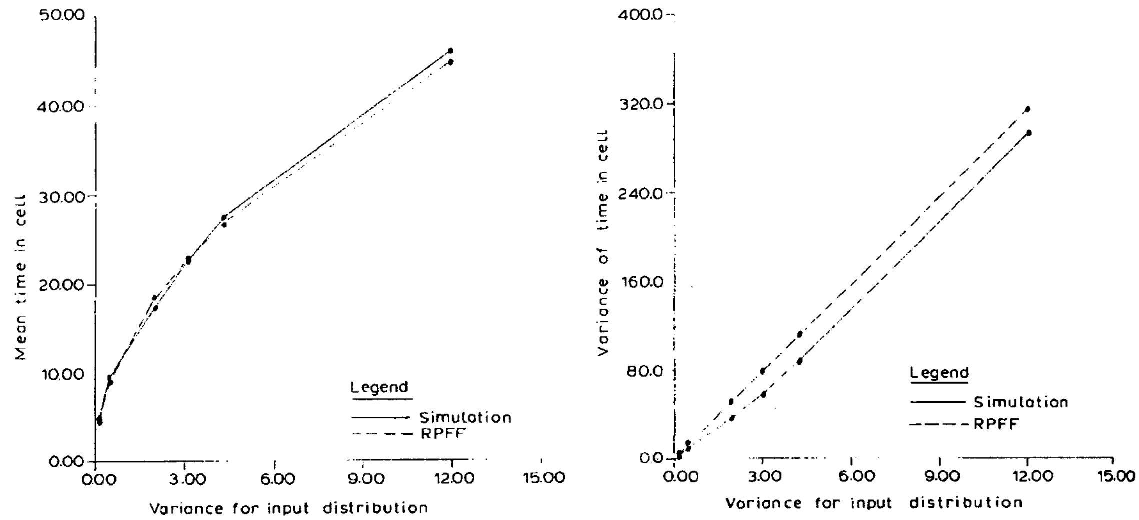


Fig. 6c. Single cell simulation and RPFF output for uniform distribution (batch = 5).

6. COMPARISON OF RESULTS FOR THE SINGLE-CELL MODELS

Eighteen simulations were made of single-cell systems with recirculation, using three different distributions, and the results from these simulations were compared with results from the random-product feedback flow (RPFF) model. Results are presented in Figs. 6a, 6b and 6c. RPFF results are plotted with a dashed line and simulation results are plotted with a solid line.

A probability density distribution of time spent in the system for one of the simulations is also shown in Fig. 7. The distribution was generated by using observations from one very long simulation run. As the time-in-system statistics were collected, the SLAM package automatically generated a histogram. As Seidmann and Nof have pointed out [6], this distribution for a unitary cell with feedback has oscillations with a frequency of $(\mu_{M1} + \mu_{R1})^{-1}$. As the probability of rework increases, the magnitude of the oscillations is more pronounced.

As Figs. 6a and 6c show, for the exponential and uniform distributions, the differences between RPFF and SLAM results for mean time in system were consistently within five percent of the RPFF values. For the normal distribution, the mean time results from the simulations differed from the RPFF results by an average of nine percent of the RPFF values, with a maximum of twenty percent

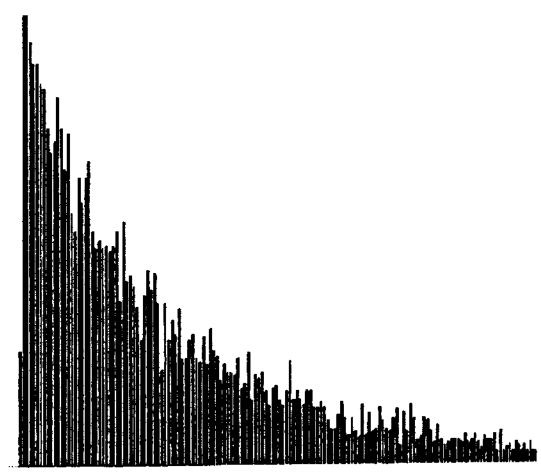


Fig. 7. Cumulative distribution for unitary cell.

difference. Most of the variation for the normal distribution results can be attributed to small sample sizes and minor errors in the random deviation routines of the simulation language [2].

