Forward Versus Spot Buying of Information Goods

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ABSTRACT: Several information goods, such as movie distribution rights or newspapers, are sold either at spot prices, or through forward subscription buying. Our paper considers a firm that offers an information good through spot buying, forward buying at a reduced price, or a combination of the two. The time lag between forward buying and spot buying brings about an uncertainty in a consumer's reservation price for the good at the time of advance purchase. We propose a consumer decision-making model that captures this fundamental feature and provides interesting insights into the key elements of consumer behavior. We establish that a consumer offered the choice between forward buying and waiting to (possibly) buy the good on spot faces the tradeoff between a lower unit price and the value of updated preferences. We also establish

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that consumers preferring forward buying have a relatively high expectation and low uncertainty in their reservation prices for the good at the time of advance purchase, while those preferring spot buying have a relatively low expectation and high uncertainty in their reservation prices for the good.

We apply the model to formulate and analyze the firm's problem when it is either a price taker or a price setter. When the firm is a price taker, the choice is whether to offer the good for only forward buying, only spot buying, or a combination of the two. With an example, we show that when both the spot price and the discount on forward buying are moderate in values, the seller chooses the mixed strategy of offering both forward and spot buying simultaneously. When the firm is a price setter, the goal is to choose the offering(s) and the price level(s). With the example, we show how firms selling information goods can increase their revenues by using a mixed offering strategy with both spot and forward offerings. This strategy lends itself to second-degree price discrimination by the seller when there are groups of customers potentially heterogeneous in terms of the distribution of their reservation prices. Our work takes significant importance in the context of information goods, which are becoming increasingly prominent and are being delivered on the Web through the mechanisms of forward and spot buying.

KEY WORDS AND PHRASES: forward buying, information goods, spot buying, subscription.

IN SEVERAL INDUSTRIES, A PRICE DISCOUNT IS GIVEN for advance purchase of goods. A well-known example is the sale of airline tickets, where advance purchase, typically more than two weeks before the start of travel, qualifies for a significant reduction in price. In this way, airlines are able to differentiate between passengers who value price over flexibility while still extracting a high price from business travelers requiring high flexibility on short notice. Maxim's bakery in Hong Kong offers a 25 percent price discount to customers ordering cakes for the mid-autumn festival at least one month before delivery [11]. Tickets to sporting events can be purchased in advance at a lower price than on the day of the event. We denote this advance purchase as *forward* buying and purchase close to delivery of a good as *spot* buying.

This phenomenon is also common to information goods. Movie distribution rights, for example, are sold either at spot prices or in advance at discounts. Cable television offerings include monthly-based subscriptions and pay-per-view spot purchases. In this paper, we study the phenomenon for information goods and lay the foundation for an analytical model investigating a market in which both spot and forward buying are possible. Digitized information goods deviate from physical goods mainly by having close to zero marginal cost and no supply limit. The latter implies that while the sales of a physical good are limited by both demand and supply, the sales of an information good is only limited by the demand. In addition, delivering digital goods over the Web means zero per-unit delivery cost. The seller's marginal costs of reproduction and distribution thus are negligible when considering information goods delivered via the Web. By accurately representing consumers' behavior, we are able to shed light on a seller's strategy in such a market.

A specific application of forward buying is the subscription to a bundle of information goods. Newspapers and magazines offer significant savings for subscriptions, typically for periods between three months and two years. This fits within the framework of forward buying, as the consumer commits to *several* issues of an information good at a lower price per issue and abandons the flexibility to buy any issue on spot. *The Wall Street Journal* offers both one-year subscriptions and daily spot buying to its online edition. This is an example of a mixed strategy, where both forward and spot buying of the issues are being offered simultaneously. What is *The Journal's* incentive for this strategy? Our work here provides clear insight into this approach. A mixed strategy is a second-degree price discrimination strategy by the seller to differentiate between consumers with a higher expectation in reservation price and consumers with a higher uncertainty in reservation price. The seller extracts the most surplus from the former group via forward buying and from the latter group via spot buying.

A consumer offered the choice between forward buying and waiting to (possibly) buy the good on spot faces the trade-off between a lower unit price and the value of updated preferences. We propose a consumer model that takes both of these factors into account and show how the model can be incorporated into the firm's decision in its offering and pricing. When the firm is a price-taker, the choice is whether to offer the good for only forward buying, only spot buying, or a combination of the two. When the firm is a price-setter, the choice is how to price the two offerings.

The paper is organized as follows. In the next section, we review the relevant literature. In the third section, we model consumer behavior in response to the offerings of forward and spot buying by a seller of an information good. In the fourth section, we define a classification of consumers that allows us to specify the aggregated behavior at the population level, such as the overall demand and the overall consumer surplus. We also study the attractiveness of forward buying for a consumer within a given class and do a comparative analysis of consumer surplus within a class for the different sales policies of the seller. In the fifth section, we examine the seller's strategy under price-taker and price-setter scenarios. Finally, we discuss our conclusions and implications for future research.

Literature Review

YIELD OR REVENUE MANAGEMENT PROBLEMS in the airline, car rental, and hotel industries embody the phenomenon of forward versus spot buying. Yet existing literature on yield management does not offer a "ground up" or microeconomic treatise of the problem; it does not identify and analyze the underpinnings of consumer behavior, as we do here. It is commonly assumed that the individual booking requests follow a stochastic arrival process, which is then used to construct the distribution of total demand [10]. We bridge this gap by proposing a consumer decision-making model that captures the market reaction to such an offering at both individual and population levels.

Tang et al. [11] address the problem of a retailer of perishable, seasonal products with uncertain demands who is unable to restock during the selling season and respond to the market demand because of long replenishment lead times and a short season. They study a program called Advance Booking Discount in which customers can precommit their orders, with guaranteed delivery during the season, at a discount price before the season begins. For this program, they establish a better matching of supply with demand during the season through more accurate forecasting and supply planning. The discount program, along with spot sales during the season, is fundamentally the same problem as we consider here. Tang et al., however, do not model consumers' reaction to the program at an individual level.

We have mentioned that subscription bundling is a specific case of forward buying. Dudley [7] addresses the question of how magazines decide on the level of discount to offer to customers for a second year's subscription in a two-year setting, but he does not recognize the key elements of consumer behavior in such a setting. Recent information systems (IS) literature has also seen attempts to apply traditional bundling theory to subscription settings [3, 6, 12]. Although subscription bundling is a form of bundling, a distinct element of the subscription setting is the time dimension arising from advance commitment to a good that is to be delivered later. The time lag brings about an uncertainty in a consumer's reservation price for the good at the time of the advance purchase. Yet all of these works fail to recognize this feature and incorporate it into their thesis. Although there is a growing body of work in IS on information goods, it deals with the topics of pricing [1, 7, 9, 12, 13], versioning [14], sharing and renting [15], price discrimination and differential pricing [5, 13], quality or product differentiation [5, 9], bundling [1, 2, 3, 4, 6, 12], and competition [4, 5, 9], among others. The mechanisms of forward versus spot buying have not yet been addressed.

