

Design and operation of an order-consolidation warehouse: Models and application

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Abstract: A warehouse is a service facility, often comprising the only view that customers actually have of a manufacturing firm. The management of this facility has significant leverage over order leadtimes and fill-rate reliability. As with other service facilities, system design and operation are decision problems that are closely interlinked. In this paper we describe and model in general terms the composite design and operating problems for a typical order-consolidation warehouse. These problems include warehouse layout, equipment and technology selection, item location, zoning, picker routing, pick list generation and order batching. The complexity of the overall problem mandates developing a new multi-stage hierarchical decision approach. Our hierarchical approach utilizes a sequence of coordinated mathematical models to evaluate the major economic tradeoffs and to prune the decision space to a few superior alternatives. Detailed simulation employing actual warehousing data is then used for validation and fine tuning of the resulting design and operating policies. We describe the application of this analytical approach to an automotive spare-parts distribution centre. The case study demonstrates substantial savings in operating costs and highlights several generic management tradeoffs.

Keywords: Warehousing, distribution centers, logistics, order consolidation, facilities planning, operations management

1. Introduction

A manufacturing firm's finished goods warehousing operation may be the only view that its customers have of the firm's operational capabilities. For several reasons, the effectiveness of this operation is likely to reflect on every other aspect of the company. First, warehousing operations generate value through the customized consolidation and packaging of multiple items. The significant role of these systems in attaining and supporting the competitive advantage of certain manufacturing, wholesale, and retail distribution companies, has been well documented recently (Clemons and Row, 1988; Wiseman, 1988). Second, the increase in labor costs means that throwing more people at any warehousing problem is not a viable solution. The result is an increasing trend towards automation of warehousing operations. Some of the technological options that are available today include large scale Automatic Storage/Retrieval Systems (AS/RS), micro-stackers and automatic carousels, powered conveyors and Automated Guided Vehicle Systems (AGVS), automated sorting of packages, pick list generation software, and bar coding. Since these modern technological systems have to be installed in an integrated way (following a substantial capital investment), incremental adjustments become difficult. Third, the information systems employed must interface with material management and production scheduling systems. These interactions again limit localized improvement.

Automation itself may not be a cost-effective answer for the small or medium sized firm, or for the large firm with very heterogeneous operations. Often a judicious use of appropriate technologies, combined with efficient conventional manual operations, can be the appropriate solution. Furthermore, the same steps that lead to efficient operation in the manual system generally carry over to eventual automated operation.

The efficiency of warehouse operations is influenced by facility design, storage and replenishment methods and picking policies. There is a fairly large literature on each of these aspects, which we cannot hope to review thoroughly; we attempt to present a sense of the major directions being pursued. For a comprehensive review of facilities design, storage and replenishment models see Kulwiec (1982), Matson and White (1982), and Ashayeri and Gelders (1985). The key point to be made in the following review, is that possibly due to the complexity of the problem, there is very little in the way of formal research on our entire problem of designing and operating order consolidation warehouses.

Order picking is considered the single largest expense in most distribution and order consolidation systems (Koenig, 1980; Ogburn, 1984). The proper selection of an order picking method can reduce costs significantly (Witt, 1987). The two basic picking methods used today are 'order picking' and 'zone picking'. In the former, each picker is responsible for picking a complete customer order and he may have to traverse the entire warehouse looking for items comprising that order. Under zone picking, each picker is assigned to a single storage zone where he is responsible for performing all the picks within that zone. Items for several orders may be picked simultaneously and then a sorting operation is required to consolidate the items for each individual order. Prominent examples of such systems are electronic assembly components, grocery, mail-orders, parts and drugs distribution centers.

Speaker (1975) and Gross (1981) describe several industrial applications of zone picking. They discuss the role of automation technology in generating pick lists and in order consolidation. Wiese (1985) presents several administrative strategies such as pre-packed units, prioritized customer orders and due-dates for deliveries and replenishments.

Several studies are devoted to order picking in manual and automated warehouses. The sequence of pick stops that minimizes the total length of a given pick tour in a rectangular warehouse is examined by Ratliff and Rosenthal (1983), Goetschalckx and Ratliff (1988a, b). Savings of up to 30% are reported using the optimal sequence over the standard 'Z-pick' path. A dynamic programming formulation minimizing the total distance traveled by the order picker is given by Pouraghabagher (1984). Armstrong, Cook, and Saipé (1979) offer an efficient mixed-integer programming formulation of the pick list generation problem. They assume a given batch size and that certain items may be retrieved from more than a single zone. Simulation methodology is used by Ogburn (1984) to examine the dynamic effects of the picker's move, pick and carry tasks on the warehouse productivity. Other operational issues such as automating the retrieval task from the subset of the fastest moving items, and the impact of carrier schedules on batch picking, are also investigated.

The impact of automation techniques on order picking has also been investigated. For example, Elsayed (1981), and Elsayed and Stern (1983) determine the optimal combination of items per batch subject to capacity constraints on the picking crane. Assuming distinct due-dates for orders, Kusiak, Hawaleshka and Cornier (1985) sequence item picks in an AS/RS in such a way as to minimize the total weighted tardiness. Graves et al. (1977), and Kim and Seidmann (1990) analyze several storage allocation policies for fully automated warehouses. Several effective heuristic rules for pick sequencing from carousel conveyors are presented by Bartholdi and Platzman (1986) and by Wen and Chang (1988). Both studies assume that item locations are given and that the bidirectional carousels travel at a constant speed, so time is linearly proportional to distance.

The interaction of travel time and pick time in end-of-aisle order picking from an AS/RS was first studied by Bozer (1986). Several extensions to this problem are presented by Foley and Frazelle (1988). Order accumulation and sortation systems are used in warehousing and distribution centers to consolidate items (masterpacks, totes, bins) associated with the same order. The performance of these systems is characterized by the simulation results of Bozer and Sharp (1985) and by the stochastic flow models of Wilhelm and Wang (1986). While some attention has been given in the literature to pick list generation

and to path planning, other key design and operational issues regarding the determination of the optimal number of zones and the optimal number of pickers have not been studied. Moreover, none of the published studies shows how to determine the optimal number of orders to be picked concurrently using zone picking (batch size), or how to take the imbalance effects of the workload variability into consideration.

In this paper, we consider the broad problem of warehouse design and operation, including warehouse layout, equipment and technology selection, item location, zoning, picker assignment, pick-list generation, and order batching. Solving the entire problem formally with a single monolithic model appears to be impractical even in a small case. This consideration has led us to formulate a comprehensive hierarchical decision structure for the problem of designing and operating order consolidation warehouses. Our hierarchical structure identifies and integrates the functional relationships among the different decision levels, with the decision variables grouped into a nested or 'hierarchical' order. This new hierarchical approach is based on a series of coordinated mathematical models which interlink the tradeoffs involved. It reduces the decision space for the order-consolidation warehouse problem and serves to identify and evaluate superior design alternatives. The application of our hierarchical approach to an extensive case study is described, including a simulation study that verifies the results of the formal analysis. The case study demonstrates substantial savings in operating costs.

The case in question is briefly described in the next section, to motivate our subsequent discussion. We then describe the general problem and a solution strategy. Models and analytical methods are developed and their role in solving the problem discussed. The results of applying these methods to the case example are described briefly, together with supporting empirical analyses. Several general management and research issues emerge from this analysis.

2. A case example: R.E. Dietz Company

The R.E. Dietz Company manufactures and distributes automotive spare parts. The company warehouse in Syracuse, NY is a 100 000 square foot facility through which an average of 40 customer orders per day for some 1 800 items are processed. Currently, items in the warehouse are stored in three types of storage: pallets, shelves and 'flow racks'. There are over 2 500 feet of storage aisles, divided into three major areas. These areas contain OEM, NAPA, and Dietz brand parts. These last two constitute the largest portion of item volume and shelf length. Stock picking is done by pickers following a predetermined pick list. Each pick list represents one customer order. The pickers walk through the entire warehouse pushing a cart onto which picked parts are placed. On completion of the pick, the order is checked for completeness and an attempt is made to complete it by searching for missing items that may have arrived in the warehouse, but are not yet on the shelves. The order is also checked by an auditor who prepares the final invoice, and it is then palletized and shrink-wrapped for shipment.

This picking scheme might be compared to a shopper filling a cart. Originally, the fastest moving items had been allocated to several prime locations so as to minimize travel distance. However, the more efficient location schemes from the viewpoint of pick distance, also result in more contention between pickers because of limited aisle widths. As a result, item placement was randomized several years ago to reduce picking contention at a price of increased pick travel distance.

The case study addresses warehouse layout, equipment choice, and operational procedures. The operational procedures considered include the implementation of zoning as well as picking schemes such as batch picking, simultaneous sorting, order consolidation and picking to different material handling media (such as carts and conveyors). Substantial savings in operating costs are demonstrated through the models developed in the following sections. These models prescribe reorganizing the layout of the warehouse in a zone picking configuration and batching a predetermined number of orders to be picked concurrently. Special attention is given to the determination of the preferred number of zones and pickers. Reallocating items to different storage technologies as a function of their relative demand rates

and physical properties leads to significant savings in aisle lengths. These models further explain how rationalizing the storage space per item reduces both the pick distance and the replenishment workload.

