

DUE DATE ASSIGNMENT FOR PRODUCTION SYSTEMS*

ABRAHAM SEIDMANN† AND MILTON L. SMITH†

This paper is concerned with the study of the constant due-date assignment policy in a dynamic job shop. Assuming that production times are randomly distributed, each job has a penalty cost that is some non-linear function of its due-date and its actual completion time. The due date is found by adding a constant to the time the job arrives to the shop. This constant time allowed in the shop is the lead time that a customer might expect between time of placing the order and time of delivery. The objective is to minimize the expected aggregate cost per job subject to restrictive assumptions on the priority discipline and the penalty functions. This aggregate cost includes 1) a cost that increases with increasing lead times, 2) a cost for jobs that are delivered after the due dates; the cost is proportional to tardiness and 3) a cost proportional to earliness for jobs that are completed prior to the due dates. We present an algorithm for solving this problem and show that the optimal lead time is a unique minimum point of strictly convex functions. The algorithm utilizes analytical procedures; computations can be made manually. No specific distributions are assumed; the distribution of total time a job is in the shop is utilized by the algorithm. This distribution can be theoretical or empirical. An example of a production system is presented.
(PRODUCTION/SCHEDULING—JOB SHOP, STOCHASTIC; QUEUES—OPTIMIZATION)

1. Introduction

Most of the studies in industrial scheduling have been characterized by random assignments of due-dates to jobs, or by some arbitrary selection of a due-date assignment policy and its parameters. It is evident that in practice the scheduler cannot prescribe due-dates of his own, and the wishes of the customers in this respect, as well as the expected shop time play an important role. Although several methods for specifying due-dates have been proposed in the literature, no analytical formulation has been made which assures the optimality of some policy. Some aspects of establishing optimal due dates are examined in this paper.

Previous simulation studies of due-date assignment procedures have been mainly concerned with the effects of different due-date assignment methods on the relative performance of some scheduling rules [1], [2], [5], [9], [11], [12], [14]. Recent studies by Eilon and Chowdury [3] and by Weeks and Fryer [15] indicate due-date assignment procedures incorporating expected job flow times perform better than rules based only on job content. Weeks [16] suggests that improved due-date performance can be achieved when assigning attainable or predictable due-dates. Dynamic programming was used to establish delivery commitments by Reinitz [10] and Heard [8]. Reinitz views the shop operation as a Markov process while Heard classified the task of determining the optimal due-date as a sequential control problem.

This study is concerned with the constant due-date assignment method. Constant due-dates are computed by adding a constant to the job arrival times. This represents

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† Texas Tech University.

situations in which an organization tries to deliver any job within a certain and constant lead time. Our objective is to study properties of the optimal due-date assignment policy which is defined as the policy which will minimize the expected aggregate cost per job.

2. Problem Formulation

2.1. The Due-Dates Assignment Model

With the constant due-date procedure each arriving job will receive a due-date d which is computed by

$$d = p + t_d \quad (1)$$

where p is the time epoch at which the job arrives to the shop and t_d is the constant lead time. Let the continuous random variable θ be the total time a job spends at the shop. Define $f(\theta)$ as the probability density function of θ where $f(\theta) > 0$, $\theta \in S$ and $S = \{\theta : 0 < \theta < \infty\}$. The construction of $f(\theta)$ can be based upon some theoretical considerations of the underlying processes, or it can be estimated statistically from historical operational data. Assume that the shop is using a priority discipline such as FCFS or EDD (job with earliest due-date first), where the distribution of time-in-shop is common to all jobs.

2.2. Economic Figures of Merit

The particular cost values to be used here are primarily opportunity costs reflecting the value associated with the potential profits which could have been realized by adopting alternative policy.

1. *Due-Date Cost*: This is a cost that represents the potential loss of sales associated with quoting long delivery dates. The due-date cost is assumed to be independent of the actual job completion time, and its value is determined by the specific due-date allocated to each individual order when it arrives at the shop. This cost component was introduced by Jones [9] and was later applied by Weeks and Fryer [15].

The due-date cost for a job whose quoted lead time was t_d is given by:

$$\lambda_d(t_d) = \begin{cases} 0 & \text{if } t_d \leq A, \\ C_1(t_d - A) & \text{if } t_d > A, \end{cases} \quad (2)$$

where A is a known, nonnegative constant representing some predetermined base value. The assumption is that customers are ready to wait for a reasonable "lead time" equal to " A " time units. When the promised lead time is less than A , there is no penalty. However, each additional unit of promised lead time beyond the base value A is associated with the due-date penalty [9], [15]. A is a constant for a given scheduling period or a planning horizon and its value may change due to changes in factors such as shop utilization or customer demands.