Basic futures and options theory in finance has some resemblance to our work here. For instance, forward buying is similar in spirit to a forward contract, and the ability to pass on forward buying and wait to buy the good on spot is similar in flavor to an option. Indeed, we adopt the terminology *forward* and *spot* straight out of that theory [8]. Having said that, we note that any attempt to further develop a connection between a forward contract and forward buying will be futile, as they are fundamentally different. A forward contract is an agreement to buy or sell an asset at a certain time in the future for a certain price, while forward buying is an agreement to buy or sell now for delivery in the future. In addition, a forward contract, by definition, has zero value to both parties at the time the contract is drawn, but this is not the case with forward buying in our setting. The parallel drawn with an option is certainly useful, and we draw on this when analyzing the consumer's behavior.

Overall, the phenomenon we address has its roots in the operations, economics, IS, and finance literatures, but the problem has not been posed, structured, and analyzed the way we do here. Our work takes on significance in the context of information goods, which are increasingly becoming prominent and are being delivered on the Web through the mechanisms of forward and spot buying.

Modeling Consumer Behavior

FOR SIMPLICITY OF PRESENTATION, we model forward and spot buying in a single-period framework. We consider a single seller of an information good with zero marginal costs of reproduction and distribution. The seller offers the spot good at unit price p. When the good is offered on spot, each consumer i will realize her reservation price, r_i. If and only if her reservation price equals or exceeds the unit price of the good, that is, $r_i \geq p$, this consumer will decide to buy the good on spot. The surplus of consumer i will then be $r_i - p$ when buying the good on spot and zero otherwise. Define

$$x^+ = \begin{cases} x \text{ if } x \geq 0, \\ 0 \text{ otherwise.} \end{cases} \tag{1}$$

Consumer i's surplus for spot buying can then be expressed as $(r_i - p)^+$.

Before realizing her reservation price, consumer i will have some *belief* about the realization of her reservation price given in terms of a known probability distribution. Hence, consumer i's *expected* surplus for spot buying is $E_{r_i}(r_i - p)^+$.

We now turn to forward buying of the information good. The seller offers a price discount $z \in [0,1]$ for forward buying, that is, offers forward buying at unit price $(1-z)p$. The expected surplus of consumer i from forward buying is given by $E_{r_i}(r_i - (1-z)p)$. As a result of the uncertainty in the reservation price, the consumer's *realized* (as well as expected) surplus when forward buying might be negative. When choosing between forward and spot buying, a rational consumer is facing the trade-off between lower price and information uncertainty. This decision is illustrated in Figure 1. At the first epoch, Time 0, the consumer decides whether to buy the good forward before observing her reservation price. At the second epoch, Time 1, the consumer might spot buy the good if she has not already committed to the good by forward buying. Note that at Time 0, the choice of waiting and buying the good on spot at Time 1 only if $r-p$ is nonnegative is like having a call option with strike price p and stock price at the exercise date r [8].

Let I_i be the indicator value for forward buying of consumer i. Formally,

$$I_i = \begin{cases} 1 & \text{if consumer } i \text{ selects forward buying,} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Similarly, let J_i be the indicator value for spot buying of consumer i:

$$J_i = \begin{cases} 1 & \text{if consumer } i \text{ selects spot buy,} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

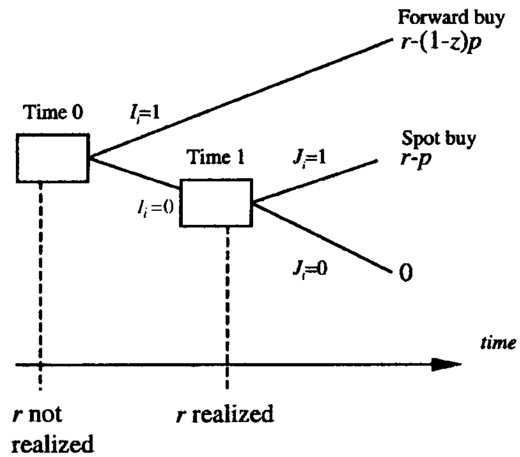

Figure 1. Consumer Decision-Making

The Consumer's Decision

When only forward buying is available to the consumers, then $I_i = 1$ if and only if $E_{r_i}(r_i - (1-z)p) \geq 0$. Similarly, when only spot buying is available, then $J_i = 1$ if and only if $r_i \geq p$. We turn to the situation when both forward and spot buying are available.

Since a consumer will never buy the good both forward and on spot, $I_i \in \{0,1\}$ and $J_i \in \{0,1\}/(I_i \cap \{1\})$. The decision problem of a rational consumer maximizing her expected surplus can then be expressed as the following simple two-stage stochastic dynamic program:

$$\max_{I_i \in \{0,1\}} E_{r_i}\left[I_i\left(r_i - (1-z)p\right) + \max_{J_i \in \{0,1\} \setminus (I_i \cap \{1\})} J_i\left(r_i - p\right)\right]. \tag{4}$$

Since the optimal second-stage decision is trivially given by

$$J_i = \begin{cases} 1 \text{ if } r_i \geq p \\ 0 \text{ otherwise,} \end{cases} \tag{5}$$

we can reformulate Equation (4) as the following one-stage program:

$$\max_{I_i \in \{0,1\}} E_{r_i}\left[I_i\left(r_i - (1-z)p\right) + (1-I_i)\left(r_i - p\right)^+\right]. \tag{6}$$

The solution is obtained in a straightforward manner as follows. Define Δ_i as the difference between forward buying and not forward buying in expectation:

$$\Delta_i = E_{r_i}\left[\left(r_i - (1-z)p\right) - \left(r_i - p\right)^+\right]. \tag{7}$$

Then, in order to maximize her expected surplus, the consumer will choose to buy at Time 0, that is, $I_i = 1$, if and only if $\Delta_i \geq 0$. The following lemma offers a rearrangement of this difference in surpluses.

Lemma 1: For consumer i, $\Delta_i = zp - E_{r_i}(p - r_i)^+$.

Lemma 1 offers a useful insight into the consumer's behavior. The term zp is the discount benefit to the consumer from forward buying. The term $E_{r_i}(p - r_i)^+$ is the risk cost to the consumer from forward buying of having her realized reservation price below the spot price. We refer to this as the *information cost* of forward buying. The consumer, thus, is trading off between the discount benefit and the information cost when deciding on forward buying.

Consumer Classes

In this section, we provide a classification of the population of consumers that will allow us to specify the aggregated behavior at the population level, such as the overall demand and the overall consumer surplus. We also study the attractiveness of forward buying for a consumer within a given class and do a comparative analysis of consumer surplus within a class for the different sales policies of the seller.

Consumers within a population will vary in terms of the form of the distribution function of r_i, denoting their belief as well as the parameters of the distribution function. For our analysis, we make a nonrestrictive assumption that these distributions can be completely characterized by the two parameters, mean μ and standard deviation σ, and a functional form that is symmetric (examples include uniform and normal distributions). Let the population consist of C classes of consumers indexed by $c = 1,\ldots,C$, where each class specifies the functional form of the probability distribution and a standard deviation σ_c. Consumers within each class may have different expectations μ_i following some distribution function. Finally, a given consumer can belong to any class c with probability λ_c, with $\lambda_1 + \lambda_2 + \ldots + \lambda_C = 1$, and there are m consumers in the population.

