Competitive delivered pricing and production

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Received November 1991, final version received January 1992

This paper studies price and production competition between profit-maximizing firms that use delivered spatial prices. Firms produce an identical good and customers buy from the firm offering the least delivered price. Each firm's transportation cost is quantity dependent and customers' demands are price elastic. This contrasts with much of the literature on competitive spatial pricing that assumes inelastic customer demand and linear firm costs, and which ignores the firm's production problem. This paper makes three contributions. First, a game-theoretic model is defined and the existence of the delivered price and production equilibrium is proved. Second, properties of the equilibrium are shown including patterns of spatial pricing. Properties of pricing and production are different from the case when transportation costs are quantity independent. For example, more than one firm may serve a single market. Third, it is shown that in equilibrium, competing firms may optimally choose delivered mill prices and have incentive to locate coincidentally. This result can explain coincident location of mill pricing firms.

Key words: Delivered pricing; Production; Competition; Transportation

JEL classification: 022; 941

1. Introduction

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A technical contribution of this paper is the incorporation of non-linear firm cost into a game-theoretic model of competitive spatial pricing. Although the microeconomic literature usually assumes diseconomies of scale in production and transportation, the competitive spatial pricing and location literature typically assumes linear production and transportation costs because of technical difficulties. However, there are some papers that study price competition between firms with quantity-dependent costs; for example, Anderson and Neven (1990) study price patterns arising from spatial Cournot competition. The incorporation of non-linear transportation cost is important from an economic perspective because empirical evidence indicates that cost for some common transportation modes is non-linear in quantity. For example, Harker and Friesz (1985) present empirical data that show that railroad transportation average cost functions are ‘U-shaped’ and water barge average cost functions are convex and monotone increasing. Fig. 1 reproduces estimates from one study done for the U.S. Department of Transportation [CACI, Inc. (1980)] of the average cost function for different railroad routes.

Research in competitive delivered pricing began with Hoover (1936) who characterized the pattern of spatial prices with quantity-independent transportation costs. Hurter and Lederer (1985) and Lederer and Hurter (1986) analyzed competitive location of firms that employ delivered prices. Thisse and Vives (1988) studied Hoover’s model with price-elastic demand examining a firm’s choice of mill or delivered pricing policies. Anderson and dePalma (1988) considered competitive location and delivered pricing of firms when customer tastes were heterogeneous. Haddock (1982) showed that ‘Pittsburgh plus’ pricing resulted from price competition between delivered pricing firms. He demonstrated that price competition may cause an isolated firm to use the site of coincident firms as a price base if the coincident firms use mill prices. Unexplained were the conditions under which the coincident firms set mill prices.

This paper makes a distinction between mill pricing systems recognized by practitioners but not economists. Logistics practitioners have recognized that the sender may have an economic advantage in transportation technology or in negotiating favorable carrier rates. Therefore delivered mill pricing may be advantageous to both parties. Logistics managers refer to ‘F.O.B. destination’ pricing, meaning that the sender ‘… is liable for transportation charges … and controls carrier selection and routing of a given shipment’ [Talley (1983, p. 93)]. Delivered mill pricing is F.O.B. destination pricing where the delivered price is equal to the sum of the price at the shipping point plus the
sender's actual transportation cost. This contrasts with 'F.O.B. origin' pricing where the '... receiver selects carriers and specifies routings and the receiver is liable for all transportation charges'.

This distinction between delivered mill pricing and F.O.B. origin pricing has not been made in the economics literature. For example, Greenhut et al. (1980) studied spatial pricing patterns of U.S. firms with significant transportation costs and asked randomly sampled firms to report examples for the
total price paid by customers. They did not ask whether firms used F.O.B. origin or F.O.B. destination pricing. Their results report that only 32.7% of firms used mill prices of either sort exclusively. This implies that over two-thirds of the firms used discriminatory delivered prices of some sort, indicating the potential of these firms to use delivered mill prices.

To simplify exposition and to stay consistent with the current use of terminology we refer to the two mill pricing systems as 'delivered mill pricing' and 'mill pricing', respectively. This paper shows that competition may cause delivered pricing firms to use delivered mill prices. Delivered pricing firms may have incentive to locate coincidentally in equilibrium and to use delivered mill prices. These results contrast with location analysis of mill pricing firms: D'Aspremont et al. (1979) show that for the Hotelling (1929) model a price equilibrium does not exist with quantity-independent transportation cost if firms locate too closely. In a more general model with quantity-independent costs, D'Aspremont et al. (1983) show that firm's equilibrium locations cannot be coincident even if a price equilibrium were to exist. A conclusion of these models is that mill pricing firms cannot locate coincidentally in equilibrium. In contrast with these oligopolistic models, Anderson and Engers (1989) examined location competition of mill pricing firms with quantity-dependent production cost and show coincident location can occur if mill prices are perfectly competitive.

This paper also helps to clarify some unresolved issues about 'Pittsburgh plus' prices: coincident mill pricing firms can be delivered pricing firms that have incentive to agglomerate and set mill prices.

