THROUGHPUT MAXIMIZATION IN FLEXIBLE MANUFACTURING SYSTEMS

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Maximizing the throughput (or revenue generation) rate has become one of the most important criteria in the design and management of Flexible Manufacturing Systems (FMSs). This paper develops dynamic part-allocation policies for FMSs having finite storage capacity at each work station. Maximizing the throughput rate means that the resulting queueing network model has a state-dependent arrival process; therefore, product-form solutions do not hold. Consequently, several alternative modeling approaches are described and formulated for deriving the optimal part-routing policies. One of these optimal policies is based on a new initiated-suspension part-routing strategy. This strategy results in a reduced load on the material handling system while increasing the expected throughput (or revenue generation) rates of the manufacturing work stations. We also propose several efficient closed-loop heuristic policies that exploit the response structure of the optimal policies. These heuristic policies are of practical significance because they are extremely easy to compute and to implement, while the resulting FMS performance is nearly optimal.

Flexible Manufacturing Systems (FMS) are automated manufacturing systems consisting of computer-controlled machines linked together with an automated material handling system. These manufacturing systems are capable of simultaneously processing multiple part types. An FMS is a highly complex system that requires sophisticated computer hardware and software to monitor and control its activities [3], [8], [28].

The system development process for an FMS is composed of multiple phases. The conceptual design and functional analysis phases [20], [33], [34] of an FMS are probably the most important phases of the system development cycle. During the conceptual design phase, the corporate manufacturing strategy can result in many planning options. During the functional analysis phase the goal is to quantitatively eliminate inferior alternatives and to focus on the best system design. If this evaluation is done badly, subsequent operational problems might be difficult, if not impossible, to correct. The functional analysis phase is certainly the time during which the most significant cost savings are possible [6], [35]. During these phases, analytical models are used to set up and explore a wide range of design options quickly. Various studies by Seidmann et al. [27] and by Suri et al. [34], [35] indicate that aggregated analytical models come within 5% to 15% of the values obtained by very detailed and time-consuming simulations. More important, several leading researchers, such as Solberg [31], Gershwin et al. [8], Buzacott and Yao [4], Kusiak [18], Stecke [32], Suri et al. [35], Co and Wysk [5], and Shanbhikumar and Yao [29], indicate that the overall insights these models provide about the system dynamics, and the decisions that result from using these models, are clearly appropriate for the early system design phases. Since real-time scheduling may improve performance appreciably [20], [39], [40], [41], it should, therefore, be incorporated in the modeling effort at this phase. In this paper, we consider the case where the FMS management objective is weighted throughput (or revenue) maximization. Smith et al. [30] show that this is currently one of the most important criteria used in managing industrial FMS's. This finding is supported by another industrial case study recently presented by Stecke [33].

Our analytical framework extends the earlier studies of Hahne [11] and of Seidmann and Schweitzer [25] by explicitly formulating the weighted throughput maximization problem for any number of work stations. This new formulation also simplifies the computations of the expected one transition rewards in the stochastic model. In addition, it permits temporary suspension of material flow activities at certain decision epochs, even though some input buffers are not full; this new option allows for improved FMS performance while reducing the material handling system utilization. Moreover, this contribution includes provisions for distinct Erlang distributions of the part-routing times as well as two new closed-loop heuristic control policies. Breakdowns and repairs of the stations and the material handling system are studied in this context. We also present and evaluate several efficient heuristic solution policies which are easy to compute and which lead to near optimal FMS performance.

The FMS model is conceptualized as having a set of work stations numbered $1, 2, \ldots, M$. These stations have identical functions but operate with distinct process characteristics due to their unique tooling setups. Work station $i$ completely processes part type $i$, $1 \leq i \leq M$. The central material handling system (MHS) uses an automated guided vehicle system to deliver individual parts into the input buffers of the various stations. It has a common storage area large enough to accommodate all the $N$ parts, or pallets, that circulate throughout the system. Each work station has an associated local input
buffer space. The local buffer associated with station $i$ can hold $B_i$ type-$i$ parts ($B_i \geq 1$). It is occupied only by those jobs whose next operation is at station $i$.

In order to insure proper utilization of the station it is safe to assume that

$$N \geq \sum_{k=1}^{n} B_k.$$  

(1)

The MHS travel time to the next station, $k$, as well as the time needed for the necessary preparatory work, has an Erlang distribution with $E_k$ stages ($E_k = 1, 2, \ldots, \infty$) and a mean of $1/\mu_k$. We made this assumption about the MHS travel time in order to capture a broad spectrum of travel-time distributions. Notice that if $E_k = 1$, the above p.d.f is reduced to the exponential p.d.f. Deterministic times can be accommodated using a large value for $E_k$. Upon completing a delivery task, the MHS is released immediately for the next delivery. The manufacturing time at work station $k$ is assumed to be exponentially distributed with a mean of $1/\lambda_k$, $1 \leq k \leq M$. A special “return conveyor” carries the processed parts back to the central buffer (see [13] for technical details). Since the return conveyor is relatively fast the time required to bring a job back from the work stations can be ignored, and Equation (1) guarantees that there will always be at least one raw part in the central storage area [40], [41]. Figure 1 presents the parts flow through the system.