We provide a simple example to exemplify the nuances of this specification of the consumer population. There are $C = 2$ classes of consumers within the population. A consumer i within the first class, denoted by subscript $i|1$, has a belief distribution given by the two-point discrete (and symmetric) distribution: $\Pr[r_{i|1} = \mu_{i|1} - \sigma_1] = 1/2$, $\Pr[r_{i|1} = \mu_{i|1} + \sigma_1] = 1/2$. The standard deviation of the distribution is σ_1 and is constant across consumers within the class. The mean value of the distribution, $\mu_{i|1}$, however, is different for different consumers within the class and is a draw from the uniform distribution $U(\sigma_1 + \delta_1, \sigma_1 + \delta_1 + \varepsilon_1)$. The belief distribution of a consumer j within the second class, denoted by subscript $j|2$, is given by the uniform distribution $U(\mu_{j|2} - \sqrt{3}\sigma_2, \mu_{j|2} + \sqrt{3}\sigma_2)$. The standard deviation of the distribution is σ_2 and is constant across consumers within the class. The mean, $\mu_{j|2}$, is different for different consumers within the class and is a draw from the triangular distribution $T(\sqrt{3}\sigma_2 + \delta_2, \sqrt{3}\sigma_2 + \delta_2 + \phi, \sqrt{3}\sigma_2 + \delta_2 + \phi + \varepsilon_2)$, where the three parameters are the lower limit, the mode, and the higher limit of the distribution, respectively. Any given consumer belongs to the first class with probability $\lambda_1 = 3/4$ and the second class with probability $\lambda_2 = 1/4$.

It is important to note from the example that both the functional form of the symmetric distribution representing a consumer's belief and the standard deviation value are fixed across all consumers within a class, but they may vary between classes. The expectation value of the belief distribution may vary across consumers within a class and is itself a draw from some distribution. We make no restrictions on this distribution. In the previous example, for instance, the distribution representing the $\mu_{j|2}$ values is nonsymmetric.

To motivate consumer behavior under this classification, consider the following simplified example of consumer demand for tickets to a game of the Rochester Rhinos, a major league soccer team. The team plays in a baseball stadium with, often, empty seats. We, therefore, assume there is no supply-side capacity constraint. One class consists of consumers who are ardent fans but do not have much spending power. Their reservation price is \$8 if the weather is not nice and \$14 if the weather is nice. The other class consists of consumers who are not ardent fans but have greater spending power (for instance, movie stars, celebrities, corporate executives). Their reservation price is \$0 if the weather is not nice and \$16 if the weather is nice.[1] If the probability of nice weather is one-half, then the first class of consumers can be captured with any price of forward buying less than \$11, while the second class can be captured with probability one-half with a spot buying price less than \$16. Note that this would be a very efficient way to price differentiate between the two consumer groups—one emphasizing expectation, the other emphasizing information uncertainty. We revisit and analyze this problem in depth in the section "The Seller's Problem" when evaluating the seller's strategy formally.

Attractiveness of Forward Buying

We now establish some useful properties of consumer behavior with respect to the distribution parameters. Later, we do a comparative analysis of the expected consumer surplus from alternate sales policies.

Proposition 1: For a consumer i in an arbitrary class c,

$$\frac{\partial \Delta_i}{\partial \mu_i} \geq 0,$$

with the equality holding only for larger values of μ_i for which $p \leq l_i$, l_i being the smallest feasible value of r_i.

Proof: See appendix.

This result tells us that for smaller values of μ_i (for which $p > l_i$) the attractiveness of forward buying is increasing in μ_i. That is, the greater a consumer's expectation of her reservation price, the greater is her preference for forward buying. Recall that the choice of waiting and buying the good on the spot at Time 1, only if $r–p$ is nonnegative, is like having a call option at Time 0 with strike price p and stock price at the exercise date r. Option theory tells us that with an increase in μ, the value of the option increases [8]. Yet in our setting the value of a forward buy increases as well. It is then not immediately clear that consumers will prefer forward buying. How do we explain this result? Note that the standard deviation value is fixed. Therefore, with increasing mean value, the consumer's realized reservation price falls above the spot price with increasing probability—that is, the consumer's information cost is decreasing in the mean.

When μ_i is large (for fixed σ_c), so that $p \leq l_i$, the information cost to the consumer from forward buying is zero, so $\Delta_i = zp$. That is, Δ_i is invariant in μ_i for larger values of μ_i—the consumer prefers forward buying almost surely.

Proposition 2: For each class c there exists a unique $\bar{\mu}_c$ such that

$$I_i = \begin{cases} 1 \text{ if } \mu_i \geq \bar{\mu}_c \\ 0 \text{ otherwise.} \end{cases}$$

Alternatively, there exists a unique solution $\bar{\mu}_c$ to the equation $\Delta_i = 0$ in μ_i such that $\Delta_i > 0$ if $\mu_i > \bar{\mu}_c$ and $\Delta_i < 0$ if $\mu_i > \bar{\mu}_c$.

Proof: See appendix.

This proposition tells us whether there are any consumers in a class that prefer forward buying at Time 0, and if so, who they are. Consumers with higher values of the mean will prefer forward buying, while consumers with lower values do not prefer forward buying. The latter will wait until Time 1 to purchase the good on spot.

We note that our conceptualization of consumer classes is mainly motivated by the above two propositions. For a fixed standard deviation, we know how a consumer's preference for forward buying varies with mean. It is, then, natural to group all consumers with the same standard deviation value into one and know easily which of them prefer forward buying at Time 0 and which do not. Since the propositions also assume a functional form of the consumer's belief distribution, the specification that the functional form of the belief distribution is the same across all consumers within a class is appropriate.

The question now is whether we can state a similar result on a consumer's preference for forward buying with respect to the standard deviation value, leading us to the next proposition:

Proposition 3 (Normal or Uniform Distribution): If the distribution representing consumer i's belief about her reservation prices is either uniform or normal, then

$$\frac{\partial \Delta_i}{\partial \sigma_c} \leq 0,$$

with the equality holding only for smaller values of σ_c for which $p \le l_i$, l_i being the smallest feasible value of r_i.

Proof: See appendix.

This restrictive result suggests that for larger values of σ_c (for which $p > l_i$), the attractiveness of forward buying is decreasing in σ_c. Given our earlier comparison that at Time 0 the choice of waiting and buying the good on the spot only if $r - p$ is nonnegative is like having a call option, this result can also be explained as follows. With an increase in σ, option theory suggests that the expected value of this option may be expected to increase [8]. But the expected value of forward buying does not change with σ_c, so as σ_c increases the attractiveness of forward buying decreases.

When σ_c is small (for fixed μ_i), so that $p \le l_i$, the information cost to the consumer from forward buying is zero, and therefore $\Delta_i = zp$. That is, Δ_i is invariant in σ_c for larger values of σ_c—the consumer almost surely prefers forward buying.

To summarize Propositions 1 and 3, consumers preferring forward buying have a relatively high expectation and low uncertainty in their reservation prices for the good at the time of advance purchase, while those preferring spot buying have a relatively low expectation and high uncertainty in their reservation prices for the good.

