

# Operational Analysis of an Autonomous Assembly Robotic Station

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**Abstract**—This paper presents operational analysis models of autonomous assembly robotic stations. Robotized assembly systems are programmable and therefore provide a cost-effective solution for the assembly of small batch sizes. Assembly tasks completions and quality considerations require task repetition and rework of a certain portion of the assembled items within the station. Concepts from stochastic processes are used to investigate the structural properties governing the probabilistic relationships of the total and functional batch times and the number of reworks. Further, a new model of industrial controlled rework is developed for stations with a bounded number of rework attempts and distinct rework rates at each trial. Explicit performance and cost measures of the robot operations at the *task*, *product*, and at the *assembly system* levels are derived. Capacity design examples illustrate various manufacturing planning considerations such as production throughput, operational efficiency, robot speed, rework rates, and station size.

## I. INTRODUCTION

ROBOTIC production facilities are designed with the concept of cell or station to provide modularity, flexibility, and controllability. A unitary robotic assembly station allows only a single work-order to be processed in the robotic station at a time. The basic premise in applying the unitary concept to process robot stations is that all the station resources should be devoted to assemble a high-quality product before it is allowed to exit. Thus inspection is included in the robotic work program, as well as rework and recovery from errors and poor quality. It results in zero defects production with minimal work-in-process inventories. Industrial applications of a unitary work station are described in [2], [5], [14], [18] and in recent articles in the *Journal of Assembly Engineering* [1983 and on].

The concept of a unitary robotic station and flow model to analyze its characteristics were introduced in [18]. Further theoretical developments are presented in [19] and [20], and comparisons of the model results to empirical simulations are presented in [23].

The purpose of this paper is to extend the operational analysis of a flexible assembly robotic station based on the above model and theory, and to analyze several important issues involved in the design of such stations. First, let us briefly describe the unitary cell model.

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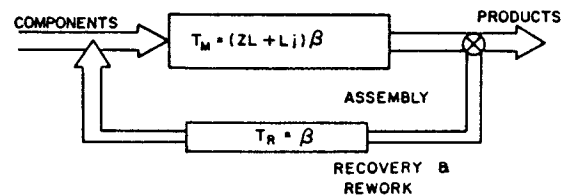


Fig. 1. Parts flow in a unitary cell.

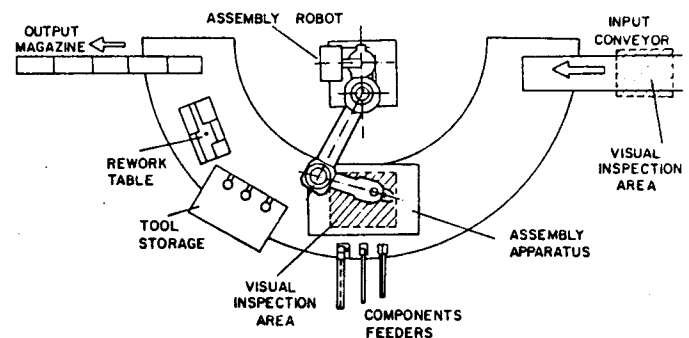


Fig. 2. Schematic structure of a SCARA robotic assembly station with inspection and rework capabilities.

Given the inherent difficulties of the industrial assembly process, (e.g., alignment, clearance, and orientation problems) the unitary assembly station model assumes that each assembly station includes *two types of activities*: 1) Regular assembly tasks, including inspection, considered as the *main tasks*, designated  $M$ ; 2) the other type of activities includes the *rework or error recovery tasks*, designated  $R$ , that are required occasionally. When an assembly has completed the regular assembly tasks it is also inspected. It will leave the station if it passes inspection. Otherwise, it will be recirculated for rework  $R$  and then return to repeat the main assembly tasks  $M$ . No other assembly can enter the station until a successfully finished system leaves it. Denote by  $q$  ( $0 \leq q \leq 1$ ) the probability to pass inspection and by  $p = 1 - q$  the probability to remain for rework.

Fig. 1 illustrates the parts flow inside the cell. The layout of a unitary cell designed for the assembly of odd-shaped components is given by Fig. 2.

In the next section we present a summary of notations and previous research results. This is followed by the development of new common relationships in Section III. The analysis of cell operations with limited number of reworks and distinct rework probabilities is carried out in Section IV. These results are then applied to the performance evaluation of flexible assembly robotic stations in Section V, and Section VI presents

TABLE I  
SUMMARY OF GENERAL MEASUREMENTS

Measure	Mean Value	Variance Value
1) No. of visits in area $M$ :	$\mu_N = B/q$	$\sigma_N^2 = Bp/q^2$
2) Total time in area $M$ :	$\mu_{\tilde{M}} = B\mu_M/q$	$\sigma_{\tilde{M}}^2 = B(p\mu_M^2/q + \sigma_M^2)/q$
3) Total time in area $R$ :	$\mu_{\tilde{R}} = Bp\mu_R/q$	$\sigma_{\tilde{R}}^2 = B(p\mu_R^2/q + p\sigma_R^2)/q$
4) Total time in the station:	$\mu_\theta = B(\mu_M + p\mu_R)/q$	$\sigma_\theta^2 = Bp(\mu_M + \mu_R)^2/q^2 + B(\sigma_M^2 + p\sigma_R^2)/q$
5) Covariance of $T_{\tilde{M}}$ and $T_{\tilde{R}}$ : $\text{COVR}(T_{\tilde{M}}, T_{\tilde{R}}) = Bp\mu_M\mu_R/q^2$		
6) Correlation coefficient of $T_{\tilde{M}}$ and $T_{\tilde{R}}$ : $\rho(T_{\tilde{M}}, T_{\tilde{R}}) = \text{COVR}(T_{\tilde{M}}, T_{\tilde{R}})/(\sigma_{\tilde{M}} \cdot \sigma_{\tilde{R}})$		

two industrial robotic cell design examples. Section VII concludes the paper.

## II. NOTATIONS AND SUMMARY OF PREVIOUS RESULTS

The model treats the case of producing a batch of  $B$  identical parts by the station. Let  $T_M$  and  $T_R$  denote the actual time for working one part *once* at  $M$  or  $R$ , respectively, and by  $T_{\tilde{M}}$  and  $T_{\tilde{R}}$  the *total* time at  $M$  and  $R$  for the entire batch. The total batch time is  $\theta = T_{\tilde{M}} + T_{\tilde{R}}$ . We let  $H_M(t)$ ,  $H_R(t)$ ,  $H_{\tilde{M}}(t)$ ,  $H_{\tilde{R}}(t)$ , and  $H_\theta(t)$  denote the cumulative probability functions for  $T_M$ ,  $T_R$ ,  $T_{\tilde{M}}$ ,  $T_{\tilde{R}}$ , and  $\theta$ , respectively, where the first two distributions are given and the last three are desired. The corresponding density functions are  $h_M(t)$ ,  $h_R(t)$ ,  $h_{\tilde{M}}(t)$ ,  $h_{\tilde{R}}(t)$ , and  $h_\theta(t)$ . The mean and variance of these distributions are denoted by  $\mu_M$  and  $\sigma_M^2$ ,  $\mu_R$  and  $\sigma_R^2$ , etc.