2. *The Tardiness Cost*: This is a cost experienced each time an order is delivered after its due-date. Frequently this cost represents possible loss of goodwill, loss of future sales and contractual penalties for late deliveries. The tardiness cost penalty

incurred by the finished job is given by:

$$\lambda_r(\theta, t_d) = \begin{cases} 0 & \text{if } \theta \leq t_d, \\ C_2(\theta - t_d) & \text{if } \theta > t_d. \end{cases} \quad (3)$$

3. Earliness Cost: This cost is associated with finished goods inventory carrying costs when jobs cannot be shipped out at their moment of completion. The earliness cost can be evaluated as follows:

$$\lambda_e(\theta, t_d) = \begin{cases} C_3(t_d - \theta) & \text{if } \theta < t_d, \\ 0 & \text{if } \theta \geq t_d. \end{cases} \quad (4)$$

All these cost functions measure the costs in terms of monetary quantities per time unit. It is assumed here that $C_1(\cdot)$, $C_2(\cdot)$ and $C_3(\cdot)$ are all monotone increasing, strictly convex functions that vanish at the origin, and they are twice continuously differentiable.

2.3. The Aggregate Cost

The total cost for each job as a function of its actual flow time θ and its assigned lead time t_d is given by:

$$C_c(\theta, t_d) = \begin{cases} g_I(\theta, t_d) = \lambda_r(\theta, t_d) + \lambda_e(\theta, t_d) & \text{if } t_d \leq A \\ g_{II}(\theta, t_d) = \lambda_d(t_d) + \lambda_r(\theta, t_d) + \lambda_e(\theta, t_d) & \text{if } t_d > A \end{cases} \quad (5)$$

where $g_I(\theta, t_d)$ and $g_{II}(\theta, t_d)$ are the total costs per job if it was assigned a lead time less than or greater than A , respectively.

The optimal lead time, t_d^* , should minimize the expected cost per job. The cost per job is a function of the random variable θ which is total time in shop; the expected cost $G(t_d)$ is given by:

$$\begin{aligned} E\{C_c(\theta, t_d)\} &= G(t_d) = \int_0^{\infty} C_c(\theta, t_d) f(\theta) d\theta \\ &= \begin{cases} \int_0^{\infty} g_I(\theta, t_d) f(\theta) d\theta & \text{if } t_d \leq A, \\ \int_0^{\infty} g_{II}(\theta, t_d) f(\theta) d\theta & \text{if } t_d > A. \end{cases} \end{aligned} \quad (6)$$

This lead time t_d^* is optimal if $G(t_d^*) \leq G(t_d)$ for every $t_d \geq 0$. Accordingly, we need to investigate the expected cost function $G(t_d)$ over the two real intervals of t_d , namely

$$\alpha = \{t_d : 0 < t_d \leq A\} \quad \text{and} \quad \beta = \{t_d : A < t_d < \infty\}.$$

Let $G_I(t_d)$ and $G_{II}(t_d)$ denote the expected cost functions for the intervals α and β , respectively. Thus,

$$G(t_d) = \begin{cases} G_I(t_d) = \int_0^{\infty} g_I(\theta, t_d) f(\theta) d\theta & \text{if } t_d \leq A, \\ G_{II}(t_d) = \int_0^{\infty} g_{II}(\theta, t_d) f(\theta) d\theta & \text{if } t_d > A. \end{cases} \quad (7)$$

3. The Optimal Lead Time

3.1. Minimizing the Expected Cost Function $G_I(t_d)$

From (7) we have:

$$\begin{aligned} G_I(t_d) &= \int_0^\infty g_I(\theta, t_d) f(\theta) d\theta \\ &= \int_{t_d}^\infty C_2(\theta - t_d) f(\theta) d\theta + \int_0^{t_d} C_3(t_d - \theta) f(\theta) d\theta. \end{aligned} \quad (8)$$

Let \tilde{t}_{dl} be the point which satisfies $G'_I(\tilde{t}_{dl}) = 0$ and $G''_I(\tilde{t}_{dl}) > 0$ for $\tilde{t}_{dl} \in \alpha \vee \beta$ and define by t_{dl}^* the minimum point of $G_I(t_d)$ over interval α only such that $G_I(t_{dl}^*) \leq G_I(t_d)$ where $t_d \in \alpha$.