Comparative Analysis of Consumer Surplus from Alternate Sales Policies

We now do a comparative analysis of the expected surplus for consumers within an arbitrary class. Define S^b as the surplus of a consumer within class c when both forward and spot buying are available:

$$S^b = \begin{cases} r-(1-z)p \text{ if } \mu \ge \bar{\mu}_c, \\ (r-p)^+ \text{ otherwise.} \end{cases} \tag{8}$$

Similarly, define S^f as the surplus of a consumer within class c when only forward buying is offered and S^s as the surplus when only spot buying is offered:

$$S^f = \begin{cases} r-(1-z)p \text{ if } \mu \ge (1-z)p, \\ 0 \text{ otherwise} \end{cases} \tag{9}$$

and

$$S^s = (r-p)^+. \tag{10}$$

The expected surplus of any consumer in class c taken over the possible μ's is

$$\begin{aligned} E_\mu S^b &= Pr\left[\mu < \bar{\mu}_c\right] E_{\mu|\mu<\bar{\mu}_c} E_{r|\mu}\left[(r-p)^+\right] \\ &+ Pr\left[\mu \ge \bar{\mu}_c\right] E_{\mu|\mu\ge\bar{\mu}_c} E_{r|\mu}\left[r-(1-z)p\right]. \end{aligned} \tag{11}$$

We can compare this with the expected consumer surplus over consumers within class c for the cases when only forward buying or only spot buying is offered. For instance, denoting $p_f = (1-z)p$, we can rewrite

$$\begin{aligned} &E_\mu S^b \\ &= Pr[\mu < \bar{\mu}_c] E_{\mu|\mu<\bar{\mu}_c} E_{r|\mu}[(r-p)^+] + E_\mu S^f - Pr[p_f \le \mu < \bar{\mu}_c] E_{\mu|p_f \le \mu < \bar{\mu}_c} E_{r|\mu}[r - p_f] \\ &= E_\mu S^f + Pr[\mu < \bar{\mu}_c] E_{\mu|\mu<\bar{\mu}_c} E_{r|\mu}[(r-p)^+ - (r-p_f)] + Pr[\mu < p_f] E_{\mu|\mu \le p_f} E_{r|\mu}[r - p_f] \\ &= E_\mu S^f + Pr[\mu < \bar{\mu}_c] E_{\mu|\mu<\bar{\mu}_c} E_{r|\mu}[-\Delta] + Pr[\mu < p_f] E_{\mu|\mu<p_f} E_{r|\mu}[r - p_f]. \end{aligned} \quad (12)$$

In the first step here we have used the fact that $\bar{\mu}_c \ge p_f$, since, from Equation (7), $\Delta = -E_r[(r-p)^+] \le 0$ for $\mu = p_f$. The first term on the right-hand side is the expected surplus of a consumer in class c when offered only forward buying. The second term is nonnegative by way of Proposition 2. The third term is nonnegative as well. The consumer's surplus is thus nondecreasing when offered spot buying in addition to forward buying.

Alternatively, we can rewrite Equation (11) as

$$\begin{aligned} &E_\mu S^b \\ &= Pr[\mu \ge \bar{\mu}_c] E_{\mu|\mu\ge\bar{\mu}_c} E_{r|\mu}[(r-(1-z)p) - (r-p)^+] + E_\mu E_{r|\mu}[(r-p)^+] \\ &= E_\mu E_{r|\mu} S^s + Pr[\mu \ge \bar{\mu}_c] E_{\mu|\mu\ge\bar{\mu}_c} E_{r|\mu}[\Delta]. \end{aligned} \quad (13)$$

The first term is the expected surplus of consumers in class c when offered only spot buying, while the second term is the additional surplus for consumers in class c due to the offering of forward buying in addition to spot buying. Note that the second term on the right-hand side is nonnegative since, from Proposition 2, $\Delta \ge 0$ if $\mu \ge \bar{\mu}_c$. The consumer's surplus is nondecreasing when offered forward buying in addition to spot buying.

The expected surplus per consumer over all consumer classes in the population is given by

$$ES = \sum_{x=1}^{C} \lambda_x E_\mu S^b. \quad (14)$$

Based on the preceding analysis, we obtain the following result.

Proposition 4: The expected surplus of an arbitrary consumer in the population from the mixed offering (both spot buying and forward buying) is at least as much as that from a pure offering (spot buying only or forward buying only).

Proof: Straightforward from Equation (14) and the preceding analysis of the surplus of a consumer within an arbitrary class.

The expression in Equation (14) is made feasible by our conceptualization of the classification of consumers. We are able to derive Equation (11) by knowing exactly which consumers within a class prefer forward buying and which do not.

Our modeling of consumer behavior has the potential for several interesting applications. In the following section, we illustrate its usefulness by evaluating the seller's sales strategy under price-taker and price-setter scenarios for the two-consumer-classes example we presented at the beginning of this section to motivate consumer behavior under our classification of consumers.

The Seller's Problem

WE FORMULATE AND ANALYZE THE FIRM'S PROBLEM involving policy decisions for selling information goods. We consider two scenarios. In the first, the firm is a price-taker and must choose whether to offer the consumers only forward buying, only spot buying, or a combination of the two. In the second, the firm is a price-setter and sets the unit price for spot buying and the discount for forward buying in addition to choosing its offering in order to maximize its expected profit.

The price-taker scenario applies when there are multiple sellers behaving competitively and offering close, if not perfect, substitutes. There can be competition for information goods both online (e.g., different news services, or in the realm of legal research, Lexis-Nexis vs. Westlaw vs. others) and in print (in those cities that still have multiple newspapers, or in small towns whose news may be covered by more than one larger city paper). The price-taker scenario may also apply when there is only one seller in the market but one who is behaving competitively, that is, who believes his actions will have no impact on the market price. In practice such a seller may be rare, but in this case we study the scenario mainly because it forms the stepping stone to the more relevant price-setter problem. It provides insights into the consumers' and the seller's preferences at various price points, which are all feasible decision points for the seller in the price-setter scenario.

The seller's objective is to maximize his profit in either scenario. As noted at the beginning of the paper, the seller's variable costs per unit of the information good are negligible, and any significant costs are fixed. Fixed costs are sunk and do not affect the seller's policy decisions. Hence, the firm's objective is maximizing its expected revenue.

When the firm offers only forward buying, its expected revenue is given by

$$\Pi^f = m \sum_{x=1}^{C} \lambda_x Pr[\mu_{i|x} \geq (1-z)p](1-z)p. \tag{15}$$

The summation term is the expected revenue to the seller per consumer i. Its summand is the probability that the consumer belongs to class x multiplied by the seller's expected revenue from that consumer. The subscript $i|x$ is used to explicitly indicate consumer i in class x. Similarly, when the firm offers only spot buying, its expected revenue is given by

$$\Pi^s = m \sum_{x=1}^{C} \lambda_x E_\mu Pr[r_{i|x} \geq p]p. \tag{16}$$

The summation term is the expected revenue to the seller per consumer. Its summand is the probability that a consumer belongs to class x multiplied by the seller's expected revenue from that consumer. When the firm offers both forward and spot buying, its expected revenue is given by

$$\Pi^b = m \sum_{x=1}^{C} \lambda_x \left(Pr\left[\mu_{i|x} \geq \bar{\mu}_c\right](1-z)p + Pr\left[\mu_{i|x} < \bar{\mu}_c, r_{i|x} \geq p\right]p\right). \tag{17}$$

The summation term is the expected revenue to the seller per consumer. Its summand is the probability that a consumer belongs to class x multiplied by the seller's expected revenue from that consumer. This expected revenue has two parts. The first part represents the expected revenue if the consumer prefers forward buying, and the second part is the expected revenue if the consumer prefers spot buying.