Each one of the completed parts has a unique value (weight). This value depends on the station producing that part type. The production control problem discussed here is how to determine which station should be fed next by the MHS in order to maximize the overall expected value of the parts processed per unit time.

The FMS layout described represents those flexible facilities where the same raw substance is used for manufacturing a variety of distinct part types (e.g., metal sheets in punch pressing). The model is admittedly simplified, and it ignores, for instance, such issues as parts rework, precedence constraints, and transient production-mix targets. Investigating the throughput maximization problem, Hahne [11], Seidmann and Schweitzer [25], Yao and Buzzacott [40], [41], Stecke [33], and Afentakis [1]—among others—also utilized such simplified FMS structures as a first step in attempting to develop more detailed system models and to provide a better intuition about the complex interactions between the FMS facility design parameters, the scheduling policy employed, and the overall system performance. Other dynamic control policies for manufacturing systems are given by Akella, Choong and Gershwin [2], Hodgson, King and Wilson [14], Mirchandani and Xu [19], Glassy [9], Glassy and Resende [10], Han and McGinnis [12], and Kimemia and Gershwin [17].

Yao and Buzzacott [41] published a paper in 1985 describing a novel Probabilistic Shortest Queue (PSQ) scheduling heuristic for a model similar to the one outlined above. In their paper they state that “the latter (deterministic) routing scheme is, however, not analytically tractable.” Our paper presents the semi-Markovian model generating such an optimal (deterministic) routing scheme as well as some closed-loop heuristic policies to be used when the state space of the exact model becomes computationally prohibitive.

Briefly, this paper is organized as follows: The FMS throughput maximization problem is formulated, and numerical analyses of the optimal routing policy and several heuristics are presented. The conclusions highlight pertinent operational insights. The expanded model formulation, treating the case of an unreliable MHS and stations, is detailed in the Appendix.

The Throughput Maximization Problem

Let $n_k$ ($1 \leq k \leq M$) denote the number of parts at station $k$ at the input buffer (or possibly the one part being processed). The state space is then:

$$\Omega = \{n = (n_1, n_2, \ldots, n_M) | 0 \leq n_k \leq B_k, (k = 1, 2, \ldots, M)\}.$$  

(2)

Our model uses the following notation in formulating the throughput maximization problem:

$${\bf e}^i = \text{the unit vector in the } i\text{-th direction, } 1 \leq k \leq M.$$  

$$g(k) = \text{the long-run expected weighted throughput value per unit time following a given control policy } k \text{ (the "gain rate" } \text{[15]). For brevity we use } g \text{ and } g(k) \text{ interchangeably.}$$

Figure 1. Parts flow within the FMS

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\( v(n) \) = the relative value of state \( n \), \( n \in \Omega \).

\( k \) = the control variable indicating the next machine designation for the MHS, \( k = 0, 1, \ldots, M \).

\( p(I, n, k) \) = the probability that following policy \( k \) the system will step directly from state \( I \in \Omega \) to \( n \in \Omega \).

\( ETP(J[n], L_s, \mu_s) = (J = j_1, j_2, \ldots, j_m) \)

\( = \) the joint Erlang transition probability that station \( i \) uses up \( j \), of its \( n_i \) \( (j_i \leq n_i) \) parts during the time that the MHS moves one part to station \( k \) for all \( i = 1, 2, \ldots, M \). This travel time has Erlang distribution with \( L_s \) stages (see [37] for algorithmic details).

\( D(n) \) = the set of stations having nonfull buffers; i.e., \( D(n) = \{i | n_i < B_i\} \).

\( W_i \) = the value (weight) assigned to a part produced at station \( i \), \( W_i \geq 0 \), \( i = 1, 2, \ldots, M \).

\( W(n, k) \) = the immediate reward incurred at state \( n \in \Omega \) following policy \( k \).

\( U(n, k) \) = the expected time until the next decision epoch if the system is at state \( n \in \Omega \) following policy \( k \).

\( \lambda(n) = \sum_{i \in \mathcal{A}(n)} \lambda_i \) (the cumulative production rate of all active stations in state \( n \)), and \( \mathcal{A}(n) = \{n(n_1, n_2, \ldots, n_m) | 0 < n_i \leq B_i \text{ for } 1 \leq i \leq M \} \).