Next, we denote by  $N$  the number of visits at  $M$  to complete a batch of  $B$  items. Since the number of visits by *one* part to  $M$  is a Bernoulli trial with success probability  $q$ ,  $N$  has a negative binomial distribution

$$\Pr [N = n] = \binom{n-1}{n-B} p^{n-B} q^B, \quad n = B, B+1, \dots \quad (1)$$

with mean  $\mu_N = B/q$  and variance  $\sigma_N^2 = Bp/q^2$ .

Given  $h_M(t)$ ,  $h_R(t)$ , and  $p$ , the probability density functions of  $T_{\tilde{M}}$ ,  $T_{\tilde{R}}$ , and  $\theta$  are

$$h_{\tilde{M}}(t) = \sum_{n=B}^{\infty} \binom{n-1}{n-B} p^{n-B} q^B h_M(t)^{* (n)} \quad (2)$$

$$h_{\tilde{R}}(t) = \sum_{n=B}^{\infty} \binom{n-1}{n-B} p^{n-B} q^B h_R(t)^{* (n-B)} \quad (3)$$

$$h_\theta(t) = \sum_{n=B}^{\infty} \binom{n-1}{n-B} p^{n-B} q^B [h_M^{* (n)} * h_R^{* (n-B)}](t) \quad (4)$$

where  $*$  denotes convolution,  $^{* (n)}$  denotes an  $n$ -fold convolution, and  $h^{* (0)}(t) = \delta(t)$ , the Dirac delta function.

Finally, the coefficient of variation of  $N$  (= total number of passes through  $M$ ) and  $\theta$  (= total station time) are given by

$$\begin{aligned} \text{COV}_n &= \sigma_N/\mu_N \\ &= (p/B)^{0.5} \end{aligned} \quad (5)$$

$$\text{COV}_\theta = \sigma_\theta/\mu_\theta$$

$$\begin{aligned} &= (p(\mu_M + \mu_R)^2 \\ &+ q(\sigma_M^2 + p\sigma_R^2))^{0.5} / (B^{0.5}(\mu_M + p\mu_R)). \end{aligned} \quad (6)$$

Table I presents a summary of several other general measures, developed previously in [18], [19].

## III. COMMON RELATIONSHIPS

Investigating the relations between the number of reworks and the cell throughput several new expressions are derived; these include the joint distribution functions, the covariance, and the correlation coefficients.

### A. The Joint Distribution Functions

We first discuss a single product case ( $B = 1$ ). Recall that the probability that a single assembly passes only once through area  $M$  ( $N = 1$ ), while it sojourns in the cell no longer than time  $t$ , is given by

$$\omega_1(t, 1) = \Pr (N = 1, t < T) = qH_M(T) \quad (7)$$

and for  $N = 2$ , is equal to

$$\omega_1(t, 2) = \Pr (N = 2, t < T) = qp[H_M^{*2} * H_R](t). \quad (8)$$

Similarly, the joint distribution function of  $t$  and  $N$  is given by

$$\omega_1(t, N) = qp^{N-1} [H_M^{*N} * H_R^{* (N-1)}](t), \quad N \geq 1. \quad (9)$$

The expression given by (9) is difficult to evaluate and the following mixed transform is used for computing the moments of key functional relationships:

$$\tilde{\omega}_1(s, z) = \int_0^\infty \sum_{N=1}^{\infty} z^N e^{-st} \omega_1(t, N) dt, \quad |z| < 1 \quad (10)$$

$$s > 0.$$

Denoting by  $\tilde{H}_M(t)$  and  $\tilde{H}_R(t)$  the Laplace transforms of  $H_M(t)$  and  $H_R(t)$ , respectively, leads to

$$\tilde{\omega}_1(s, z) = q\{z\tilde{H}_M(s) + \sum_{N=2}^{\infty} z^N p^{N-1} \tilde{H}_M(s)^N \tilde{H}_R(s)^{N-1}\}. \quad (11)$$

Fortunately, the sum of the infinite geometric series in (11) converges for any given  $|z| < 1$  and  $s > 0$ ; then we can

write:

$$\tilde{\omega}_1(s, z) = \frac{qz\tilde{H}_M(s)}{1 - zp\tilde{H}_M(s)\tilde{H}_R(s)}, \quad s > 0, |z| > 1. \quad (12)$$

The moments of  $t$  and  $N$  are computed by the partial derivatives of (12) with respect to  $s$  and  $z$  and by setting  $z$  equal to one and  $s$  equal to zero, respectively.

In the general case,  $B = 1, 2, \dots$  and  $N = B, B + 1, \dots$ . The joint distribution of  $\theta$  and  $N$  is now

$$\omega_B(\theta, N) = \Pr(\theta < T, N = n), \quad t > 0$$

$$n = B, B + 1, \dots \quad (13)$$

From (13) and from [19, eq. (18)] one can verify that

$$\omega_B(\theta, N) = \binom{N-1}{N-B} p^{N-B} q^B [H_M^{*(N)} * H_R^{*(N-B)}](t). \quad (14)$$

Since the convolution operator is associative and commutative we use the general properties of the Laplace transform of (14) to obtain

$$\tilde{\omega}_B(s, z) = \left[ \frac{qz\tilde{H}_M(s)}{1 - zp\tilde{H}_M(s)\tilde{H}_R(s)} \right]^B. \quad (15)$$

Expression (15) is used next to compute the covariance and the correlation coefficients of  $\theta$  and  $N$ .

#### B. The Covariance of the Cell Times: $\theta$ and $N$

In order to compute the covariances of  $\theta$  and  $N$  we first define by  $\theta_i$  and  $N_i$  the total time in the cell and the total number of visits to  $M$  by the  $i$ th assembly,  $1 \leq i \leq B$ . Since

$$\sum_{i=1}^B \theta_i = \theta \quad \text{and} \quad \sum_{i=1}^B N_i = N$$

the covariance of  $\theta$  and  $N$  is given by

$$\begin{aligned} \text{COVR}(\theta, N) &= E\{(\theta - E(\theta))(N - E(N))\} \\ &= E\left\{ \left( \sum_{i=1}^B \theta_i - E\left( \sum_{i=1}^B \theta_i \right) \right) \right. \\ &\quad \cdot \left. \left( \sum_{j=1}^B N_j - E\left( \sum_{j=1}^B N_j \right) \right) \right\} \\ &= E\left\{ \left( \sum_{i=1}^B (\theta_i - E(\theta_i)) \right) \left( \sum_{j=1}^B (N_j - E(N_j)) \right) \right\} \\ &= E\left\{ \sum_{j=1}^B [(\theta_j - E(\theta_j))(N_j - E(N_j))] \right. \\ &\quad \left. + 2 \sum_{j < i} [(\theta_i - E(\theta_i))(N_j - E(N_j))] \right\}. \end{aligned}$$

The term

$$E\{(\theta_j - E(\theta_j))(N_j - E(N_j))\}$$

is the covariance of  $\theta_j$  and  $N_j$ . For each assembly  $j \in B$  we have

$$\text{COVR}_j(\theta_j, N_j) = E(\theta_j N_j) - E(\theta_j)E(N_j) \quad (17)$$

and then

$$E(\theta_j N_j) = \int_0^\infty \sum_{N_j=1}^\infty N_j \theta_j \omega_1(\theta_j, N_j) d\theta_j. \quad (18)$$

