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Performance evaluation of a flexible manufacturing cell with random multiproduct feedback flow

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This paper presents a predictive model for describing the productive capacity of special multiproduct manufacturing cells with stochastic activity times and random feedback flow. Two cases are considered, single-product type batch production and multiproduct type interleaved production. The major performance measures which are evaluated are the total batch processing time, number of product recycles and the distribution of interdeparture times for each product type. Expressions are given for the distribution functions, means and variances of these quantities, for *any* distribution of manufacturing time. Operational analysis of a flexible manufacturing cell is discussed with an illustration of the oscillatory phenomenon of the probability density function of its total batch time.

Introduction

A manufacturing cell is a part of a production line designed to fabricate families of parts having similar geometry and process requirements. Black (1983), Schonberger (1983) and Zisk (1983) have presented a recent overview of cellular manufacturing technology. In this paper we study the *unitary manufacturing cell* and evaluate the nature of its performance. Such a cell contains several flexible (i.e. programmable) work stations which are tended by a materials handling robot (Ottinger 1982). It produces one single product at a time. Produced parts are examined in an automatic inspection station which is in the cell and if necessary are recycled and reworked until they pass inspection. This design is applied in light assembly lines with relatively short cycle times at each work station (for example insertion of solid-state chips to printed-circuit boards). The operation of the unitary cell differs from the common single-machine sequencing problems since it operates under continuous load with stochastic processing times and no set-up costs.

In a unitary manufacturing cell, each station is idle for part of the cycle. In many cases, however, no attempt would be made to utilize this excess capacity, because of the extreme difficulty that would arise in controlling the flow of parts between the stations. Another common reason for unitary cell design is to avoid frequent changes in tooling and accessories, for example, paint colour on a mixed-model production facility.

The industrial applications of unitary cell design include ceramic mold-making systems for investment foundry casting (Tanner 1981), robotic assembly of

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typewriter ribbon cassettes or trimming injection-molded parts with the aid of drilling and milling units (Warnecke and Schraft 1982). As an example, consider a unitary cell with several assembly stations that perform a series of component-mating, screw-driving, nut-running and staking operations (figure 1). The base parts arrive at the cell on a conveyor and the robot is used to transfer the base parts between the conveyor and the assembly stations. The series of assembly tasks terminates at an inspection station. At this station, the poor assemblies are detected; the robot will strip the faulty items from the base parts and palletize them for further manual inspection. The base parts are taken by the robot for another assembly pass through the main assembly area. Good assemblies are delivered to a subsequent production cell by another, outgoing conveyor.

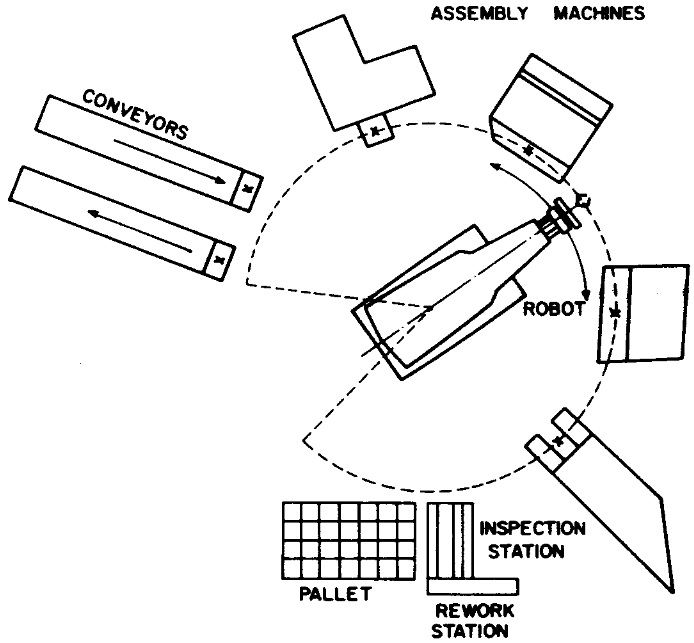

Figure 1. Layout configuration of a robotic assembly cell.

In an earlier paper (Seidmann and Nof 1985), the stochastic flow of parts is described for the case of a unitary manufacturing cell composed of two active areas, as shown in Fig. 2. One area, denoted M, is the *main* production line while the other area, denoted R, is the *recycle*, or reworking area. Parts produced in area M either exit the system, with probability q, or will be routed to the reworking area R and then again to M, with probability $P=1\text{-}q$; p is the probability of detecting a manufacturing defect. A similar application is described by Elmaghraby (1977, p. 336). Seidmann and Nof (1985) consider the analysis of a *single* item of a known type and present the operational analysis of a robotic assembly cell.

