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# Part Selection Policy for a Flexible Manufacturing Cell Feeding Several Production Lines

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*Abstract:* Analysis of the part selection policy in the case of a flexible manufacturing cell producing different parts for several emanating lines is presented. The discrete response control problem is formulated as an undiscounted semi-Markovian decision problem whose decision epochs occur when the cell completes a part and must decide what type of part to fabricate next. Design objective is to minimize the expected shortage penalty per unit time incurred by the production lines when they run out of inputs. Optimal production policies are characterized, and a discussion of related industrial implementation issues is given.

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■ This paper investigates a new production control problem arising in automated manufacturing systems. We consider a flexible (i.e., programmable) manufacturing cell producing different parts for several single-model emanating lines. The cell is only capable of manufacturing one part at a time [21, 28, 29]. This cell may be considered as a special case of a flexible manufacturing system (FMS). It fabricates  $R$  types of parts and forwards them to  $R$  emanating lines, each with finite input buffer space. When the cell completes a part, its controller (e.g., a microprocessor) determines the part type to produce next. The controller has full knowledge of the current buffer stocks at each line. The control problem at the cell is to select which part to make next in order to minimize the expected shortage penalty per unit time which is incurred by the lines when they run out of input parts.

Potential applications of this model may extend beyond the specific system discussed here. They include the case of a material handling robot tending several machines and the case of a general-purpose machine tool feeding base parts to several other production units.

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## RELATED STUDIES

A flexible manufacturing system can be regarded as a cluster of computer numerical control (CNC) machining centers, special machine tools, inspection machines, and some automated material handling devices, such as shuttle pallets, carts, and industrial robots [8, 14]. The loading and unloading of machines, the adaptive route selection, part sequencing, and the determination of various manufacturing variables (i.e., feed rates and spindle speeds) are conducted under the real-time control of a host computer. The typical FMS can sequence parts randomly and respond quickly to the needs of the assembly floor because its setup time is minimal [1]. A major design issue for FMS is the generation of an appropriate operational control strategy. Various aspects of this problem have been recently studied by several researchers. For example, Solberg [22], Nof, Barash, and Solberg [17], Hildebrant [10], Stecke [23], and Suri [26] have used closed queueing network theory [3] to predict the system throughput under several internal flow control policies. Most of the studies based on closed queueing network models assume first come, first served queue discipline, no machine breakdowns, exponential production times, and unlimited buffer spaces (no blocking).

A different approach has been taken by Buzacott and Shanthikumar [5], who studied storage options and modeled

the FMS as a finite-capacity open network of queues [19]. A network flow optimization scheme and a three-level hierarchical production control algorithm were discussed by Kimemia and Gershwin [15, 16].

Suri and Cao [27] applied perturbation analysis to the optimization of multiclass queueing networks with general service time distributions. Preliminary experimentation with three-server networks have indicated that the method is computationally efficient and reasonably accurate. Other studies using simulation [25], elementary Markov decision models [9], and deterministic loading and scheduling models [11, 12, 24] are reviewed in [6].

No FMS study has given consideration to the problem of buffer stocks in the context of interfacing the FMS with other facility operations [6, p. 25]. In our model, buffer stocks are essential for interfacing the cell with the various production lines. Scheduling of the cell differs from the common single machine scheduling problems since we assume continuous load, stochastic production times, and no setup costs.

## THE MODEL

The model assumes one manufacturing cell producing parts, one at a time, for  $R (\geq 1)$  assembly or production lines. The time for the cell to produce one part for line  $k$  is exponentially distributed with mean  $1/\mu_k$ ,  $1 \leq k \leq R$ . Line  $k$  has a production time which is exponentially distributed with mean  $1/\lambda_k$ ; the line takes parts one at a time from its input buffers as long as parts are available. In reality, manufacturing times are not always expected to be exponentially distributed. Several empirical studies, however, have concluded that the assumption of exponential time distributions is not always critical [2]. For example, Solberg [22] and Suri [26] have indicated that the assumption of exponentially distributed processing times, when used, gave good predictions of the *real production rate* and other performance measures in certain flexible manufacturing facilities.