Using Leibnitz's rule for differentiation under the integral sign we get the first derivative of $G_I(t_d)$,

$$G'_I(t_d) = - \int_{t_d}^\infty C'_2(\theta - t_d) f(\theta) d\theta + \int_0^{t_d} C'_3(t_d - \theta) f(\theta) d\theta. \quad (9)$$

There is no relative minimum at $t_d = 0$ since

$$G'_I(0) = - \int_0^\infty C'_2(\theta - t_d) f(\theta) d\theta < 0. \quad (10)$$

The second derivative of $G_I(t_d)$ is:

$$G''_I(t_d) = \int_{t_d}^\infty C''_2(\theta - t_d) f(\theta) d\theta + \int_0^{t_d} C''_3(t_d - \theta) f(\theta) d\theta \quad (11)$$

and for this model $G''_I(t_d) > 0$.

Since $G_I(t_d)$ is a strictly convex function of t_d its minimum joint is unique for any total shop time density $f(\theta)$.

Setting the first derivative of $G_I(t_d)$ equal to zero we get:

$$G'_I(\tilde{t}_{dl}) = - \int_{\tilde{t}_{dl}}^\infty C'_2(\theta - t_d) f(\theta) d\theta + \int_0^{\tilde{t}_{dl}} C'_3(t_d - \theta) f(\theta) d\theta = 0 \quad (12)$$

where \tilde{t}_{dl} is the absolute unique minimum of the unconstrained expected cost function $G_I(t_d)$. The solution of (12) can always be made numerically. However, since we have shown that $G_I(t_d)$ is unimodal it makes it easier to conduct the numerical search.

Having obtained \tilde{t}_{dl} it is clear that if $\tilde{t}_{dl} \leq A$ then $t_{dl}^* = \tilde{t}_{dl}$ would be the minimum point of $G_I(t_d)$ over α .

However, we need to investigate what will be the minimum expected cost point for interval α when $\tilde{t}_{dl} \in \beta$; observing the first derivative of $G(t_d)$ it can be seen that

$$\lim_{t_d \rightarrow +\infty} \{G'_I(t_d)\} > 0 \quad \text{and} \quad \lim_{t_d \rightarrow 0^+} \{G'_I(t_d)\} < 0.$$

This means that the function $G_I(t_d)$ is monotone decreasing for all $t_d < \tilde{t}_{dl}$, and it is monotone increasing for all $t_d > \tilde{t}_{dl}$.

Thus, if \tilde{t}_{dl} is greater than A the function $G_I(t_d)$ will be monotone decreasing for $t_d \in \alpha$ and t_{dl}^* is set equal to A . Such a case is plotted in Figure 1.

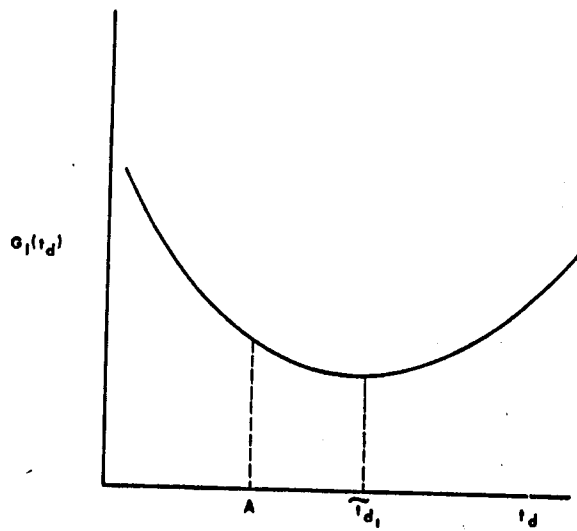


FIGURE 1.