The Firm Is a Price-Taker

Here the firm takes the prices for forward buying and spot buying as given. With prices fixed, the firm has three possible binary decisions, denoted by the decision vector $K = (k_f, k_s, k_b)$, where the restrictions $k_j \in \{0,1\}$ for $j = f,s,b$ and $k_f + k_s + k_b = 1$ apply. The decisions are the following: offer only forward buying ($k_f = 1$), offer only spot buying ($k_s = 1$), and offer both forward and spot buying ($k_b = 1$). The decision problem then becomes

$$\max_K \; k_f \Pi^f + k_s \Pi^s + k_b \Pi^b. \tag{18}$$

We illustrate the seller's decision by considering an example of consumer demand for an information good. The consumer classification and the consumer belief distribution are same as that described in the example of demand for tickets in the last section. In that example, a consumer's belief on her reservation price is represented by a two-point discrete distribution irrespective of the class to which she belongs. To facilitate our analysis, we first address the seller's decision assuming there is exactly one consumer in the population and that the consumer belongs to one class with certainty. We then extend this analysis to the overall population in the example.

Consider a single consumer with belief distribution given by $\Pr[r_{i|c} = l_{i|c}] = \Pr[r_{i|c} = h_{i|c}] = 1/2$. Let the mean of the distribution be μ_i and standard deviation be σ_c.

Assuming both forward and spot buying are offered simultaneously by the seller, the cutoff mean value for the different values of p and z is given by

$$\bar{\mu}_c = \begin{cases} (1-2z)p + \sigma_c, & \text{if } zp > \sigma_c \\ (1-z)p, & \text{if } zp < \sigma_c. \end{cases} \tag{19}$$

This is obtained as follows. For the consumer,

$$E_{r_{i|c}}\left[(p - r_{i|c})^+\right] = \begin{cases} 0, & p \le l_{i|c} \\ \dfrac{p - l_{i|c}}{2}, & l_{i|c} < p \le h_{i|c} \\ p - \dfrac{l_{i|c} + h_{i|c}}{2}, & h_{i|c} < p. \end{cases} \tag{20}$$

Also,

$$\begin{aligned} l_{i|c} &= \mu_{i|c} - \sigma_c \\ h_{i|c} &= \mu_{i|c} + \sigma_c. \end{aligned} \tag{21}$$

Substituting this into the earlier expression and rearranging its inequalities,

$$E_{r_{i|c}}\left[(p - r_{i|c})^+\right] = \begin{cases} 0, & \mu_{i|c} \ge p + \sigma_c \\ \dfrac{p - \mu_{i|c} + \sigma_c}{2}, & p - \sigma_c \le \mu_{i|c} < p + \sigma_c \\ p - \mu_{i|c}, & \mu_{i|c} < p - \sigma_c. \end{cases} \tag{22}$$

We now have $E_{r_{i|c}}(p - r_{i|c})^+$ as a function of $\mu_{i|c}$ for any p and σ_c. By extension, $\Delta_{i|c}$ is a function of $\mu_{i|c}$ for any p and σ_c. A plot of the same to identify its zero, shown in Figure 2, yields the above result for the cutoff mean in a straightforward manner.

Having derived the consumer's cutoff mean value when both forward and spot buying are offered, we map in Figure 3 the consumer's preference at Time 0 for forward buying in the (p, p_f) space for all three of the seller's strategies, where $p_f = (1-z)p$ is the forward buy price. In this space, $p_f < p$ is the relevant region, as the forward buy price must be less than the spot price. Within this region, the consumer's preference at Time 0 under the three different sales strategies differs across the four different subregions whose boundaries are indicated by solid lines. The consumer's preference for forward buying is indicated as the triplet (a,b,c), corresponding to the seller's strategies of offering only forward buying, offering only spot buying, and offering a combination of both, respectively. Note that the consumer could prefer forward buying (f), or not prefer forward buying (n) at Time 0.

The line $p_f = 0.5(p + \mu_{i|c} - \sigma_c)$ in the interval $\mu_{i|c} - \sigma_c < p < \mu_{i|c} + \sigma_c$ is the contour $\mu_{i|c} < \bar{\mu}_c$, where $\bar{\mu}_c$ is given in Equation (20). Above the contour, $\mu_{i|c} < \bar{\mu}_c$ and the

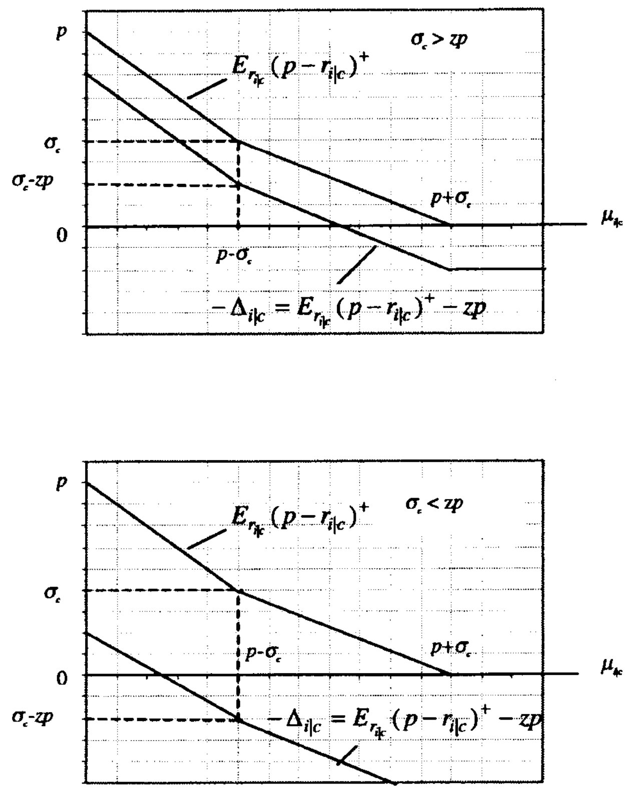

Figure 2. Identifying the Cutoff Mean

consumer's preference is *n,* and below the contour $\mu_{i|c} \geq \bar{\mu}_c$, her preference is *f,* when offered both forward buying and spot buying. Similarly, we have the line $p_f = \mu_{i|c}$ for $p > \mu_{i|c} + \sigma_c$, using Equation (20).

Suppose now that this is the only consumer in the population. When the consumer's preference is *f,* the seller's revenue at Time 0 is p_f. When the consumer's preference is *n* (and the seller offers spot buying), then the seller's expected revenue at Time 0 is $\Pr[r_{i|c} \geq p]p$. Therefore, in each region the seller will compare his *forward buy revenue* and *expected spot buy revenue* at Time 0 to determine which is larger and select optimally that strategy. The bold curve in Figure 3 is the iso-revenue $p_f = \Pr[r_{i|c} \geq p]p$. Above the iso-revenue, the seller's forward-buy revenue is greater, and below it, the seller's expected spot-buy revenue is greater.

The seller's optimal strategy is indicated in the figure for the different subregions, next to the consumer's preference. Because of the iso-revenue, the subregion below the line $p_f = 0.5(p + \mu_{i|c} - \sigma_c)$ and the line $p_f = \mu_{i|c}$ is further divided into two. That is, there are four distinct subregions (numbered 1, 2, 3, and 4 in the figure) where the seller's strategy is different.