Production line  $k$  and its input buffers have room for  $B_k \geq 1$  parts, one in processing plus up to  $B_k - 1$  in the buffers. Production line  $k$ , when idle because of the absence of parts, incurs a shortage penalty at a rate of  $C_k$  dollars per hour, where  $C_k$  is a surrogate for idle time and lost production opportunities. These numbers must be provided by management as opportunity costs, and their relative magnitude indicates the relative importance of the lines. The control problem is to select the production sequence at the cell in order to minimize the expected shortage penalty per hour. Each production decision is made with a full knowledge of the buffer status at all  $R$  lines.

The above control problem may be formulated as a finite-state infinite-horizon undiscounted semi-Markovian decision process [13] with the following structure:

**State-space**  $\Omega = \{n = (n_1, n_2, \dots, n_R) \mid 0 \leq n_k \leq B_k \text{ for } 1 \leq k \leq R\}$ , where  $n_k$  is the total number of parts at line  $k$ ,

either in the input buffers or the possible one part in process. The number of states is  $NS \equiv (B_1 + 1)(B_2 + 1) \dots (B_R + 1)$ .

### Decision epochs and set of alternatives in state $n$ .

(a) Unblocked case:  $n \neq B \equiv (B_1, B_2, \dots, B_R)$ . The cell has just completed a part and forwarded it to the appropriate input buffer or line. It must now decide what type of part to produce next. This type must be selected from those lines having nonfull buffers, i.e., from  $A(n) \equiv \{k \mid n_k < B_k\}$ .

Note that the cell cannot produce for a blocked line (whose buffers are all full) and cannot remain idle if an unblocked line exists.

(b) Blocked case:  $n = B$ . (All buffers are full.) The cell remains idle until one buffer becomes empty, and the cell must then begin production of a part for that line. Note that since all  $R$  lines are busy, the mean idle time is exponentially distributed with a mean  $1/\lambda_{\text{sum}}$ , where  $\lambda_{\text{sum}} \equiv \sum_{i=1}^R \lambda_i$  is the transition rate until the first buffer becomes empty.

The dynamic programming functional equations [13] for minimizing the expected shortage penalty cost per unit time are

$$v(n) = \text{Min}_{k \in A(n)} \left[ q(n, k) - g/\mu_k + \sum_{0 \leq j \leq n} P(j \mid n, k) v(n - j + e^k) \right] \quad n \in \Omega, n \neq B \text{ (unblocked case),} \quad (1)$$

$$v(B) = -g/\lambda_{\text{sum}} + \sum_{i=1}^R \frac{\lambda_i}{\lambda_{\text{sum}}} v(B - e^i) \text{ (blocked case),} \quad (2)$$

$$v(0) = 0, \quad (3)$$

where

$v(n)$  is defined as the relative value of state  $n$ ,  $n \in \Omega$ .

$g$  is defined as the long-run expected shortage penalty cost per unit time following the optimal production policy (the "gain rate" [13]).

$e^k$  is defined as the unit vector in  $k$ th direction,  $1 \leq k \leq R$ .

$P(j \mid n, k)$ , with  $j = (j_1, j_2, \dots, j_R)$  and  $n = (n_1, n_2, \dots, n_R)$ , is defined as the joint probability that line  $i$  uses up  $j_i$  of its  $n_i$  parts during the time that the cell manufactures one part of type  $k$ , for all  $i = 1, 2, \dots, R$ . [See Appendix A.]

$q(n, k)$  is defined as the expected penalty cost incurred by all  $R$  lines, starting with state  $n$  and extending over the time interval needed by the cell to produce one part of type  $k$ . It is the mean one-transition cost. [See Appendix B.]

In Equation (1),  $q(\mathbf{n}, k)$  and  $1/\mu_k$  are the one-transition expected cost and mean holding time, respectively, while the sum gives the expected value of the next state:  $\mathbf{n}$  drops by  $j$ , and the cell adds one part to line  $k$ . In Equation (2), the one-transition expected cost is zero (no shortage costs since no lines are starving), the mean holding time is  $1/\lambda_{\text{sum}}$ , and  $\lambda_i/\lambda_{\text{sum}}$  is the probability that line  $i$  will be the first line to become unblocked. Equation (3) is a convenient way to fix an otherwise arbitrary additive constant in the relative values.

Equations (1), (2), and (3) comprise  $NS + 1$  equations for the  $NS + 1$  unknowns  $g$  and  $v(\mathbf{n})$ ,  $\mathbf{n} \in \Omega$ . They possess a unique solution [18] since, under any policy, the resulting Markov chain is aperiodic and has a single ergodic class (all states can reach state  $\mathbf{0}$ ). The solution of these equations gives the minimal expected shortage penalty cost per unit time  $g$ , the value of each state  $v(\mathbf{n})$  relative to the value  $v(\mathbf{0}) = 0$  of the state with no parts present, and the optimal control policy [the minimizing  $k$  in Equation (1) as a function of  $\mathbf{n}$ ].