In many industrial situations, however, the performance prediction of manufacturing cells is related to either *batch* production runs, or to the *model mix* manufacturing case where the cell receives, from the supervising computer, random

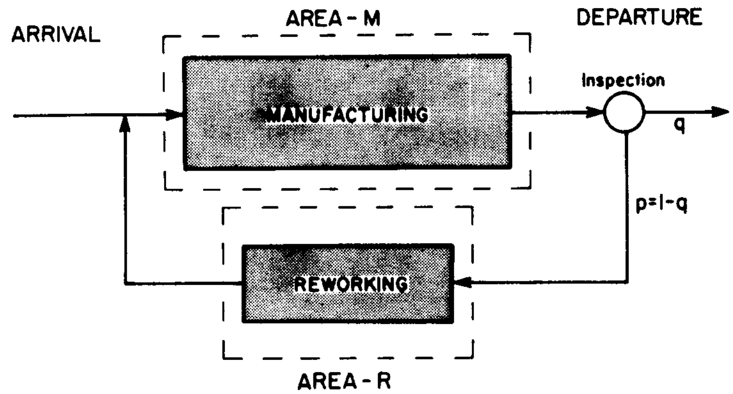

Figure 2. Feedback flow in a flexible manufacturing cell.

calls for the production of diverse product types (Bjorke 1979, Doyal *et al.* 1980). This paper extends the previous results to handle the case of batch productions as well as to the case of interleaved manufacturing of several product types.

Analysis of batch production

Suppose that the unitary cell has to deliver a batch of B identical parts to another unit in the plant. This batch can be a pallet, a unit load, an assembly kit or a specific customer's order. It is assumed that there are no inserted idle times or pre-emptions. Thus, once processing begins on the first part, processing continues uninterrupted until all B parts are completed. Since the cell processes only one part at a time, it is realistic to assume that manufacturing times for all the parts have identical and independent distribution functions.

Let T_{M} and T_{R} denote the actual times for working one part *once* at M and R, respectively, and by $T_{\bar{\mathrm{M}}}$ and $T_{\bar{\mathrm{R}}}$ the *total* time at M and R for the entire batch. The total batch time is $\theta = T_{\bar{\mathrm{M}}} + T_{\bar{\mathrm{R}}}$. Let $H_{\mathrm{M}}(t)$, $H_{\mathrm{R}}(t)$, $H_{\bar{\mathrm{M}}}(t)$, $H_{\bar{\mathrm{R}}}(t)$, and $H_\theta(t)$ denote the cumulative probability functions for T_{M}, T_{R}, $T_{\bar{\mathrm{M}}}$, $T_{\bar{\mathrm{R}}}$ and θ, respectively, where the first two distributions are given and the last three are desired. The corresponding density functions are $h_{\mathrm{M}}(t)$, $h_{\mathrm{R}}(t)$, $h_{\bar{\mathrm{M}}}(t)$, $h_{\bar{\mathrm{R}}}(t)$ and $h_\theta(t)$. The mean and variance of these distributions are denoted by μ_{M} and σ^2_{M}, μ_{R} and σ^2_{R}, etc. The moment-generating functions are denoted by $m_{\mathrm{M}}(s)$, $m_{\mathrm{R}}(s)$, $m_{\bar{\mathrm{M}}}(s)$, $m_{\bar{\mathrm{R}}}(s)$ and $m_\theta(s)$ where, for example, using Hogg–Tanis (1977) notation

$$m_{\mathrm{M}}(s) = E[\exp(sT_{\mathrm{M}})] = \int_0^\infty dH_{\mathrm{M}}(t)\exp(st) = \int_0^\infty \exp(st) h_{\mathrm{M}}(t)\,dt \qquad (1)$$

These moment-generating functions always exist in practice since the manufacturing times are strictly bounded.

Let N denote the number of visits at M to complete a batch of B items. Since the number of visits by *one* part to M is a Bernoulli trial with success probability q, N has a negative binomial distribution where

$$\Pr[N=n] = \binom{n-1}{n-B} p^{n-B} q^B, \quad n = B, B+1, \ldots \qquad (2)$$

with mean $\mu_{\mathrm{N}} = B/q$ and variance $\sigma^2_N = Bp/q^2$ (Feller 1968, p. 164).

Given N, $T_{\bar{M}}$ consists of the sum of N independent times at M, $T_{\bar{R}}$ consists of the sum of $(N\text{-}B)$ independent times at R and θ is the sum of both. Consequently, the conditional expectations are

$$E[\exp(sT_{\bar{\mathbf{M}}})/N]=[m_{\mathbf{M}}(s)]^{N} \tag{3}$$

$$E[\exp(sT_{\bar{\mathbf{R}}})/N]=[m_{\mathbf{R}}(s)]^{N-B} \tag{4}$$

$$E[\exp(s\theta)/N]=[m_{\mathbf{M}}(s)]^{N}[m_{\mathbf{R}}(s)]^{N-B} \tag{5}$$

Unconditioning on N, we obtain the following moment-generating functions

$$\begin{aligned} m_{\bar{\mathbf{M}}}(s) &= \sum_{n=B}^{\infty} \Pr(N=n)\, E[\exp(sT_{\bar{\mathbf{M}}})/n] \\ &= (qm_{\mathbf{M}}(s)/[1-pm_{\mathbf{M}}(s)])^{B} \end{aligned} \tag{6}$$

$$\begin{aligned} m_{\bar{\mathbf{R}}}(s) &= \sum_{n=B}^{\infty} \Pr(N=n)\, E(\exp(sT_{\bar{\mathbf{R}}})/n] \\ &= (q/[1-pm_{\mathbf{R}}(s)])^{B} \end{aligned} \tag{7}$$