In all three cases, the variance of time in system, as predicted by the RPFF model, was consistently higher than the results from the simulation. Differences ranged from ten to thirty percent of the RPFF values. It should be noted that the simulations were stopped according to a batch means test on the mean time in the system. No test was used to guarantee the accuracy of the variance. Differences in variance commonly occur when simulation results are compared to analytical results or historical data. In many cases, this behavior is caused by positive correlation and by minor numerical errors in the random deviation routines of the simulation language [2]. Considering the assumptions of the RPFF model and the inherent behavior of a simulation model, the results seem reasonable.

The single-cell results suggest that the time in system is most dependent on the mean process time at the work station. This is even more apparent when the equations for performance measures are considered. Variance also depends most directly on the mean process times for work and rework. As the probability of rework increases, the mean rework time must decrease to keep the production rate from dropping dramatically. In general, a single-cell system with rework will never produce at as fast a rate as a system without rework. Recirculation can only be justified by comparing production costs or costs resulting from perturbations that errors in production can cause.

7. RESULTS FOR THE TWO-CELL MODEL

Eighteen simulations were also made of two-cell systems with recirculation using three distributions. Results for each of the distributions are shown in Figs. 8a, 8b and 8c. The two-cell model appears to behave very much like the one-cell model when the results for time in system are considered. Of particular interest are the mean and variance for time spent in the rework task. For the normal distribution, the rework mean and variance converge very quickly to a constant value. For the other distributions, the rework mean and variance increase only marginally for higher values of input variance.

Also of interest is the average percentage of the time spent on the rework task. As Table 5 shows, the percentage of time in rework constantly stays near 17% for these particular parameters. The single-cell model yielded similar results, as shown in Table 4.

A larger system with six cells and recircula-

TABLE 4

Percentage of time spent on each task in one-cell systems

Distribution	Case	Station 1	Rework
Exponential	1	85	15
	2	84	16
	3	84	16
	4	83	17
	5	83	17
	6	84	16
Average		*84*	*16*
Normal	1	83	17
	2	83	17
	3	84	16
	4	83	17
	5	84	16
	6	84	16
Average		*83*	*17*
Uniform	1	82	18
	2	82	18
	3	84	16
	4	83	17
	5	83	17
	6	82	18
Average		*83*	*17*

tion, as shown in Fig. 9, has also been simulated. Results showed that, for many parameter sets where probabilities of rework were large, the system cycle time did not converge to a stable steady-state value even after a very long period of time. Cycle time stability depended on the cell time variance and the probabilities of rework. As cell time variance increased, probabilities of rework had to be reduced to insure stability. Figure 10 shows input data and results for a six-cell system with exponentially distributed cell times. The input data was the same for each cell and for rework. The output was stable since cell-time variances were between 0.8 and 2.0 times the cell mean times.