In the initial case, where the MHS is idle, the buffers are empty, and a decision \( k \), \( 1 \leq k \leq M \), is made, the mean time for the system to reach the next state \( (e^k) \) is \( 1/\mu_k \).

When the system is in state \( n \neq B \) and the decision to produce an additional part type \( k \), \( 1 \leq k \leq M \), \( k \in D(n) \), is made, the expected benefit is given by \( W_k \) ($/ per part produced). During that time interval, the system evolves from the state \( n \) to state \( (n - J + e^k) \). This transition reflects a random decrement of \( J \) parts in the input buffer of the active stations and of a single part increment at the \( k \)-th buffer. The mean time to reach state \( (n - J + e^k) \) is \( 1/\mu_k \). Alternatively, the MHS may remain idle, sparing its capacity until the next change in the system state occurs (denote this decision as \( k = 0 \)). If \( k = 0 \) then the mean time that the system will hold in this state is \( 1/\lambda(n) \); the next decision epoch will be at an end of processing event at station \( i \), \( i \in \mathcal{A}(n) \), with probability \( \lambda_i/\lambda(B) \).

The MHS controller can initiate part delivery only after its destination station is unblocked. The mean holding time in the blocked case is \( 1/\lambda(B) \), \( n = B \), and since the MHS is blocked there is no transition "reward." The probability that station \( i \) will be the first to become unblocked is \( \lambda_i/\lambda(B) \).

Our objective is to identify the stationary control policy \( k^* \) which routes the MHS in such a way that the long-run weighted average of the FMS throughput rate \( g(k^*) \) is maximized:

\[
\max_k \left\{ g(k^*) = \frac{\sum_{n \in \Omega} \pi(n, k) W(n, k)}{\sum_{n \in \Omega} \pi(n, k) U(n, k)} \right\}
\]  

(3)

The equilibrium distribution, \( \{\pi(n, k), n \in \Omega\} \), is uniquely determined by the following system of linear equations:

\[
\pi(n, k) = \sum_{I \in \Omega} p(I, n, k) \pi(I, k), \quad n \in \Omega
\]  

(4)

\[
\sum_{I \in \Omega} \pi(n, k) = 1
\]  

(5)

where

\[
W(n, k) = \begin{cases} w_k & \text{if } k \in \{1, 2, \ldots, M\} \\ 0 & \text{otherwise}, \end{cases}
\]  

(6)

and

\[
U(n, k) = \begin{cases} 1/\mu_k & \text{if } k \in \{1, 2, \ldots, M\}, \quad n \neq B \\ 1/\lambda(n) & \text{if } k = 0, \quad n \neq B \\ 1/\lambda(B) & \text{if } k \in \{1, 2, \ldots, M\}, \quad n = B. \end{cases}
\]  

(7)

Following the above notations and problem formulation, the following continuous-time, semi-Markovian dynamic programming equations [16] are used to reveal the optimal policy maximizing the throughput rate of the FMS:

\[
v(\emptyset) = 0 \quad \text{(arbitrary fixed value)}
\]  

(8)

\[
v(\emptyset) = \max_{k=1, \ldots, M} \{W_k - g/\mu_k + v(e^k)\} = 0,
\]

(9)

\[
v(n) = \max_{k=1, \ldots, M} \left\{ \begin{array}{l}
W_k - g/\mu_k \\
+ \sum_{e \geq e^k} ETP(J[n], L_s, \mu_s) v(n - J + e^k)
\end{array} \right\} + \sum_{i \in \mathcal{A}(n)} (\lambda_i/\lambda(B)) v(B - e^k)
\]

(10)

\[
v(B) = -g/\lambda(B) + \sum_{i=1}^M (\lambda_i/\lambda(B)) v(B - e^k)
\]

(11)

It can be shown that under any policy the associated semi-Markovian process on \( \Omega \) is ergodic, and the control policy is stationary since identical decision rules are used at each decision epoch [38]. This means that Equations (8)-(11) possess a unique solution. Equation (8) is used to fix the arbitrary additive component of

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the relative-value vector and to determine the desired routing \((k)\) at \(r=0\). Schweitzer's [21], [22] value-iteration approach was adapted to solve these equations.

Several numerical techniques including a heuristic initial guess for the relative value and the gain rate, variable step size, step skipping, and value extrapolations are implemented to accelerate the solution time. Convergence is typically observed within one hundred iterations, and the computational efforts tend to increase quadratically with the number of states. Memory requirements are linear with the number of states. We also found that a ten-stage \((L_n = 10)\) Erlang distribution can be used for modeling the task times of MHS's with deterministic processing times. The relative error in the computed gain rates is typically less than 5% when a ten-stage Erlang distribution is used for representing deterministic MHS's times. The computational effort is not affected much by the increase in the number of stages of the MHS times since the transition probabilities are computed only once, and the state space doesn't increase with \(L_n\), as decision epoches are imbedded at the end-of-travel states. These computations are performed off line only once. The optimal decisions are then stored in a look-up table for the actual real time retrieval by the MHS controller. In general, the average CPU time for a problem having one thousand states is roughly 15 minutes on an IBM/PC model 80 and ten times as fast on a CDC 990 mainframe using a vector processor facility. We expect even better performance using the newly developed aggregation methods for large-scale Markov chains (cf. Schweitzer, Puterman and Kindel [23] or Takahashi [36]). Other advancements supporting the efforts to derive optimal solutions to large-scale problems include the potential usage of RISC workstations or mainframe computers having high-performance supercomputing vector facilities.