A general conclusion of this work is that care must be taken in selecting models to study location competition of firms that are empirically observed to use mill prices, and to make a distinction between the delivered mill pricing and mill pricing systems. Studying competitive location using the Hotelling model is inappropriate for delivered mill pricing firms. A firm that sets delivered mill prices may not set mill prices when located elsewhere or under different demand conditions. Therefore, use of the Hotelling model will result in incorrect predictions of competitive locations for these firms.

This paper is organized as follows. Section 2 presents our basic model with delivered prices. Section 3 explains how competition between firms is modeled. Section 4 describes the production and price equilibrium, with patterns of spatial prices characterized in section 5. Section 6 presents extensions.

2. The model with delivered pricing

Assume that $N = \{1, \ldots, n\}$ firms locate in a compact convex set $Z$, a subset of $R^2$, in order to produce and sell a homogeneous good to customers located in $Z$. Without loss of generality, assume that each firm owns a single
facility in \( Z \) at \( z_i \), which may be thought of as plant or warehouse. Customers are found at exactly \( m \) market locations given by the set \( M \). The \( m \) market locations are \( z_1, \ldots, z_m \); at market \( z \in M \) there are \( \rho(z) \) customers. Although a discrete distribution of customers is assumed for simplicity, a continuous distribution of customers could alternatively be assumed. All of the results of this paper can be extended to this case.

Each customer's demand for the good, \( d(p) \), is a function of its delivered price, \( p \). For simplicity, the demand function is assumed identical for all customers, but our results generalize to the case of non-identical demands. It is assumed that \( d(p) \) is continuous, equal to zero at a finite price \([d(\alpha) = 0 \text{ for some } \alpha > 0]\) and monotone decreasing below this price \([d'(p) < 0 \text{ for } \alpha \geq p \geq 0]\). These assumptions are relatively general. Another important assumption is that each customer buys exclusively from the firm offering the least delivered price.

Firms compete by setting prices and production allocations for each market. It is assumed that firms set delivered prices to customers. Delivered pricing can be justified whenever firms' transportation technology is at least as efficient as any customer's. [See Lederer and Hurter 1986] for a full justification of this assumption, and Phlips (1983) for a discussion of such markets.] Each firm's problem is to decide what delivered prices to set and what production to make for each market.

The set of delivered prices by a single firm to all the markets is its 'price schedule'. Firm \( i \)'s price schedule, \( p_i \in R^m_+ \), specifies a delivered price, \( p_i(z) \), for each market \( z \in M \). \( P \) is the space of all price schedules of a single firm and a vector of price schedules for all firms is written \( \bar{p} = (p_1, \ldots, p_n) \) with \( p_i \in P \). \( \bar{P} \) is the space of price schedules for all firms.

The set of delivered quantities by a single firm to all the markets is its 'production decision'. Firm \( i \)'s production decision is given by a vector \( q_i \in R^m_+ \) specifying a quantity \( q_i(z) \geq 0 \) to be shipped from \( z_i \) to \( z \) for each \( z \in M \). The space of all production decisions for a single firm is written \( Q \) and the vector of production decisions for all firms is written \( \bar{q} = (q_1, \ldots, q_n) \) with \( q_i \in Q \). \( \bar{Q} \) is the space of production decisions for all firms.

If a firm does not satisfy customer demand at announced prices, customers may usually search for an alternative supply. However, this paper assumes that customer demand that cannot be satisfied by a least pricing firm is not filled by another firm pricing above this level. This assumption can be justified and motivated in four ways. First, it guarantees the existence of a pure strategy Bertrand equilibrium which may not exist if unsatisfied demand is provided by another firm; see Tirole (1989) for a discussion of this issue. At the Bertrand equilibrium all demand is satisfied. Second, the purpose of this paper is to study patterns of pricing. Price patterns generated using the assumption are similar to those induced with more complicated market rationing rules. For example, Gerner (1986) analyzes and numerically
computes the mixed strategy equilibrium for Bertrand competition for the so-called 'proportional rationing rule' described by Tirole. An analysis of mixed strategy equilibria for the spatial Bertrand problem could theoretically be done for the model of this paper since our model decomposes into separable local Bertrand games. The analysis is computationally complex. However, the qualitative insights gained would be similar to those presented here differing only for markets where more than one firm serves customers. At these markets competing firms use mixed price strategies, pricing above competitive prices (marginal cost) but less than monopoly prices, with the mean and support of the distributions rising with firms' costs. This pattern is like that found here and the price pattern will be identical at other markets. Third, our assumption may be reasonable for markets where all customers adopt the policy: 'buy only from the least price firm'. This policy is attractive to customers because, as we will see in Lemma 4.2, it forces competing firms to price at marginal cost in equilibrium if it is efficient for two or more firms to serve a market. It is also informationally efficient: customers need not know firms' cost structures to implement the strategy. (Unfortunately, it is not a subgame perfect strategy for a single play game but it can be shown that it is subgame perfect in a repeated version of the game.) This policy is certainly attractive in markets where firm cost is unknown or uncertain to customers because of changing customer demands or firm costs. Fourth, our assumption is equivalent to assuming that firms are price-takers, at least for markets where it is efficient for two or more firms to serve. For these markets perfect competition reigns and our analysis extends existing research on competitive location of price-taking firms. For example, Anderson and Engers (1989) study location competition of mill pricing firms when prices are perfectly competitive.