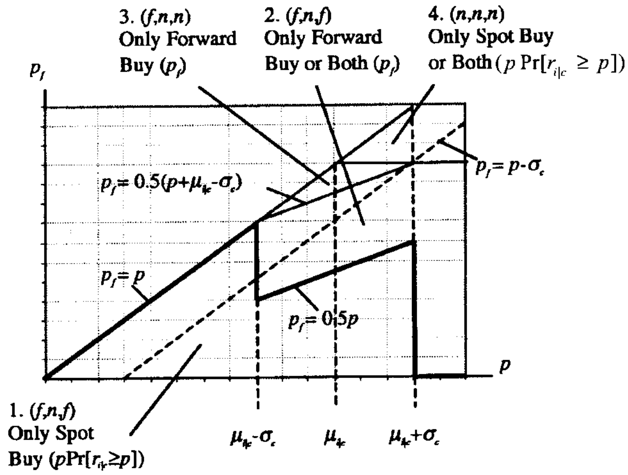

Figure 3. A Consumer's Preference at Time 0 and the Seller's Optimal Strategy for That Consumer

In region 1, the discount from forward buying is *relatively* high in that the seller "loses" from offering forward buying. Since the discount level is high, the consumer will choose forward buying if offered. The seller's expected revenue from spot buying by the consumer is more. Hence, the seller chooses the strategy of offering only spot buying.

Region 2 provides the *pareto optimal* conditions under which the consumer prefers forward buying and the seller "gains" from it.

In region 3, the consumer will not prefer forward buying if spot buying is offered as well, but prefers forward buying if only that is offered. Since in this region the seller gains from forward buying when compared to spot buying, the seller will select the strategy of offering only forward buying. Basically, the discount level from forward buying is small enough that the seller does not lose much revenue from offering forward buying.

In region 4, the consumer does not prefer forward buying if it is available.

We now return to the example and see how the seller's strategy for an individual consumer translates into an overall strategy for the entire population. Note that, in this example, there is no uncertainty in the mean value for a consumer in a class. In addition, to complete the specification of the population in the example, we assume that there are $m = 2$ consumers in the population and that each consumer belongs to either of the two classes with equal probability, that is, $\lambda_c = 1/2$, $c = 1,2$. The mean and standard deviation values for a consumer, depending on her class, are

$$\begin{aligned} \mu_{i|1} &= 11, & \mu_{i|2} &= 8, \\ \sigma_1 &= 3, & \sigma_2 &= 8, \end{aligned} \tag{23}$$

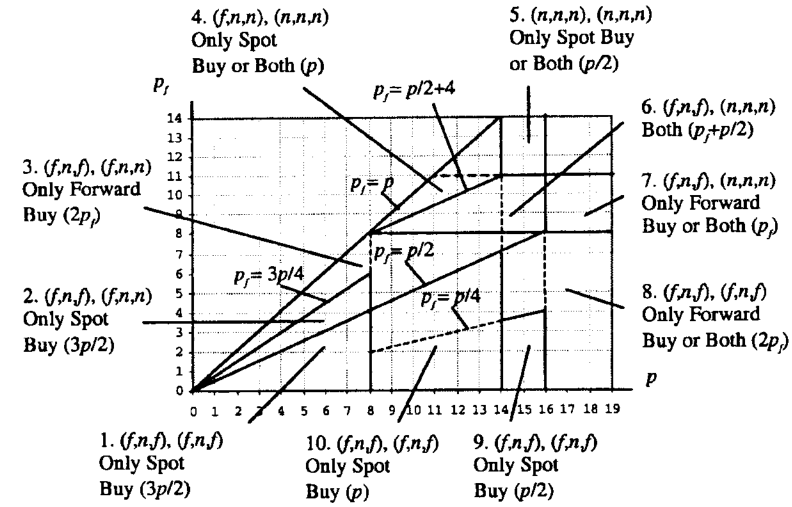

Figure 4. Both Consumer's Preferences at Time 0 and the Seller's Overall Optimal Strategy

where we have used the subscript $c = 1,2$ to indicate the consumer class and the subscript $i|c$ to indicate consumer i in class c.

Referring to Equations (15), (16), and (17), we see that from the seller's revenue perspective this population can be viewed as having exactly one consumer in the first class and exactly one in the second. Merging the preferences for the two consumers in the (p, p_f) space, then, we get Figure 4. We have identified all the regions (with solid boundaries) for which either the seller's strategy or the seller's revenue is different. The two consumers' preferences for forward buying at Time 0 are indicated as (a,b,c), (d,e,f), where the triplet (a,b,c) is the preference of consumer 1 with mean value 11 and standard deviation value 3, and the triplet (d,e,f) is the preference of consumer 2 with mean value 8 and standard deviation value 8. The seller's strategy is indicated next to the consumers' preferences. The seller's revenue from the strategy is also indicated within brackets.

In all, there are 10 regions where the seller's revenue is nonzero. Note that this excludes the region at the top right-hand corner of the chart, where $p > 16$ and $p_f > 11$, since both the forward-buy revenue and expected spot-buy revenue to the seller from both consumers will be zero.

Some of the regions are further partitioned by dotted lines because, although the seller's strategy and revenue are the same across the partitions, the reasoning for optimality differs. Consider region 3, for instance, where the seller's strategy is to offer only forward buying and his revenue is $2p_f$. The dotted lines $p = 8$ and $p = 14$ partition the region into three. In the left partition, the seller's (expected) revenue at Time 0 from the three strategies is given by the triplet $(2p_f, 3p/2, p_f + p/2)$. It is $3p/2$ for the strategy of offering only spot buying because, for $p < 8$, $\Pr[r_{i|1} \geq p] = 1$ for consumer 1 and $\Pr[r_{i|2} \geq p] = 1/2$ for consumer 2. In this partition, the seller's optimal strategy is to offer only forward buying, since $2p_f > 3p/2$ and $p_f > p/2$. In the middle partition, the seller's (expected) revenue at Time 0 from the three strategies is given by the triplet $(2p_f, p, p_f + p/2)$. In this partition, where $8 < p < 14$, $\Pr[r_{i|1} \geq p] = 1/2$ for consumer 1 and $\Pr[r_{i|2} \geq p] = 1/2$ for consumer 2. The seller's optimal strategy is to offer only forward buying since $p_f > p/2$. In the right partition, the seller's (expected) revenue at Time 0 from the three strategies is given by the triplet $(2p_f, p/2, p_f + p/2)$. In this partition, with $14 < p < 16$, $\Pr[r_{i|1} \geq p] = 0$ for consumer 1 and $\Pr[r_{i|2} \geq p] = 1/2$ for consumer 2. The seller's optimal strategy is to offer only forward buying, since again $p_f > p/2$.

The reasoning behind the other regions in the figure is similar to that for region 3, so for brevity we omit their explanations here. In what follows, we briefly offer insights on the different conditions for which the seller chooses different strategies.

When the discount level on forward buying is high but the spot price is relatively low (regions 1, 2, 9, and 10), the seller avoids offering forward buying, since he gives up too much to induce the consumers to commit to forward buying.

When the spot price is high but the discount level on forward buying is also high (regions 7 and 8), the seller can effectively use forward buying to extract some surplus from the consumers who would otherwise not buy at all. Note that region 8 provides the *pareto* optimal conditions under which the two consumers prefer forward buying and the seller gains from it over spot buying (since it is above the iso-revenue curves for both consumers).

When the spot price is small and the discount level is not high (region 3), the seller can again benefit from offering forward buying without giving up much and having the consumers commit to the good.