For simplicity of exposition, the possibility of station or MHS failures is not explicitly included in our initial formulation. However, station and MHS availability can be adjusted downwards to reflect their expected fraction of down times. In the case of infrequent and lengthy station breakdown, one can resolve Equations (8)-(11) for a smaller topology including only the active machines. Appendix A details the value equations for the case of an unreliable MHS and stations. These equations seem to be more appropriate in the case of short (but frequent) breakdown and repair cycles. They lead to a significantly larger state space and to increased computational effort.

The next section presents three heuristic control policies that characterize the optimal control structure given by the semi-Markovian decision process formulation. These heuristics produce good results with minimal computational effort. Industrial applications of these heuristics should facilitate improvements in the dynamic routing performance of large FMS facilities.

Numerical Analyses and Heuristics

In this section, the performance of the optimal routing policies and three heuristics are illustrated and compared for several FMS measures. For brevity, only small systems are detailed. The observations outlined are, however, similar to those obtained during our empirical investigation of larger and more involved problems [37] depicting systems with up to six machines. The six instances presented here focus on deriving basic insights about the key structural properties of the optimal policy. These insights are later used in identifying several heuristic policies.

The first instance represents an FMS having two work stations \((M=2)\). Each work station has three buffer spaces \((B_1=B_2=3)\) and production rates of 50 and 100 parts per hour for the first and second work stations respectively \((\lambda_1=50, \lambda_2=100)\). Routing times are exponential, and the MHS can deliver parts to these stations at a rate of 100 parts per hour \((\mu_1=\mu_2=100)\). The value assigned to each part produced is 50 dollars \((W_1=W_2=50)\). Denote the above data set as Instance 1. The second instance uses modified buffer sizes: \(B_1=B_2=6\). The third instance assumes the same data set as instance one, except that the weight is modified: \(W_1=20\) and \(W_2=100\).

Next, several heuristic feedback control policies are proposed and compared with the optimal policies. The following heuristics were used in this study:

1. The Fastest Shortest Queue (FSQ) policy routes parts to the work station having the smallest number of parts in the input buffer at the decision epoch. In case of a tie, the station having the fastest processing rate is selected. Since production throughput is maximized when all work stations are busy, it is anticipated that FSQ will perform well with respect to that objective.

2. The Probabilistic Shortest Queue (PSQ) policy routes parts with the highest probability to the shortest queue. Introduced and studied by Yao and Buzacott [41], this heuristic policy serves as an analytically tractable approximation for the FSQ policy since product form conditions are satisfied only under PSQ routing.

3. The Weighted Shortest Queue (WSQ) policy is a new deterministic extension of the SQ policy and of Foschini's Worktime Balance heuristic [7], which routes parts to the work station having the minimal work load on queue at the decision epoch. The current work load on queue \(i\) is given by \((n_i/\lambda_i)\), \(i=1, 2,\ldots, M\). WSQ is proposed in order to account for the differences in routing times, processing rates, and products values among the various stations and part types.

Specifically, WSQ computes the ratios \(R(n_i)\) for all

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unblocked work stations at each decision epoch \( (n \neq B) \):

\[
R(n_i) = \frac{\mu_i n_i}{\sum_{i \in D(n)} \mu_i}, \quad i \in D(n).
\]  

\( \text{(12)} \)

The WSQ policy routes the MHS to the work station associated with the smallest value of \( R(n_i) \). FSQ policy is used in case of a tie.

Tables 1 and 2 present results of the optimal policies and the heuristics described above. Performance measures for the FSQ and the WSQ policies were computed analytically by prescribing \( k(n) \) according to the selected heuristic and then inserting it into the functional equations of the model. The conditional transition probabilities and the mean values of the state-holding times were also computed exactly according to the relevant control policy. The PSQ results are taken from Yao and Buzacott [41], which presents results only for Instance 1. The performance measures presented are: work station throughputs (\( r_i \)), work station utilizations (\( U_i \)), number of starvation periods at each station (\( N_{SP,i} \)), expected duration of each starvation period at a given station (\( SD_i \)), expected buffer occupancy at the decision epochs (\( MBO_i \)), expected duration of each buffer occupancy at the MHS (\( BD \)), MHS utilization (\( UH \)), and the mean weighted production value (\( g \)).