Next the production and transportation functions of the firms are described. It is assumed that the firms have their own transportation resource or have contracted for supply of transportation services from a carrier at a favorable rate. The cost structure is relatively general. Firms have location-dependent production and transportation costs given by $K_i(z_i, q_i)$:

$$K_i(z_i, q_i) = C_i'(z_i) \sum_{z \in M} q_i(z) + \sum_{z \in M} F_i(z_i, z, q_i(z)).$$

If firm $i$ is located at $z_i$, its production cost is a location-dependent unit cost $C_i'(z_i)$ times total output. Production costs are assumed to exhibit constant marginal cost but these results can be generalized to the case of convex increasing marginal cost: see Lederer (1992) for details. The simpler case is presented to ease notation and reduce the length of this paper.

Firm $i$'s transportation cost is separable by market. The cost of shipping the quantity $q_i(z)$ from $z_i$ to $z$ is $F_i(z_i, z, q_i(z))$. It is assumed that $F_i(z, z, q_i(z))$ is continuous with respect to $q_i(z)$, $F_i(z_i, z, 0) = 0$, $F_i'(z_i, z, q_i(z)) > 0$. 


and $F'_i(z, z, q_i(z)) > 0$ for $q_i(z) \geq 0$, where the primes indicate the first and second derivative of $F_i$ with respect to $q_i(z)$ respectively. These assumptions state that there is a positive marginal transportation cost, indicating that there is a loading charge for each unit shipped, and that transportation costs are monotone and strictly convex in volume shipped. Note that the transportation cost may arise because of diseconomies of scale in loading, and be unrelated to actual geographical movement. This transportation cost function contrasts to linear transportation cost functions usually assumed in location models. Assumptions on $F_i$ are consistent with evidence cited by Harker and Friesz (1985): the average transportation cost function is monotone convex and increasing. If a firm-specific fixed cost was assumed, then average transportation cost would be U shaped. We ignore such fixed costs for simplicity.

**Firm demand and profit under production and price decisions**

Customers purchase from the firm with the least delivered price. Given any $\bar{p} \in \bar{P}$, we can specify the demand accruing to each firm at each market. For $z \in M$, let $N(z) = \{i \in N | p_i(z) = \min_{k \in N} [p_k(z)]\}$; $N(z)$ is the set of firms that set the lowest price at $z$. If $N(z) = \{i\}$, firm $i$ serves the customers at $z$ alone and can sell $d(p_i(z))p(z)$ units.

If the cardinality of $N(z)$ is greater than 1, several firms can serve market $z$. The demand accruing to firm $i \in N(z)$ is a function of price schedules and production decisions. This function is given by $r$, which we refer to as a ‘market sales rule’.

**Definition 2.1.** A market sales rule is a mapping $r : P \times Q \to R^+ \times M$ having the properties:

(a) If $i \notin N(z)$, $r_i(p_1, q_1, \ldots, p_n, q_n, z) = 0$.

(b) If $\sum_{i \in N(z)} q_i(z) \leq d(p_j(z))p(z)$ for $j \in N(z)$, then $r_j(p_1, q_1, \ldots, p_n, q_n, z) = q_j(z)$ for all $j \in N(z)$.

Market sales rule $r$ awards $r_j(p_1, q_1, \ldots, p_n, q_n, z)$ units of sales to firm $j$ at market $z$. Restriction (a) says that only low pricing firms can sell to a market, and (b) says that if the total deliveries of low pricing firms to a market is less than market demand, each low pricing firm sells its entire production. Note that a firm may send more of a good to a market than it sells if the total quantity sent by firms exceeds market demand. These two restrictions are reasonable, and relatively weak. To simplify notation, we write the market sales rule as $r_j(\cdot, z)$.

The profit for firm $i$ for firms’ price schedule and production decisions is

$$\Pi'_i(p_1, q_1, \ldots, p_n, q_n) = \sum_{z \in M} \{[p_i(z) - C_i(z_i)]r_i(\cdot, z) - F_i(z_i, z, q_i(z))\}. \quad (2.1)$$
Firm profit in equilibrium will be shown to be independent of the market sales rule selected.

3. Modeling competition

Competition is modeled as a game. Firms privately and simultaneously choose price schedules and production decisions, these become known, and payoffs are made according to (2.1). This formulation is appropriate in markets where pricing and production changes can both occur frequently. The simultaneous selection of prices (or production) by different firms is appropriate when firms have similar costs and time horizons to set and change prices. Simultaneous choice by a single firm of prices and production is reasonable when either can be set or changed in roughly the same time frame and with similar cost.