Once the functional equations [(1), (2), and (3)] are solved, it is possible to compute other quantities of interest. Appendix C outlines the derivation of  $A_t$ , the output rate of line  $t$  ( $t = 1, 2, \dots, R$ ). The utilization of line  $t$  is given by

$$U_t = A_t/\lambda_t. \quad (4)$$

Also, the utilization of the cell is

$$CU = \sum_{t=1}^R A_t/\mu_t. \quad (5)$$

Note that  $A_t/\mu_t$  is the fraction of time the cell is making parts for line  $t$ , and  $(1 - CU)$  is the fraction of time that the cell is blocked (idle).

## COMPUTATIONAL SCHEME PROCEDURE

For simplicity of storage, the  $NS$  states  $\mathbf{n} \in \Omega$  were mapped into the integers  $\{1, 2, \dots, NS\}$  via the function  $\psi: \Omega \rightarrow \{1, 2, \dots, NS\}$ , where

$$\begin{aligned} \psi(\mathbf{n}) = & 1 + n_R + n_{R-1}(B_R + 1) + n_{R-2}(B_{R-1} + 1)(B_R + 1) + \dots \\ & + n_1(B_2 + 1)(B_3 + 1) \dots (B_R + 1). \end{aligned} \quad (6)$$

The  $NS + 1$  equations were solved by successive substitution, using the value-iteration scheme in [18] with a stepsize

$$\tau \equiv \text{Min}[1/\lambda_{\text{sum}}, \text{Min}_{1 \leq k \leq R} 1/\mu_k],$$

and an initial guess  $v(\mathbf{n}) = 0$  for all  $\mathbf{n}$ . The termination criterion was to exit when  $g$  could be estimated within  $\pm(1-10)\%$ . Convergence typically took 20 to 60 iterations

and neither computer time nor main memory requirements were excessive for the problems we addressed, up to a few thousand states. For example, solving a model with 216 states required 465 CPU sec and 0.25 M bytes of main memory on a CDC 855 computer with the NOS 2.1 operating system.

## An Example

Consider a cell feeding  $R = 3$  production lines. All three lines have production rates of 6 units per hour ( $= \lambda_1 = \lambda_2 = \lambda_3$ ) and four buffer zones ( $= B_1 = B_2 = B_3$ ). The cell can produce parts for any of these lines at a rate of 21 units per hour ( $= \mu_1 = \mu_2 = \mu_3$ ). The lines are identical except for having distinct shortage penalties \$120, \$370, and \$210 per idle hour ( $= C_1, C_2, C_3$ ). The state space for this problem has  $NS = (4 + 1)^3 = 125$  states. The utilization of the cell would be

$$\text{Min} \left\{ 1.0, \sum_{k=1}^3 \lambda_k/\mu_k \right\} = 0.857 \quad (7)$$

if there were infinite buffers, and will be somewhat less for the present case where finite buffers cause blocking of the cell.

The value iteration [18] algorithm for generating the optimal solution converged after 46 iterations with a relative precision of  $\pm 1\%$ . The expected shortage penalty cost per unit time,  $g$ , was found to be \$23.94/hr. This represents the steady-state average shortage cost per hour associated with the optimal policy. Table 1 illustrates several states along with their optimal decisions and associated relative value function. When the system is empty [ $\mathbf{n} = (0, 0, 0)$ ;  $\psi(\mathbf{n}) = 1$ ], the cell will produce product type 2, and if the state is  $\mathbf{n} = (0, 2, 0)$ ,  $\psi(\mathbf{n}) = 11$ , it will produce a part type 3. This relative preference of line 2 to all other lines and of line 3 over line 1 clearly corresponds to the relative shortage penalties of these lines (i.e.,  $C_2 > C_3 > C_1$ ). Type 2, for example, is selected whenever  $n_2 = 0$  (regardless of  $n_1$  or  $n_3$ ), or if  $n_2 \leq 1$  and  $n_3 \geq 1$ .