Table 2 presents an additional set of FMS parameters used for comparative sensitivity analysis of the heuristic control policies: design parameters considered here are the relative MHS capacity and the local buffers. Instance 4.1 serves as the base-line case for Table 3 since the MHS capacity is set equal to the average aggregate throughput of the stations. In Instance 4.2 the MHS is "under-powered," and Instance 4.3 depicts a case where the "under-powered" MHS serves smaller buffers. Similarly, the MHS is "over-powered" in Instance 4.4, and it provides even larger buffers in Instance 4.5.

Comparisons between the optimal and two heuristic policies for Instance 4 are presented in Table 4. The impacts of incrementing the buffers (e.g., Instance 4.2 vs. 4.3 in Table 4) and of increasing the MHS capacity (e.g., Instance 4.1 vs. 4.2 in Table 4) on the FMS throughput are clearly illustrated for all these policies. From the results in Tables 1, 3, and 4 it is clear that under the FSQ policy the mean buffer occupancies at decision epochs (MBO) are very close. This is an anticipated result for that policy. The two heuristic pol-

<table>
<thead>
<tr>
<th>Performance Measures</th>
<th>Policy</th>
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<tr>
<td>( i )</td>
<td>OPTIMUM</td>
</tr>
<tr>
<td>( r_i )</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>( MBO_i )</td>
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</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>( UH )</td>
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</tr>
<tr>
<td>( g )</td>
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<td>( i )</td>
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<tr>
<td>( r_i )</td>
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<tr>
<td></td>
<td>2</td>
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<tr>
<td>( MBO_i )</td>
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<td></td>
<td>2</td>
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<td>( g )</td>
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Table 3. The FMS Parameters for Instance 4

<table>
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<tr>
<th>Instance #</th>
<th>Buffer Spaces</th>
<th>MHS Capacity</th>
<th>Comments</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$B_1$</td>
<td>$B_2$</td>
<td>$\mu_1$</td>
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<tr>
<td>4.1</td>
<td>3</td>
<td>3</td>
<td>140</td>
</tr>
<tr>
<td>4.2</td>
<td>3</td>
<td>3</td>
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</tr>
<tr>
<td>4.3</td>
<td>2</td>
<td>2</td>
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</tr>
<tr>
<td>4.4</td>
<td>3</td>
<td>3</td>
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</tr>
<tr>
<td>4.5</td>
<td>6</td>
<td>6</td>
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</tr>
</tbody>
</table>

All these configurations assume: $W_1 = 45, W_2 = 20, \lambda_1 = 50, \lambda_2 = 100$

Table 4. Comparative Sensitivity Analysis of the Heuristic Control Policies: Instance 4

[ΔFSQ(%) and ΔWSQ(%) denote the relative deviation from the results obtained by the optimal policy]

<table>
<thead>
<tr>
<th>Instance</th>
<th>Performance Measure</th>
<th>Policy</th>
<th>Relative Performance</th>
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<tbody>
<tr>
<td></td>
<td>OPTIMUM</td>
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<td>WSQ</td>
</tr>
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<td>4.1</td>
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<td>0.82</td>
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<tr>
<td></td>
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<td>3628</td>
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<td>4.2</td>
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<td>0.89</td>
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<tr>
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<td></td>
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<td>0.85</td>
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<td>4233</td>
<td>4228</td>
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Figures 2, 3, and 4 depict the dynamic decision policies for Instances 1, 2, and 3 respectively. Consider, for example, state $n=\{1,2\}$ in Figure 2: the optimal decision for that state is to route a part to work station 1, $(k^*(1,2)=1)$. These figures seem to indicate that in these cases the MHS is never kept idle (except for the blocked states $(n=B)$), all buffers are utilized, and the optimal decision regions in each state space are simple connected sets. These figures also show that the boundary curves between the two decision regions are non-decreasing as a function of $n_1$ and $n_2$. This means that, if the optimal decision for a given state $\{n=\vec{n}_1, \vec{n}_2\}$ is, say, $k=2$, then it will be the same $k$ for all unblocked states having $\vec{n}_1 \leq n_1 \leq \vec{B}_1$ and $n_2 = \vec{n}_2$, or vice versa for $k=1$.

These figures show that FSQ and WSQ have a decision structure similar to that of the optimal policy. In Figure 3 we see that the response structure of WSQ differs from that of the optimal policy by just a single state. This explains why the WSQ policy provides response patterns and performance measures which are close to those of the optimal policy. The state in which the policies are different is an interior joint: both buffers are non-empty and the stations are actively producing parts under the optimal or the WSQ policy.