3. Problem statement and solution approach

The general problem of warehouse design can be broadly described in terms of the decision variables, objectives, constraining factors, and relationships between decision variables and other parameters of the problem. This problem is structured hierarchically, by dividing the major decision variables into levels. We first describe the overall problem formulation and then present our hierarchical solution approach. The formulation is presented in qualitative rather than symbolic terms. This is because we do not subsequently use a symbolic formulation either for deriving qualitative results or for developing algorithms. Furthermore, it will be seen that many of the crucial relationships in the formulation are highly dependent on the particular situation being considered. For example, the possible layout configurations and aisle length restrictions depend on the shape and construction of the warehouse building. Similarly, pick-cycle times depend on a variety of physical measurements peculiar to a given facility, as well as to the size and mix of items being handled, the design of carts or conveyors, and a host of other factors. Thus, it appears that developing a very detailed formulation capable of general use is not viable; features have to be included that would be too specific to apply to other problems. Nevertheless, it is still possible to understand the general structure of these problems, and to set out a general and practical strategy for solving them.

Objective

Minimize the annualized incremental initial costs plus the warehouse operating costs. The incremental initial costs include:

- (i) Warehouse layout rearrangements.
- (ii) New storage hardware purchasing and installation.
- (iii) Additional material handling systems purchasing and installation.
- (iv) Defining, procuring and installing new information systems (data collection, software and computer hardware).

The operating costs include:

- (i) Labor costs for:
 - picking;
 - sorting;
 - replenishing;
 - checking, auditing, and invoicing;
 - picklist generation and exception handling.
- (ii) Inventory holding costs.

Problem parameters

- (i) Number of line items (SKU's).
- (ii) Expected number of orders per day.
- (iii) Demand per item per order.
- (iv) Item dimensions.
- (v) Labor costs (\$/person/day).
- (vi) Available storage facilities.
- (vii) Current storage allocation policy.
- (viii) Replenishment costs per storage technology.
- (ix) Picking costs.

- (x) The required mean length of a working day.
- (xi) Picker pick times.
- (xii) Picker walk times.
- (xiii) Picker grab times.
- (xiv) Sorting times.
- (xv) Packaging and invoicing times.

Decision variables

The decision variables (grouped into levels) are as follows:

- (i) Warehouse floor plan: Warehouse dimensions and the physical layout of aisles, racks and shelves. This determines overall material flow in replenishment as well as picking.
- (ii) Equipment and technology selection: The choice of specific types of racks and shelving, storage equipment including automated storage, material handling vehicles, conveyor types and layouts, bar coding and reading equipment, and sorting equipment.
- (iii) Allocation of items to racks and storage types.
- (iv) Determination of item facings and total length of racks and shelves of each type. 'Facing' is the width of storage assigned to an item, typically expressed as multiples of a physical dimension of the item.
- (v) Assignment of items and storage types to zones.
- (vi) Number of pickers and number of zones.
- (vii) Number of orders batched per pick cycle.
- (viii) Number of sorters or sorting equipment.
- (ix) Pick list generation method and sorting procedure.

Problem constraints

- (i) All items must be assigned to storage facilities.
- (ii) Each item must be assigned to a) at least one storage technology, and b) a unique position within one zone. Each storage position can hold at most one item.
- (iii) Expected pick time per zone per day must be less than or equal to a given limit.
- (iv) The number of pickers must be greater than the number of zones. (Each zone is assigned at least one picker.)
- (v) The assignment variables, the number of pickers, and the number of zones must be integers.
- (vi) Sorting effort = f_1 (picking method, batch size, material handling hardware, sortation hardware, product identification scheme).
- (vii) Pick time = f_2 (number of zones, number of pickers per zone, picking method, batch size assignment of items to storage technology, number of facings per item).
- (viii) Replenishment effort = f_3 (assignment of items to storage type, number of facings per item).
- (ix) Picking cycle time = f_4 (synchronization method used, number of zones, batch size, imbalance effect).
- (x) Storage hardware cost = f_5 (assignment of items to storage technology, required number of facings per item, currently available storage hardware, the marginal cost per unit of additional storage capacity per storage technology).
- (xi) Aisle length = f_6 (number of zones, assignment of items to storage type, number of facings per item, allocation of items among zones).
- (xii) Aisle length \leq Building constraints.

While this problem formulation is complex, we propose a hierarchical decomposition into three decision levels that are not very tightly coupled. The levels are

- Facility Design & Technology Selection,
- Item Allocation, and
- Operating Policy.

The relationships between these decision levels are shown in Figure 1. In practice, the *Facility Design & Technology Selection* level often represents a very large number of choices. In many cases, existing facility constraints will determine the scope of possible choices. The major tradeoff in the selection of automated equipment is in comparing initial costs against operating cost reductions, usually in terms of reduced labor costs and customer lead times. The total viability of any automation strategy must be estimated by solving the entire problem.

The second level, *Item Allocation*, refers to the way in which items are deployed in aisles and zones. While the detailed allocation depends on the number of zones, a layout which divides the picking workload equally between zones is usually desirable. This means that the number of zones need not be known at this level. However, at the previous level, the aisle layout must accommodate the impact of changes in the number of zones. The choice of item 'facings' refers to the storage width assigned to each item. This in turn determines the total aisle length. This calculation requires the order batch size to be known.

The *Operating Policy* level determines two major parameters: the order batch size and the number of zones. The former is the number of customer orders that are batched together and picked in one picking cycle. A zone is a picking region to which one or more pickers are restricted. The order batching calculation requires the total aisle length to be known, which in turn depends on the batch size. The operating level problem is cast in terms of minimizing the picking and sorting cost. This is done by formulating the more detailed Number of Zones and Pickers analysis as shown in Figure 2 and further detailed in the next section.

Formally, solution of the problem requires iterating between these three decision levels to converge upon a solution that minimizes total cost. The major exogenous data requirements are demand data, item attributes (volume and package size), picking and sorting times, and cost information. The *Facility Design & Technology Selection* decisions are made first and impose constraints within which more detailed decisions about *Item Allocation* and *Operating Policy* are made. These detailed decisions, in turn, provide the necessary feedback for evaluating the merits of the higher level decisions. We develop an iterative procedure of using a series of coordinated analytical models as a first cut at determining design and operating parameters. The result of the analysis is then taken as a base scenario, which directs further search to a small neighborhood of superior alternatives. The final search is carried out by using a detailed simulation, which is able to account for the many secondary issues which are simplified away in the analytical process. The use of analytical models drastically reduces the number of alternatives to be simulated; this makes the use of detailed simulation a viable procedure for fine-tuning the final solution. Furthermore, the analytical models, though stylized, provide estimates against which to assess and validate the results of the detailed simulation.

4. Decision and evaluation models

The analysis schematically outlined in Figures 1 and 2 could be carried out in various ways and at many levels of detail and complexity. For example, in choosing a particular Physical Layout and Aisle Design, the costs of stock replenishment and picking effort have to be considered in addition to the cost of storage. In this case, there may be an upper limit on the space available with each storage technology. Several established models are available for dealing with the Facility Design & Technology Selection level issues (cf., Hax and Candea, 1984; Kusiak, 1990; Nahmias, 1989). However, we found a lack of structured treatment relevant to the Item Allocation and Operating Policy levels. The models and analyses that are developed here are described with brief numerical examples. These models optimize, or evaluate the subproblems at each level of the hierarchy. The description below follows the organization of Figures 1 and 2.