3.2. *Minimizing the Expected Cost Function $G_{II}(t_d)$*

For interval β we get from (7) the expected cost per job if the quoted lead time t_d is greater than A . This gives

$$G_{II}(t_d) = \int_0^\infty g_{II}(\theta, t_d)f(\theta)d\theta$$

$$= C_1(t_d - A) + \int_{t_d}^\infty C_2(\theta - t_d)f(\theta)d\theta + \int_0^{t_d} C_3(t_d - \theta)f(\theta)d\theta. \quad (13)$$

The minimum of $G_{II}(t_d)$ at interval β , t_{dII}^* , has to satisfy $G_{II}(t_{dII}^*) \leq G_{II}(t_d)$ for $t_d \in \beta$, and let \tilde{t}_{dII} be defined as the minimum point of $G_{II}(t_d)$ where $G'_{II}(\tilde{t}_{dII}) = 0$, $G''_{II}(\tilde{t}_{dII}) > 0$ and $t_d \in \alpha \cup \beta$.

Differentiating under the integral sign we get the first and the second derivatives of $G_{II}(t_d)$.

$$G'_{II}(t_d) = C'_1(t_d - A) - \int_{t_d}^\infty C'_2(\theta - t_d)f(\theta)d\theta + \int_0^{t_d} C'_3(t_d - \theta)f(\theta)d\theta \quad (14)$$

and

$$G''_{II}(t_d) = C''_1(t_d - A) + \int_{t_d}^\infty C''_2(\theta - t_d)f(\theta)d\theta + \int_0^{t_d} C''_3(t_d - \theta)f(\theta)d\theta. \quad (15)$$

The first derivative of $G_{II}(t_d)$ will vanish once at \tilde{t}_{dII} since $G_{II}(t_d)$ is strictly convex. It can be shown that if $\tilde{t}_{dII} \in \alpha$ then t_{dII}^* , the minimum cost point for interval β , will be equal to A . On the other hand, if $\tilde{t}_{dII} \in \beta$ then $t_{dII}^* = \tilde{t}_{dII}$.

3.3. *The Overall Minimum Cost*

Based on the preceding analysis we want to find the optimal lead time t_d^* , such that

$$G(t_d^*) < G(t_d), \quad 0 < t_d < \infty,$$

or

$$G(t_d^*) \leq G(t_d) \quad \text{for } t_d \in \alpha$$

and

$$G(t_d^*) \leq G_{II}(t_d) \quad \text{for } t_d \in \beta.$$

The following theorem will help us to establish general relationships between the two cost functions, $G_I(t_d)$ and $G_{II}(t_d)$.

THEOREM 1. *If the first derivatives of $G_I(t_d)$ and $G_{II}(t_d)$ vanish at \tilde{t}_{dI} and at \tilde{t}_{dII} , respectively, then $\tilde{t}_{dI} \geq \tilde{t}_{dII}$.*

PROOF. This theorem is proved by contradiction. Suppose that we have $\tilde{t}_{dI} < \tilde{t}_{dII}$. Recall that $G_{II}(\tilde{t}_{dI}) = C_1(\tilde{t}_{dI} - A) + G_I(\tilde{t}_{dI})$ and since \tilde{t}_{dII} is the minima point of $G_{II}(t_d)$ it has to satisfy

$$G_{II}(\tilde{t}_{dII}) = C_1(\tilde{t}_{dII} - A) + G_I(\tilde{t}_{dII}) < C_1(\tilde{t}_{dI} - A) + G_I(\tilde{t}_{dI}). \tag{16}$$

However, $C_1(\tilde{t}_{dII} - A) > C_1(\tilde{t}_{dI} - A)$ since it is a monotone increasing function and we assumed here $\tilde{t}_{dII} > \tilde{t}_{dI}$. Also, $G_I(\tilde{t}_{dII}) \geq G_I(\tilde{t}_{dI})$ because \tilde{t}_{dI} is defined as the unique minima of $G_I(t_d)$. Thus (16) cannot hold for $\tilde{t}_{dI} < \tilde{t}_{dII}$ and \tilde{t}_{dI} must always be greater than or equal to \tilde{t}_{dII} .

In order to determine the value of t_d^* , the optimal lead time, we must consider three cases:

Case 1: $\tilde{t}_{dI} < A$.

In this case \tilde{t}_{dI} is at region α and by theorem 1 $\tilde{t}_{dII} \in \alpha$. This implies that the minimum of $G_{II}(t_d)$ over region β is at $t_{dII}^* = A$.

Thus, $G_{II}(A) < G_{II}(t_d)$ for $t_d \in \beta$. Having \tilde{t}_{dI} at region α we can write

$$G_I(\tilde{t}_{dI}) < G_I(t_d) \quad \text{for all } t_d \in \alpha$$

and therefore,

$$G_I(\tilde{t}_{dI}) < G_I(A) = G_{II}(A) < G_{II}(t_d) \quad t_d \in \beta. \tag{17}$$

For case 1 the expected cost is thus minimized by having $t_d^* = \tilde{t}_{dI}$.