In the upper partition of region 4, and in region 5, the price for forward buying is high enough that neither consumer will choose forward buying. At the same time, the spot price is not too high to eliminate both from purchasing the good on the spot as well.

In region 6, where both the spot price and the discount level on forward buying are moderate in values, the seller chooses the strategy of offering both forward and spot buying simultaneously. The seller captures consumer 1 through forward buying and captures consumer 2 with probability one-half with spot buying.

The Firm Is a Price-Setter

The decision problem when the firm is a price-setter is

$$\max_{p,z,K} \; k_f \Pi^f + k_s \Pi^s + k_b \Pi^b .$$

The seller chooses the price-point on Figure 4 and the corresponding offering. We turn to our example again to see how the seller's strategy decision plays out in the price-setting scenario. Using the optimal revenue expression identified for each of the regions in Figure 4, the seller will optimally set $p_f = 11$ and $p = 16$ and offer both forward and spot buying simultaneously, while operating in region 6. As explained previously, the seller will capture consumer 1 with forward buying and consumer 2 with a probability one-half with spot buying. The seller's expected revenue will be 19.

If the seller chooses to offer only forward buying, he will optimally set $p_f = 8$, and both consumers will choose forward buying. The seller's expected revenue is 16. If the seller chooses to offer only spot buying, he will optimally set $p = 14$ and capture each consumer with probability one-half. The seller's expected revenue will be 14.

The seller can effectively use forward buying and spot buying to price discriminate between consumer classes with different characteristics. This result helps us understand to some extent *The Wall Street Journal's* two-pronged strategy of offering both one-year subscriptions and daily spot buying. *The Journal* must be targeting readers like consumer 1 with the former and targeting readers like consumer 2 with the latter.

Contracting Implications for Information Goods

We develop a model of consumer behavior reflecting forward and spot buying and the combination of the two. This model allows several insights into the behavior of rational consumers and has the potential for interesting applications. We formulate and evaluate the problem of optimal selling and offering of information goods for the firm as both price-taker and price-setter.

The core of our work lies in recognizing the market mechanisms of forward and spot buying and accurately capturing the underpinnings of the problem of consumer behavior in response to those mechanisms. The time lag between forward and spot buying brings about an uncertainty in a consumer's reservation price for the good at the time of the advance purchase. Our modeling recognizes this fundamental aspect and is by and large driven by the structure of the problem. The central ideas or elements of the problem arise in a variety of contexts in the areas of operations, economics, finance, and IS. We contribute to existing literature in these areas by identifying, formulating, and evaluating the consumer behavior problem. The problem takes significance when dealing with information goods that are being offered increasingly through these market mechanisms over the Web. The paper addresses the strategy of a seller who markets information goods when there are potentially heterogeneous groups of consumers in terms of their reservation prices for the goods. We shed light on this with our modeling of consumer behavior and our analysis of the seller's decision-making process over time.

Based on our modeling, we offer several insights into consumer behavior and the seller's strategy under both the price-taker and price-setter scenarios. We show that a consumer offered the choice between forward buying and waiting to (possibly) buy the good on the spot faces the trade-off between a lower unit price and the value of updated preferences and maybe learning more about the nature of the offered good. We then offer a clear characterization of the consumers who prefer forward buying and consumers who prefer spot buying for given price levels. Those who prefer forward buying have a relatively high expectation and low uncertainty in their reservation prices for the good that is to be delivered in the future. Those who prefer spot buying, on the contrary, have a relatively low expectation and high uncertainty in their reservation prices for the good.

When the firm is a price-taker, the choice is whether to offer the good for only forward buying, only spot buying, or a combination of the two. We show that when both the spot price and the discount level on forward buying are moderate in value, the seller chooses the mixed strategy of offering both simultaneously. When the spot price is not high but the discount level on forward buying is high, the seller offers only spot buying. Likewise, when the discount level is small, the seller chooses to offer only spot buying. When both the spot price and the discount level on forward buying are high, the seller effectively uses forward buying to extract some surplus from consumers who would otherwise not buy at all.

When the firm is a price-setter, we show that its optimal strategy is to always offer both forward and spot buying in order to price discriminate between the two kinds of consumers. The seller offers a discount on forward buying to target the first group of consumers, while offering spot buying to target the second group. We believe this result provides insight into the mixed offering strategy of several providers of information goods, such as MP3.com, which offers annual and monthly subscriptions to its music channels and also spot purchases of individual CDs.

Our work lays the foundation for further research on the sale of information goods through the mechanisms of forward and spot buying: (a) We have demonstrated the seller's decision for a specific form of the belief distribution function. For general symmetric distributions, analyzing the firm's decision-making under the two scenarios presented is an important and natural extension. (b) Subscription bundling as a form of forward buying is common in the digital economy. It would then be interesting to extend our analysis to incorporate bundling of several information goods. (c) While we have assumed that marginal costs are negligible and fixed costs are sunk, fixed costs, including setup costs and the costs of producing the first unit of a good, are often significant in the context of information goods. Recovering fixed costs, therefore, is the key to a seller's strategy. For instance, it costs a magazine publisher more to generate and publish two different articles than a single one. Content aggregators such as MP3.com and Yahoo.com invariably incur costs for setup and acquiring information. It would be necessary to study such settings to understand the impact of fixed costs on the seller's decision-making.

Our research framework may also be used to address physical goods that are being sold through the mechanisms of forward and spot buying. Yield management problems differ from the problem of information goods because of the additional capacity constraints on the supply side. We are looking at ways to reflect supply constraints in this same framework in order to develop a common platform for analyzing forward and spot buying of both information and physical goods.

Notes

1. This is an example where a "non-fan" with greater spending power who is almost indifferent to the product is willing to pay more for it than the ardent fan who values the product to

some significant measure but is limited by budget constraints. Although the reserve utility of the ardent fan may be greater than that of a non-fan, the willingness to pay is bounded from above by the lower of the reserve utility and the budget constraint. The budget constraint may be a result of the income and the valuation of the two types of fans for all other goods (a Hicksian composite good). Therefore, a non-fan with a lower reserve utility may well exhibit a higher willingness to pay, should her budget constraint be different from that of a fan.

2. Since, technically, r_i can only admit nonnegative values, we cannot allow the possibility that $l_i < 0$, but by considering this possibility we are able to enrich our analysis to include cases in which, say, a normal density function with a negligible left tail approximates the consumer's belief on her realized reservation prices.

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Appendix: Proofs

Lemma 1: Using the definition in Equation (1), we can rewrite Equation (7) as follows:

$$\Delta_i = E_{r_i}\left[(r_i - p)^+ - (p - r_i)^+ + zp - (r_i - p)^+\right]. \tag{A1}$$

Canceling equal terms gives

$$\Delta_i = zp - E_{r_i}(p - r_i)^+, \tag{A2}$$

which completes the proof. QED

Proposition 1: Let $Y_i = E_{r_i}(p-r_i)^+$. Let $f(r_i)$ be the density of the distribution function, and let the support be the interval $[l_i, h_i]$. Since we have assumed that f is symmetric, let $g(r_i-\mu_i)=f(r_i)$, where $g(-t_i)=g(t_i)$ and t_i is the normalized r.v. with mean 0, to explicitly capture the dependency on the mean. The support of $g(t_i)$ is the interval $[l_i-\mu_i, h_i-\mu_i]$. Then,

$$\begin{aligned} Y_i &= \Pr[r_i < p]E_{r_i|r_i<p}(p - r_i) \\ &= \Pr[t_i < p - \mu_i]E_{t_i|t_i<p-\mu_i}(p - t_i - \mu_i), \end{aligned} \tag{A3}$$

where because of symmetry we have teased out the explicit dependency on the mean through a variable change to the normalized variable.