In some cases, the optimal policy will be to produce *more* for a line having parts in its buffer rather than to begin production for an idle line. This is clearly depicted by  $\psi(\mathbf{n}) = 8, 9, 10$ , where it was preferred to reduce the chances of starvation at line 2 as compared to reducing the recurrent penalty of line 1. This policy contradicts the common practice of giving highest priority, in parallel service systems, to using idle capacity. That practice is not optimal when the servers are nonidentical. Since the optimal policy in this example is to select type 1 when  $n_1 < 3$ ,  $n_2 \geq 2$ , and  $n_3 \geq 2$ , this means that the last two buffers in line 1 have only a limited value. They may be used if  $n_2 = n_3 = 4$ . Similar observation holds for the last buffer space on line 3, which may be utilized only if  $n_2 = 4$  and  $n_1 > 2$ . The decision rule, as a function of  $n_1$  and  $n_3$ , is illustrated in

$\Psi(n)$	$n$			$v(n)$	$k(n) = \text{Decision}$
1	0	0	0	0.00	2
2	0	0	1	-27.11	2
3	0	0	2	-48.01	2
4	0	0	3	-64.37	2
5	0	0	4	-77.21	2
6	0	1	0	-32.12	2
7	0	1	1	-56.99	2
8	0	1	2	-76.11	2
9	0	1	3	-91.17	2
10	0	1	4	-103.01	2
11	0	2	0	-55.80	3
12	0	2	1	-77.13	3
13	0	2	2	-94.11	2
14	0	2	3	-107.65	1
45	1	3	4	-139.84	1
46	1	4	0	-103.81	3
47	1	4	1	-120.83	3
48	1	4	2	-131.89	1
49	1	4	3	-140.26	1
50	1	4	4	-146.17	1
51	2	0	0	-35.02	2
52	2	0	1	-61.27	2
99	3	4	3	-151.03	3
100	3	4	4	-153.73	1
101	4	0	0	-60.22	2
124	4	4	3	-153.71	3
125	4	4	4	-155.10	.

Table 2 for the case of  $n_2 = 3$ . Note that, if  $n_2 = 3$ , type 1 is selected only if  $n_3 > 1$ , and 2 is selected when  $n_1 > 1$  with  $n_3 \geq 3$ .

$n_1$	Decisions					
4	3	3	3	2	2	
3	3	3	3	2	2	
2	3	3	3	2	2	
1	3	3	3	1	1	
0	3	3	1	1	1	
	0	1	2	3	4	$n_3$

The entries of the value function  $v(n)$  give the relative value of one state versus another. Setting  $v(0) = 0$  means that all other cases will have negative values (less costs), with the blocked case having the largest value (minimal associated penalty). The values  $v(n)$  can be employed in comparing the relative return on increased buffers at each line; see Table 3 for the effect of additional parts at line 3. This table shows that one would be willing to pay up to \$27.11 to instantaneously procure one part for line 3 if the system is empty, but only up to \$12.84 if the system

$n$	$v(0, 0, n) - v(0, 0, n + 1)$
0	27.11
1	20.90
2	16.36
3	12.84

already has three parts at line 3. The diminishing marginal value of having extra parts at the buffers is evident from these differences in the  $v$ 's. This diminishing marginal value is also intuitive.

Using the computational procedure of Appendix C, the output rate for the three lines is 5.45, 5.9, and 5.81 units per hour ( $A_1$ ,  $A_2$ , and  $A_3$ , respectively). From Equations (4) and (5), the associated line utilizations are 90.8%, 98.3%, and 96.8%. ( $U_1$ ,  $U_2$ , and  $U_3$ ); manufacturing cell utilization is 81.71%.

The shadow price, or the marginal economic penalty, of reducing all buffer sizes from 4 to 3 is computed by comparing the shortage cost per hour of the two configurations. The new cost is \$43.45/hr. The difference in cost (\$19.51/hr) is the marginal economic value of the last space at all three buffers. This reduction of the buffers reduces the line utilizations to 86.8%, 95.9%, and 93.9% ( $U_1$ ,  $U_2$ , and  $U_3$ ) and the cell utilization to 79.02%. Similar sensitivity analysis studies can be conducted in order to investigate the relative effects of other factors such as changing the production rates, additions or deletions of lines, and varying the relative shortage penalties.