This term is directly evaluated from

$$\begin{aligned} E(\theta_j N_j) &= -\frac{\partial}{\partial z} \cdot \frac{\partial}{\partial s} \tilde{\omega}_1(s, z) \\ &= ((1+p)\mu_M + 2p\mu_R)/q^2, \quad s = 0, z = 1. \end{aligned} \quad (19)$$

Thus the covariance term in (17) is

$$\text{COVR}_j(\theta_j, N_j) = p(\mu_M + \mu_R)/q^2. \quad (20)$$

The second summation term in (16) vanishes since  $\text{COVR}(\theta_i, N_j) = 0$  for every  $j < i$  (due to the independence of  $\theta_j$  and  $N_j$ ). Substituting (20) into (16) leads to the desired final result

$$\text{COVR}(\theta, N) = Bp(\mu_M + \mu_R)/q^2. \quad (21)$$

The correlation coefficient of  $\theta$  and  $N$  is

$$\begin{aligned} \rho(\theta, N) &= \text{COVR}(\theta, N)/(\sigma_\theta \cdot \sigma_N) \\ &= \left[ 1 + \frac{(\sigma_M^2 + p\sigma_R^2)q}{p(\mu_M + \mu_R)^2} \right]^{-1/2} \end{aligned} \quad (22)$$

Equation (22) proves that the correlation coefficient  $\rho(\theta, N)$  is independent of  $B$ . It increases with the means of  $T_M$  and  $T_R$  (namely,  $\mu_M, \mu_R$ ) and decreases as their respective variances (namely,  $\sigma_M^2, \sigma_R^2$ ) increase.

#### C. The Covariance of the Areal Times: $T_M, T_R$ , and $N$

The values of  $\text{COVR}(T_M, N)$  and  $\text{COVR}(T_R, N)$  can be derived from the joint distributions of the areal times ((2) and (3)) or directly from (21). This leads to

$$\text{COVR}(T_M, N) = Bp\mu_M/q^2 \quad (23)$$

and

$$\text{COVR}(T_R, N) = Bp\mu_R/q^2. \quad (24)$$

#### D. The Correlation Coefficients of the Areal Times: $T_M, T_R$ , and $N$

Following similar analysis the relevant correlation coefficients are derived as

$$\rho(T_M, N) = \mu_M(\mu_M^2 + q\sigma_M^2/p)^{-1/2} \quad (25)$$

and

$$\rho(T_R, N) = \mu_R(\mu_R^2 + q\sigma_R^2)^{-1/2}. \quad (26)$$

The relationship between these correlation coefficients and the rework probability at the cell is illustrated by Fig. 3 for

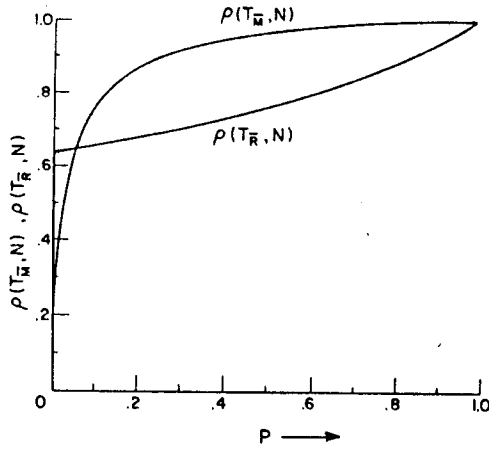


Fig. 3. The correlation coefficients as a function of the rework probability.

the case of  $\mu_M = 141$ ,  $\sigma_M = 40$ ,  $\mu_R = 10$ , and  $\sigma_R = 12$ . These numbers represent the time (in seconds) of a water pump assembly station designed for a major manufacturer of domestic dishwashers. Notice the relative differences between these functions for the small values of  $p$ . While the curves generated by those equations vary greatly according to the input data they do illustrate that for small values of  $p$  the actual number of reworks is loosely correlated with the times in  $M$  and  $R$ , and conversely for larger values of  $p$ . The figure also illustrates the interesting fact that unlike  $\rho(T_M, N)$  the curve of  $\rho(T_R, N)$  does not start at zero and as a result has a smaller change in magnitude as  $p$  increases.

#### IV. LIMITED NUMBER OF REWORKS WITH UNEQUAL REWORK PROBABILITIES

The first model below assumes distinct rework rates at each trial. It is followed by an analysis of three common cases: Constant, Decreasing, and Increasing rework rates.

##### A. Notation

The previous studies of the unitary stations assumed that in order to obtain a perfect yield each produced item is recycled and reworked until it passes inspection. In certain designs, however, the number of trials is limited for practical reasons. This section analyzes those cases in which the number of visits at  $M$  is limited to  $J$  ( $J > 1$ ): The rework probability after the  $i$ th visit to  $M$  is  $p_i$  where  $p = (p_1, p_2, \dots, p_J)$  is the rework probability vector. For brevity, we present here only the single part case ( $B = 1$ ).

##### B. The Distribution of the Number of Reworks

In this station we get that

$p_i = \Pr$  [ $i$ th visit to  $M$  fails/all ( $i - 1$ ) previous visits to  $M$  are failures] and  $k'(n) = \Pr$  [ $n$  visits to  $M$ , where only the  $n$ th visit is successful]

$$k'(n) = \begin{cases} 1 - p_1, & n = 1 \\ p_1 p_2 \cdots p_{(n-1)} (1 - p_n), & 2 \leq n \leq J. \end{cases} \quad (27)$$

Since the number of reworks is limited define

$$\hat{k} = \Pr [J \text{ visits to } M, \text{ all failures}] \quad (28)$$

$$= p_1 p_2 \cdots p_J.$$

The probability density function of  $k(n)$  is given by  $k(n) = \Pr [N = n]$

$$k(n) = \begin{cases} k'(n), & n = 1, 2, \dots, J-1 \\ k'(J) + \hat{k} = \prod_{i=1}^{J-1} p_i, & n = J. \end{cases} \quad (29)$$

Consider, for instance, a unitary station with  $J = 3$ ; the evaluation of its operational performance characteristics follows the general scheme of the previous sections. Assuming, for generality, distinct rework probabilities leads to

$$\begin{aligned} k(1) &= 1 - p_1 \\ k(2) &= p_1(1 - p_2) \\ k(3) &= p_1 p_2 (1 - p_3) + p_1 p_2 p_3 \\ &= p_1 p_2. \end{aligned} \quad (30)$$

The mean value of  $N$  is

$$\mu_N = \sum_{j=1}^J j k(j) = 1 + \sum_{i=1}^{J-1} \left( \prod_{j=1}^i p_j \right) \quad (31)$$

and the variance of  $N$  is

$$\sigma_N^2 = \sum_{z=1}^J z^2 k(z) - \mu_N^2. \quad (32)$$

The total cell time probability density functions for  $J = 3$  is

$$\begin{aligned} h_\theta(t) &= k(1)h_M(t) + k(2)[h_M^{*(2)} * h_R](t) \\ &\quad + k(3)[h_M^{*(3)} * h_R^{*(2)}](t). \end{aligned} \quad (33)$$