Since $T_{\bar{M}}$ and $T_{\bar{R}}$ are independent once N is specified

$$E[\exp(s\theta)/n]=E[\exp(sT_{\bar{\mathbf{M}}})/n]E[\exp(sT_{\bar{\mathbf{R}}})/n]$$

This leads to

$$\begin{aligned} m_{\theta}(s) &= \sum_{n=B}^{\infty} \Pr(N=n)\, E[\exp(s\theta)/n] \\ &= (qm_{\mathbf{M}}(s)/[1-pm_{\mathbf{M}}(s)m_{\mathbf{R}}(s)])^{B} \end{aligned} \tag{8}$$

The corresponding means and variances are given by

$$\mu_{\bar{\mathbf{M}}}=B\mu_{\mathbf{M}}/q \tag{9}$$

$$\sigma^2_{\bar{\mathbf{M}}}=B(p\mu^2_{\mathbf{M}}/q+\sigma^2_{\mathbf{M}})/q \tag{10}$$

$$\mu_{\bar{\mathbf{R}}}=Bp\mu_{\mathbf{R}}/q \tag{11}$$

$$\sigma^2_{\bar{\mathbf{R}}}=B(p\mu^2_{\mathbf{R}}/q+p\sigma^2_{\mathbf{R}})/q \tag{12}$$

$$\mu_{\theta}=B(\mu_{\mathbf{M}}+p\mu_{\mathbf{R}})/q \tag{13}$$

$$\sigma^2_{\theta}=Bp(\mu_{\mathbf{M}}+\mu_{\mathbf{R}})^2/q^2+B(\sigma^2_{\mathbf{M}}+p\sigma^2_{\mathbf{R}})/q \tag{14}$$

The average percentage of the total batch time θ spent in reworking (PR) is defined by

$$\begin{aligned} \mathrm{PR} &= 100\mu_{\bar{\mathbf{R}}}/\mu_{\theta} \\ &= 100p\mu_{\mathbf{R}}/(\mu_{\mathbf{M}}+p\mu_{\mathbf{R}}) \end{aligned} \tag{15}$$

This is, understandably, independent of B.

The probability density functions may be extracted from the moment-generating functions as

$$h_{\bar{\mathbf{M}}}(t)=\sum_{n=B}^{\infty}\binom{n-1}{n-B}p^{n-B}q^{B}h_{\mathbf{M}}(t)^{*(n)} \tag{16}$$

$$h_{\bar{R}}(t)=\sum_{n=B}^{\infty}\binom{n-1}{n-B}p^{n-B}q^{B}h_{R}(t)^{*(n-B)} \tag{17}$$

$$h_{\theta}(t)=\sum_{n=B}^{\infty}\binom{n-1}{n-B}p^{n-B}q^{B}\,[h_{M}^{*(n)}*h_{R}^{*(n-B)}]\,(t) \tag{18}$$

where * denotes convolution ($*(n)$ denotes an n-fold convolution) and $h*^{(0)}(t)=\delta(t)$, the Dirac delta function. These sums of convolutions are not convenient for numerical use unless $h_M(t)$ and $h_R(t)$ have simple forms. Instead, it is simpler to numerically invert the closed expressions given here for $m_{\bar{M}}(s)$, $m_{\bar{R}}(s)$ and $m_\theta(s)$ using a standard software package for inverting Laplace transforms. For example, the IMSL software package, which applied the inversion method by Crump (1976), has been used here.

Numerical experimentation has shown that in many cases $h_\theta(t)$ shows oscillations (for example, Fig. 4). Knowledge of this stochastic behaviour is essential for interpreting operational (or simulated) data and in the design of material flow control policies. The significance of these oscillations has, apparently, not been investigated previously. This phenomenon is demonstrated later.

The covariance of $T_{\bar{M}}$ and $T_{\bar{R}}$ is computed, using $\theta=T_{\bar{M}}+T_{\bar{R}}$, as

$$\begin{aligned}\mathrm{COV}(T_{\bar{M}},T_{\bar{R}})&=E[(T_{\bar{M}}-E(T_{\bar{M}})(T_{\bar{R}}-E(T_{\bar{R}})]\\&=(\sigma_\theta^2-\sigma_{\bar{M}}^2-\sigma_{\bar{R}}^2)/2\\&=Bp\mu_M\mu_R/q^2\end{aligned} \tag{19}$$

The covariance is linear in the batch size B. Note that if $T_{\bar{M}}$ and $T_{\bar{R}}$ are independent, then $\mathrm{COV}(T_{\bar{M}},T_{\bar{R}})=0$; the converse, however, is *not* true (Feller 1968, p. 230). The correlation coefficient of $T_{\bar{M}}$ and $T_{\bar{R}}$ is

$$\rho(T_{\bar{M}},T_{\bar{R}})=\mathrm{COV}(T_{\bar{M}},T_{\bar{R}})/(\sigma_{\bar{M}},\sigma_{\bar{R}}) \tag{20}$$

The correlation coefficient $\rho(T_{\bar{M}},T_{\bar{R}})$ is independent of B.