4.1. Assignment of Items to Zones (AIZ)

Assuming a certain zone configuration as a working model, items must be assigned to zones. Broadly speaking, there are two alternatives. First, to assign items from each demand class (and storage type) to

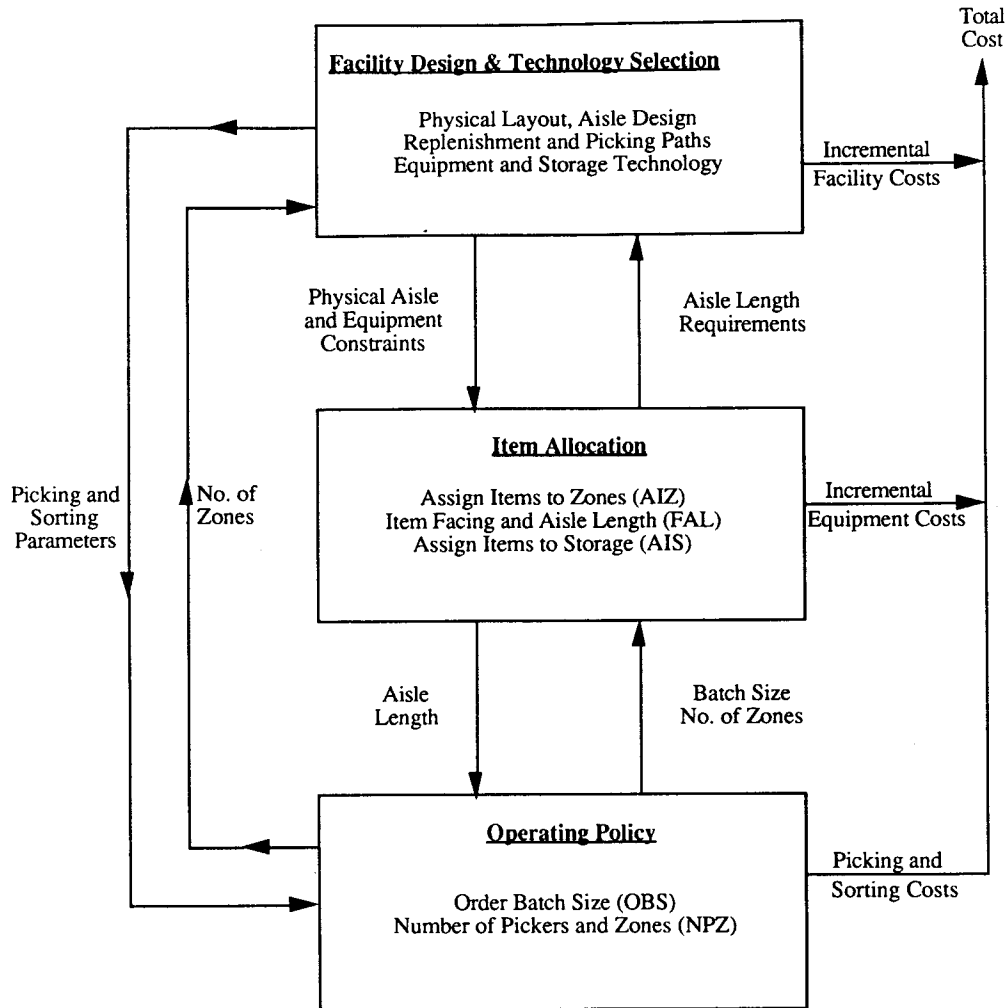


Figure 1. Hierarchical analysis of Warehouse Design and Operating Policies

obtain a uniform workload across zones (*interleaved balanced load*). The second, to assign storage types entirely to certain zones, and to balance workloads across zones to the extent possible. Following this alternative we can expect, for instance, all pallets in one zone, and all gravity racks in another. The first option leads to shorter average pick times, comparability of productivity across zones, similar jobs for pickers, and simplicity of balancing work loads and pick cycles. The latter has potential benefits in terms of specialization of picking equipment, development of special picking skills, concentration of similar storage types, and possibly more efficient replenishment of stock. A model of the effect of assignment on pick rates, shows that separating high pick rate items from low pick rate items leads to lower average pick productivity. It further indicates that adjacent items should be stored in ascending pick frequencies. Items with the highest pick frequency should be placed as close to the head of the aisle as possible.

If each item must be assigned to one and only one zone then the problem is one of partitioning the M available items into Z distinct subsets and determining the relative assignment of the various items. Since pick times are independent of the zone allocation the evaluation criterion used is the minimization of the expected distance travelled by the pickers. Assume a system with Z one aisle zones where items can be placed and assume that zones operate with identical pickers. For simplicity we also assume identical width of storage facing per item. Let P_i denote the probability that item i will be picked in a given cycle

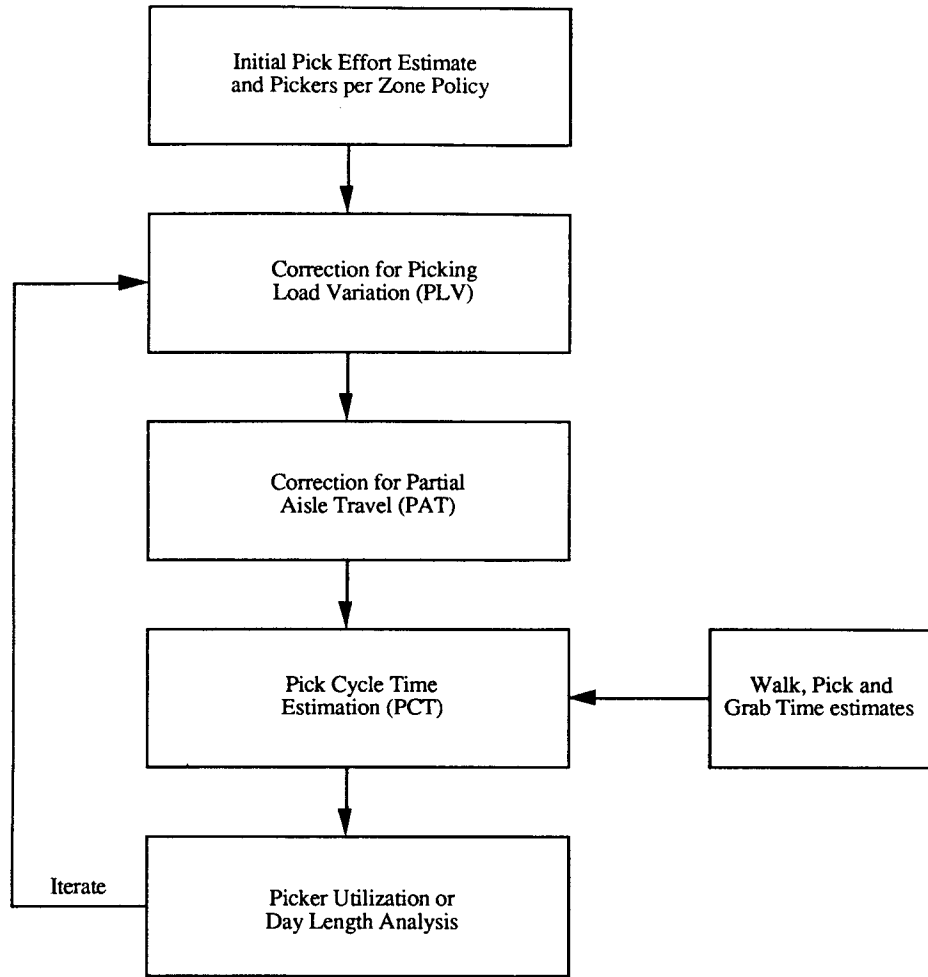


Figure 2. Determining the Number of Pickers and Zones (NPZ)

(fast moving items will be associated with higher P_i values). The picker starts his pick cycle at the head of the aisle and moves into the aisle until the innermost item in the pick list is reached; then the actual picking process begins with a move back to the head of the aisle.

Let n_k be the number of items assigned to zone k ($1 \leq k \leq Z$), with

$$\sum_{k=1}^Z n_k = M. \quad (4.1)$$

The probability of picking the item assigned to the j -th position away from the aisle in zone k is $P_{k[j]}$.

Observing zone k , note that the picker walks the entire aisle length if the n_k -th item has to be picked; this happens with probability $P_{k[n_k]}$. The picker cycle is one unit shorter if the inner item to be picked is in position n_{k-1} ; this happens with probability $P_{k[n_{k-1}]}(1 - P_{k[n_k]})$.

In general, the probability that the pick cycle for zone k is i units long ($i \leq n_k$) is given by

$$PL_{ki} = P_{k[n_i]} \prod_{q=n_i+1}^{n_k} (1 - P_{k[q]}). \quad (4.2)$$

Table 1

i	1	2	3	4	5	6	7	8	9	10
P_i	0.1	0.6	0.4	0.2	0.5	0.6	0.4	0.2	0.4	0.9

The expected distance travelled by a picker is

$$ED = \frac{2}{Z} \sum_{k=1}^Z \sum_{i=1}^{n_k} iP_{k[n_i]} \prod_{q=n_i+1}^{n_k} (1 - P_{k[q]}). \quad (4.3)$$

The denominator of the above expression is constant while the numerator has a sum of M cross products. Each cross product multiplies the PL_{ki} values by the corresponding distance. This sum of M elements is minimized by arranging the PL_{ki} values in a nonincreasing sequence and the i values in a nondecreasing sequence. The P_i (and PL_{ki}) values are bounded in the interval $(0, 1)$. It means that the expected distance travelled by a picker is minimized if the Z items with the largest P_i values are stored at the first storage position of each zone. The next Z items with the highest P_i values are assigned next and so forth. (This allocation is not indifferent to the actual assignment of items to zones within each of these assignment rounds.) Note that this model captures the impact of the number of zones, the allocation of items to zones, the batch size (through P_i), and the assignment of items to the relative storage priorities on the expected picker workload.

These results mean that:

(i) The 'interleaved balance load' assignment is preferred and each zone will have both fast and slow moving items.

(ii) The items in each zone will be stored in a *decreasing* order of their P_i values.

As an example consider $Z = 3$ zones with $M = 10$ items and the values of P_i as in Table 1.