##### C. Total Cell Times

The mean  $\mu_\theta$  and the variance of  $\sigma_\theta^2$  of  $\theta$  can be computed from  $h_\theta(t)$  above. Since this function is not convenient for direct numerical evaluation these parameters are derived here using the well-known distribution independent results for the mean and variance of a random sum of random variables [4]. Recalling that each item passes  $N$  times through  $M$  and  $(N - 1)$  times through  $R$  one gets that

$$\mu_\theta = \mu_M \mu_N + \mu_R (\mu_N - 1) \quad (34)$$

and

$$\sigma_\theta^2 = (\mu_N - 1)(\sigma_M^2 + \sigma_R^2) + (\mu_M + \mu_R)^2 \sigma_N^2 + \sigma_M^2. \quad (35)$$

The expected throughput of good items per time unit is, in general, equal to

$$TP(J, p) = \left( 1 - \prod_{i=1}^J p_i \right) / \mu_\theta \quad (36)$$

and if  $J = 3$ , then

$$TP(3, p) = (1 - p_1 p_2 p_3) / \mu_\theta.$$

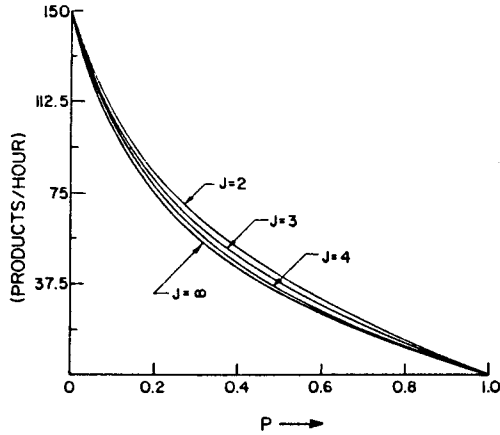


Fig. 4. Station's throughput with varying rework limits.

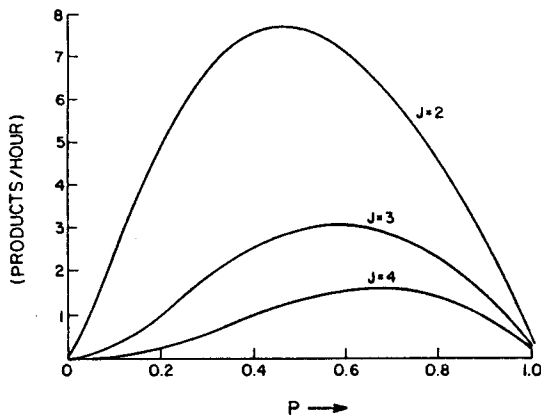


Fig. 5. Absolute differences in throughputs between unlimited and limited number of reworks.

The expected "throughput" of rejected items is

$$RI(J, p) = \left( \prod_{i=1}^J p_i \right) / \mu_0$$

$$= (p_1 p_2 p_3) / \mu_0. \quad (37)$$

Following this illustration, the interested reader can construct the other performance characteristics similar to the complete set of Table I above. It should be pointed out that in certain industrial cases (i.e.,  $J > 3$ ), the expected differences between the results of the unlimited rework model (Section II) and the limited rework model (Section IV) are significant. The latter model is more accurate in practice.

For examples, consider two models of operating such an assembly station with  $\mu_M = 25$ ,  $\sigma_M = 6$ ,  $\mu_R = 56$ , and  $\sigma_R = 49$ . One mode of operations allows *unlimited* number of reworks attempts and other assumes a limited number of reworks with  $J = 2, 3$ , and 4. Fig. 4 depicts the expected hourly throughput of *good* products as a function of the rework probability  $p$  for these four cases ( $J = 2, 3, 4$ , and  $\infty$ ). The *absolute* hourly difference in throughput between the cases of limited and the unlimited number of rework attempts [ $TP(J, p) - TP(\infty, p)$ ] and the *relative* differences in throughputs [ $(TP(J, p) - TP(\infty, p)) / TP(\infty, p)$ ] are presented by Figs. 5 and 6, respectively.

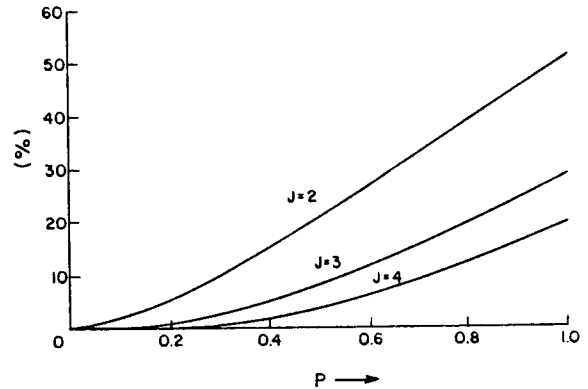
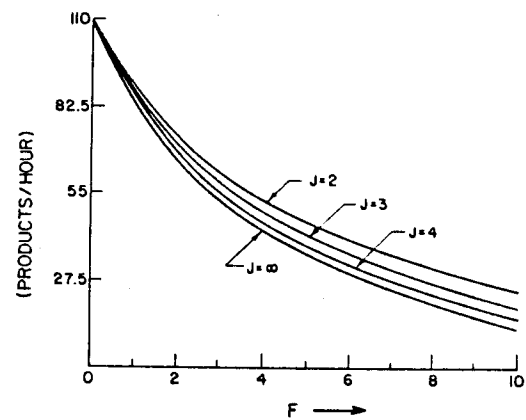
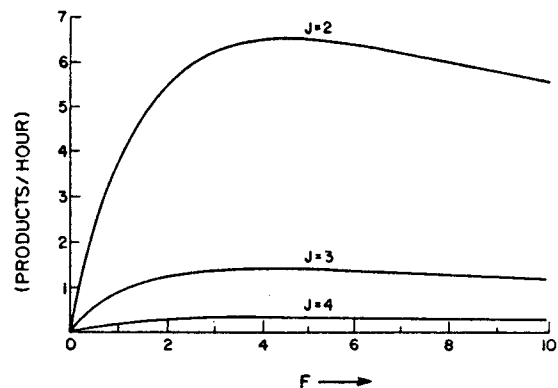


Fig. 6. Relative differences in throughputs between unlimited and limited number of reworks.

Fig. 7. Station's throughput with varying rework times ( $\mu_R = F\mu_M$ ).Fig. 8. Absolute differences in throughputs between unlimited and limited rework attempts with varying rework times ( $\mu_R = F\mu_M$ ).

These figures clearly indicate that as  $p_i$  increases limiting the number of rework attempts the result is a relative increase of the cell-throughput. These differences in the station's throughput become more pronounced as the limit on the number of rework attempts  $J$  reduces.

The impact of the *relative* changes in  $\mu_R$  with respect to  $\mu_M$  was also studied. Figs. 7-9 depict performance measures similar to those of Figs. 4-6 but assuming  $p = 0.25$  and varying  $\mu_R$  from zero to ten times  $\mu_M$  (here  $\mu_R = F\mu_M$  and  $0 \leq F \leq 10$ ). The results prove that as  $\mu_R$  increases the designer may be more inclined to augment the cell's throughput by enforcing a limited rework control policy.