The relative variability can be measured by the coefficient of variation, namely, the ratio of the standard deviation to the mean. The coefficient of variation of N (number of passes through M) and θ (total cell time) are given by

$$\begin{aligned}\mathrm{COV}_N&=\sigma_N/\mu_N\\&=(p/B)^{1/2}\end{aligned} \tag{21}$$

and

$$\begin{aligned}\mathrm{COV}_\theta&=\sigma_\theta/\mu_\theta\\&=[p(\mu_M+\mu_R)^2+q(\sigma_M^2+p\sigma_R^2))^{1/2}/(B^{1/2}(\mu_M+p\mu_R)]\end{aligned} \tag{22}$$

Both of these decreases with $B^{-1/2}$, namely, when B increases. Therefore a larger batch size will display a relatively smaller process variability. This effect is illustrated by Fig. 3 for the case of $\mu_M=10$, $\sigma_M^2=6{\cdot}1$, $\mu_R=18$ and $\sigma_R^2=2{\cdot}3$. The greatest relative variability in total time is shown to for intermediate values of p.

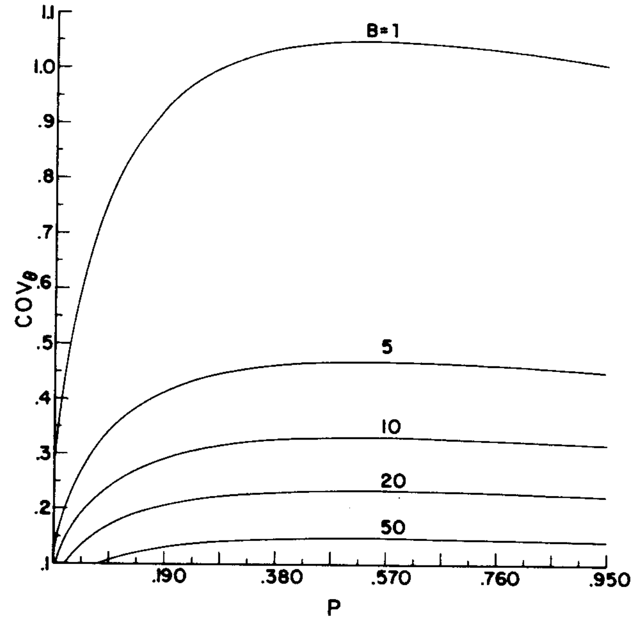

Figure 3. Total batch time coefficient of variation, COV_θ.

Example

Consider an automatic machine cell operating with $L>1$ assembly operations in series. Each assembly operation time is exponentially distributed with mean $1/\mu$, and an additional fixed material-handling travel time is α. Therefore

$$h_M(t)=\begin{cases}\exp[-\mu(t-\alpha)]\,\mu[\mu(t-\alpha)]^{L-1}/(L-1)! & t>\alpha\\ 0 & \text{otherwise}\end{cases} \tag{23}$$

and, for $n>1$

$$h_M^{*(n)}(t)=\begin{cases}\exp[-\mu(t-n\alpha)]\,\mu[\mu(t-n\alpha)]^{nL-1}/(nL-1)! & t>n\alpha\\ 0 & \text{otherwise}\end{cases} \tag{24}$$

The rejection rate is p and each rejected assembly needs a fixed time β for reworking. Hence

$$h_R(t)=\delta(t-\beta) \tag{25}$$

with

$$h_R^{*(n-B)}(t)=\delta[t-(n-B)\beta] \tag{26}$$

The probability density of the total batch time is given by eqn. (18) as

$$h_\theta(t)=\begin{cases}\displaystyle\sum_{n=B}^{n_{max}}\binom{n-1}{n-B}p^{n-B}q^B\exp[-\mu(t-n\alpha-(n-B)\beta))\mu^{nL} \\ \qquad (t-n\alpha-(n-B)\beta)]^{nL-1}/(nL-1)! \\ \qquad\qquad \text{if } t>B\alpha \\ 0 \qquad\qquad \text{otherwise}\end{cases} \tag{27}$$

where n_{max} is the largest integer such that $t>n\alpha+(n-B)\beta$. Following eqn. (13), the mean batch production time is

$$\mu_\theta = B(\alpha + L/\mu + p\beta)/q \tag{28}$$

Production process variability, eqn. (14), is given by the variance

$$\sigma_\theta^2 = Bp(\alpha + L/\mu + \beta)^2/q^2 + B(L/\mu)^2/q \tag{29}$$

and by the coefficient of variation

$$\mathrm{COV}_\theta = \sigma_\theta/\mu_\theta \tag{30}$$

Numerical example

Suppose that $p = 0{\cdot}15$, $\alpha = 5$ s, $\beta = 28$ s, $L = 6$ and $1/\mu = 5$. The probability density function $h_\theta(t)$ for $B = 1$, $B = 3$, and for $B = 10$ is illustrated in Figs. 4, 5 and 6. The local peaks on the curve for $B = 1$ represent completions after 0, 1, 2, 3, . . . , reworks. As the batch size increases, these peaks are less pronounced and the curve ultimately approaches a unimodal bell shape. This finding contradicts the general assumption that the total time in the assembly facility is normally distributed as assumed, for