Next the strategy space for this game is specified. A strategy for a firm is a duple \( \{p_i, q_i\} \), where \( p_i(q_i) \) specifies firm \( i \)'s price schedule (production). The space of all strategies of a single firm is \( P \times Q \). When each firm chooses a strategy from \( P \times Q \) the payoff to firm \( i \) is \( \Pi_i(p_1, q_1, \ldots, p_n, q_n) \).

Solution concept for the game

We seek a non-cooperative Nash equilibrium for this game, a vector of strategies for the firms \( (p_1^*, q_1^*, \ldots, p_n^*, q_n^*) \) such that for each firm \( i \):

\[
\Pi_i(p_1^*, q_1^*, \ldots, p_i^*, q_i^*, \ldots, p_n^*, q_n^*) \geq \Pi_i(p_1^*, q_1^*, \ldots, q_i, p_i, \ldots, p_n^*, q_n^*)
\]

for all \( \{p_i, q_i\} \in P \times Q \).

A few additional restrictions to the 'solution' of the game are made because the Nash equilibrium is not unique. These 'refinements' ensure the existence of a unique price and production equilibrium.

First, we require firms not to price below their marginal cost at any market. It is irrational for a firm to price below marginal cost at a market where it is serving any customers as the firm could reduce production or raise price, and increase its profit. If the firm is serving no customers, the firm need not price below marginal cost at any market as the firm could raise its price and its profit would be unchanged. Pricing below marginal cost is seen to be a weakly dominated strategy, and we restrict firms from using these weakly dominated strategies. It is important to note, however, that Nash equilibria for this game exist with some firms using these weakly dominated strategies. In equilibrium a firm could price below its marginal cost, understanding a competitor will match its price and serve all demand. Using weakly dominated strategies lowers competitors' profits but does not raise the firm's.
Second, we require that in equilibrium supply equals or exceeds demand. This is an important Nash equilibrium refinement because there are Nash equilibria where customer demand is not satisfied. There are two justifications for this refinement: first, other Nash equilibria are Pareto dominated by the one we propose, and second, continued shortages will encourage entry to the market when fixed costs are low enough. We will refer to a vector of strategies for the firms where customer demand is satisfied as 'demand satisfying'.

These two restrictions parallel those introduced in Lederer and Hurter (1986). The interested reader is directed there for further discussion. We will refer to a demand satisfying Nash equilibrium where firms price above marginal cost as a 'price–production equilibrium'.

4. Price–production equilibrium

In this section we study the properties of the price–production equilibrium. The price–production equilibrium will be shown to be (essentially) unique.

To characterize an equilibrium, the following problems must be defined.

Definition 4.1. Fix a vector of price schedule and production decisions for all firms: \((p^*, q^*)\). For each firm \(i \in N\) define problem \(P(i)\):

\[
\max_{p_i, q_i \in R^n} \sum_{z \in M} \left\{ \left[ p_i(z_i) - C_i(z_i) \right] q_i(z) - F_i(z_i, z, q_i(z)) \right\}
\]

subject to (1) \(0 \leq q_i(z) \leq d(p_i(z))\), and (2) \(p_i(z) - \min_{j \neq i} [p_j^*(z)] \leq 0\),

if \(q_i(z) > 0\).

Problem \(P(i)\) maximizes firm \(i\)'s profit with respect to its price schedule and production decisions when faced by competitors' price schedules assuming that if firm \(i\) has the least price at a market, firm \(i\) can supply the market up to its demand.

Three properties are necessary and sufficient for \((p^*, q^*)\) to be a price–production equilibrium:

Property A: For each \(i \in N\), \((p^*_i, q^*_i)\) solves \(P(i)\).

Property B: For each \(i \in N\) and \(z \in M\),

\[
p^*_i(z) \geq F_i(z_i, z, q^*_i(z)) + C_i(z_i).
\]

Property C: For each \(z \in M\),

\[
\sum_{i \in N} q^*_i(z) = d \left( \min_{j \in N} [p^*_j(z)] \right) \rho(z).
\]

Property A states that each firm responds optimally to its competitors' prices. Property B requires firms' to price above marginal cost. Property C
states that supply equals demand at each market. We now have the following lemma.

Lemma 4.1. \((\tilde{p}^*, \tilde{q}^*)\) is a price–production equilibrium if and only if Properties A, B and C hold.

Proof. Suppose \((\tilde{p}^*, \tilde{q}^*)\) is an equilibrium. Property B holds. If Property A does not hold for firm \(i\), then this firm can change its price schedule and production decision and increase its profits, contradicting the fact that \((\tilde{p}^*, \tilde{q}^*)\) is an equilibrium. Firm \(i\) can increase its profits by reducing its price a small amount \(\varepsilon > 0\) below the lowest price of competitors at each market and then choosing production decisions to maximize profit. Choices \((\tilde{p}^*, \tilde{q}^*)\) are demand satisfying, so that if Property C does not hold, then some firm produces and ships more to a market than it sells, contradicting the assumption of an equilibrium, therefore Properties A, B and C hold in equilibrium.