The optimal assignment of items to zones is shown in Table 2.

4.2. Item Facings and Aisle Length (FAL)

Prior to the assignment of items to storage type (AIS), we determine the quantity of each item to be stored in the picking area. This determines the total aisle length required. In general, items arrive on pallets from the production floor to some backup storage and marshalling storage area. Depleted items in the pick area are immediately replenished. The replenished amount (to be determined) is identical to the capacity of the storage space dedicated for this item in the pick area. Given this storage space and the item geometry we now have to find the number of 'facings' per item.

The larger the number of facings, the larger the quantity of the item stored. For a given storage type, the depth of storage is usually fixed. The larger the quantity stored, the less is the replenishment cost, since the latter includes a large fixed component due to travel. However, picking costs increase with the number of facings, because of increased aisle lengths. Clearly, large fast-moving items require the most storage volume and aisle length.

Assuming items on the shelves are stored in flat (i.e., side by side) bins we define

V = Speed of a picker (ft/day).

PC = Cost of a picker (\$/day).

Table 2

Zone	Items		
1	10	7	9
2	6	3	8
3	2	5	4

↑
Head of aisle

- N = Number of orders (day^{-1}).
 R = Number of orders per batch pick (orders/cycle).
 FRC_j = Fixed replenishment cost for item j (\$/replenishment).
 VRC_j = Variable replenishment cost for item j (\$/unit).
 BW_j = Bin width required to store one facing of item j (ft/unit).
 NOP_j = Mean number of orders per day for item j (day^{-1}).
 OS_j = Number of item j units required by a single order (units).
 AC = The daily pick aisle cost (racks, floor space etc.) per foot of facing (\$/ft/day).
 L_j = The number of storage layers for item j .
 BC_j = Number of item- j units stored in a single facing. It is equal to the depth of the storage bin divided by the item length.
 NOF_j = Number of parallel facings of item j (the *decision variable*).

During each day (assuming a round trip for each pick cycle) the picker passes $2N/R$ times in front of each item. The contribution of item j to the Total Daily Cost (TDC_1) of the picker is

$$\text{TDC}_{1j} = (\text{PC}) * (2N/R) * (\text{NOF}_j) * (\text{BW}_j)/V. \quad (4.4)$$

The mean number of item j replenishments per day is given by

$$\frac{(\text{OS}_j) * (\text{NOP}_j)}{(\text{BC}_j) * (\text{NOF}_j)}. \quad (4.5)$$

The total daily replenishment cost is

$$\text{TDC}_{2j} = \frac{(\text{FRC}_j) * (\text{OS}_j) * (\text{NOP}_j)}{(\text{BC}_j) * (\text{NOF}_j)} + (\text{VRC}_j) * (\text{OS}_j) * (\text{NOP}_j). \quad (4.6)$$

The facility cost contribution due to item j storage is

$$\text{TDC}_{3j} = (\text{AC}) * (\text{NOF}_j) * (\text{BW}_j)/L_j. \quad (4.7)$$

Adding these three cost components we get

$$\text{TDC}_j = \text{TDC}_{1j} + \text{TDC}_{2j} + \text{TDC}_{3j}. \quad (4.8)$$

Solving $(\partial \text{TDC}_j / \partial \text{NOF}_j) = 0$, gives the optimal value of NOF_j :

$$\text{NOF}_j^* = \sqrt{\frac{(\text{FRC}_j)(\text{OS}_j)(\text{NOP}_j)}{(\text{BC}_j)[2(\text{PC})(N)(\text{BW}_j)/((R)(V)) + (\text{AC})(\text{BW}_j)/L_j]}}. \quad (4.9)$$

The total aisle length, TAL , is

$$\text{TAL} = \sum_j (\text{NOF}_j^*)(\text{BW}_j)/L_j. \quad (4.10)$$

For illustration, consider the following Dietz data for a particular item: $V = 40000$ ft/day, $P = 150$ \$/day, $N = 40$ orders/day, $R = 3$ orders/day, $\text{FRC}_j = 10$ \$/replenishment, $\text{BW}_j = 2''$, $\text{NOP}_j = 5$ orders/day, $\text{OS}_j = 4$ units, $\text{AC} = 0$ (there is an ample supply of shelves for storage), $L_j = 5$ layers, $\text{BC}_j = 40$. Implementing (4.9) gives $\text{NOF}_j^* = 5$ facings.

The above model can be easily extended to handle additional problem features such as:

- Minimal and/or maximal number of aisle facings per item.
- Imposing an upper bound on the total aisle length.
- Imposing an upper bound on the total number of items replenishments per day.
- Cost-based consideration for differentiated storage modes.

These extensions require a mathematical programming solution approach and are not detailed here.

4.3. Assignment of Items to Storage Types (AIS)

In order to evaluate picking and replenishment costs for each storage technology option, the volume of storage as well as the aisle length required for each item must be selected. This requires knowledge of the number of facings of an item which would be required with each storage technology. The size and package configuration of an item may place constraints on the choice of storage because of physical dimensions, as well as picking efforts.

The common storage technologies used for manual item picking are:

- (i) pallets;
- (ii) gravity flow racks;
- (iii) conventional shelves.

Items are assigned to various storage technologies to minimize replenishment and picking effort, while considering the items' physical dimensions. The fastest moving items are generally assigned to pallet storage due to ease of replenishment. The average volume movers are assigned to gravity flow rack storage and the slowest moving items are assigned to shelf storage. Certain 'bulky' items, however, are best stored in pallet racks and some items are received from outside suppliers in pallets or loose cartons, making it more convenient to store these items in this manner in the warehouse. Thus, a line item by line item check is necessary to validate the final allocation.

The general formulation of this problem requires incorporation of the calculation of facings based on replenishment and picking effort. This, together with various constraints on space and feasibility, leads to a non-linear assignment problem with side constraints. The mathematical model needed in this case is straightforward and is thus omitted. Alternatively, the tradeoff between picking and replenishment can be approximately solved by setting an upper bound on replenishment frequency, so as to not exceed replenishment resource availability (personnel and equipment). The number of facings is then selected to minimize the picking effort subject to this constraint.

4.4. Order Batch Size (OBS)

The benefit of zone picking is the minimization of the picker travel distance per item picked through the maximization of the number of picks along the travel path. However, this method requires sorting, accumulation and consolidation of the individual items arriving from the various zones into complete customer orders. The number of customer orders batched for a single pick cycle determine the 'pick density' of the picker, the processing capacity at the central accumulation/sortation unit, the number of divert points, and the number of accumulation lanes.

Customer orders contain several line items. If one order is picked by one picker at a time, the entire aisle length associated with an item class must be traversed to pick the order. This causes the distance travelled per pick to be very large. As orders are batched to be picked, the pick density increases, and the distance per pick decreases. In addition there may be a secondary benefit from multiple picks (or grabs) at a single stop. A countervailing factor is that under many systems, the order is being assembled as it is picked. With batching, picked items must be sorted and assembled into orders. This work increases as the batch size increases.

The distance per pick decreases asymptotically with the width of an item's shelf 'facing', and sorting costs increase linearly with the number of orders aggregated in a pick cycle. As an approximation, it is assumed that the total length is independent of the batch size. This assumption is not a binding one as we iterate several times through the analytical models depicted by Figure 1. Let

L = Length of all aisle facing (ft(')).

E = Mean number of items per order.

N = Mean number of orders per day.

B = Accumulation lane width at the sortation system (ft(')).

R = Batch size for picking (or the number of customer orders picked simultaneously).

The total daily distance travelled by the pickers under the current order picking mode is given by the Order Picking Distance:

$$OPD = 2 * L * N. \quad (4.11)$$

The total daily distance traveled by all the pickers under the proposed zone picking mode is given by the Zone Picking Distance:

$$ZPD = 2 * L * N/R. \quad (4.12)$$

Under zone picking the sorter gets $R * E$ items in each pick cycle. These are processed into R customer orders. These items will arrive on a conveyor to the center of the outgoing accumulation lanes. Assuming the sorting effort increases linearly with the batch size the mean sorter walking distance per item is $R * \frac{1}{2}B$, and his total distance per order is

$$S = \frac{(R * E) * R * \frac{1}{2}B}{R} = \frac{1}{2}E * R * B. \quad (4.13)$$

Adding up the picking and sortation efforts per day leads to the Total Daily Distance (TDD):

$$TDD = \frac{2 * L * N}{R} + \frac{1}{2}N * E * R * B. \quad (4.14)$$

The optimum value of R is found by solving

$$\frac{\partial TDD}{\partial R} = \frac{-2 * N * L}{R^2} + \frac{1}{2}N * E * B = 0. \quad (4.15)$$

The solution of this equation gives

$$R = \sqrt{\frac{4 * L}{E * B}}. \quad (4.16)$$

As expected, the batch size increases with the aisle length (L) and decreases with the number of items per order (E) and with the sortation effort measure (B). Note that this model assumes no physical volume constraints on the picker (i.e., picking from the shelves to in-aisle conveyors). Other administrative issues such as accumulation requirements by carrier, or carrier pick schedule are assumed to be handled at the picklist generation stage.