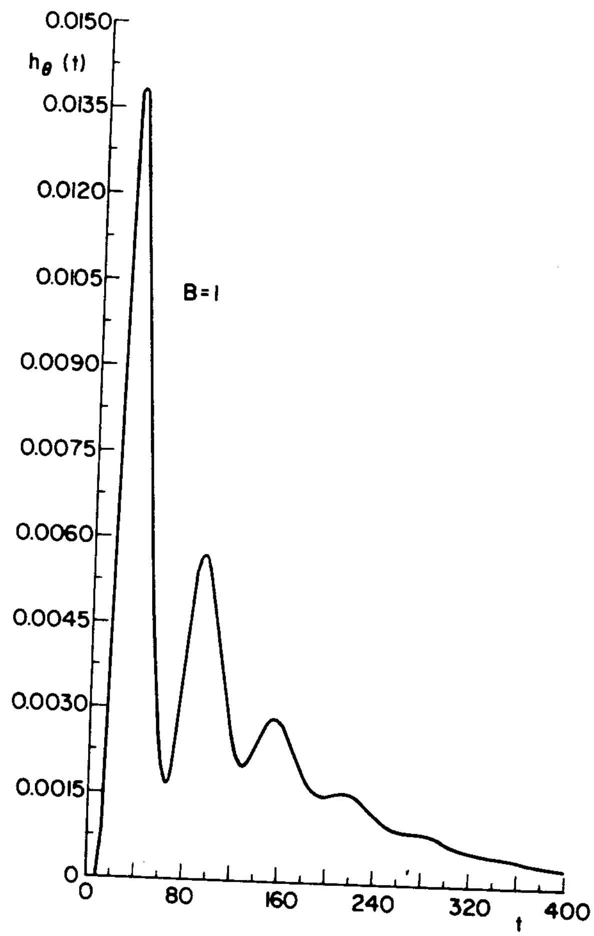

Figure 4. The density function $h_\theta(t)$ for batch size 1.

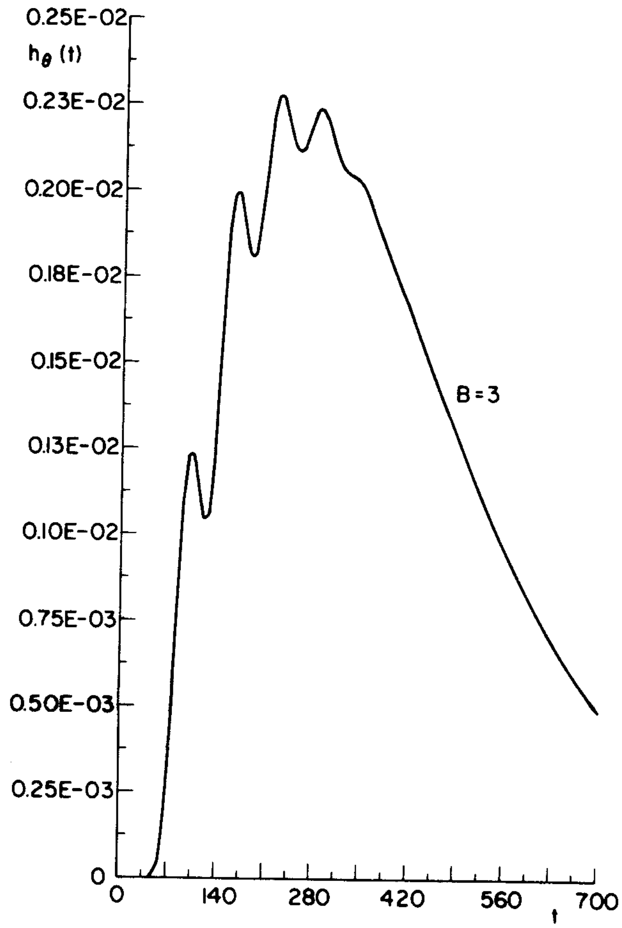

Figure 5. The density function $h_\theta(t)$ for batch size 3.

instance, by Wilhelm and Ahmadi-Marandi (1982). This assumption would be acceptable *in our case* only when $B \simeq 10$. The mean batch time as a function of p and B can be observed in Fig. 7.

The computations of the production capacity, operational efficiency and assembly cost for this example follow the general scheme of Groover (1980, Chap. 5) or Seidmann and Nof (1985) and are not detailed here.

Interleaved multiple product types

This section considers the case where the unitary manufacturing cell is providing z distinct types of parts, for continuous consumption by z assembly lines (see, for example, Hutchinson and Wynne (1973)). This configuration is depicted in Fig. 8. When the cell is finished making one part, the next part it makes will be of type i (for assembly line i) with probability y_i, $1 \leqslant i \leqslant z$, where