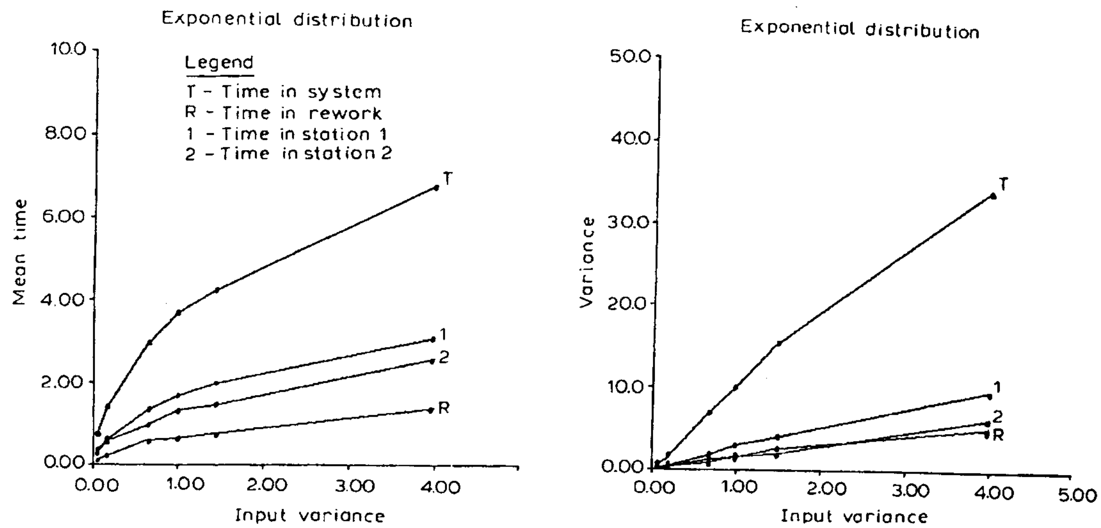


Fig. 8a. Two cell simulation output for exponential distribution (batch = 1).

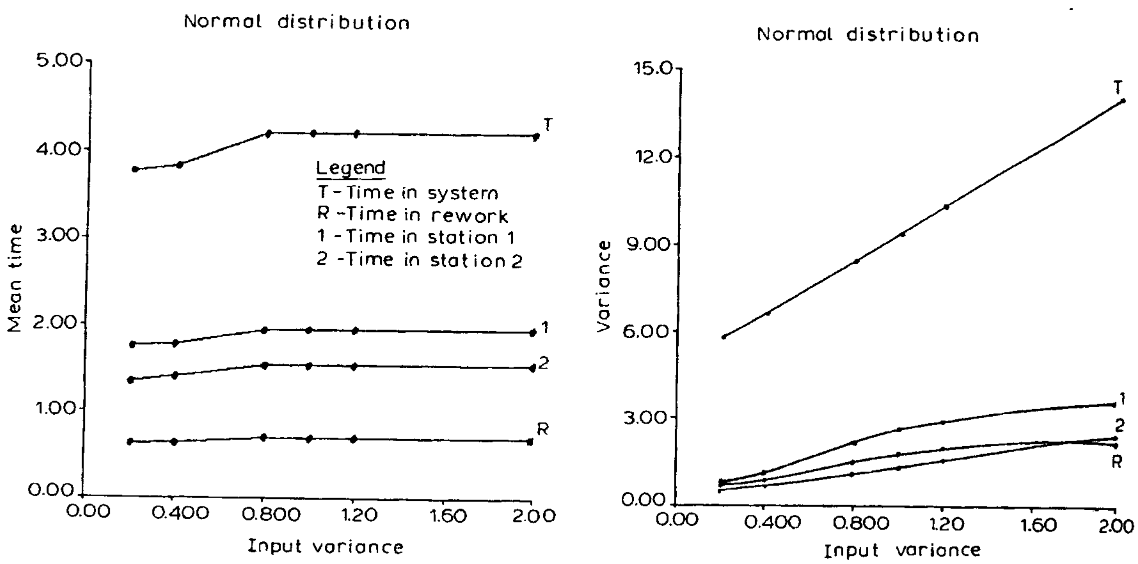


Fig. 8b. Two cell simulation output for normal distribution (batch = 1).