As an example, in the Dietz layout $L = 800'$ and $E = 48$ items/customer-order. Assuming $B = 6'$ results in a recommended batch size of $R = \sqrt{(4(800))/(48(6))} = 3.33$ orders/cycle.

4.5. Number of Pickers and Zones (NPZ)

The number of pickers required for a given batch size can be computed from an estimate of the pick (and walk) time and the number of pick cycles to be completed. This initial estimate can be improved to include refinements such as the effects of imbalances across pick zones. The number of storage zones cannot exceed the number of pickers, but there can be multiple pickers per zone. With fewer zones the imbalance in pick loads across zones is reduced, and pick cycles are likely to be more uniform. However, multiple pickers create issues of contention and coordination within zones which can reduce picking productivity. In general, space utilization within picking aisles is likely to be worse due to the allowances that have to be made. Furthermore, since it is desirable for zones to be similar, the degree of flexibility in the design is reduced with multiple pickers. In cases where the density of picks is high or where the rate of picking from a bin is very high, multiple pickers per zone may be needed to get enough picking capacity per foot of aisle length. In such a case, automation also tends to become a viable alternative. The benefits from assigning one picker per zone include more compact travel areas for each picker, no

contention, independence from other pickers in terms of performance achieved, 'ownership' of a zone, and the potential for learning about the items assigned to a zone.

The utilization of pickers is computed based on total work load estimated from demand data, pick cycle time estimates, the number of pickers assigned to each zone, and the number of minutes of picking per day. Equivalently, given a target utilization rate, it would be possible to determine the minimum number of pickers and zones. However, the other approach facilitates adjustments of workload due to *work imbalance*, and *partial traversal* of aisles, which are described below. Note that in the simulation conducted for the Dietz case study, it proved to be most convenient to select the number of pickers and to estimate the number of hours per day required to complete the orders accumulated for that day. A sample computation for a single picker per zone application goes as follows:

Using the results from one iteration of the hierarchical analysis, assume that the following are current values for the various decision parameters: Number of zones = $Z = 4$ zones, Aisle length = $L = 880'$, and Batch size = $R = 4$ orders/pick-cycle. The operating data is as follows: Number of orders per day = $N = 40$ orders/day, Number of items per order = $E = 48$ items/order, Number of units ordered of one item = $IP = 3.3$ items/pick, Net Work Day = $WD = 400$ min/day (net pick time), Walking speed = $V = 50$ ft/min, and Pick Time = $PT = 0.29$ min/item. These are average values obtained by direct measurements and analysis of marketing data files.

The workload computations proceed as follows:

$$\text{No. of cycles/day} = Y = [N/R]^+ = 40/4 = 10.$$

$$\text{Available time} = WD/(N/R) = 400/10 = 40 \text{ [min/cycle]}.$$

$$\text{Walk distance} = (L/Z) * 2 = (880/4) * 2 = 440' \text{d.}$$

$$\text{Number of stops/day} = (R/Z)E = (\frac{1}{4})4 * 48 = 48.$$

Then

$$\text{Cycle pick time} = (R/Z)E * PT = 48 * 0.29 = 13.92 \text{ min.}$$

$$\text{Cycle walk time} = (L/Z) * 2/V = 440/50 = 8.8 \text{ min.}$$

$$\text{Unload and get new pick list} (\approx \text{constant}) = 2.0 \text{ min.}$$

$$\text{Total cycle time:} \qquad \qquad \qquad 24.72 \text{ min/cycle.}$$

Accounting for an imbalance effect of 24.5% (see the pick load variation correction described below), we get

$$\text{Picker Utilization} = \frac{24.72 * 1.245 * 100}{40} = 77\%.$$

A more accurate estimate of picking effort in terms of Pick Cycle Time (PCT) is described below.

4.6. Correction for Picking Load Variation (PLV)

While allocation of items to zones will be done so as to equalize average picking loads, on any given occasion random variations in mix will cause pick loads to vary from the average. This variation will lengthen pick cycles, which are determined by the maximum pick time across zones. This imbalance must be accounted for in evaluating the feasibility of a given design alternative. The model takes the probability that an item on an order falls in a given zone as uniform. Then, given the total number of items in a batch of orders, the mean and variance of the number of items falling in a given zone can be estimated. Using a Normal probability model as an approximation, the excess pick capacity necessary to meet the maximum load with 95% probability is computed. This capacity stated in terms of a percentage above the average load, is taken as a safety factor in calculating the utilization of pickers.

At each pick cycle there are $I = (N * E)/Y$ items to be picked. Assuming that items are stored uniformly among the Z zones the probability that a single pick will be directed at a particular zone is $1/Z$. Let k_i be the number of items to be picked at zone i , $i = 1, 2, \dots, Z$, in a given cycle where a total

of I picks are performed. The joint distribution of the number of picks is given by the multinomial distribution:

$$f(k_1, k_2, \dots, k_z) = \frac{I * Z^{-I}}{k_1! k_2! \dots k_z!}. \quad (4.17)$$

A new cycle can begin only when all Z zones have completed picking items for the current batch. The sum of the times required by the Y pick cycles determines the distribution of the number of hours required to clear the daily work load. The length of each cycle is constrained by the zone having the largest number of picks assigned to it as all zone start each cycle picking simultaneously. (This is an approximation based on the assumption that the zone workload is monotone with the number of picks.) The expected value of the maximal term of the above distribution is given by (the first order statistic)

$$\text{EVMT} = Z * \sum_{k_i \geq k_j \forall j \neq i} k_i * \text{Probability}(k_i \geq k_j \text{ for all } j \neq i). \quad (4.18)$$

The explicit expression for EVMT is fairly involved. Instead, we approximate the probability that a zone will be assigned a measurable amount of picks beyond its expected value ($= I/Z$) for each cycle. Detailed simulations (described below) validates the accuracy of this approximation. Picking I items per cycle, the probability of having to handle n ($n = 0, 1, 2, \dots, I$) such picks from one zone follows a binomial distribution:

$$\text{Probability}(n \text{ out of } I \text{ picks}) = \binom{I}{n} \left(\frac{1}{Z}\right)^n \left(1 - \frac{1}{Z}\right)^{n-I}, \quad (4.19)$$

with mean $\bar{k} = I/Z$ and variance $v(k) = I(Z-1)/Z^2$. The mean value satisfies $I/Z \gg 5$ and the normal approximation to the binomial distribution is invoked. Using a standard normal value for, say, the 97.5% load protection, we need to add 1.96 standard deviations to the mean values. Note that the relative expansion of the mean is given by

$$\beta = \frac{1.96\sigma(\bar{k})}{\bar{k}} = 1.96\sqrt{\frac{Z-1}{I}}. \quad (4.20)$$

This illustrates the fact that the imbalance effects *increase* with the number of zones and *decrease* with the batch size R . As an example, consider the following data set: $N = 40$ orders per day, $Y = 10$ cycles a day, $E = 48$ items per order, and $Z = 4$ zones.

The mean number of picks per cycle per zone is $\bar{k} = (N * E)/(Y * Z) = (40 * 48)/(10 * 4) = 48$ items; the variance is $V(\bar{k}) = (40 * 48/10) * (4-1)/4^2 = 36$; and the standard deviation, $\sigma(\bar{k}) = 6$. The Planned WorkLoad (with 97.5% protection) is

$$\text{PWL} = \bar{k} + 1.96\sigma(\bar{k}) = 48 + 1.96 * 6 = 59.76 \text{ picks/cycle}.$$

Based on these computations, an allowance for 24.5% ($= \{[\text{PWL}/(\bar{k})] - 1\}100\%$) excess picking capacity beyond the expected load per zone should be made because of imbalance effects.

4.7. Correction for Partial Aisle Travel (PAT)

Once we complete the first iteration at the operating policy level of the decision hierarchy, we have initial estimates for the order batch size and the number of pickers per zone. These estimates are further refined by estimating the pick load variation, partial aisle travel, and the pick cycle time (cf. Figure 2). The results of this refinement process are then fed back into the facilities Design & Technology selection decision level (cf. Figure 1).

The subsection deals with one portion of the refinement process taking place within the operating policy level, the correction for partial aisle travel. This correction is motivated by the observation that in

the case of slower moving items, it may not always be necessary for a stock picker to traverse the entire shelf length assigned to these items. The aisle length actually covered depends on the location of the *farthest* item that must be picked in the cycle. The possible savings due to a shorter travel distance along the aisle can be estimated probabilistically for a given distribution of item numbers and orders frequency.