Here, $G(t_d)$ must be as shown in Figure 2.

Case 2: $\tilde{t}_{dII} < A < \tilde{t}_{dI}$.

In this situation we get that $\tilde{t}_{dI} \in \beta$ and $\tilde{t}_{dII} \in \alpha$. We have shown in 3.1 and 3.2 that

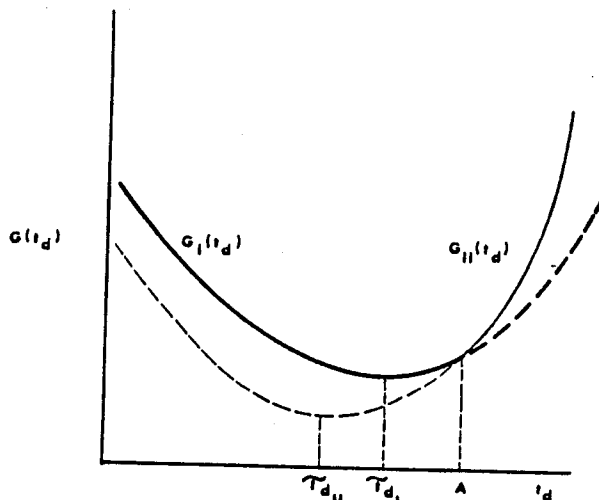


FIGURE 2.

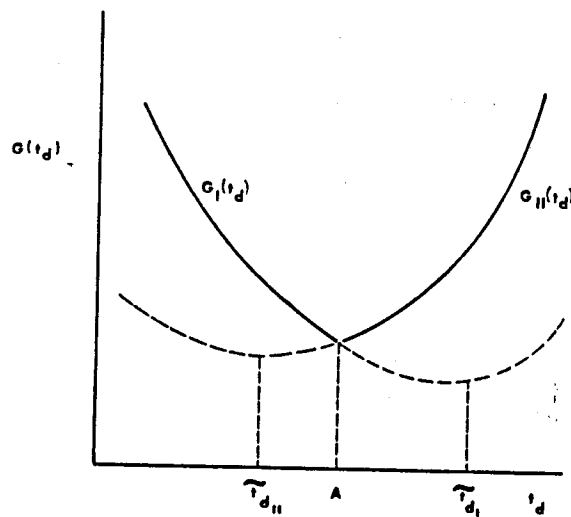


FIGURE 3.

in this case both t_{dl}^* and t_{dII}^* should be equal to A . The optimal lead time is attained for case 2 by setting $t_d^* = A$.

This case is illustrated by the graph of $G(t_d)$ in Figure 3.

Case 3: $A < \tilde{t}_{dII}$.

Finally, the two minimum points \tilde{t}_{dl} and \tilde{t}_{dII} are at region β . Since $\tilde{t}_{dl} > A$, the function $G_I(t_d)$ is monotone decreasing for $t_d \in \alpha$, such that

$$G_I(A) < G_I(t_d), \quad t_d \in \alpha$$

From $\tilde{t}_{dII} \in \beta$ we get

$$G_{II}(\tilde{t}_{dII}) < G_{II}(t_d), \quad t_d \in \beta$$

and so

$$G_{II}(\tilde{t}_{dII}) < G_{II}(A) = G_I(A) < G_I(t_d), \quad t_d \in \alpha \tag{18}$$

which means that having $t_d^* = \tilde{t}_{dII}$ will minimize the expected cost function $G(t_d)$.

In Figure 4 the relation between $G_I(t_d)$ and $G_{II}(t_d)$ is shown for this case.

