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Observations on the normality of batch production times in flexible manufacturing cells

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Several observations regarding the normality of the total batch completion times in flexible manufacturing are presented. It is shown that normality holds under some but not all conditions. Other conditions, prevalent in flexible manufacturing and leading to multimodal batch completion times, are discussed.

Introduction

Proper identification of the probability density function of the total batch completion times is required for the design and operational control of many manufacturing and assembly systems. Wilhelm (1986) points out that operation times are (approximately) normally distributed in small-lot assembly systems. In this he reacts to an earlier statement by Seidmann *et al.* (1985) regarding the limitation of the normality assumptions to certain manufacturing systems. This note reviews the assumption of normal batch completion times in flexible manufacturing and assembly cells with random product feedback flow. Several necessary conditions required for the validity of this assumption are identified. An illustrative example demonstrates that batch completion times need not be normally distributed even when task times are normally distributed.

The total batch times

Following the notations and the definition of the flexible manufacturing cell in our earlier paper (Seidmann *et al.* 1985) consider first a batch of B items produced one at a time. Let L denote the number of necessary reworks, and by T_M and T_R denote the actual times for processing one part once at the manufacturing area and once at the rework area. The total elapsed time to make one item in the batch is

$$\theta = (L + 1) \text{ copies of } T_M + L \text{ copies of } T_R \quad (1)$$

We can now consider three general cases:

Case 1. Large batches

If the task processing times (T_M, T_R) are statistically independent, then the central limit theorem shows that for large batches ($B \rightarrow \infty$) the total time to produce a batch is approximately normally distributed.

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Hence, in the case of large batches, our results concur with the observation of Wilhelm (1986) that batch completion times are approximately normal. Moreover, Wilhelm (1986) agrees with our empirical observation that $B \geqslant 10$ provides a reasonable threshold for this case.

Case 2. Small batches, large number of reworks

Consider next the case of a cell producing small batches, i.e. $B \rightarrow 1$. Two asymptotic cases are discussed, namely: large or small number of reworks.

If B is small and L is large, then we concur again with the observation of Wilhelm (1986): the distribution of T will be bell-shaped and approximately normal via the application of the central limit theorem to eqn. (1).

Case 3. Small batches, small number of reworks

Finally, consider the case where B and L are both small. It seems that this case is overlooked in the analysis of Wilhelm (1986). He presents good reasons why, in general, the actual completion times T_M and T_R are approximately normally distributed. For instance, this may happen if there is uncertainty about part entry times, or subassembly ready times, or if T_M and T_R represent a sequence of many tasks, such as occurs on an assembly line. In contrast, Fig. 4 in the paper by Seidmann *et al.* (1985) depicts a common case for $B = 1$ where θ is clearly multimodal and not normally distributed.

We now bring a stronger counter-example to the idea of normal batch times in the case where the values of B and L are both small. In this example θ is not normally distributed even though both T_M and T_R are normally distributed. Suppose in this example that $L = 0$ with probability 0·65 and $L = 1$ with probability 0·35. Suppose also that the distributions of T_M and T_R are given by

$$\begin{bmatrix} T_M \sim N(\mu_M = 30,\ \sigma_M^2 = 3) \\ T_R \sim N(\mu_R = 10,\ \sigma_R^2 = 3) \end{bmatrix}$$

The resulting distribution of θ is then

$$\theta \sim \{\cdot 65N(30, 3) + \cdot 35[N(30, 3) + N(10, 3) + N(30, 3)]\}$$

or

$$\theta \sim \{\cdot 65N(30, 3) + \cdot 35N(70, 9)\}$$

The probability density function of θ, $h_\theta(t)$, which is bi-modal, is shown in the Figure. Note that the right mode of $h_\theta(t)$ bulges because

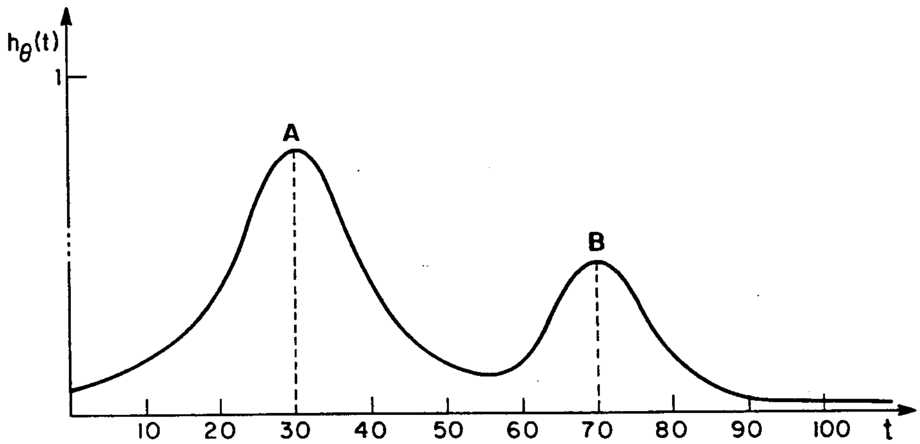
$$2E(T_M) + E(T_R) \gg E(T_M) + 3\sigma(T_M) \tag{2}$$