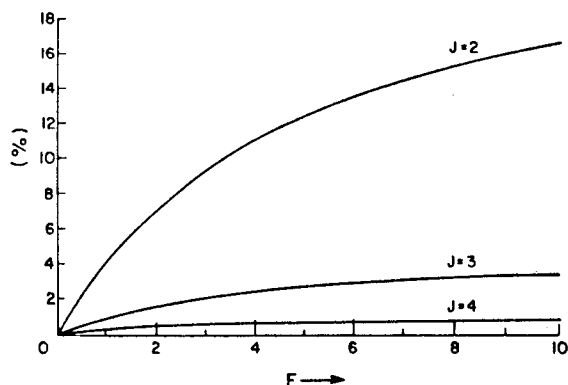


Fig. 9. Relative differences in throughputs between unlimited and limited rework attempts with varying rework times ( $\mu_R = F\mu_M$ ).

TABLE II  
SAMPLE DISTRIBUTIONS OF REWORK PROBABILITIES

Rework Pattern	Possible Structure	
	Geometric	Exponential ( $\alpha > 0$ )
Constant:	$p_i = p$	
Decreasing:	$p_i = p^i$	$p_i = pe^{-\alpha(i-1)}$
Increasing:	$p_i = 1 + p^{i-1}(p-1)$	$p_i = p + (1-p)(1 - e^{-\alpha(i-1)})$

D. The Three Rework Probability Patterns

The rework probabilities need not be identical for all the rework attempts. These probabilities depend on the assembly process technology as well as on the nature of the rework task. The three general patterns are as follows:

1) *Constant Rework Rates:* All the rework cycles possess the same probability. A typical example is the robotic assembly cell for high-pressure water pumps used in dishwashers. Following the assembly stage the robot conducts a leak test; rejected products are disassembled and then reassembled again using another set of gaskets and washers. Constant rework rates were observed for this operation.

2) *Decreasing Rework Rates:* The rework rates decrease from cycle to cycle as the assembled system performance improves. A typical example is in-circuit robotic test and rework cells used in a printed circuit board assembly line. Incoming boards are positioned on the tester by the robotic arm. Rejected boards indicate to the robot the location of the electronic chip to be replaced and the robot then replaces the faulty components with new ones during the rework cycle. Another test is conducted following these component replacements. These cells tend to exhibit decreasing rework rates.

3) *Increasing Rework Rates:* The rework rates increase from cycle to cycle as a result of deteriorating product performance. Such a case should be avoided at the design stage. In practice, however, some instances cannot be eliminated and the number of rework attempts is limited by the system controls. Typical examples include forced realignment attempts at the assembly stage or the jamming of insertions sockets. Those incidents result in increasing work rates.

Table II presents several optional mathematical characterizations of these processes. Geometric and exponential decay (growth) functions are illustrated for the case of decreasing (increasing) rework rates. In both, function  $p_i = p$  for  $i = 1$

and the decay (growth) functions approach zero (one) as  $i$  approaches infinity.

V. PERFORMANCE OF UNITARY ASSEMBLY ROBOTIC STATIONS

A. Evaluation Levels

Evaluating the performance of *unitary* assembly robotic stations, one must consider the fact that they have to produce a variety of subassemblies with a range of processes. The robotic nature implies that the station has the ability to recover from errors and rework faulty assemblies automatically. These two characteristics may cause the performance to be highly variable, depending on product, process, and task variations. Certainly, there is a *tradeoff* between the number of rework and recovery attempts, and the rate of successful, quality products exiting the station. These considerations led Wilhelm [26] to suggest that the performance of programmable assembly systems be evaluated by certain measures at three levels, namely, at the *task*, *product*, and *system levels*. Extending his basic hierarchical decomposition approach to the model discussed above results in these functional level descriptors:

*At the task level*, the mean and variance of task time indicate the required total process time and the stability of the assembly operation in terms of finite production (assembly) times. Also at the task level, the probability of successful assembly completion indicates the efficiency of the station at performing given tasks.

*At the product level*, the product throughput or cycle time per product unit indicates the production capacity requirement for given products. The yield, or ratio of good product output to product input indicates the station's efficiency at the product level.

*At the assembly system level*, measures of utilization and cost can be used to evaluate automatic assembly plans, layouts, and operating strategies for a programmable assembly system design.

Measures at these different levels are not independent, but allow one to focus on relevant design aspects of the assembly station and assess the merit of flexibility. Following the equations developed above, the performance measures for our model are computed as follows.

B. Task Level

First, consider the case of unlimited rework attempts. For each assembly task programmed at the robotic station, the mean total time and its variance can be computed as shown in Table I. With regard to the probability of success, in this hypothetical case it is assumed that only good products leave the station. For the case of limited number of rework attempts, the mean and variance of the total number of assembly task trials are given by (31) and (32).

C. Product Level

Product mean throughput with *unlimited* rework is denoted as

$$TH(p) = 1/\mu_0 = \frac{q}{\mu_M + p\mu_R} \tag{38}$$

and with *limited* rework we recall from (36) that

$$TP(J, p) = \left(1 - \prod_{i=1}^J p_i\right) / \mu_\theta.$$

Clearly, when  $J \rightarrow \infty$  and when all rework probabilities are identical (i.e.,  $p_i = p, i = 1, 2, \dots$ ) then  $TP(\infty, p) \rightarrow TH(p)$ .

In terms of efficiency, different measures are needed depending on whether rework is limited or not. If rework is *unlimited* then all output is considered good quality, and we can define *operational efficiency* as follows:

$$E = \frac{\mu_M}{\mu_\theta} = \mu_M TH(p) = \frac{q\mu_M}{\mu_M + p\mu_R}. \quad (39)$$

This is the ratio of the mean ideal cycle time when no rework (or recovery) is needed ( $\mu_M$ ) to the mean product assembly time including rework ( $\mu_\theta$ ).

The operational efficiency when rework is *limited* can be computed similarly

$$E = \mu_M TP(J, p) = \frac{\mu_M}{\mu_\theta} \left(1 - \prod_{i=1}^J p_i\right). \quad (40)$$

#### D. System Level

1) *Variable Production Costs*: Performance measures at the system level take into consideration general factors comprising the integrated activities of the station. Operational efficiency or yield at the system level can be computed similarly to those measured at the product level. To compute the cost of an assembly station design, several models can be used. Following the recent study by Naidish [10] these cost elements are derived below as a function of the product quality features, in-process inspection, and rework. The cost per assembled unit is the total of setup, tooling, material, and operating cost [1], [3], [6], [10].

Suppose the *setup cost*, for a batch  $B$  of the same products, is  $C_s$ . This cost includes mainly the time spent on station layout changeovers, feeders and fixtures reconfiguration, sensors and testers changes that are required during part-type changeovers. This cost is divided per assembled unit, relative to the batch size, or, on the average:

$$\text{setup cost per assembled unit} = C_s/B.$$

The *tooling cost* per assembly trial is  $C_t$ . Tooling costs cover replacement of worn or broken tools and fixtures, disposable kitting trays, and the time required for on-going adjustments of the station equipment. For example, Csakvary [3] indicates that a major cost factor in assembly systems are part jamming and related tools or feeder breakdowns. Therefore tooling cost per completed unit depends on the number of repeated trials. On the average, it leads to

$$\text{tooling cost per assembled unit} = C_t \cdot \mu_N.$$

The *material cost*  $C_m$  also depends on the number of repeated visits at the station since it can be assumed that at each visit some parts and materials are replaced.