Finally, the economic merit of the optimal control policy can be measured by benchmarking it against some reasonable heuristic operating rules. We believe that good heuristics can be designed using the insights gained by examining the optimal policy. Such a heuristic may select type 2 if  $n_2 < 4$  and select type 3 if  $n_2 = 4$  and  $n_3 < 4$ ; type 1 is selected in all other cases. The shortage cost per hour,  $g'$ , obtained from solving Equations (1), (2), and (3) with  $k$  as the above function of  $n$ , will depict the mean penalty per unit time under the heuristic policy. The difference ( $\Delta g = g - g'$ ) is the per-hour benefit from following the optimal rather than a heuristic policy.

## IMPLEMENTATION ISSUES

Implementing the control system discussed here involves two steps (see Fig. 1). The first step is the off-line policy generation step which includes the data collection, application of the semi-Markov-decision-process algorithm, and the tabulation and recording of the resulting optimal policy. This step is a one-time operation which is carried out on a "supermini" or a mainframe computer. Memory requirements are linear in NS while time complexity is a quadratic function of NS. The off-line policy generation step illustrated here is generated by a standard FORTRAN IV computer program called FMDP. An optimal policy file generated by FMDP is transferred to another program (MDX) which

computes the output rate for each line. Both programs seem to have the speed and small memory requirements necessary for the solution of medium-size problems (for example: four lines with seven buffers at each).

## CONCLUSIONS

This paper demonstrates the feasibility of applying semi-Markovian decision processes to generate new manufacturing control policies. Production sequencing in the manufacturing cell was formulated as a Markov decision problem with product type as a decision variable and instantaneous buffer states as the state variable. The approach may also be applied to more complex industrial systems with a variety of operating structures. For example, one may consider a different cost structure which penalizes deviations from a specified set of production rates; then the cell can remain idle even though the buffers are not full. In this connection, the present model with exponentially distributed production times can be extended to phase-type or general time distributions and to unreliable machines. The anticipated computational complexity of exploding state spaces prohibits the practical use of such models. Current research in approximation and aggregation theory of large scale Markovian processes [20] may lead to practical computational procedures for these more complex production systems.

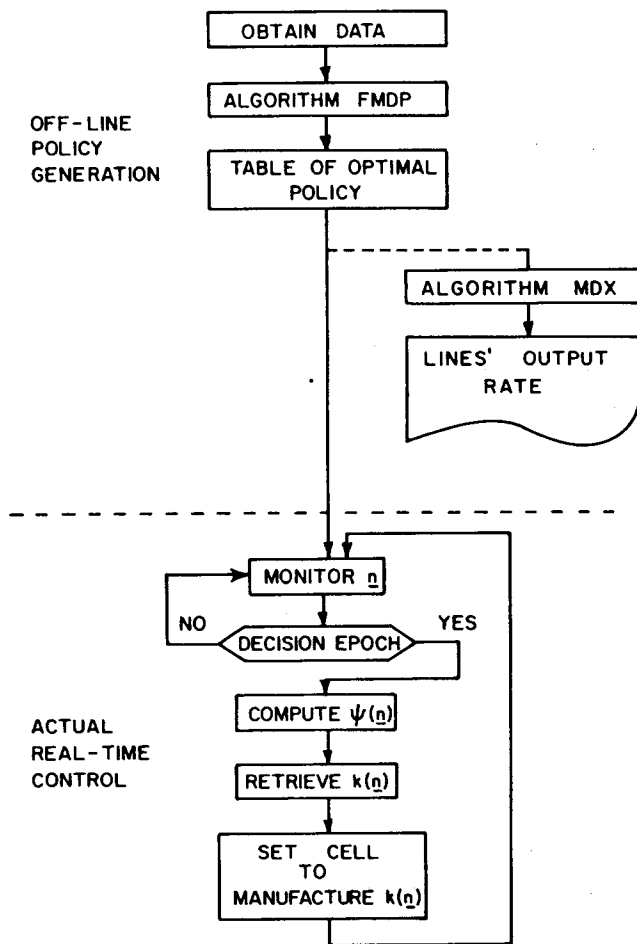


Fig. 1. An overview of the optimal control system.

The second step is the actual implementation of the control policy. This can be conducted either manually or by a programmable controller (i.e., a microcomputer). If a microcomputer is used to control a CNC manufacturing cell, the instantaneous inventory state  $\mathbf{n}$  is constantly monitored by standard devices such as limit switches or photo cells connected to A/D converters. At each decision epoch, the microcomputer computes the mapping transformation function  $\psi(\mathbf{n})$  and uses the results to retrieve the already tabled optimal decision  $k(\mathbf{n})$ . Minimal storage in the microcomputer is accomplished by storing only NS integer records in a file having the optimal decisions. This means, for example, that a 16K RAM (or EPROM) microcomputer chip can handle models of industrial systems having 16,000 states. Given  $k(\mathbf{n})$ , the system may call up the required numerical control program as the pallets are shuttled into the machines.