If Properties A and C hold, then each firm responds optimally to the others’ price schedule and production decision and all output is sold. Property B guarantees that firms price above marginal cost. Therefore \((\tilde{p}^*, \tilde{q}^*)\) is an equilibrium. Q.E.D.

Lemma 4.1 implies that in equilibrium each firm will sell all of its output no matter which market sales rule is assumed. Restriction (b) from Definition 2.1 shows that this implies that equilibrium price schedules and production decisions are independent of the market sales rule chosen. Therefore, firm profits using price–production equilibrium decisions are the same for all market sales rules. This enables us to drop \(r\) from our notation, understanding that a market sales rule is being used.

Each firm will choose a price schedule and production decision that maximizes its profit against the others’ price schedules and production decisions. At each market, each firm will undercut competitors’ prices and increase production when current prices are higher than the firm’s marginal cost of production and transportation. This process will result in market prices equal to the second most efficient firm’s marginal cost when it is not optimal for the firm to cut prices further to set a monopolist’s price. The following lemma formalizes this result. Notation required includes firm \(i\)’s monopoly price at \(z\): \(p_{M_i}(z)\). If customer demand is linear in delivered price, \(d(p) = a - bp\), then

\[
p_{M_i}(z) = \frac{1}{2} \left[ \frac{a}{b} + F_i(z_i, z, q_i(z_i)) + C_i'(z_i) \right].
\]
Lemma 4.2 Let \((\hat{\phi}^*, \hat{q}^*)\) be a price schedule–production decision equilibrium. Then, for all \(i \in N\) and \(z \in M\),

(A) If \(q_i^*(z) > 0\) and \(q_j^*(z) = 0\), \(\forall j \neq i\), then

\[
F_i(z_i, z, q_i^*(z)) + C_i(z_i) \leq F_j(z_j, z, 0) + C_j(z_j), \quad \forall j \neq i.
\]

Also,

\[
p_i^*(z) = \min \left[ p_{M_i}(z), \min_{k \neq i} \left[ F_i(z_k, z, 0) + C_k(z_k) \right] \right],
\]

and if \(p_{M_i}(z) > p_i^*(z)\), then there exists some \(k \in \{N - i\}\) such that

\[
p_i^*(z) = F_k^*(z_k, z, 0) + C_k^*(z_k) = \min_{j \neq i} \left[ F_j^*(z_j, z, 0) + C_j^*(z_j) \right].
\]

(B) If \(q_i^*(z) > 0\), \(q_k^*(z) > 0\) and \(k \neq i\), then

\[
F_i(z_i, z, q_i^*(z)) + C_i(z_i) = F_k(z_k, z, q_k^*(z)) + C_k(z_k)
\]

\[
= \min_{j \in N} \left[ F_j(z_j, z, q_j^*(z)) + C_j(z_j) \right]
\]

and

\[
p_i^*(z) - p_k^*(z) = \min_{j \in N} \left[ F_j(z_j, z, q_j^*(z)) + C_j(z_j) \right].
\]

(C) If \(q_i^*(z) = 0\), then \(p_i^*(z) \geq F_i(z_i, z, q_i^*(z)) + C_i(z_i)\).

The details of a formal proof follow the above arguments and are found in Lederer (1992). Part (A) states that a firm that serves a market alone has the lowest marginal cost of all firms to serve that market, and sets a price equal to the minimum of: its monopoly price and the least marginal cost of its competitors. If the serving firm does not set a monopoly price, a least marginal cost competitor must set its price to its marginal cost. Part (B) states that if two or more firms jointly serve a market, each serving firm has the lowest marginal cost of all firms to serve that market and prices at its marginal cost. Part (C) states that for all markets, each firm prices at or above the marginal cost it has when it serves no customers. These results are similar to those of Hoover (1936), and Lederer and Hurter (1986) for the case of inelastic demand and constant marginal cost, and Thisse and Vives (1988) for the case of elastic demand and constant marginal cost.

Lemma 4.2 implies an important efficiency property that holds in equilibrium: each market will be served by the firms with the least marginal cost. The most efficient firms will price at either the marginal cost of its most efficient rival or its monopoly price.

Although this lemma characterizes equilibrium price schedules, this result is meaningless without proving the existence of a price–production equili-
For any price schedule equilibrium let $p^* \in \mathbb{R}^n_-$ be the vector of least prices at each market.

**Theorem 4.1.** A price-production equilibrium exists. The production equilibrium is unique and the vector of least prices at each market, $p^*$, is also unique.

A constructive proof that can be used to solve for the equilibrium is found in the appendix.