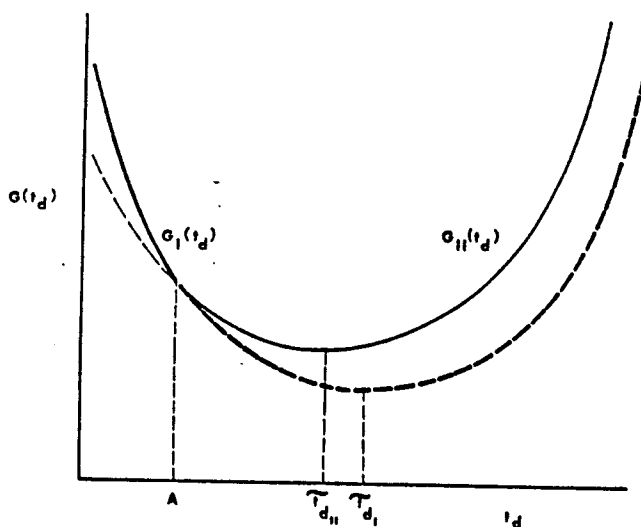


FIGURE 4.

4. Linear Cost Model

Frequently, linear cost functions may be applicable for the three cost functions given in §2 as indicated by Jones [9] and others [3], [4], [7], [13].

Here we assume that the due-date cost is:

$$\lambda_d(t_d) = \begin{cases} 0 & \text{if } t_d \leq A, \\ K_1(t_d - A) & \text{if } t_d > A; \end{cases} \quad (19)$$

the tardiness cost is given by

$$\lambda_t(\theta, t_d) = \begin{cases} 0 & \text{if } \theta \leq t_d, \\ K_2(\theta - t_d) & \text{if } \theta > t_d, \end{cases} \quad (20)$$

and the earliness cost is

$$\lambda_e(\theta, t_d) = \begin{cases} K_3(t_d - \theta) & \text{if } \theta < t_d, \\ 0 & \text{if } \theta \geq t_d \end{cases} \quad (21)$$

where A , K_1 , K_2 and K_3 are known nonnegative constants, not all zero.

The expected cost for $t_d \in \alpha$ is

$$\begin{aligned} G_I(t_d) &= \int_0^\infty (K_2 \cdot \max(0, \theta - t_d) + K_3 \cdot \max(0, t_d - \theta)) f(\theta) d\theta \\ &= K_2 \int_{t_d}^\infty (\theta - t_d) f(\theta) d\theta + K_3 \cdot \int_0^{t_d} (t_d - \theta) f(\theta) d\theta. \end{aligned} \quad (22)$$

Denoting the distribution function of θ by $F(\theta)$ we can write

$$G'_I(t_d) = -K_2(1 - F(t_d)) + K_3 \cdot F(t_d) \quad (23)$$

and

$$G''_I(t_d) = (K_2 + K_3) \cdot f(t_d). \quad (24)$$

Setting the first derivative equal to zero will lead to $F(\tilde{t}_{dI}) = K_2/(K_2 + K_3)$ where \tilde{t}_{dI} is the absolute unique minimum of the unconstrained expected cost function $G_I(t_d)$. When there is no tardiness cost, i.e.: $K_2 = 0$, then $G_I(t_d)$ is a monotone increasing function with a minimum at $t_{dI}^* = 0$.

The expected cost for $t_d \in \beta$ is given by

$$G_{II}(t_d) = K_1 \cdot (t_d - A) + K_2 \cdot \int_{t_d}^\infty (\theta - t_d) f(\theta) d\theta + K_3 \cdot \int_0^{t_d} (t_d - \theta) f(\theta) d\theta \quad (25)$$

and its first two derivatives are

$$G'_{II}(t_d) = K_1 - K_2(1 - F(t_d)) + K_3 F(t_d) \quad (26)$$

and

$$G''_{II}(t_d) = (K_2 + K_3) f(t_d). \quad (27)$$

When the due-date cost is greater than the tardiness cost, i.e.: $K_1 \geq K_2$, the first derivative is greater than zero and thus $G_{II}(t_d)$ will be a monotone increasing function. This will imply that the minimum cost over interval β be obtained by setting t_{dII}^* equal to A .

When $K_1 < K_2$ the first derivative of $G_{II}(t_d)$ will vanish once at \tilde{t}_{dII} where $F(\tilde{t}_{dII}) = (K_2 - K_1)/(K_2 + K_3)$.