## APPENDIX A: THE TRANSITION PROBABILITIES

Let  $\mathbf{n} = (n_1, n_2, \dots, n_R)$ ,  $\mathbf{j} = (j_1, j_2, \dots, j_R)$ , and  $P(\mathbf{j} | \mathbf{n}, k)$  denote the joint probability for each line  $i = 1, 2, \dots, R$  that line  $i$  consumes  $j_i$  parts during the time interval that the cell makes one part of type  $k$  ( $k = 1, 2, \dots, R$ ), given that this interval begins with  $n_i$  parts at line  $i$  and its buffers.

Let  $z$  denote the time for the cell to produce one part of type  $k$ , with probability density

$$f_k(z) = \mu_k e^{-\mu_k z}, \quad z \geq 0. \quad (8)$$

By conditioning and unconditioning on  $z$ , we have

$$P(\mathbf{j} | \mathbf{n}, k) = \int_0^{\infty} dz f_k(z) \prod_{i=1}^R h_i(j_i | n_i, z), \quad (9)$$

where  $h_i(j | n, z)$  is the probability that line  $i$  consumes  $j$  parts during the time interval  $z$ , given that this time interval starts with  $n$  parts at line  $i$ . Note that *once  $z$  is specified*,  $k$  no longer enters the  $h$ 's, and that the  $j_i$  for distinct  $i$  are independent random variables.

We find the  $h$ 's as follows. If  $n = 0$ , we must have

$$h_i(j | 0, z) = \delta_{j0}, \quad n = 0. \quad (10)$$

If  $n \geq 1$ , we must have

$$h_i(j | n, z) = \begin{cases} 0 & \text{if } j < 0 \text{ or } j > n. \\ 1 - \sum_{m=0}^{n-1} h_i(m | n, z) & \text{if } j = n. \end{cases} \quad (11)$$

The remaining case is  $n \geq 1, 0 \leq j < n$ . Since line  $i$  consumes input parts as a Poisson process with rate  $\lambda_i$  until its inputs are depleted, the probability that it consumes exactly  $j$  parts in a time interval  $z$  is

$$h_i(j|n, z) = (\lambda_i z)^j e^{-\lambda_i z} / j!, \quad n \geq 1, 0 \leq j \leq n-1. \quad (12)$$

We may combine cases (10), (11), and (12) into

$$h_i(j|n, z) = \sum_{m=1}^s x(i, m) z^{l(i, m)} e^{-w(i, m)z} \quad 1 \leq i \leq R, 0 \leq j \leq n, z \geq 0. \quad (13)$$

where

$$\text{If } n = 0, \quad (14a)$$

$$s = 1$$

$$l(i, 1) = 0$$

$$w(i, 1) = 0$$

$$x(i, 1) = \delta_{j0}.$$

$$\text{If } n > 1 \text{ and } 0 \leq j \leq n-1, \quad (14b)$$

$$s = 1$$

$$l(i, 1) = j$$

$$w(i, 1) = \lambda_i$$

$$x(i, 1) = (\lambda_i)^j / j!.$$

$$\text{If } n \geq 1 \text{ and } j = n, \quad (14c)$$

$$s = n + 1$$

$$l(i, m) = \begin{cases} m-1 & 1 \leq m \leq n \\ 0 & m = n+1 \end{cases}$$

$$w(i, m) = \begin{cases} \lambda_i & 1 \leq m \leq n \\ 0 & m = n+1 \end{cases}$$

$$x(i, m) = \begin{cases} -(\lambda_i)^{m-1} / (m-1)! & 1 \leq m \leq n \\ 1 & m = n+1 \end{cases}$$

Here the dependence of the  $l$ 's,  $w$ 's and  $x$ 's upon  $n$  and  $j$  has been suppressed.