$$y_i > 0 \quad \text{and} \quad \sum_{i=1}^{z} y_i = 1$$

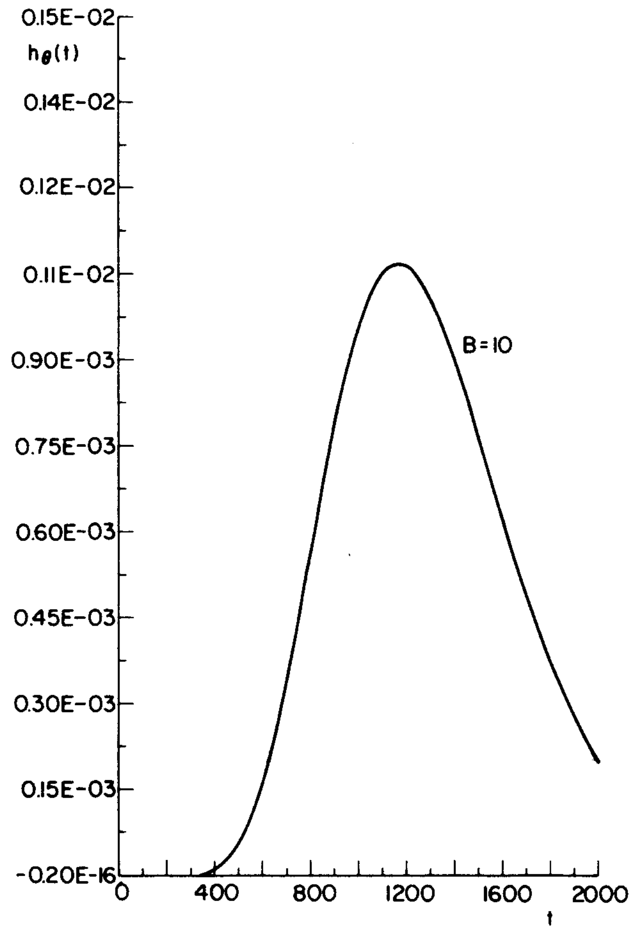

Figure 6. The density function $h_\theta(t)$ for batch size 10.

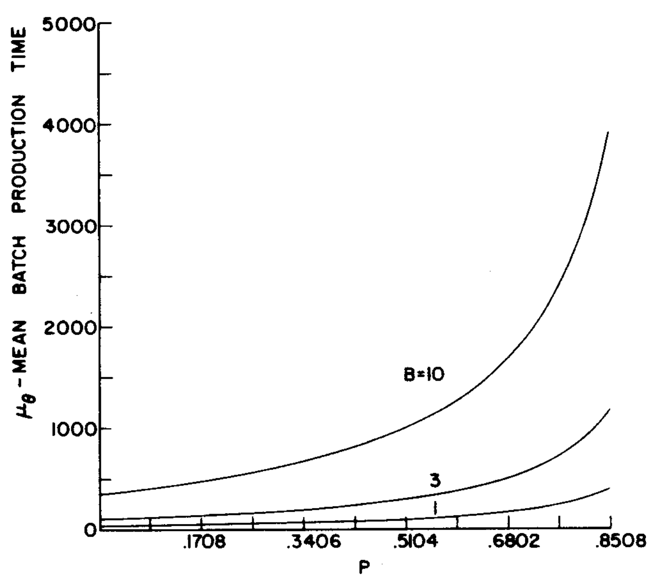

Figure 7. Mean total batch production time μ_θ for the illustrative system.

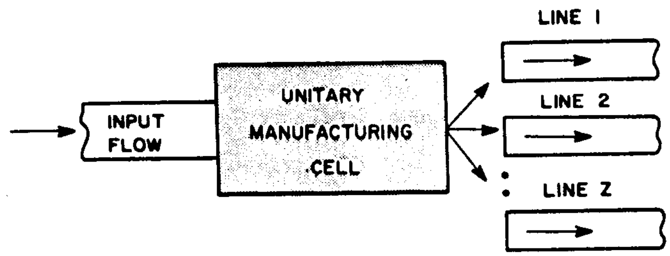

Figure 8. A cell feeding several production lines.

A recent study by Seidmann and Schweitzer (1984) discusses the optimal part-selection policy for this interleaved unitary cell operation.

In what follows, the subscript i denotes parameters for part type i: the probability density functions of the time for one visit at M and R are $h_{Mi}(t)$ and $h_{Ri}(t)$, respectively, and $p_i = 1 - q_i$ is the probability a part made at M needs reworking. The probability density function and the moment generating function of the time for the cell to successfully produce one part of type i are denoted by $h_{\theta i}(t)$ and $m_{\theta i}(s)$, respectively. The expressions for these are obtained by specializing the second section to unit batch sizes

$$h_{\theta i}(t) = \sum_{n=1}^{\infty} p_i^{n-1} q_i [h_{Mi}^{*(n)} * h_{Ri}^{*(n-1)}](t), \quad 1 \leqslant i \leqslant z$$

$$m_{\theta i}(s) = q_i m_{Mi}(s)/[1 - p_i m_{Mi}(s) m_{Ri}(s)], \quad 1 \leqslant i \leqslant z$$

The mean and the variance to successfully make one type i item are given by

$$\mu_{\theta i} = (\mu_{Mi} + p_i \mu_{Ri})/q_i$$

and

$$\sigma_{\theta i}^2 = p_i(\mu_{Mi} + \mu_{Ri})^2/q_i^2 + (\sigma_{Mi}^2 + p_i \sigma_{Ri}^2)/q_i$$

where μ_{Mi}, σ_{Mi}^2, $[\mu_{Ri}, \sigma_{Ri}^2]$ are the mean and variance associated with the distribution $h_{Mi}[h_{Ri}]$.