Market prices are unique in equilibrium but price schedules are not. Lemma 4.2 parts (A) and (C) imply that equilibrium price schedules are not unique. Part (A) indicates that a firm that does not serve customers can set any price at or above its marginal cost with one restriction: if a serving firm prices below its monopoly price, a competitor with the least marginal cost must price at its marginal cost. There may be more than one such competitor. Part (C) implies that other firms price above marginal cost. Equilibrium production decisions and price schedules are uniquely defined up to an equivalence class and for any element of this class, market prices and firm profits are the same. Given equilibrium production decisions, we can select a representative equilibrium price schedule from this equivalence class:

$$
p^*_i(z) = \begin{cases} 
\min \left[ p_{im}(z), \min_{j \neq i} \left[ F_i(z_i, z, q^*_i(z)) + C_i(z_i) \right] \right], \\
\text{if } F_i(z_i, z, q^*_i(z)) + C_i(z_i) < F_j(z_j, z, 0) + C_j(z_j), \text{ for all } j \neq i; \\
F_i(z_i, z, q^*_i(z)) + C_i(z_i), \\
\text{if } F_i(z_i, z, q^*_i(z)) + C_i(z_i) = F_j(z_j, z, q^*_j(z)) + C_j(z_j) \\
= \min_{k \neq N} F'_k(z_k, z, q^*_k) + C'_k(z_k), \text{ for some } j \neq i; \\
F'_i(z_i, z, 0) + C'_i(z_i), \text{ otherwise.}
\end{cases}
$$

Finally, we conclude that firm profits can be uniquely expressed as a function of firm locations: $\Pi_i(z_1, \ldots, z_n) = \Pi_i(p^*_i q^*_i, \ldots, p^*_n q^*_n)$. Using (4.1),

$$
\Pi_i(z_1, \ldots, z_n) = \sum_{z \in M} \left\{ \min \left[ p_{im}(z), \min_{j \neq i} \left[ F'_i(z_j, z, q^*_j(z)) + C'_j(z_j) \right] \right] \\
- C'_i(z_i) \right\} q^*_i(z) - F'_i(z_i, z, q^*_i(z)).
$$
Discussion of results

Non-linear transportation costs generate several results that do not occur with linear costs. First, several firms may share service to a single market and the firms earn positive contribution for this service. In previous models with linear costs [for example, Lederer and Hurter (1986) and Thisse and Vives (1988)] if several firms serve a single market, the firms price at marginal cost, which implies that firms earn zero contribution for the market. Second, the pattern of spatial prices is quite different than with linear cost models. When transportation costs are quantity independent and linear in distance, a firm's marginal cost to serve a market is a linear function of its distance to the market. If transportation costs are quantity dependent, a firm's marginal cost to a market is a function of its transportation cost function, its competitors' costs and the market size. This implies that a firm's marginal cost (and its equilibrium price) is a complicated function of these factors. The economic interpretation of price patterns is discussed next.

5. Patterns of spatial pricing

We study patterns of spatial prices using the results of Lemma 4.2. For any market \( z \) there are three possible cases. Case one occurs when a single firm serves \( z \) alone and prices at the marginal cost of its most efficient competitor. Case two occurs when a single firm serves at its monopoly price. Case three occurs if two or more firms serve \( z \). In this instance, these firms' (say, firms \( i \) and \( j \)) marginal costs must be equal at \( z \):

\[
C_i(z) + F_i(z_i, z, q_i^*(z)) = C_j(z_j) + F_j(z_j, z, q_j^*(z)).
\] (5.1)

The first case corresponds to basing point pricing: basing point prices occur when a firm's price (say firm \( i \)'s) is equal to a fixed charge at a competitor's location plus a transportation charge based upon a competitor's transportation cost (say firm \( j \)'s). This is precisely firm \( i \)'s price: a fixed charge, \( C_i(z_j) \), plus the competitor's marginal transportation cost, \( F_j(z_j, z, 0) \). (Firm \( j \) delivers zero to \( z \) since firm \( j \) does not serve \( z \).) The second case is just monopoly pricing.

The third case corresponds to delivered mill pricing. The firm sets a delivered price to each market but the price is equal to a charge at the firm's site, \( C_i(z_j) \), (referred to as the 'mill price') plus the shipping cost generated by the sale which is the marginal transportation cost, \( F_i(z_i, z, q_i^*(z)) \). What is distinctive about this type of mill pricing is that the customer buys both the good and transportation from a single firm and pays a single charge. In other contexts, mill pricing means that the firm sets a mill price and customers purchase transportation separately. If transportation markets are
The pattern of prices generated by the three cases is analyzed next. For the following discussion we assume particular demand and transportation cost functions. Consider a linear market where the firms' transportation cost is equal to \( F_i(z_i, z, q_i^*(z)) = (1 + cq_i(z))q_i(z)(|z_i - z| + \varepsilon) \), where \( c, \varepsilon > 0 \) and \( i \in N \). The parameter \( c \) is a measure of the diseconomies of scale in transportation cost, and the parameter \( \varepsilon \) can be interpreted as a loading charge for each unit transported. The parameter \( \varepsilon \) is necessary to guarantee that \( F_i(z_i, z, q_i(z)) > 0 \). If \( \varepsilon = 0 \), then \( F_i(z_i, z, q_i(z)) = 0 \) for any \( q_i(z_i) > 0 \), violating assumptions on the transportation cost of section 2. For simplicity, assume firms' marginal production costs are location independent. Suppose the demand function is \( d(p) = a - bp \) for \( a, b > 0 \). In the following examples, the parameters are chosen as follows (unless indicated otherwise): \( a = 10, b = 1, \) and \( \varepsilon = 0.001 \).