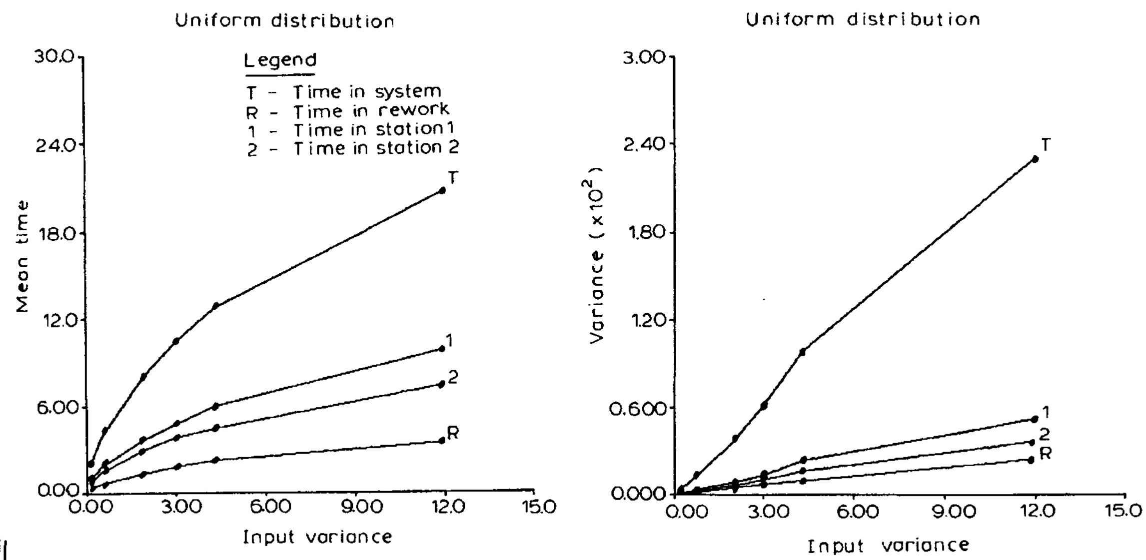


Fig. 8c. Two cell simulation output for uniform distribution (batch = 1).

TABLE 5

Percentage of time spent on each task in two-cell systems

Distribution	Case	Station 1	Station 2	Rework
Exponential	1	44	39	17
	2	43	42	15
	3	46	33	20
	4	46	37	18
	5	46	35	18
	6	46	38	21
Average		*45*	*37*	*18*
Normal	1	46	37	17
	2	46	37	17
	3	46	37	17
	4	47	36	17
	5	46	37	17
	6	47	37	16
Average		*46*	*36*	*17*
Uniform	1	47	38	15
	2	46	38	16
	3	46	37	17
	4	46	37	17
	5	47	35	18
	6	47	36	17
Average		*46*	*37*	*17*

Recirculation of this magnitude may not be acceptable for robotic systems but many manufacturing processes depend on multiple recirculation loops. As mentioned before, integrated-circuit fabrication [1] often requires many rework operations. Painting and coating operations may also behave like the system shown in Fig. 9. The results of the six-cell model suggest that, for robotic cells where operation is essentially unmanned, large amounts of recirculation represent a poor system design since the cell time may not be predictable. To be useful then, a cell with large amounts of recirculation must either have small probabilities associated with rework, or the recirculation loops must be independent.

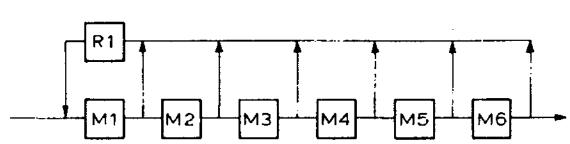


Fig. 9. Six cell system with recirculation.