Let η denote the elapsed time between two successive production completions at the cell. We seek the probability density function $h_\eta(t)$ and moment generating function $m_\eta(s)$ of η. By conditioning and then unconditioning on the type of part being manufactured, we see

$$h_\eta(t) = \sum_{i=1}^{z} y_i h_{\theta i}(t), \quad t > 0 \tag{31}$$

$$m_\eta(s) = \sum_{i=1}^{z} y_i m_{\theta i}(s) \tag{32}$$

The mean and variance of η are therefore given by

$$\mu_\eta = \sum_{i=1}^{z} y_i \mu_{\theta i} \tag{33}$$

$$\sigma_\eta^2 = \sum_{i=1}^{z} y_i(\sigma_{\theta i}^2 + (\mu_{\theta i})^2) - \mu_\eta^2 \tag{34}$$

$$= \sum_{i=1}^{z} y_i[\sigma_{\theta i}^2 + (\mu_{\theta i})^2] - \sum_{i=1}^{z}\sum_{j=1}^{z} y_i y_j \mu_{\theta i}\mu_{\theta j} \tag{35}$$

The average overall production rate of the cell, in parts per unit time, is then $1/\mu_\eta$. Next, fix i and let ηi denote the elapsed time between two successive production completions of type i parts at the cell; this also corresponds to the interarrival time of parts to assembly line i from the cell. Seek the probability density function $h_{\eta i}(t)$ and the moment generating function $m_{\eta i}(s)$ of ηi. To find these, suppose that two successive productions of a type i part are separated by $K \geqslant 0$ productions of parts of the other types. Then

$$\Pr(K=j) = (1-y_i)^j y_i \tag{36}$$

and the probability density function of the time for *each* of these K productions is

$$\hat{h}_i(t) = \sum_{j=1, j\neq i}^{z} [y_j/(1-y_i)]h_{\theta j}(t) \tag{37}$$

with moment generating function

$$\hat{m}_i(s) = \sum_{j=1, j\neq i}^{z} [y_j/(1-y_i)]m_{\theta j}(s) \tag{38}$$

By conditioning and then unconditioning on K, we find

$$\begin{aligned} h_{\eta i}(t) &= \sum_{j=0}^{\infty} \Pr(K=j)[\hat{h}_i^{*(j)} * h_{\theta i}](t) \\ &= y_i \sum_{j=0}^{\infty} (1-y_i)^j [\hat{h}_i^{*(j)} * h_{\theta i}](t), \quad 1 \leqslant i \leqslant z \end{aligned} \tag{39}$$

which converges geometrically. Taking transforms

$$\begin{aligned} m_{\eta i}(s) &= y_i m_{\theta i}(s) \sum_{j=0}^{\infty} [1-y_i)^j (\hat{m}_i(s)]^j \\ &= y_i m_{\theta i}(s)/[1-(1-y_i)\hat{m}_i(s)] \\ &= y_i m_{\theta i}(s)/[1 - \sum_{j=1, j\neq i}^{z} y_j m_{\theta j}(s)] \end{aligned} \tag{40}$$

The mean and variance of ηi are then given by

$$\mu_{\eta i} = \sum_{j=1}^{z} (y_j/y_i)\mu_{\theta j} = \mu_\eta / y_i \tag{41}$$

$$\begin{aligned} \sigma_{\eta i}^2 &= 1/y_i \sum_{j=1}^{z} y_j(\sigma_{\theta j}^2 + \mu_{\theta j}^2) + (2/y_i)\mu_{\theta i}(\mu_\eta - y_i\mu_{\theta i}) \\ &\quad + (2/y_i^2)(\mu_\eta - y_i\mu_{\theta i})^2 - (\mu_{\eta i}(y_i)^2 \\ &= (\sigma_\eta^2 + \mu_\eta^2)/y_i - 2\mu_\eta\mu_{\theta i}/y_i^2 + (\mu_\eta/y_i)^2 \end{aligned} \tag{42}$$

From eqns. (33), (34) and (42), the variance $\sigma_{\eta i}^2$ is very large when y_i is small, and decreases to $\sigma_{\theta i}^2$ as y_i increases to unity.

The average production rate of type i parts, in parts per unit time, R_i, is then

$$R_i = 1/\mu_{\eta i} = y_i/\mu_\eta \tag{43}$$

Cell design and control

Several design issues of the automatic assembly cell example that are described were explored by Seidmann and Nof (1985). They include the selection of robot speed and size determination of the main area as a function of the rate of required rework. The results that are obtained here with regard to batch and interleaved production provide additional design and control insights.