The spatial pattern of delivered mill prices generated by the model is quite different from the pattern of mill prices in models where transportation costs are not quantity dependent. Suppose there are two firms on the linear market, with \( N = \{ A, B \} \). If both firms serve customers at market \( z \), delivered mill prices are set, and the equilibrium price is

\[
\begin{align*}
    p^*(z) &= \frac{2(|z_A - z| + \varepsilon)(|z_B - z| + \varepsilon) + C_A(|z_A - z| + \varepsilon) + C_B(|z_B - z| + \varepsilon) \rho(z)}{(|z_A - z| + \varepsilon) + (|z_B - z| + \varepsilon) + 2bc(|z_A - z| + \varepsilon)(|z_B - z| + \varepsilon) \rho(z)}.
\end{align*}
\]

(5.2)

The price is derived by expressing \( q_A^*(z) \) in terms of \( q_B^*(z) \) using eq. (5.1) after substituting the form of \( F \) assumed, and then equating total production to demand. After \( q_B^*(z) \) is found, it is substituted into (5.1) to get prices. The pattern of delivered mill prices is a complicated function of the parameters \( a, b, c, C_A, C_B \) and \( \rho(z) \). Delivered mill prices are not linear in distance \( |z_A - z| \) or \( |z_B - z| \). By differentiating (5.2) twice with respect to \( |z_A - z| \) (or \( |z_B - z| \)), it can be shown that delivered mill prices are concave increasing functions of \( |z_A - z| \) (or \( |z_B - z| \)) is \( \rho(z) \) is identical for all markets. The delivered mill price at \( z \) is sensitive to market size \( \rho(z) \).

Consider the case when \( \varepsilon = 0.9, C_A = C_B = 1, c = 9 \) and \( \rho(z) = 1 \) for all \( z \). If the firms are located at \( z_A = 0 \) and \( z_B = 3 \), the pattern of equilibrium prices and the quantities delivered by the firms as a function of customer location is shown in fig. 2. All prices shown are delivered mill prices and all customers are served by both firms. Prices are concave in the region between the firms; outside this interval the prices are concave as well.
Fig. 2. Two non-coincident firms located at 0 and 3 set delivered mill prices everywhere.

This pattern of delivered mill prices is striking because price is not linear and increasing in \( |z_A - z| \) as is the case with quantity-independent transportation cost. There is also 'cross hauling': each firm delivers to markets that are geographically closer to the other firm.

Delivered mill prices occur everywhere because it is most efficient for both firms to serve all customers. Diseconomies of scale for transportation make it efficient for both firms to serve every customer, even customers that are closer to a competitor. By varying the parameters of this example, situations can be generated where it is most efficient for only a single firm to serve some of the customers. For example, if the parameter \( c \) is decreased so that \( c = 0.75 \), the pattern of prices and production is as shown in fig. 3. Now, at some markets demand is served by a single firm which bases its price upon the marginal cost of the competitor: for example in the interval \( [-0.4, 0.2] \), firm A serves at B's marginal cost. There is also 'cross hauling' in this example.

An important case of delivered mill pricing occurs when two or more identical firms (having the same production cost and transportation cost functions) locate coincidentally. In this case, it is most efficient for the firms to jointly serve all markets, and thereby set delivered mill prices to them. As an example, consider a linear market with two identical firms, A and B, located coincidentally at the origin. Because \( |z_A - z| \) and \( |z_B - z| \) are equal, at any market where \( q_A^*(z) > 0 \), eq. (5.2) specifies a delivered mill price:

\[
p^*(z) = \frac{C' + \left( |z_A - z| + \varepsilon (1 + a \rho(z)) \right)}{1 + (|z_A - z| + \varepsilon)2bc \rho(z)}.
\]  (5.3)
Fig. 3. Two non-coincident firms located at 0 and 3 set delivered mill prices to some markets and basing point prices to others.

Fig. 4. Two identical coincident firms set delivered mill prices everywhere.

By differentiating twice with respect to \(|z_A - z|\), it is shown that delivered mill prices are concave and increasing in distance from the firms as long as deliveries to customers are positive and consumer density is constant. Note that delivered mill prices need not be monotone in distance if \(\rho(z)\) is not constant. Fig. 4 shows the pattern of delivered mill prices and each firm's deliveries when the firms are located at the origin, \(c = 0.1\), \(C_A = C_B = 1\), and
Delivered mill pricing can occur if two coincident firms with identical transportation cost functions have different production costs. In this case, if the production costs are not too different, it will be most efficient for the firms to jointly serve some of the markets. The lower cost firm can serve customers alone at some markets, setting a basing point price or a monopoly price at these other markets.

A firm uses a basing point price at a market when its marginal costs are strictly lower than that of any other firm and it does not find it advantageous to set a monopoly price. The firm sets its price equal to the least marginal cost of any competitor. If transportation cost is as assumed above, a basing point price is the base competitor's marginal cost plus the distance from the base. Basing point prices have been noted in fig. 3.