Since the distribution function of the total time in the shop $F(\theta)$ is a nondecreasing function of θ it is clear that $F(\theta_2) \geq F(\theta_1)$ if and only if $\theta_2 > \theta_1$. Using that property and applying the analysis of section 3.3 the corresponding algorithm for computing the optimal lead time t_d^* is developed as follows:

- Step 1: Check if $K_3 = 0$
 If yes: Go to step 3
 if no: go to step 2.
- Step 2: Check if $F(A) > K_2/(K_2 + K_3)$
 If yes: Set $t_d^* = \tilde{t}_d$
 where $F(\tilde{t}_d) = K_2/(K_2 + K_3)$
 If no: Go to Step 3.
- Step 3: Check if $F(A) < (K_2 - K_1)/(K_2 + K_3)$
 If yes: Set $t_d^* = \tilde{t}_d$
 where $F(\tilde{t}_d) = (K_2 - K_1)/(K_2 + K_3)$
 If no: Set $t_d^* = A$.

An Example. Suppose that all jobs to be produced arrive at the shop according to a Poisson process with density λ and that the actual production time is exponentially distributed with mean $1/\mu$. The priority rule being used is FCFS. It is well known from queuing theory that the p.d.f. of the total time in this shop is given by

$$f(\theta) = (\mu - \lambda)\exp(-(\mu - \lambda)\theta), \lambda < \mu.$$

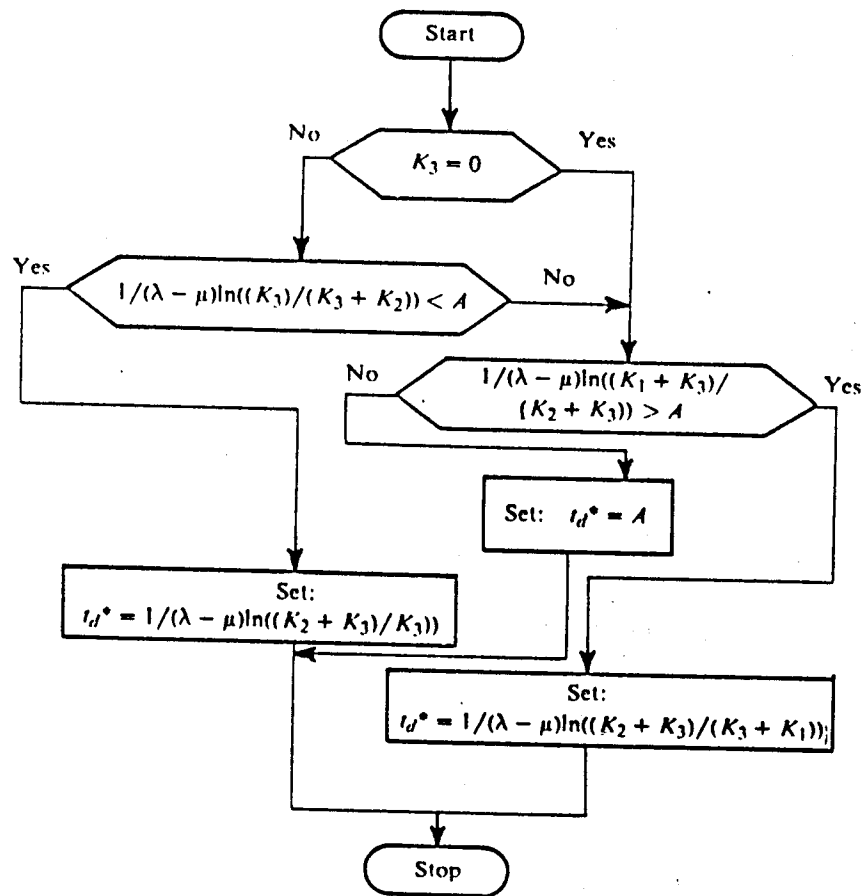


FIGURE 5.

Thus, the total time in the shop is exponentially distributed with mean $1/(\mu - \lambda)$. Figure 5 depicts the optimal procedure for the case of linear costs that results from incorporating the exponential probability function in the algorithm of this section.

It is of interest to note that the optimal lead time t_d^* is either the expected time in shop multiplied by a constant or is equal to A . The expected time in the system is here $1/(\lambda - \mu)$; the constant is either $\ln((K_3 + K_2)/(K_3))$ or $\ln((K_1 + K_3)/(K_2 + K_3))$ and closely resembles the Total Work and Mean Queue Time due-date assignment procedure introduced by Eilon and Chowdhury [3].