Batch size and rework rate

It is clear that operations requiring a higher rate of rework will produce less products, or smaller batches, within a given period of time. With regard to manufacturing cells with feedback flow for rework, the trade-off between batch size B and rework rate P can be specified as follows. Assuming a lower value of mean total production time per batch is desired, and consequently a higher batch size, select plan (B_1, P_1) rather than (B_2, P_2) if

$$\mu_\theta(B_1, P_1) \leqslant \mu_\theta(B_2, P_2)$$

where $\mu_\theta(B, P)$ is given by eqn. (13). For the assembly cell example, assume $\alpha = 4$, $L/\mu = 15$ and $\beta = 1$. Comparing rework rates of $P_1 = 0{\cdot}1$ and $P_2 = 0{\cdot}3$ it is found from eqn. (28) that for the same production rate of the machines (but with less defects) B_1 can be as high as 30% greater than B_2. Under the same assumptions, if $P_2 = 0{\cdot}4$, then B_1 can be as high as 52% greater than B_2, and so on.

Optimal number of interleaved products

An important advantage of flexible manufacturing cells is the ability to provide concurrent processes of different variety to produce the same or different product types. In a number of studies (for example, Nof and Moodie (1981)) it has been shown that such mixed operation is usually preferred because less busy equipment can be utilized for alternative processing, and as a result significant gains in productivity can be achieved. However, this may not be the case in interleaved production as modelled here.

Example

Suppose three product types A, B, and C require a mean process time of 54, 45, and 53 minutes in the main area M and 4, 3, and 4 minutes in the rework area R, respectively. The mean time to successfully produce one product, μ_θ, is 60·44 min for types A and C, and 50·33 min for type B. Assuming interleaved production with $y_A = y_B = 0{\cdot}4$ and $y_C = 0{\cdot}2$, the overall mean production time per unit of any type is (from eqn. (33)) 56·40 min. The result is, of course, a weighted average of the results obtained for the individual type production.

This example shows that there is no advantage in terms of increased productivity in the interleaved production, as defined in our model. However, there is a clear advantage in terms of the capability to switch from one type to another in the same facility. The design objective, then, will be to *minimize the number of cells* to reduce excess capacity. In addition to technological considerations of cell design, we can formulate the following design problem:

Minimize the number of cells N_c by assigning subsets of product types to individual cells, subject to the constraint

$$\sum_{i=1}^{z_c} \mu_{\theta i} Q_i \leqslant T \quad \text{for } c=1,2,\ldots,N_c \tag{44}$$

where z_c is the number of product types assigned to cell c; Q_i is the total quantity derived from product type i during the period T. The standard deviation σ_η^2 (from eqn. (34)) can be used to test for worst-case conditions.

Concluding remarks

A stochastic model for multiproduct manufacturing cells with random feedback flow is developed for the cases of batch and interleaved production. Since the analysis is based upon the distribution of single passage times at each area in the cell, the model recognizes the probabilistic aspects of the system explicitly. Results hold for *any type* of proper single-passage distribution function. The operation of the cell is characterized in terms of the total batch processing time, real times for production and rework, interproduction times, productive capacity and several other variability measures.

The analytic expressions given can be applied to predict mean values and variances of major operational variables such as the number of product recycles, the length of time that a given batch will stay in the facility, or the expected aggregate time spent in each area. Using the central limit theorem, approximate confidence intervals can be drawn for the anticipated cell performance and desired values for capacity planning can be obtained as well as for material flow planning and control. The effects of changes in batch size, reworking rate and product mix can be investigated in order to provide information as to the sensitivity of the expected manufacturing cell design to these parameters. Finally, the modelling approach can be used as a 'building block' to handle more complex systems such as the case of several cells in a cascade or as a part of a general production model (Koenigsberg and Mamer 1982).

Dans det article est présenté un modèle prédictif qui sert à décrire la capacité de production de cellules spéciales de fabrication à produits multiples avec des durées d'activité stochastiques et un flux de renvoi d'information aléatoire. Deux cas sont examinés, une production par lot du type à simple produit et une production intercalée du type à plusieurs produits. Les mesures les plus importantes de la performance qui sont évaluées sont les suivantes: durée totale de traitement du lot, nombre de recyclages du produit et distribution des durées entre les commencements de chaque type de produit. Des expressions sont données pour les fonctions de distribution, les moyennes et les variances de ces quantités pour *toute* distribution de la durée de fabrication. L'analyse opérationnelle d'une cellule de fabrication flexible est examinée, avec une illustration du phénomène oscillatoire de la fonction de densité de probabilité de sa durée de lot totale.

Diese Arbeit präsentiert ein prädikatives Modell für eine Beschreibung der produktiven Kapazität von speziellen Multiproduktherstellungszellen mit stochastischen Aktivitätszeiten und Zufallsrückführfluß. Zwei Fälle werden betrachtet: einzelproduktartige Chargenherstellung und multiproduktartige verschachtelte Herstellung. Die Hauptleistungsgrößen. die begutachtet werden. sind die Gesamtchargenverarbeitungszeit, die Zahl der Produktrückflüsse und die Verteilung der Zwischenabgangszeiten für jeden Produkttyp. Ausrücke werden für die Verteilerfunktionen, die Durchschnittswerte und Varianzen dieser Größen für *jede* Zuteilung der Herstellungszeit gegeben. Die analytische Arbeitsuntersuchung einer flexiblen Herstellungszelle wird mit einer Darstellung des Schwingphänomens der Häufigkeitsfunktion der Gesamtchargenzeit illustriert.

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