Next we studied another FMS model having four heterogeneous work stations ($M=4$). The local buffer spaces, the MHS delivery rates, the production rates and products' values are given by $(B_1=B_2=B_3=B_4=2)$, $(\mu_1=200, \mu_2=340, \mu_3=400, \mu_4=260)$, $(\lambda_1=30, \lambda_2=60, \lambda_3=70, \lambda_4=120)$, and $(W_1=50, W_2=30, W_3=25, W_4=20)$, respectively. Denote this configuration as Instance 5. Table 5 presents a sample of several system states and the associated policy decisions. It shows the differences between the response patterns of the optimal and the heuristic policies. The weighted values are 6393, 6334 and 6363 $/hour for the optimal, FSQ, and WSQ policies, respectively. In this instance the per-
Performance ranking of all three policies is almost the same as in the preceding cases. The FSQ and WSQ are as good as the optimal policy to two significant digits, even though they do not always follow the general response pattern of the optimal policy: apparently, those decision epochs in which the heuristic control policies do not coincide with the optimal responses are reached with smaller probabilities, as designated by the values of \( \pi(n, k) \) in Table 5. Overall, the linear correlation coefficient between the equilibrium state probabilities of the optimal policy and FSQ is 0.968, and it is 0.934 between the optimal policy and the WSQ heuristic.

Finally, in Instance 6 we studied a set of FMS topologies with two stations and Erlang distributed MHS times. We assumed the mean delivery rate to stations one and two is 100 parts per hour \( (\mu_1 = \mu_2 = 100) \) with three- and four-stage Erlang distributions \( (E_3 = 3, E_4 = 4) \) respectively. The production rates of the stations are 50 and 100 parts per hour \( (\lambda_1 = 50, \lambda_2 = 100) \), and the values assigned to the parts produced are 20 and 250 dollars each \( (W_1 = 20, W_2 = 250) \). We considered two operational modes for the MHS. Under the first mode, denoted as CO (continuous operations), the MHS must remain active as long as there are some empty buffers \( (E = B) \). Under the second operational mode, denoted as TS (temporary suspension), the MHS can remain idle at certain system states even if some buffers are not completely full. In doing so, the MHS "hedges" its capacity for future, more rewarding, delivery calls. This last mode is the one implemented in all of the preceding optimal solutions. The sample results presented in Table 6 demonstrate that in all but the last case the temporary suspension option (TS) improved the FMS operations because it increased the value of the output stream \( (g) \) while reducing the MHS utilization \( (UH) \). The benefits generated by this option tend to diminish with the increase in the average buffer capacity \( (E) \). The superiority of the temporary suspension option is based on increasing the value contribution of each MHS delivery, rather than on increasing the MHS utilization. This is an important feature, as heavily utilized MHS's tend to become bottlenecks which adversely offset the FMS throughput. Given the clear benefits of the temporary suspension option, it is worthwhile investigating the applicability of this approach for scheduling other key resources in automated manufacturing.

Conclusions

A new version of the throughput maximization problem in FMS's is defined, formulated, and solved. The structure and performance of optimal allocation policies are investigated and compared with closed-loop heuristic control policies. The proposed Weighted Shortest Queue (WSQ) policy is of practical significance because the resulting weighted throughputs are approximately only one percent below the optimum, and in addition it is easier to compute and to implement. Under WSQ, dynamic allocation decisions can be determined for each state individually by performing very simple algebraic operations on the observable state variables. The optimal policy, on the other hand, can only be generated by solving the complete set of state-transition dynamic programming equations.

Regarding the performance of the FMS, we see that,
Table 5. Optimal Dynamic Routing Decisions Generated by the Optimal and Two Heuristic Control Policies: Instance 5