Thus on the average material cost per assembled unit =  $C_m \cdot \mu_N$ .

Finally, suppose the *operating cost* per time unit is  $C_o$ . This cost is relative to the amount of time that each unit occupies the station.

On the average, operating cost per assembled unit =  $C_o \cdot \mu_\theta$ .

The *total average cost* per assembled unit  $C$  is the sum of the above four cost components, as follows:

$$C = [C_s/B + C_t \cdot \mu_N + C_m \cdot \mu_N + C_o \cdot \mu_\theta]. \quad (41)$$

In (41) only variable costs are considered, and it is assumed that the revenue for rejected items and parts (or materials) due to rework is negligible. Conceptually, it is a straightforward task to add more variables and modify some of the above assumptions so that these models are made more appropriate for a particular firm. For example, the model can be expanded by adding the costs of special mechanical feeding devices and the part magazine, or some inventory lot sizing considerations.

2) *Due-Date Delivery*: The probability of meeting the prescribed *due date*  $T_D$  for a given batch of size  $B$  is an important performance measure at the system level. Using the Central Limit Theorem, the probability distribution of the total batch time  $\theta$  is approximately normal with mean  $\mu_\theta$  and variance  $\sigma_\theta^2$  computed from (34) and (35), respectively.

The recent papers by Hira and Pandley [7], Seidmann *et al.* [18], [19], and by Wilhelm and Ahmadi-Marandi [25] describe various generic cases in which the normal distribution is an acceptable approximation for small-batch assembly systems.

Following these results, the probability that the batch requires less than  $T_D$  time units is computed from

$$\Pr [\theta < T_D] = \Pr \left[ Z < \frac{T_D - \mu_\theta}{\sigma_\theta} \right] \quad (42)$$

where  $Z$  is the standard normal random variable.

It is also possible to compute an approximate  $100(1 - \alpha)$ -percent confidence interval for the batch completion time. Since  $\theta$  is the actual batch completion time, then

$$\Pr [\mu_\theta - Z_{\alpha/2}\sigma_\theta \leq \theta \leq \mu_\theta + Z_{\alpha/2}\sigma_\theta] = 1 - \alpha \quad (43)$$

approximately, where  $Z_{\alpha/2}$  denotes a percentage point of the standard normal distribution such that

$$\Pr [Z > Z_{\alpha/2}] = \alpha/2. \quad (44)$$

Thus the approximate  $100(1 - \alpha)$ -percent confidence interval is

$$\{\mu_\theta - Z_{\alpha/2}\sigma_\theta \leq \theta \leq \mu_\theta + Z_{\alpha/2}\sigma_\theta\}. \quad (45)$$

For example, consider a batch with  $B = 41$ ,  $\mu_\theta = 242$  h,  $\sigma_\theta^2 = 119$  h<sup>2</sup>, and  $T_D = 256$  h. The probability of meeting this due date and a 95-percent confidence interval on the batch completion time are

$$\Pr [\theta < 256] = \Pr \left[ z < \frac{256 - 242}{\sqrt{119}} \right] = 0.899$$

and the confidence interval is

$$\{242 - (1.96) \sqrt{119} \leq \theta \leq 242 + (1.96) \sqrt{119}\}$$

or

$$\{220.6 \leq \theta \leq 263.4\}.$$

## VI. APPLICATION TO FLEXIBLE ASSEMBLY ROBOTIC STATION DESIGN

### A. The Assembly Station

To illustrate the application of our model in station design, we describe two examples: A printed circuit board assembly station and combined machinery and assembly station. The performance measures defined earlier, relating to the station productivity, are calculated and analyzed for these illustrations. Two major design issues are discussed: selection of *robot speed*, and *station size* in terms of the number of tasks planned for the station.

Various references describe the integrated circuit production and assembly process (i.e., [12], [21]). The example of the printed circuit board assembly station, which is quite common in industry [16], [17], [22] is as follows (see Fig. 2). The robot picks up several odd-shaped components out of a feeder, orients each component in turn at the programmed position above the pre-drilled holes in the board, and inserts the component. Consider the sequence of motions from moving to the feeders, picking up components, moving and orienting them, and finally inserting them in the board as the main assembly tasks of the robot. Suppose that  $L$  is the number of main assembly tasks and  $\alpha$  is the total robot travel time during the main assembly tasks, assumed to be deterministic.

In a flexible robotic station dissimilar tasks are anticipated; therefore, the task times are distributed in our model and can have a relatively large variance. Note also, that if the number of main assembly tasks  $L$  also varies, then repeated analysis for several values of  $L$  should be carried out. Obviously, once the station is built it will usually accommodate a certain maximum  $L$ , for which the analysis should be carried out.

Several practical problems may occur, such as feeder jamming, wrong components, wrong position in the robot program, or insertion holes in the board that are too tight. Such problems are usually cleared and removed during the initial start-up of the station either by improved equipment operation such as the addition of a force sensor to the gripper which helps search for the proper insertion holes, or strict quality control of components and boards prior to operation at the station.

One typical problem that is difficult and costly to remove is the alignment of the component pins that have to be inserted precisely in the proper sockets, even when the robot's basic resolution unit is smaller than the tolerances of the mating parts [8]. An acceptable solution for this problem is a realignment device. The robot control program, when realizing that an insertion attempt has failed, will automatically activate a recovery procedure, bring the component with faulty pins to the realignment device, insert the component for a brief realignment operation in the device, and then attempt another insertion. Usually, this recovery procedure is sufficient, but

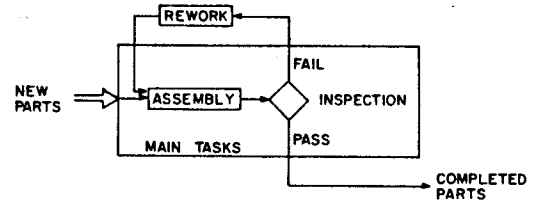


Fig. 10. The assembly station operational parameters.

if the predrilled holes are a little tight, additional realignment attempts may be needed.

Suppose the recovery procedure of using the realignment device requires the total time  $\beta$ , where  $\beta$  is the expected rework or recovery time at the assembly station (*exponentially distributed*).

To further specify the assembly station operation assume constant rework probability  $p$ , and suppose the operation time for each of the  $L$  assembly tasks in the station has a *log-normal distribution* with mean  $\mu_m$  and variance  $\sigma_m$ . Assume that  $\mu_m$  can have a  $\beta$  related to the rework and recovery time, as follows:

$$\mu_m = j\beta, \quad j > 0 \quad (46)$$

where  $\beta$  is the expected time for the rework functions as defined above, and  $j$  is some known constant. With regard to  $\alpha$ , the robot's total travel time in the station's main tasks, it is reasonable to assume that  $\alpha$  is relative to the number of tasks  $L$ , and that it can be expressed as

$$\alpha = zL\beta, \quad z > 0 \quad (47)$$

where  $z$  is a known technical factor, relating  $\alpha$  to the rework time. The above relations are summarized in Fig. 10. Equation (34) can now be rewritten

$$\mu_\theta = q^{-1}\beta(zL + Lj + p) \quad (48)$$

and from (48), measures at the product level can be written as follows:

$$TH(p) = q[\beta(zL + Lj + p)]^{-1} \quad (49)$$

and

$$E = \frac{\beta L(z + j)q}{\beta L(z + j + p/L)} = q[1 + p/(L(z + j))]^{-1}. \quad (50)$$

### B. Robot Speed and Rework Rate

An important decision in the design of the assembly station is the selection of the robot. This decision often depends on the velocity range at which the robot is required to operate. Fast arm acceleration is very important for assembly robots. In assembly, the traveling paths are relatively short and therefore the joints either move at their maximum speed for short periods only, or they do not reach their speed limit at all. Another typical design issue is whether to configure the station with careful attention given to the assembly work in order to avoid rework; alternatively, relatively quick assembly operations can be planned, however, as a result, the amount of rework and recovery will typically have to increase [2], [16]. The latter issue can further be viewed as a question of how

many rework attempt should be allowed in the station. These two design issues will now be explored.