For simplicity, we assume that in each cycle there is at least one item to be picked from the slow moving item subgroup; this subgroup is assigned to the back-end of the aisles. We further assume that each slow moving item is assigned a single pallet. We use continuous representations for the item percentiles and distance requirements. Empirically, it is found that a small percentage of the items represents a significant portion of the total (cumulative) demand. As a convenient modeling approximation, the demand rate distribution can be represented by the cumulative density function given by Rosenblatt and Eynan (1989):

$$S(i) = i^\omega, \quad 0 < \omega \leq 1 \quad [\text{cumulative density function}], \quad (4.21)$$

$$s(i) = \omega i^{\omega-1}, \quad 0 < \omega \leq 1 \quad [\text{probability density function}], \quad (4.22)$$

with

$$\int_0^1 s(i) \, di = 1. \quad (4.23)$$

The value of ω is varied to represent the shape of the actual curve. For instance, if 80% of the demand requires only an r -portion ($0 < r < 1$) of the items stored, then:

$$\omega = \log(0.8)/\log(r).$$

Given the $S(i)$ curve and the expected cumulative percentage of the item demands we can compute the expected partial distance to be covered. Since i has a continuous distribution $S(i)$, then $S(i)$ has a uniform distribution on the interval (0, 1). The expected cumulative percentage of the items demand is given by the *largest order statistic* from a sample size of n on the interval (0, 1). The value of n is a function of the number of zones (Z), batch size (R), the number of items per order (E), and the number of orders per day (N).

Denoting this order statistic by Z_n , it is well known that the expected value of the *largest* order statistic from sample size n of a uniform distribution is:

$$E(Z_n) = \frac{n}{n+1}. \quad (4.24)$$

Given Z_n , we need to solve the equation

$$Z_n = S(i) \quad (4.25)$$

for the corresponding value of $i = S^{-1}(Z_n)$. This value is the percentage of the total length of the slow moving items storage area walked by the picker during each pick cycle. Multiplying this value by the length of the slow moving storage area per aisle reveals the distance walked by this picker in that subzone. Note that an improved computation of n would take into consideration imbalance effects and the probability $f(n)$ of n items per cycle being picked from the slow moving zone. In such a case one would use

$$E(i) = \sum_{n=0}^{\text{UB}} f(n) * S^{-1}\left(\frac{n}{n+1}\right) \quad (4.26)$$

where UB is a finite estimate of the upper bound on the value of n (say $\text{UB} = R * E/Z$).

In the Dietz case, the sample data indicates that 16% of the items account for walking 80% of the aisle length of the slow moving items (Figure 3). This leads to

$$0.8 = 0.16^\omega \Rightarrow \omega = \log(0.8)/\log(0.16) = 0.122.$$

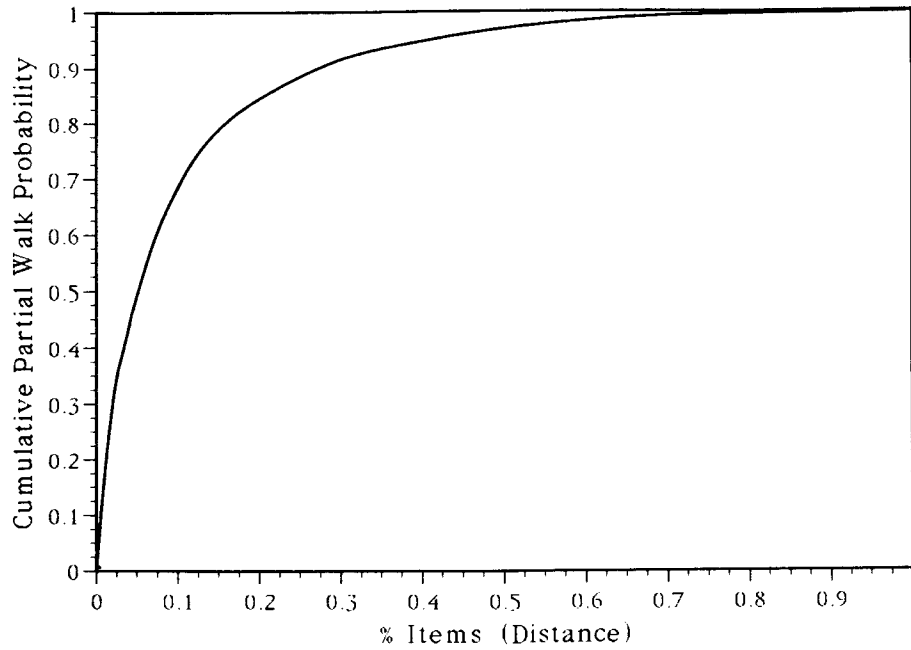


Figure 3. Partial walk distribution in slow moving pallets area

Using the results from one iteration of the hierarchical analysis, assume that the following are current values for the various decision parameters: $Z = 3$ zones, $R = 4$ zones/batch, $N = 40$ orders/day, $E = 48$ items/order, $p =$ percentage of the total demand directed at slow moving items = 5.3%, $Y = 10$ cycles/day. The total number of items to be picked from the slow moving pallets is given by: $n = p * E * R/Z = 0.053 * 48 * 4/3 = 3.39$ items. The first-order statistic defined above yields

$$E(Z_n) \cong \frac{n}{n+1} = \frac{3.39}{3.39+1} = 0.722.$$

Solving (4.25) results in $i = 12\%$, of the total distance for the slow moving item subzone.

The results obtained from a detailed simulation of the Dietz dataset indeed indicate that $E(Z_n) = 0.77$. The actual fractional distance estimated by the simulation is 15%. The analytical approximation gives 12% due to the estimation error in the shape of the $S(i)$ -curve. Using the actual data would have resulted in a 15% estimate as well. However, during the initial design phase this data may vary (as a function of the item allocation) or may not be accessible at all. Hence, we choose to use the more compact $S(i)$ -curve approximation.

4.8. Pick Cycle Time Estimation (PCT)

The pick cycle time computations give the expected number of hours required by each picker to handle the daily workload. This time also includes a compensation factor to account for imbalance effects. At this stage of the iterative process it is assumed that all decisions regarding item allocation, aisle length, batch size and the number of zones have been tentatively made.

To estimate the pick cycle time, we first define several new data parameters:

- ST = Stop time (min).
- GTPT_k = Grip time per unit using technology k (min).
- RFPT_k = Relative frequency for picking an item stored using technology k (item⁻¹).
- NUPT_k = Mean number of units picked per item stored in technology k .

TECH = The number of different storage technologies in use (here: $k = 1, 2, \dots, \text{TECH}$).
 β = The imbalance effect due to the variation in zones workload.
 UL = Constant time to Unload the cart at the conveyor, to get new boxes and a new pick list (min).

The picker activities during each cycle are partitioned into *walking*, *stopping*, *grabbing items* and *unloading* the cart. Assuming identical aisle lengths in all zones and assuming the picker goes the full zone length during each cycle, the Walk Time is given by:

$$\text{WT} = 2 * L / (V * Z). \quad (4.27)$$

The Pick Time is given by

$$\text{PT} = [(R * E) / Z] * \left(\text{ST} + \sum_{k=1}^{\text{TECH}} \text{GTPT}_k * \text{RFPT}_k * \text{NUPT}_k \right). \quad (4.28)$$

Considering the imbalance effect as well, we get that the Picker Cycle Time is

$$\text{PCT} = (\text{WT} + \text{PT} + \text{UL}) * (1 + \beta). \quad (4.29)$$

Consider the following sample data: $V = 50$ ft/min, $E = 48$ items/order, $\text{ST} = 0.29$ min/stop, $\text{GTPT}_{\text{pallets}} = \text{GTPT}_{\text{shelves}} = 0.0383$ min/unit, $\text{RFPT}_{\text{pallets}} = 0.5$, $\text{RFPT}_{\text{flowrack}} = 0.3$, $\text{RFPT}_{\text{shelves}} = 0.2$, $\text{NUPT}_{\text{pallets}} = 3$, $\text{NUPT}_{\text{flowrack}} = 4$, $\text{NUPT}_{\text{shelves}} = 3$, $\text{UL} = 2$ min. In addition, we assume that the current values for the parameters generated by the iterative process are $L = 880$ ft, $Z = 4$ zones, $R = 4$ orders.

This leads to $\text{WT} = 2 * 880 / (50 * 4) = 8.8$, with $\text{PT} = [(4 * 48) / 4] * (0.293 + 0.194) = 23.34$. If we assume $\beta = 0.245$ (see the PLV model), then

$$\text{PCT} = (8.8 + 23.34 + 2) * (1 + \beta) = 42.5 \text{ min/cycle}.$$

Since $Y = [N/R]^+$, we get 40/4 or 10 cycles/day which require (on the average):

$$\frac{42.5 * 10}{60} = 7.08 \text{ hours}.$$

This is based on the value of β for satisfying the requirement that each picker will have a probability of 97.5% to complete each cycle within 42.5 min.