<table>
<thead>
<tr>
<th>State [n] n n n n</th>
<th>State Number</th>
<th>( k^*(n) ) OPTIMUM FSQ WSQ</th>
<th>( x(n,k^*) ) OPTIMUM FSQ WSQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0</td>
<td>1</td>
<td>3 4 4</td>
<td>0.000 0.000 0.000</td>
</tr>
<tr>
<td>0 0 0 1</td>
<td>2</td>
<td>3 3 3</td>
<td>0.003 0.007 0.004</td>
</tr>
<tr>
<td>0 0 0 2</td>
<td>3</td>
<td>3 3 3</td>
<td>0.000 0.000 0.000</td>
</tr>
<tr>
<td>0 0 1 0</td>
<td>4</td>
<td>2 4 4</td>
<td>0.001 0.002 0.000</td>
</tr>
<tr>
<td>1 1 0 0</td>
<td>37</td>
<td>3 4 4</td>
<td>0.008 0.006 0.012</td>
</tr>
<tr>
<td>1 1 0 1</td>
<td>38</td>
<td>3 3 3</td>
<td>0.007 0.016 0.017</td>
</tr>
<tr>
<td>1 1 0 2</td>
<td>39</td>
<td>3 3 3</td>
<td>0.002 0.006 0.008</td>
</tr>
<tr>
<td>1 1 1 0</td>
<td>40</td>
<td>3 4 4</td>
<td>0.011 0.012 0.015</td>
</tr>
<tr>
<td>1 1 1 1</td>
<td>41</td>
<td>3 4 4</td>
<td>0.020 0.033 0.033</td>
</tr>
<tr>
<td>1 1 1 2</td>
<td>42</td>
<td>3 1 3</td>
<td>0.005 0.024 0.020</td>
</tr>
<tr>
<td>1 1 2 0</td>
<td>43</td>
<td>4 4 4</td>
<td>0.019 0.007 0.002</td>
</tr>
<tr>
<td>1 1 2 1</td>
<td>44</td>
<td>2 4 4</td>
<td>0.032 0.019 0.009</td>
</tr>
<tr>
<td>1 1 2 2</td>
<td>45</td>
<td>2 2 1</td>
<td>0.010 0.025 0.007</td>
</tr>
<tr>
<td>1 2 0 0</td>
<td>46</td>
<td>3 4 4</td>
<td>0.003 0.002 0.003</td>
</tr>
<tr>
<td>1 2 0 1</td>
<td>47</td>
<td>3 3 3</td>
<td>0.008 0.006 0.007</td>
</tr>
<tr>
<td>1 2 0 2</td>
<td>48</td>
<td>3 3 3</td>
<td>0.003 0.004 0.004</td>
</tr>
<tr>
<td>1 2 1 0</td>
<td>49</td>
<td>4 4 4</td>
<td>0.009 0.004 0.004</td>
</tr>
<tr>
<td>1 2 1 1</td>
<td>50</td>
<td>3 4 4</td>
<td>0.012 0.015 0.013</td>
</tr>
</tbody>
</table>

Table 6. The FMS Performance Under the Continuous Operations (CO) and the Temporary Suspension (TS) Modes: Instance 6 (Process times follow Erlang distribution \( \lambda_1 = 3, \lambda_2 = 4 \))

<table>
<thead>
<tr>
<th>Buffer Allocation ( B )</th>
<th>Weighted Throughput Value ($/hour$)</th>
<th>MHS Utilization ( g ) CO TS</th>
<th>MHS Utilization ( UH ) CO TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11,527</td>
<td>0.734</td>
<td>0.499</td>
</tr>
<tr>
<td>2</td>
<td>16,704</td>
<td>0.932</td>
<td>0.710</td>
</tr>
<tr>
<td>2</td>
<td>20,944</td>
<td>0.974</td>
<td>0.849</td>
</tr>
<tr>
<td>3</td>
<td>19,395</td>
<td>0.979</td>
<td>0.801</td>
</tr>
<tr>
<td>4</td>
<td>20,898</td>
<td>0.993</td>
<td>0.849</td>
</tr>
<tr>
<td>6</td>
<td>10,910</td>
<td>0.882</td>
<td>0.499</td>
</tr>
</tbody>
</table>

It may be worthwhile to examine whether similar closed-loop heuristics work well for other types of FMS's, particularly those involving unreliable work stations, general transportation and processing times, and multiple part-routing options. Another important unresolved problem is the derivation of heuristic rules which can capture the temporary suspension feature of the optimal policy. Furthermore, there is a need to analyze the interaction between the part-routing policies created here and the higher-level production planning scheme which determines the desired processing rates, the manufacturing mix, and the scheduled deliveries of finished products [8], [24], [25], [41].

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REFERENCES


Appendix

Throughput Maximization for an Unreliable FMS

In this appendix we present the semi-Markovian extension of (8)-(11) aimed at modeling part-routing decisions in an unreliable FMS. It is assumed that the MHS or the stations can be either up or down, and that they are subject to breakdowns only when they are active. The MHS decisions are evaluated whenever a station is repaired or when it breaks down. The constant breakdown and repair rates of the stations are $\xi$ and $\eta$, $(i = 1, 2, \ldots, M)$, and those of the MHS are $\xi_0$ and $\eta_0$, respectively. The stations’ MHS status is given by $\alpha_i$ [$\alpha_0$], where the values of 0 or 1 refer to the down and up states. The content of buffer $i$ can be determined by $j_i$ units if $0 \leq j_i \leq n_i$ and $\alpha_i = 1$. Hence the set of future buffer states is

$$TB(n, \alpha) = \{ j | 0 \leq j_i \leq n_i, \cap \alpha_i = 1 \} \quad (A1)$$

The ergodic transition rate following decision $k$ can be derived from:

$$\tau(k) = \begin{cases} 
\sum_{i \in A(g)} \left( (1 - \alpha_i) \eta_i + \alpha_i (\xi + \mu) \right) + \alpha_0 (\mu_0 + \xi_0) + (1 - \alpha_0) \eta_0 \\
\text{if } k = 1, \ldots, M \\
\sum_{i \in A(g)} \left( (1 - \alpha_i) \eta_i + \alpha_i \xi_i + \alpha_i \lambda_i \right) \\
\text{if } k = 0.
\end{cases} \quad (A2)$$