Insertion of Equations (8) and (13) into Equation (9) gives

$$P(j|n, k) = \mu_k \int_0^\infty dz e^{-\mu_k z} \sum_{m_1=1}^{s_1} \sum_{m_2=1}^{s_2} \dots$$

$$\sum_{m_R=1}^{s_R} x(1, m_1) x(2, m_2) \dots x(R, m_R) z^{lsum} e^{-wsum} (z)$$

where

$$lsum = \sum_{i=1}^R l(i, m_i),$$

$$wsum = \sum_{i=1}^R w(i, m_i),$$

and the dependence upon  $n$  and  $j$  has been suppressed. Performing the integral over  $z$  leads to the final result

$$P(j|n, k) = \mu_k \sum_{m_1=1}^{s_1} \sum_{m_2=1}^{s_2} \dots \sum_{m_R=1}^{s_R} \frac{x(1, m_1) x(2, m_2) \dots x(R, m_R) (lsum)!}{(\mu_k + wsum)^{1+lsum}} \quad (15)$$

## APPENDIX B: THE ONE-TRANSITION EXPECTED COST

Set

$$q(n, k) = \sum_{i=1}^R C_i U(i, n_i, k), \quad (16)$$

where  $C_i$  is the shortage cost per unit time for line  $i$  and  $U(i, n_i, k)$  is the mean duration within the time interval that the cell is making one part of type  $k$ , that line  $i$  is idle (starved), given that this interval began with  $n_i$  parts at line  $i$ .

Let  $z$  denote here the time for the cell to produce one part of type  $k$ , with the probability density  $f_k(z)$  given in Appendix A.

If  $n_i = 0$ , then clearly

$$U(i, 0, k) = 1/\mu_k, \quad i = 1, 2, \dots, R. \quad (17)$$

If  $n_i \geq 1$ , let  $t$  denote the time for line  $i$  to consume all  $n_i$  of its parts. Then  $t$  has an Erlang probability density

$$h(t) = (\lambda_i)^{n_i} t^{n_i-1} e^{-\lambda_i t} / (n_i-1)!, \quad t \geq 0,$$

and  $U$  is the expected value of  $\text{Max } [0, z - t]$ :

$$U(i, n_i, k) = \int_0^\infty dt h(t) \int_t^\infty dz f(z)(z - t).$$

With the change of variables  $z = t + w$ , this becomes, after evaluation of the rightmost integral,

$$U(i, n_i, k) = \frac{1}{\mu_k} \int_0^\infty dt h(t) e^{-\mu_k t} \\ = \frac{1}{\mu_k} \left( \frac{\lambda_i}{\lambda_i + \mu_k} \right)^{n_i}, \quad B_i \geq n_i \geq 1. \quad (18)$$

Comparison with Equation (17) shows that Equation (18) also holds when  $n_i = 0$ . Note that the mean idle period  $U(i, n_i, k)$  cannot exceed the mean cell manufacturing time  $1/\mu_k$ .

Combination of Equations (16), (17), and (18) leads to the final result

$$q(n, k) = \sum_{i=1}^R \frac{C_i}{\mu_k} \left( \frac{\lambda_i}{\lambda_i + \mu_k} \right)^{n_i} \quad 1 \leq i \leq R, \\ 0 \leq n_i \leq B_i. \quad (19)$$

## APPENDIX C THE OUTPUT RATE OF EACH LINE

This appendix shows how to compute the output rate  $A_t$ ,  $1 \leq t \leq R$ , of line  $t$  under any given policy  $k(n)$ . We will use it with  $k(n)$  being the optimal control policy found from Equations (1), (2), and (3). Policy  $k(n)$  defines a semi-Markov reward process which earns a unit reward whenever the cell produces a part of type  $t$ . The expected reward per unit time for this process is just  $A_t$ . The value equations, in analogy to Equations (1), (2), and (3), are

$$v(n) = q(n) - A_t/\mu_k(n) + \sum_{0 \leq j < n} P[j|n, k(n)] \\ \times v(n - j + e^{k(n)}) \quad (20)$$

$$n \in \Omega, \quad n \neq B,$$

$$v(B) = -A_t/\lambda_{\text{sum}} + \sum_{i=1}^R \frac{\lambda_i}{\lambda_{\text{sum}}} v(B - e^i), \quad (21)$$

$$v(0) = 0, \quad (22)$$

where the one-transition reward is

$$q(n) = \begin{cases} 1 & \text{if } k(n) = t, \\ 0 & \text{otherwise.} \end{cases} \quad (23)$$

These equations are also solved by successive substitution, using the value-iteration scheme of [18].

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