1) *System Measures for Alternate Designs*: A fast robot will have a relatively small  $\alpha$ . In order to compare two alternative station plans, suppose the plans have different robots but the same number of main assembly tasks at the station,  $L$ , the same rework time  $\beta$ , and the same technical factors  $j$  and  $z$ . One can arbitrarily set

$\beta_1$ , the value of  $\beta$  in the first station plan, as

$$\beta_1 = 1 \text{ time unit.}$$

First, consider the model with *unlimited* rework and denote by  $p_1$  and  $p_2$  the rework rates at the first and second station plans, respectively. For the two stations to yield the same level of production throughput  $TH(p)$ , it will be necessary that

$$TH(p_1) = TH(p_2)$$

or

$$(1 - p_1)[\beta_1(zL + Lj + p_1)]^{-1} = (1 - p_2)[\beta_2(zL + Lj + p_2)]^{-1}. \quad (51)$$

From here the mean recovery time at the second plan (for equal throughput) should be

$$\beta_2 = 1(zL + Lj + p_1) \cdot (1 - p_2)/((zL + Lj + p_2)(1 - p_1)). \quad (52)$$

Suppose, for example, that  $L = 3$ ,  $j = 5$ ,  $z = 0.92$ , and  $p_1 = 0.3$ . Then from (52)  $\beta_2$  is given by

$$\beta_2 = 25.8(1 - p_2)/(17.26 + p_2). \quad (53)$$

The first plan's throughput is  $TH(0.3) = (0.7)[0.92(3) + 3(5) + 0.3]^{-1} = 0.039$  products per time unit.

Thus in the same cell size ( $L = 3$ ) and operation ( $j = 5$ ,  $z = 0.92$ ) a different robot speed is associated with a different value of  $p$ . Since  $p_2$  is greater than zero, it can be shown in this case that  $\beta_2 < 1.45$  is required in the second plan to yield equal throughput in both plans. In other words, in order to obtain similar throughput (0.039) with a slower robot a smaller rework rate  $p_2$  will be allowed. For instance, with a 40-percent slower robot, i.e.,  $\beta_2 = 1.40$ , a rework rate of only  $p_2 = 3.4$  percent can be tolerated. Achieving such a low rate of rework will require a major effort to reduce all factors of variability at the station. This may be unjustified compared to the savings generated by installing a slower robot.

This analysis implies that when robot speed is decreased by 40 percent, the rework probability or task reliability must be changed by one order of magnitude.

It is also interesting to note that the operational efficiency of the two station plans compared above does not directly depend on the value of  $\beta$ . Applying (50) for the parameters specified above,  $E_1 = 0.69$  for the station with the faster robot and higher probability of rework, and  $E_2 = 0.96$  for the other plan. Although the two robots will yield the same production throughput, the station with smaller rework rate and slower robot will have significantly higher operational efficiency.

Up to this point, we have two station plans that provide the

same throughput of assemblies, but have different robots. Let us now compare the two plans at the task and system level.

2) *Task Measures for Alternate Designs*: At the task level, the two station plans will compare as follows (assume  $B = 1$ ):

Plan 1: ( $\beta = 1.0, p_1 = 0.3$ )

$$\mu_\theta = [(0.92)(3) + (3)(5) + 0.3]/0.7 = 25.800 \quad (54)$$

and since  $\sigma_m^2 = L\sigma_M^2 = 3\sigma_m^2$  and  $\sigma_R^2 = \beta^2$  we obtain

$$\sigma_\theta^2 = 215.900 + 4.285\sigma_m^2. \quad (55)$$

Plan 2: ( $\beta = 1.4, p_2 = 0.034$ )

$$\mu_\theta = [((0.92)(3) + (3)(5))1.4 + (0.034)1.4]/0.966 = 25.800 \quad (56)$$

$$\sigma_\theta^2 = 25.201 + 3.105\sigma_m^2. \quad (57)$$

Note that the variance of  $\sigma_\theta^2$  in the second plan is relatively smaller owing to the smaller rework rate. This attribute of Plan 2 is highly desirable in certain production facilities, where flow must be tightly controlled.

3) *Operating Costs for Alternate Designs*: The cost of each plan is analyzed at the system level. While the operating unit cost  $C_o$  and material cost  $C_m$  can be assumed equal for both plans, to cater for tighter tolerances the tooling cost  $C_t$  can be *higher* for the plan with *lower* rework probability  $p$ . Suppose  $C_t$  (Plan 2) =  $K \cdot C_t$  (Plan 1) where  $K > 1$ . Then, the mean total unit cost per assembly following (41) and assuming  $B = 1$

$$C_1 = 25.800C_o + 1.429(C_m + C_t), \quad \text{for Plan 1} \quad (58)$$

$$C_2 = 27.788C_o + 1.035(C_m + C_t), \quad \text{for Plan 2.} \quad (59)$$

For illustration, suppose  $C_m = D$ ,  $C_o = 5D$ ,  $C_t = 10D$ , and  $D > 0$ . Comparing the unit cost in the two plans, the second plan will be less costly when  $C_2 < C_1$ , or when  $K < 1.425$ . It means that in this case a 42.5-percent increase can be allowed for tooling costs in the second plan in order to sustain smaller rework probabilities.

4) *Limited Rework Plans*: To complete this design analysis, let us now consider the more realistic use of limited rework controls. Suppose the number of rework attempts is limited to  $J = 3$ . Here the rework probabilities reduce from trial to trial such that the rework vectors are  $p_1 = [0.3, 0.15, 0.075]$  and  $p_2 = [0.034, 0.017, 0.0085]$ , for Plans 1 and 2, respectively. The production throughput of the two plans remain the same if

$$TP_1(3, p_1) = TP_2(3, p_2).$$

From (31), (34), and (36) we get for Plan 1

$$\mu_N = 1.345$$

$$\mu_\theta = 24.232$$

$$TP_1(3, p_1) = (1 - (0.3)(0.15)(0.075))/24.232 = 0.0411.$$

TABLE III  
TWO ALTERNATIVE PCB ASSEMBLY PLANS WITH SIMILAR THROUGHPUT

	Plan 1	Plan 2
No. of assembly tasks, $L$ :	3	3
Coefficient of mean operation time, $j$ :	5	5
Coefficient of robot travel time, $z$ :	0.92	0.92
Rework time, $\beta$ (time units):	1.00	1.40
Mean operation time, $\mu_m$ (time units):	5.00	7.00
Robot travel time, $\alpha$ (time units):	2.760	3.864
Rework probability, $p$ :	0.300	0.034
Mean throughput, $TP(p)$		
unlimited rework:	0.039	0.039
limited rework ( $J = 3$ ):	0.041	0.041
Operational efficiency, $E$		
unlimited rework:	0.690	0.960
limited rework ( $J = 3$ ):	0.729	0.992
Mean assembly time		
unlimited rework:	25.800	25.800
limited rework ( $J = 3$ ):	24.232	24.232
Variance assembly time, $\sigma_t^2$		
unlimited rework:	$215.9 + 4.285\sigma_m^2$	$25.201 + 3.105\sigma_m^2$
Mean no. of visits in $M$ , $\mu_N$		
unlimited rework:	1.429	1.035
limited rework ( $J = 3$ ):	1.345	1.033