The time to pick all items on a pick list is divided into three components: (i) A 'walk time' associated with the *distance* travelled by the picker, (ii) a 'stop time' associated with the number of different line items picked (i.e., the number of cart *stops*), and (iii) a 'grab time' associated with the total *number of master packs* picked at each cart stop. We found that the degree of collinearity between the explanatory variables 'Distance' and '#Stops' renders standard sequential regressions unstable. As expected, a higher value for the distance walked is generally found with a higher number of stops, (i.e., a longer pick list). Using a Bayesian formulation one can estimate the regression parameters via the posterior mean, assuming a multivariate normal spherical prior distribution with a constant variance. This formulation separates the effect of the two variables 'Distance' and '#Stops' on the 'Total Cycle Time' using a ridge regression technique (e.g., Draper and Van Nostrand, 1979). Further technical details are given in our working paper (Gray et al., 1990).

5. Application to the Dietz case study

The interactions between the design, allocation and operating level decisions require iterating between the associated models described above. In the Dietz case study, an initial estimate of total aisle length was combined with demand requirements and picking time estimates to produce initial values of order batch size using the OBS model. The number of pickers (and zones) was determined next. The

batch size provides inputs to a more accurate facing and aisle length computation from the FAL model. The total aisle length and the number of zones is needed to assess the possible layouts, which have to accommodate the zones physically in a compact (preferably single line) aisle configuration. There are several alternate ways in which the models could have been sequenced to obtain a starting point for determining warehouse design and operating policy. In the Dietz case, it became apparent that the major aisle layout alternatives – *breadthwise* versus *lengthwise* – depended on the number of zones (Z) and the aisle length per zone (L) that could be accommodated in the existing building. It also became apparent that there was not a strong case to be made for complete automation (or unmanned) of storage and sorting. Once the layout and equipment technology alternatives were enumerated, the Item Allocation and Operating Policy were recomputed top-down following the hierarchical model structure. Few iterations were necessary between the three stages of the hierarchy to reach the conclusion that the best solution involves operating either three or four zones while batching three or four orders to be picked simultaneously. Additional options were simulated, however, to confirm the robustness of our conclusions.

At the current demand load it was established that three zones with single pickers would meet the picking load whether picking to carts or conveyors. Four zones would provide a margin of safety and excess capacity against growth. Upon conclusion of the Item Allocation analyses, the item facing requirements indicate that 880 aisle feet would be used, broken down approximately as:

Pallet storage for fast movers: 230 ft.

Gravity flow racks: 170 ft.

Shelves for slow movers: 200 ft.

Pallet storage for slow movers: 280 ft.

For the three-zone case, this implies a pick aisle length of 150 ft, and the four-zone case 110 ft. These requirements dictated a lengthwise layout. Note that the largest share of aisle length goes to slow moving items, which triggered an interest in reviewing the slow movers.

Items were assigned to zones to balance workloads as described by the AIZ model. To yield similar zones, the storage types were also divided equally between zones. This simplifies work assignment and scheduling. The final conclusion of the batching model was that three or four orders should be batched together for one pick cycle. The number of orders per batch (R) affects the sorting station design and cost as well as the length of the conveyors needed to accumulate the items picked in one cycle, and the time offset between pick cycles and sort cycles.

The zone and picker analysis indicated that the total day length necessary to complete the load at existing levels would on average be

Three zones: 6.46 hours.

Four zones: 5.12 hours.

In terms of picker utilization, based on picking to carts and assuming a 400-minute picking day, the final rates yielded by the PCT model were

Three zones: 97%.

Four zones: 77%.

The subsequent detailed simulation runs corroborated these results with an extremely high degree of correspondence.

6. Simulation model and results

In order to evaluate the final alternatives and to select between them, a simulation model was developed using the SIMSCRIPT language. The major input variables to the simulation were:

- Demand in terms of picks per item class per order.
- Walking time, stop time and grab time per package type.
- Aisle length by item category.
- Cart unloading time.
- Length of master pack per sub-zone.

Details of the statistical methodology used for the estimation of pick activity times from field data are given in our more detailed working paper by Gray et al. (1990). The outputs from the simulation included:

- Average pick cycle time.
- Maximum pick cycle time across zones.
- Average length of day (time taken to pick all orders).
- Maximum length of day.
- Average size of picked item queue by item length measure.
- Average number of pick cycles per day.
- Average number of master packs picked per day.

Based on the exploratory and evaluation models, the simulation was conducted on twelve cases as defined by

- three, four, and five zone configurations;
- two, three, four and five orders per batch.

In each case, the average day length measure was used as an index of feasibility and performance, and 100 simulations were conducted for each case to determine this average. The results are displayed in Table 3 and Figure 5. Not surprisingly, the more the zones (pickers) and the larger the batch size, the shorter the day length. However, it is desirable to minimize the number of zones to minimize picking labor, and to minimize the number of orders per batch to minimize sorting and checking labor. Based on the results in the table, it appears that the configurations providing adequate capacity at minimal labor cost are:

- four zones with two or three orders per batch, or
- three zones with three or four orders per batch.

The configuration with three zones and three orders per batch is lowest cost alternative that meets the required daily pick capacity. Figure 4 depicts the schematic arrangement of the two final candidate layouts. It shows several identical picking zones with a conveyor belt running at the head of the pick aisles. This conveyor leads to the order sorting and consolidation area that is close to the shipping docks.

The simulation confirms and refines estimates made with the exploratory and evaluative models. For example, it can be seen that using the PLV model to compute the allowance for imbalance in pick loads across zones provides a reasonable approximation. As expected, the *imbalance is higher with more zones and fewer batches*. An important issue confirmed by the simulation is that the picker generally travels only a small fraction of the subzone for pallet storage of slow movers. This was predicted by our PAT model and became a major factor in the ability of the three zone configuration to meet the picking loads with high reliability.

The final configurations were further analyzed to test the dependence of the results on assumptions regarding the input data. The sensitivity analysis was performed on the following scenarios:

- Reduced stop time in picking, representing reduced acceleration or deceleration time. This might be realized by picking to a conveyor instead of carts. A 50% reduction was considered.
- Heavy demand load. The expected number of orders per day were increased to 50 (25% above the base case) and the variability was increased to a range of $\pm 20\%$.
- Mix changes resulting in an increase or decrease in total aisle length. Changes of $\pm 15\%$ were considered.

These changes were imbedded in three scenarios:

- I. The *optimistic* case considered a reduction in aisle length and an improvement in stop time.
- II. The *high load* case considered the load increase and aisle length increase, but with decreased stop time.
- III. The *worst* case was the same as the high load case, but with no improvement in stop time.

The results are given in Table 4. They show that the 3-zone configuration with a batch size of 3 is still adequate for the high load case, but not for the worst case. In the former, the batch size can be increased, to improve the situation. In the worst case, the 3-zone configuration would be inadequate even with a batch size of 5. However, the conditions of that case seem unlikely to arise. If they did, a change

to a 4-zone layout would be forced. The detailed simulation model is written in the SIMSCRIPT II.5 programming language. It uses Release 2.20 of PC-Simscript available from CACI. The typical run time on an IBM PC/XT is 45–50 minutes.

7. Conclusions and future work

In this paper we present a new analytical approach to the problem of determining the design and operating policy for a finished goods warehouse. We develop a hierarchical framework that considers the item allocation and operating policy levels in detail, and shows the interactions between these levels, as well as with the design and layout level. In addition to presenting several coordinated mathematical models for important subproblems, we propose a thrust that uses these analytical results to limit the number of alternatives being considered to a small subset of superior options. As a result, detailed simulation study with a search over the limited decision space becomes a reasonable alternative. Furthermore, the simulation model is validated using the analytical results, and in turn, the simulation results provide support for the approximations made in developing the analytical models. A comparison of simulation results with the outcomes of the analytical models shows that the analytical approach is robust and accurate.

The approach is driven by and illustrated using a case study of the R.E. Dietz Company. The analysis develops a layout for the facility, enables an evaluation of some proposed new storage and material handling technologies and generates a new operating policy based on zoning, item deployment, and batched order picking. The analysis is supported by empirical studies of demand and item characteristics, as well as a detailed empirical study of pick activity times based on regression analysis. The detailed