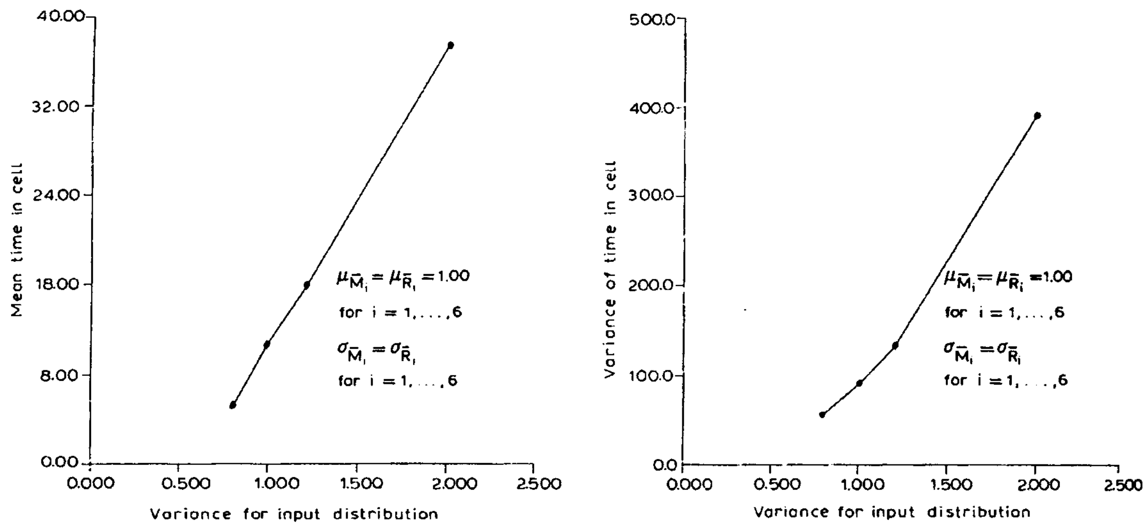


Fig. 10. Six cell simulation output for exponential distribution (batch = 1).

8. APPLICATION OF MEASURES

The models have all demonstrated the behavior of a production system with stochastic operating parameters. The performance measures derived are useful not only in the planning of such a system but also in the control and scheduling of tasks.

An example of a system where these measures would be relevant is a programmable assembly cell where tasks could be scheduled automatically, according to the materials and tools available. In the simplest case, a task might be to insert a part which could be picked by a robot from 1 of 3 feeders. Jammed or empty feeders would be attended to on a periodic basis, but the robot could continue operating as long as at least one feeder was functioning. The robot would insert the part into a workpiece and, if the insertion failed, the part would be discarded and a new part picked from a feeder. Insertion would then be attempted again. On encountering a bad part or a jammed feeder, the robot could move to another feeder and pick another part.

Since feeders will be located in unique locations, travel time for the robot will be different for each feeder. Insertion failures may have several causes—absence of part, faulty part, or faulty workpiece. All of these uncertainties will add to the variance of total task time. Since there is a fair amount of variability to this system, a deterministic calculation would be likely to underestimate the preferred cycle time for performing the task.

By using a model which accounts for variance in task times, an adjusted cycle time can be predicted for, say, 95% of all tasks and this cycle time will ensure that work is completed before another task is started. The variance of time in a station or in the cell is the most useful parameter for predicting this adjusted cycle time.

These measures have greater applicability in the design of programmable assembly systems, where in-process inspection and rework are used to ensure almost 100% quality in finished parts. Quality is surely a goal of all production systems, but some trade-off between quality and production rate must be made to arrive at the optimal production cost.

The insertion task example described previously can also be used to show how recirculation performance meaures can be used to

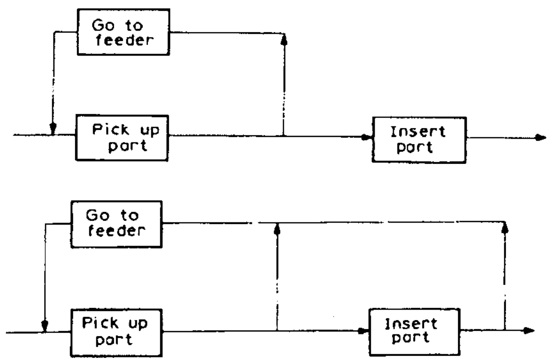


Fig. 11(a). System 1; (b) System 2.

predict output rates and the impact of quality considerations for different system designs. One might consider two designs for a given task. One where insertion was attempted until it was successful, and another where insertion was attempted only once and bad workpieces, where insertion failed, would leave the system and either be discarded or reworked later. In both cases, part picking is attempted until successful. These two distinct designs are illustrated in Figs. 11a and 11b.

For this example, the task and rework times are assumed to follow a normal distribution with the parameters shown in Table 6.

Suppose the probability of a bad part at the feeder is 0.05 and probability of a bad insertion is 0.05.