which means that

$$\frac{E(T_M) + E(T_R)}{\sigma(T_M)} \gg 3 \tag{3}$$

As it happens, in this example this ratio is indeed

$$\frac{30 + 10}{\sqrt{3}} \gg 3$$



The bimodal function for the example cell. (Note: point A corresponds to, $L = 0$, while point B corresponds to a single rework case, $L = 1$.)

This example indicates clearly that in certain cases batch completion times need not be normally distributed in spite of having normally distributed manufacturing times (T_M) and rework times (T_R).

Discussion

It appears that batch times will be (approximately) normally distributed in the following cases:

(1) large batches, ($B \geqslant 10$);

(2) small batches with many reworks, ($B \rightarrow 1$, $L \rightarrow \infty$);

(3) small batches with normally distributed manufacturing times and no rework, ($B \rightarrow 1$, $L = 0$, $T_M \sim N(\mu, \sigma^2)$).

By contrast, it seems that batch times will not be normally distributed in the following cases:

(4) Small batches with a limited number of rework attempts (i.e. $L = 1$ or 2) and relatively small manufacturing time variability [i.e. $(E(T_M) + E(T_R))/(\sigma(T_M)) \geqslant 3$]. (see the example in the Figure).

(5) Small batches with minimal rework requirements (i.e., $L \rightarrow 0$) and manufacturing times T_M that are not normally distributed.

The above five cases are not exhaustive since the behaviour of θ also depends on the relative effect of rework times—i.e., $E(L)E(T_R)$—and whether they can be negligible. Of all these cases, it should be noted that cases (4) and (5) with relatively small batches and limited reworks are the predominant ones in flexible manufacturing.

Numerous simulation and probabilistic analyses of single and multiple robotic manufacturing cells with random feedback flow and with $B = 1$ were recently performed by Wilhelm *et al.* (1986), with task distribution times taken as exponential, uniform and normal. It was noted that for robotic cells with unmanned operations large amounts of recirculation represent a poor system design, since the cell time may be excessive and unpredictable. Thus, it was recommended that in order to be useful, a cell must either have a limited number of reworks allowed, or small probabilities of rework, or the rework should be separated from the cell. This reinforces the notion that cases (4) and (5) above (i.e. small batches with limited number of reworks) represent a common design in programmable automation.

Finally, we thank Dr Wilhelm for raising the important issue of when normality of batch completion times holds in small-lot assembly systems. This issue is vital to the design, analysis, and control of production cells. The above analyses show that (approximate) normality holds under some but not all conditions. It is sensitive to, at least, the following variables:

(1) batch size,

(2) number of reworks,

(3) the ratio $(E(T_M) + E(T_R))/\sigma(T_M)$,

(4) the shape of the distribution of T_M, and

(5) the total expected rework time: $E(L)E(T_R)$.

It is also sensitive to the probability of rework (p) and to the expected rework times $E(T_R)$.

This short note demonstrates that it is possible to isolate the key variability factors and to assess their influence on the completion time directly. Future research will attempt to formulate the relationship between these and other variables, the topology of the manufacturing system, and the probability distribution function of the total production times.

Plusieurs remarques sont faites quant à la normalité des temps de réalisation totale de lots en fabrication flexible. L'article montre que la normalité est présente dans certaines conditions seulement. D'autres conditions qui prévalent en fabrication flexible et entraînent des temps de réalisation de lot multimode sont examinées.

Es werden einige Beobachtungen, die über die Normalität der Gesamtfertigungszeiten von Partien in flexiblen Fertigungsanlagen gemacht wurden, vorgestellt. Es wird nachgewiesen, dass die Normalität für manche, jedoch nicht alle Bedingungen gilt. Andere Bedingungen werden besprochen, die bei der flexiblen Fertigung vorherrschen und die zu mehrfachen Modalwerten in Verteilungen von Partien-Gesamtzeiten führen.

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