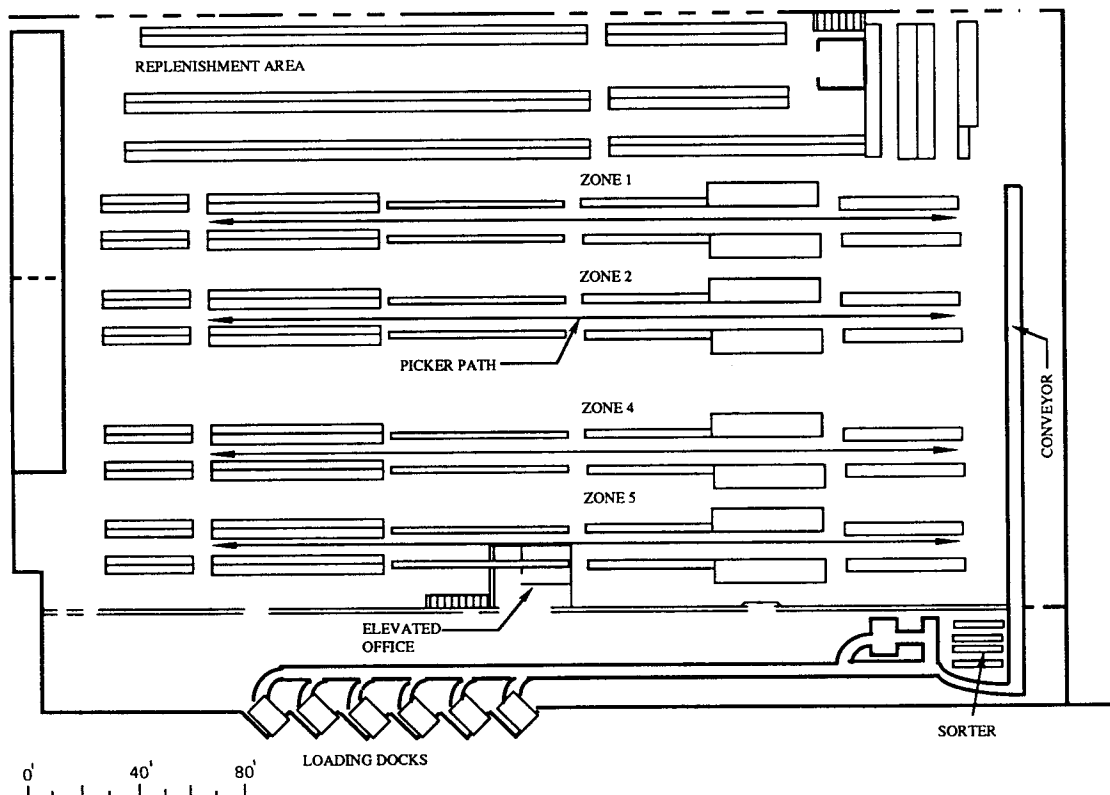


Figure 4a. Schematic arrangement of three picking aisles in the lengthwise arrangement

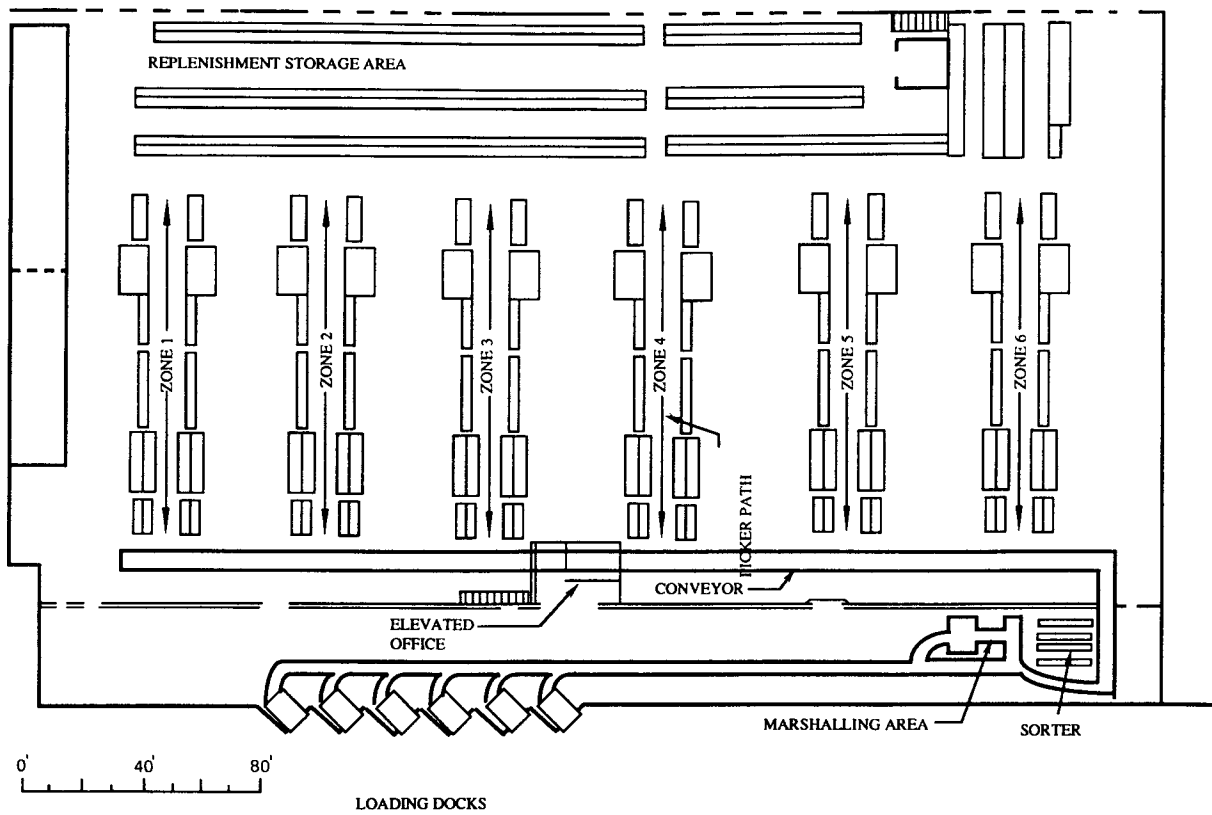


Figure 4b. Schematic arrangement of six picking aisles in the breadthwise arrangement

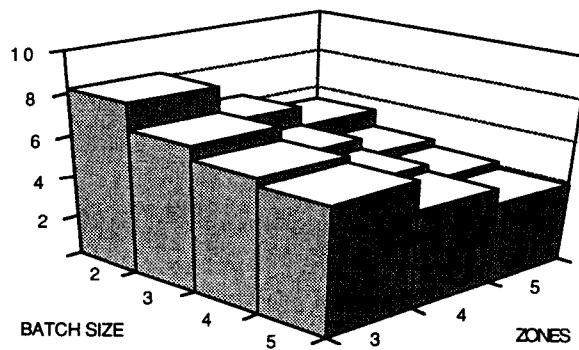


Figure 5. Mean day length (hours) as a function of the batch size and the number of zones

Table 3

Simulation output results depicting the mean day length in hours for 3, 4 and 5 zones with batch size of 2 to 5 orders (* = viable option). The table entries are mean length of day and the 95th percentile of the day length distribution, in hours

Zones	Batch size			
	2	3	4	5
3	8.16/8.38	7.03/7.22 *	6.42/6.61 *	6.06/6.23
4	6.43/6.60 *	5.51/5.66 *	5.02/5.16	4.58/4.85
5	5.41/5.55	4.61/4.74	4.18/4.29	3.92/4.04

Table 4

Simulation output results depicting the impact of changes in the aisle length, mean daily demand and the material handling system on the mean day length in hours

(Z, B) ^a	Case		
	I	II	III
(3, 3)	5.38/5.53 ^b	7.48/7.68	9.16/9.42
(3, 4)	4.86/4.99	6.65/6.48	8.33/8.57
(4, 2)	5.05/5.18	7.13/7.32	8.46/8.70
(4, 3)	4.24/4.36	5.86/6.02	7.17/7.38

Case	Aisle length	Mean daily demand	In-aisle conveyor
I. (Optimistic)	748 ft	40 orders	Yes
II. (High load)	1012 ft	50 orders	Yes
III. (Worst case)	1012 ft	50 orders	No

analysis of storage requirements results in establishing a total aisle length requirement that is 25% less than the current layout. Furthermore, the study reveals that substantial reductions (on the order of 50%) could be made in picking, sorting, and checking labor requirements, without loss of handling capacity, by using the improved layout in conjunction with the newly proposed zoning and batching policy parameters. The new design also has several advantages in terms of material flow patterns, job design, comparative quality and efficiency monitoring, and flexibility under changing load. The study highlights several potential areas for improvement, including a review of slow-moving items, inventory record accuracy, and integration of warehousing, inventory and production management systems.

Major generic tradeoffs are identified, analytically modelled and quantified in this study:

a) order batching reduces the distance travelled per pick but increases the sorting and order consolidation efforts.

b) reducing the number of facings per item minimizes the aisle length but tends to increase replenishment costs, and

c) the variation in picking loads and the imbalance effect, increase with the number of zones and decrease with the order batch size.

The impact of item volume and annual demand rate on the choice of storage technology and the impact of allocating fast and slow moving items across zones are also modeled.

There are many possible directions for future research. However, we do not believe that a formal model of the entire problem is necessary or even useful at this point. Our extensive case study as well as previous experience on such problems makes it clear that there are too many firm specific and even facility specific features that are critical to the solution. This precludes a unique formulation that is both general and useful. However, the analytical models turn out to be very effective for the case in point. Some of the most promising areas for further investigation are:

- Integration of the aisle length and order batch size models.
- Further development of the batch size and zone models.
- Analysis of sorting, material handling and replenishment costs.
- Development of pick list generation algorithms to balance pick cycles across zones, and to handle customers priorities in case of partial backorders.

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