The mean cycle time and variance for system 1 can be calculated using the RPFF equations as shown below.

$$\mu_W = (1.0 + 0.05*3.0)/0.95 + 2.0 = 3.21$$

$$\sigma_W^2 = ((1.0 + 3.0)/0.95)^2*0.05$$

TABLE 6

Example tasks and rework times

Task	Mean	Variance
Pickup	1.0	0.4
Feeder travel	3.0	1.5
Insertion	2.0	0.1

$$+ (0.4 + 0.05*1.5)/0.95 + 0.1$$

$$= 1.49$$

The mean cycle time and variance for system 2 can be found by using the two-cell simulation model or the RPFF model. Results by simulation are then: $\mu_W = 3.66$; $\sigma_W^2 = 4.5$ (compared with 3.54 and 3.78 respectively by the RPFF approximation in Fig. 5).

Applying a 95% confidence interval, accounting for cycle times within 2 standard deviations of the mean, the cycle time for system 1 could be set at 5.6 while the system 2 cycle time could be set at 7.9. System 2 will turn out parts at a rate that is only about 70% of the production rate for system 1, but the yield of good parts will be higher since system 1 would have to produce an average of 105 parts to get 100 good ones. If the workpieces are too expensive to discard when errors occur, then system 2, with more capability for recirculation, would probably be more cost effective. However, the cost effectiveness must be balanced against additional production costs.

9. CONCLUSIONS

Three models have been shown for the analysis of production systems with recirculation. For the single-cell model, the RPFF and simulated results have shown the relationship between the variance of the individual task time and the total cycle (cell) time and its variance for a variety of distributions. The RPFF relatively simple model was shown to be a very good predictor of performance.

For the two-cell model, simulated and RPFF results were shown for a variety of distributions. An RPFF approximation model was also provided.

Examples have also been shown for the use of these models in the design of production systems with recirculation. With proper design, for which the models can serve well as evaluation tools, task recirculation can increase the yield of good parts although cycle times are also increased.

The RPFF model works well for predicting performance of robotic cells with task recirculation. The model requires only a small amount of data and can be used for all types of relevant distributions. The RPFF results are most accurate for tasks with single rework operations although multiple rework operations can be grouped together within a single cell. Additionally, multiple rework loops can also be analyzed separately. Results can be quickly generated to compare different system designs and to determine the amount of inspection and machine intelligence necessary to achieve a required system performance.

The RPFF model addresses the types of questions that are important in the design of an intelligent robotic cell. In many cases errors occur during manufacturing processes and with an intelligent machine it can be more efficient to react to these errors rather than to attempt to eliminate them. RPFF results can be used to evaluate the gains associated with this strategy.

Simulation and more complex models are necessary for more detailed analysis of tasks with recirculation. Particularly, when the amount to be produced is small, transient effects are of more interest than the steady state measure provided by RPFF. Also, in some cases, the amount of rework will depend on the number of times that recirculation has occurred. In many practical situations, recirculation is limited to only a finite number of recirculation attempts. In all of these cases, the RPFF model may be used to develop rough measures of system performance but additional models are required to provide adequate information for system control.

ACKNOWLEDGEMENT

Mr. Wilhelm and Dr. Nof acknowledge partial support for this research from Purdue University's Center for Computer Integrated Design, Manufacture and Automation (CIDMAC).

REFERENCES

1 Dooley, B.J., 1983. A model for the prediction of assembly, rework and test yields, IBM J. Res. Dev., 27 (1): 59–67.

2 Law, A.M., W.D. Kelton, 1982. Simulation Modeling and Analysis, McGraw-Hill, New York.

3 Newhart, D.D., J.W. O'Leary, 1973. Integrated circuit shop simulation for capacity planning and scheduling, Proceedings, Winter Simulation Conference.

4 Pritsker, A.A.B., C.D. Pegden, 1979. Introduction to Simulation and SLAM, Wiley & Sons, New York.

5 Schriber, T.J., R.W. Andrews, 1979. Interactive analysis of simulation output by the method of batch means, Proceedings, Winter Simulation Conference.

6 Seidmann, A., S.Y. Nof, (1985) Unitary manufacturing cell design with random product feedback flow, IIE Transactions, 17 (2): 188–193.

(Received April 2, 1984; accepted August 29, 1984)