Similarly, for Plan 2 (assuming undetermined  $\beta_2 (> 0)$ )

$$\mu_N = 1.033$$

$$\mu_\theta = 18.379\beta_2.$$

Thus the following equation defines the  $\beta_2$  parameter for Plan 2 resulting in similar throughput as in Plan 1

$$TP_2(3, p_2) = (1 - (0.034)(0.017) \cdot (0.0085)) / (18.379\beta_2) = 0.0411. \quad (61)$$

From here, we obtain  $\beta_2 = 1.36$ . It is interesting to note, that the performance results are very close to the results obtained above for unlimited rework with equal rework probabilities. (This implies that the hypothetical unlimited rework model can provide useful robust results that are also easier to calculate.)

In terms of operational efficiency, the results are slightly better than in the unlimited rework case

$$E_1 = \mu_{M1} TP(3, p_1) = (17.76)(0.0411) = 0.729$$

$$E_2 = \mu_{M2} TP(3, p_2) = (24.15)(0.0411) = 0.992.$$

Table III summarizes the results of the printed circuit board assembly station design example discussed above.

### C. Station Size and Rework Rate

For this discussion consider a robotic station where components are machined and then assembled. The major design issue: the *number* of tasks to include in the station, and hence its size, as related to rework rates.

It can be observed from the earlier analysis that in general, the production throughput rate in a station increases under the following conditions:

- 1) when  $p$ , the rework probability, *decreases*,
- 2) when the robot speed, represented by  $\alpha = zL\beta$ , *increases*,

3) when the expected time spent on each assembly task, represented by  $\mu_m = j\beta$ , becomes relatively *smaller*.

On the other hand, different station's designs can yield similar production throughput rates. To illustrate this point consider the following alternative assembly plans all having ( $z = 1$ ):

*Plan A:* In order to achieve a production throughput rate of approximately 3.3 assemblies per hour, a station operation can include 3 long tasks in the main area ( $L = 3$ ), with  $p = 0.1$ ,  $j = 10$ ,  $\alpha = 1.5$  min, and  $\beta = 0.5$  min.

*Plan B:* Alternatively, a station with the same number of tasks but faster operation (or shorter tasks),  $j = 2$ , a slower robot arm,  $\alpha = 3$  min, rework time of  $\beta = 1$  min, and a rework rate of  $p = 0.5$  can yield a comparable 3.2 assemblies per hour.

*Plan C:* Also, a larger cell with ten short tasks,  $j = 0.65$ , very slow robot,  $\alpha = 10$  min,  $\beta = 1$  min, but higher production quality ( $p = 0.1$ ) will yield an average production throughput rate of 3.3 assemblies per hour.

The three plans are summarized in Table IV.

Assuming several such alternative assembly station plans are technologically feasible, they can be compared at the system level by economic analysis, as discussed before. This time, however, suppose for simplicity identical  $\alpha, \beta, C_s, C_m, C_l$ , and  $C_o$ . Plan 1 will be more economical than Plan 2 when the following occurs:

$$(C_m + C_l)(1/q_1 - 1/q_2) < C_o\beta((L_2(z_2 + j_2) + p_2)/q_2 - (L_1(z_1 + j_1) + p_1)/q_1) \quad (62)$$

where  $q_1 = 1 - p_1$  and  $q_2 = 1 - p_2$ .

Consider the case where a station is comprised of  $L = 3$  tasks and has  $\beta = 1$  min. Plan 1 calls for relatively long tasks, with  $j = 2$ , but relatively low rework probability of  $p = 0.1$ ; Plan 2 calls for fast (or short) tasks,  $j = 0.5$ , associated with a higher  $p = 0.5$ . In both plans  $z = 1$  and the production throughput rate is about 6 assemblies per hour. According to

TABLE IV  
THREE ALTERNATIVE MACHINING/ASSEMBLY PLANS WITH SIMILAR THROUGHPUT

	Plan A	Plan B	Plan C
Station size (no. of tasks, $L$ )	3	3	10
Rework rate, $p$	0.1	0.5	0.1
Rework time, $\beta$ (min)	0.5	1.0	1.0
Mean task time, $\mu_m = j\beta$ (min)	$(10)(0.5) = 5.0$	$(2)(1.0) = 2.0$	$(0.65)(1.0) = 0.65$
Robot travel time, $\alpha = zL\beta$ ( $z = 1$ ) (min)	1.5	3.0	10.0
Throughput, $TH(p)$	3.3	3.2	3.3

the economic condition of (62), Plan 1 is preferred to Plan 2 only if

$$0.11C_o < 0.89(C_m + C_l). \quad (63)$$

Considering task level performance measures in comparing between the alternative assembly station plans, as indicated above, measures of performance variability can be applied. Obviously, the higher variability of the time spent by each assembly in the station, the more erratic the performance (e.g., in terms of queue requirements), and the more difficult it is to control the station operation. In the case of the two plans above however, the variance of the total unit stay in the station is almost the same:  $102.46 \text{ (min)}^2$  in Plan 1, and  $102.00 \text{ (min)}^2$  in Plan 2. In other situations, the plan that has a relatively smaller variability would, of course, be preferred.

## VII. CONCLUSIONS

This paper is concerned with the characterization of particular automated production systems, robotic assembly stations. The analysis focuses on the product recirculation phenomenon with an individual station. The model considered in this paper assumes that in order to satisfy the required quality standards a certain proportion of the items must return, once or more, through a repeat, or reworking process before exit. Probability theory is employed to mathematically formulate the product flow aspect of interest. Results provide the distribution of total assembly time and total time in each part of the station, the distribution of total number of reworks, and sensitivity to process quality.

Other results including the joint distribution function of the total system time and the number of required reworks, the coefficients of variation for the number of reworks, and the total system time are also presented. Special attention is given to modeling stations with bounded number of rework attempts and with distinct rework probabilities.

The computations of the performance measures at the task, product, and the assembly system level are shown to be practical as no specific distributions must be assumed for manufacturing or reworking processes. These computations are very efficient since a stochastic mathematical model is used rather than digital simulation. Tractable expressions for the mean and variance of total system times are also derived.

This type of operational information is useful for system designers in order to evaluate capacity, in-process buffer spaces, and dynamic control policies. Two applications of the results to flexible robotic station design are illustrated: one in analyzing the interacting effects of robot speed and rework rate

and the second addresses the issues of station size, rework rates and their impingement on the attainment of production targets.

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