The joint transition probability that in stations’ state $\alpha$ a station $i \in A(g)$ will use $j_i$ ($0 \leq j_i \leq n_i$) of its $n_i$ parts during the time interval needed by the MHS to try and deliver a part to station $k$ is $P(j_i | n, \alpha, \tau(k))$. The derivation of $P(j_i | n, \alpha, \tau(k))$ is similar to the one discussed above with the following exceptions: i) use $j_i = 0$ whenever $\alpha_i = 0$, and ii) replace $\mu_0$ by $\tau(k)$.

The first value equation used (A3) represents an arbitrary value point in which all buffers are empty ($n = 0$) and all stations and the MHS are up ($\alpha = 1$, $\alpha_0 = 1$).

$$v(n, \alpha = 1, \alpha_0 = 1) = \max_k \left\{ W_k - \frac{\xi}{\tau(k)} + \frac{\mu_k}{\tau(k)} v(e^k, \alpha, 1) \right\} = 0 \quad (A3)$$

(arbitrary starting point) $n = 0$, $\alpha = 1$.

The next set of equations (A4) deals with the case in which a decision can be made; the MHS is up ($\alpha_0 = 1$), and the buffers are not all blocked ($n < B$). The future states depicted here are part delivery to station $k$, breakdown of line $i$, MHS breakdown, or an end of repair event in station $i$. The possibility of making no decision ($k = 0$) due to a temporary suspension of the MHS is also recognized by $u(n, \alpha, 1)$, as defined by (A7).

$$v(n, \alpha, \alpha_0 = 1) = \max_k \left\{ \max_{i \in A(g)} \left\{ W_i - \frac{\xi}{\tau(k)} \right\} + \frac{1}{\tau(k)} \sum_{j \in TB(n, \alpha_0)} P(j | n, \alpha, \tau(k)) v(n - J^e, \alpha, 1) \mu_k \right\}$$

+ \sum_{i \in A(g)} \alpha_i \xi_i v(n - J, \alpha - e^i, 1) + \xi_0 v(n - J, \alpha, 0) + \sum_{i \in A(g)} (1 - \alpha_i) \eta_i v(n - J, \alpha + e^i, 1) \right\}, u(n, \alpha, 1) \right\} \quad (A4)$$

(unblocked and operational MHS)

$0 \leq n < B$, $0 \leq \alpha \leq 1$, $\alpha_0 = 1$

When the MHS is down ($\alpha_0 = 0$), no decision can be made. The FMS may evolve to one of three possible future states: failure of one of the active stations, an end-of-repair event at one of the stations currently down, or an end of repair event at the MHS. These states are given by the following equations:

$$v(n, \alpha, \alpha_0 = 0) = -\frac{\xi}{\tau(k)} + \frac{1}{\tau(k)} \sum_{i \in TB(n, \alpha_0)} P(j_i | n, \alpha, \tau(k))$$

$$\left\{ \sum_{i \in A(g)} \alpha_i \xi_i v(n - J, \alpha - e^i, 0) + \sum_{i \in A(g)} (1 - \alpha_i) \eta_i v(n - J, \alpha + e^i, 0) + \eta_0 v(n - J, \alpha, 1) \right\} \quad (A5)$$

(MHS is down) $0 < n < B$, $0 \leq \alpha \leq 1$, $\alpha_0 = 0$.

In the blocked (A6) or in the suspended case (A7), the FMS state may evolve to one of the following future states: a unit decrement in the input buffer at one of the active stations, failure of one of the active stations, or an end-of-repair event at one of the down stations.

$$v(B, \alpha, 1) = u(B, \alpha, 1) \quad (A6)$$

(blocked case) $n = B$, $0 \leq \alpha \leq 1$, $\alpha_0 = 1$

$$u(n, \alpha, 1) = -\frac{\xi}{\tau(k)} + \sum_{i \in A(g)} \frac{\alpha_i \lambda_i}{\tau(k)} v(n - e_i, \alpha, 1)$$

+ \sum_{i \in A(g)} \frac{(1 - \alpha_i) \eta_i}{\tau(k)} v(n, \alpha + e_i, 1) + \sum_{i \in A(g)} \frac{\alpha_i \xi_i}{\tau(k)} v(n, \alpha - e_i, 1) \quad (A7)$$

$0 < n < B$, $0 \leq \alpha \leq 1$, $\alpha_0 = 1$ ($k = 0$)

(MHS is suspended temporarily)
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