Numerical Example. Let $A = 12$ days, $\lambda = 0.15$ jobs per day, $\mu = 0.2$ jobs per day, $K_1 = \$10/\text{day}$, $K_2 = \$35/\text{day}$ and $K_3 = \$12/\text{day}$. Following the procedure of Figure 5 leads to

$$\begin{aligned} 1/(\lambda - \mu)\ln((K_3)/(K_3 + K_2)) &= 27.3 > A = 12 \quad \text{and} \\ 1/(\lambda - \mu)\ln((K_1 + K_3)/(K_2 + K_3)) &= 15.2 > A = 12. \end{aligned}$$

Hence the quoted lead time t_d^* is 15.2 days where the due-date for all new jobs will be equal to the arrival date plus 15.2 days.

5. Conclusion

This research investigated a commonly used industrial policy for assigning minimum cost due-dates in a dynamic job production system. The model considered in this paper assumes that the distribution of the total time in the shop is common to all jobs. The structure of the optimal solution obtained was found to be independent of the specific probability distribution function of both the interarrival job times and the total shop time.

It was shown that the optimal lead time is a unique minimum point of strictly convex functions. This simple structure of the optimal solution has two significant implications. The unimodal function simplifies a numerical search to obtain the minimum cost lead time and having a single solution makes it easier to implement in the industrial environment. The result is that the approach is flexible since no specific distributions need be assumed and the procedure is computationally tractable.

In summary, the analysis of the basic scheduling model considered here has provided some new insights that may foster improved guidelines for the design of production control systems for more complex cases. For management, perhaps the main contribution of the above results is that the results demonstrate how, in certain production environments, an optimal due-date assignment rule can easily be derived.

References

1. BAKER, K. R., *Introduction to Sequencing and Scheduling*, Wiley, New York, 1974.
2. CONWAY, R. W., "Priority Dispatching and Job Lateness in a Job Shop," *J. Industrial Engineering*, Vol. 16, No. 4 (1965), pp. 228-237.
3. EILON, S. AND CHOWDHURY, I. G., "Due-Dates in Job-Shop Scheduling," *Internat. J. Production Res.*, Vol. 14, No. 2 (1976), pp. 223-237.
4. ELMAGHIRABY, S. E., "The One Machine Sequencing Problem with Delay Costs," *J. Industrial Engineering*, Vol. 19, No. 2 (1968), pp. 105-108.
5. ELVERS, D. A., "Job Shop Dispatching Rules Using Various Delivery Due-Date Setting Criteria," *Production and Inventory Management*, Vol. 14, No. 4 (1973), pp. 63-70.
6. FELLER, W., *An Introduction to Probability Theory and Its Applications*, Vol. 2, 2nd ed., Wiley, New York, 1971.

7. GUPTA, J. N. D. AND DUDEK, R. A., "Optimality Criteria for Flowshop Schedules," *AIEE Trans.*, Vol. 3, No. 3 (1971), pp. 199-205.
8. HEARD, E. L., "A Dynamic Programming Approach to Setting Due-Dates in a Closed Job Shop," Unpublished Ph.D. Dissertation, Indiana University, 1970.
9. JONES, C. H., "An Economic Evaluation of Job Shop Dispatching Rules," *Management Sci.*, Vol. 20, No. 3 (1973), pp. 293-307.
10. REINITZ, R. C., "On the Job Scheduling Problem," Chapter V in *Industrial Scheduling*, J. F. Muth and G. L. Thompson, eds., Prentice Hall, Englewood Cliffs, N.J., 1963.
11. REITER, S., "A System for Managing Job Shop Production," *J. Business*, Vol. 39, No. 3 (1966), pp. 371-393.
12. ROCHETTE, R. AND SADOWSKI, R. P., "A Statistical Comparison of the Performance of Simple Dispatching Rules for a Particular Set of Job Shops," *Internat. J. Production Res.*, Vol. 14, No. 1 (1976), pp. 63-75.
13. SCHILDA, A. AND FREDMAN, I. J., "On Scheduling Tasks with Associated Linear Loss Functions," *Management Sci.*, Vol. 7, No. 3 (1961), pp. 280-285.
14. WEEKS, J. K., "A Simulation Study of Due-Date Assignment Policies in a Dual-Constrained Job Shop," Unpublished Ph.D. Dissertation, University of South Carolina, 1974.
15. ——— AND FRYER, J. S., "A Methodology for Assigning Minimum Cost Due-Dates," *Management Sci.*, Vol. 23, No. 8 (1977), pp. 872-881.
16. ———, "A Simulation Study of Predictable Due-Dates," *Management Sci.*, Vol. 25, No. 4 (1979), pp. 363-373.

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