Because of the symmetry of f, there are two possibilities, $l_i=-\infty$ and $h_i=+\infty$, and both l_i and h_i are finite.[2] We call the former an *infinite-support* distribution and the latter a *finite-support* distribution. In the latter case, l_i and h_i must be treated as varying with the mean, since the standard deviation is held fixed. In fact, it may be verified that they both increase linearly with the mean.

If it is an infinite-support distribution, then

$$\frac{\partial Y_i}{\partial \mu_i} = -\Pr[t_i < p - \mu_i], \tag{A4}$$

which is clearly negative. If it is a finite-support distribution, then we can further define

$$Y_i = \begin{cases} 0, & p \le l_i \\ \Pr[t_i < p - \mu_i]E_{t_i|t_i<p-\mu_i}(p - t_i - \mu_i), & l_i < p \le h_i \\ p - \mu_i, & p > h_i. \end{cases} \tag{A5}$$

For the regime $p \le l_i$, clearly, $\partial Y_i/\partial \mu_i = 0$. For the regime $l_i < p \le h_i$, using Leibnitz's rule

$$\frac{\partial Y_i}{\partial \mu_i} = -\Pr\left[t_i < p - \mu_i\right] - (p - l_i) g(l_i - \mu_i) \frac{\partial l_i}{\partial \mu_i}, \tag{A6}$$

where we have used $\partial l_i/\partial \mu_i = 1$ for simplification in the second step. Clearly, $\partial Y_i/\partial \mu_i <$ 0 in this regime. Finally, for the regime $p > h_i$,

$$\frac{\partial Y_i}{\partial \mu_i} = -1. \tag{A7}$$

It follows that $\partial \Delta_i/\partial \mu_i > 0$ for an infinite-support distribution. For a finite-support distribution, we conclude that $\partial \Delta_i/\partial \mu_i > 0$ for all $\mu_i > \mu_0$ and $\partial \Delta_i/\partial \mu_i = 0$ otherwise. Here, since l_i increases with μ_i, there exists mean value μ_0 such that $p = l_0$, l_0 being the value of l_i corresponding to $\mu_i = \mu_0$, and $p < l_i$ for all $\mu_i i > \mu_0$. QED

Proposition 2: Consider the expression $Y_i = E_{r_i}(p - r_i)^+$. We will prove separately for the two cases: infinite-support and finite-support symmetric distributions.

For an infinite-support distribution,

$$\lim_{\mu_i \to -\infty} Y_i \to \lim_{\mu_i \to -\infty} E_{r_i}(p - r_i) = p - \mu_i = +\infty \tag{A8}$$

and

$$\lim_{\mu_i \to \infty} Y_i = 0. \tag{A9}$$

The former is true because the tail (of the distribution) to the right of p tends to zero, and the latter is true because the tail to the left of p tends to zero. Existence then follows from continuity and the facts that

$$\lim_{\mu_i \to -\infty} \Delta_i = -\infty < 0 \tag{A10}$$

and

$$\lim_{\mu_i \to \infty} \Delta_i = zp > 0. \tag{A11}$$

Uniqueness follows from Proposition 1.

For a finite-support symmetric distribution, using the definition of Y_i given in the proof to Proposition 1,

$$\lim_{\mu_i \to -\infty} Y_i \to \lim_{\mu_i \to -\infty} p - \mu_i = +\infty. \tag{A12}$$

This is true because in the limit we operate in the regime $p > h_i$. Also, $Y_i = 0$ for all $\mu_i > \mu_0$, where μ_0 is defined in the earlier proof to Proposition 1. Existence in the interval $(-\infty, \mu_0)$ then follows from continuity and the facts that

$$\lim_{\mu_i \to -\infty} \Delta_i = -\infty < 0 \tag{A13}$$

and

$$\Delta_i = zp > 0 \tag{A14}$$

for all $\mu_i > \mu_0$. Uniqueness follows from the proof of Proposition 1. QED

Proposition 3: It is sufficient to show that

$$\frac{\partial Y_i}{\partial \sigma_c} \geq 0. \tag{A15}$$

Normal

We have the following expression:

$$Y_i = \int_{-\infty}^{p} (p - r_i) f(r_i) dr_i = \sigma_c \int_{-\infty}^{\frac{p-\mu_i}{\sigma_c}} \left(\frac{p-\mu_i}{\sigma_c} - x \right) \phi(x) dx. \tag{A16}$$

By Leibnitz's rule,

$$\begin{aligned} \frac{\partial Y_i}{\partial \sigma_c} &= \int_{-\infty}^{\frac{p-\mu_i}{\sigma_c}} \left(\frac{p-\mu_i}{\sigma_c} - x \right) \phi(x) dx \\ &+ \sigma_c \left[-\frac{p-\mu_i}{\sigma_c^2} \cdot \left(\frac{p-\mu_i}{\sigma_c} - \frac{p-\mu_i}{\sigma_c} \right) \phi\left(\frac{p-\mu_i}{\sigma_c} \right) - 0 \cdot \lim_{x \to -\infty} \left(\frac{p-\mu_i}{\sigma_c} - x \right) + \int_{-\infty}^{\frac{p-\mu_i}{\sigma_c}} \left(-\frac{p-\mu_i}{\sigma_c^2} \right) \phi(x) dx \right] \\ &= - \int_{-\infty}^{\frac{p-\mu_i}{\sigma_c}} x \phi(x) dx > 0, \end{aligned} \tag{A17}$$

which completes the proof for the case of normal distribution.

Uniform

Let the reservation price for consumer i be uniformly distributed $r_i \sim U(a_i,b_i)$. Then,

$$Y_i = \begin{cases} 0, & p \le a_i \\ \dfrac{(p-a_i)^2}{2(b_i-a_i)}, & a_i < p \le b_i \\ \dfrac{(b_i-a_i)}{2}, & p > b_i. \end{cases} \tag{A18}$$

Denote the mean as μ_i and the standard deviation as σ_c, so that $b_i = \mu_i + 3^{1/2}\sigma_c$ and $a_i = \mu_i - 3^{1/2}\sigma_c$. Substituting into the previous expression, we obtain the explicit dependency of Y_i on σ_c:

$$Y_i = \begin{cases} 0, & p \le a_i \\ \dfrac{\left(p-\mu_i+\sqrt{3}\sigma_c\right)^2}{2\sigma_c}, & a_i < p \le b_i \\ \sqrt{3}\sigma_c, & p > b_i. \end{cases} \tag{A19}$$

The claim holds for the first and third regimes. For the second regime, differentiation with respect to σ_c yields

$$\begin{aligned} \frac{\partial Y_i}{\partial \sigma_c} &= \frac{\left(p-\mu_i+\sqrt{3}\sigma_c\right)\left(\mu_i+\sqrt{3}\sigma_c-p\right)}{2\sigma_c^2} \\ &= \frac{(p-a_i)(b_i-p)}{2\sigma_c^2}. \end{aligned} \tag{A20}$$

Since $a_i < p \le b_i$, this expression is nonnegative, which completes the proof for the uniform case. QED