If transportation cost is relatively linear in quantity ($c$ is close to zero) and demand is relatively inelastic (so that no regions of monopoly pricing occur) basing point prices can be set by all firms to almost all markets. If $z_A = 1$, $z_B = 3$, and $c = 0.1$, $C_A' = C_B' = 1$, fig. 5 displays a pricing pattern very similar to the pattern of basing point prices first described by Hoover (1937) and more recently by Lederer and Hurter (1985) and Thisse and Vives (1988) where one firm uses the other firm as a base, and prices at the other's marginal cost. However, with quantity-dependent transportation costs, there always is an interval of shared markets and delivered mill prices for customers as long as $c > 0$. In this example sharing occurs in the interval (0.8, 2.2). This interval

\[ \rho(z) = 1 \text{ for all } z. \]

The figure shows prices/deliveries for customers to the 'right' of the firm, those to the left are symmetric.

Fig. 5. Two non-coincident firms located at 1 and 3 set basing point prices to most markets, setting delivered mill prices in the interval (0.8, 2.2).
will decrease as \( c \) decreases and disappear when \( c = 0 \). As long as there is sharing there is cross hauling.

Monopoly pricing occurs when a firm's monopoly price is less than any competitor's marginal cost. If the assumptions above are made, firm \( i \)'s monopoly price is

\[
P_i^M(z) = \frac{(a/b) + C_i + (1 + 2ac\rho(z))(|z_i - z| + \epsilon)}{2(1 + bcc\rho(z)(|z_i - z| + \epsilon))}.
\]

Monopoly prices need not be monotone in distance if market sizes vary. If all markets are of the same size, monopoly prices are increasing and concave in distance. Fig. 6 presents an example of monopoly pricing, with the firm located at the origin and \( c = 0.1 \) and \( C_A = C_B = 1 \). In this figure prices and deliveries are shown for customers to the 'right' of the firm; to the left prices/deliveries are symmetric.

The pattern of spatial prices in markets can be complicated, with firms pricing at mill, basing point and monopoly prices at different markets. To demonstrate how complicated the pattern can be, the existence of 'Pittsburgh Plus' pricing is demonstrated. In Pittsburgh Plus pricing, several coincident firms use delivered mill prices to all markets, while some distant firms use the multiple firm site as a price base.

Consider three firms located on a linear market: two firms, referred to as A and B, locate at the origin, and the third, denoted by C, locates at 2. Customers are uniformly distributed with density equal to 1, the demand function is linear and firms' transportation function is in the form assumed

---

Fig. 6. Monopoly prices for a firm located at the origin.
above. Let $c = 0.01$ and $C_A^c = C_B^c = 1$. Fig. 7 displays the pattern of equilibrium spatial prices and each firm's deliveries to customers. At markets in the interval $[-4, 0.8]$, the coincident firms serve exclusively and use delivered mill prices that increase in distance from them. In the interval $(0.8, 1.6)$, all three firms serve and prices increase and then decrease. In the interval $[1.6, 4]$, firm C serves alone, basing its prices upon firm A's and B's marginal costs. In the interval $[4, 5]$, all three firms set delivered mill prices and again jointly serve. The pattern is more complicated than usually assumed in Pittsburgh Plus pricing (by Haddock, for example). Firm C sets delivered mill prices to some markets and basing point prices to others. The spatial pattern of prices is generally monotone in distance from firms A and B, but not always. There is also cross hauling.

An important conclusion previously presented (and shown in fig. 4 and in the last example) is that delivered mill pricing occurs if two firms with identical transportation and production costs locate coincidentally. This makes the question of equilibrium firm location significant in predicting delivered mill pricing or more complicated patterns such as Pittsburgh Plus. In particular, if firms are free to relocate to maximize profit, will firms with identical transportation costs ever locate coincidentally?

To answer this question, the concept of a location equilibrium must be defined. Assume that the firms are free to relocate their plants, or alternatively, have not yet located any plants. The firms are assumed to choose locations simultaneously, and understand that once all locations are chosen,
the firms engage in a price–production game as described in this paper. Locations \((z_1^*, \ldots, z_n^*)\) are equilibrium locations if and only if for all \(i \in N\),

\[
\Pi_i(z_1^*, \ldots, z_i^*, \ldots, z_n^*) \geq \Pi_i(z_1^*, \ldots, z_i, \ldots, z_n^*), \quad \text{for all } z_i \in Z.
\]

The next example shows that coincident location can occur in equilibrium, and the firms earn positive profits.

**Example 5.1.** Consider two identical firms, denoted A and B, that are deciding where to locate on a linear market with just one market at \(z=0\), and \(p(0) = 1\). Assume that \(C_A = C_B = C'\). Suppose \(d(p) = a - bp\) such that \(C' < a/b\), so that positive demand can occur in equilibrium if the firms are close enough to the market. The firms' transportation cost function is as assumed above.

It is not hard to show that location at the origin is a location equilibrium. Suppose firm B is at the origin, and firm A locates to the right of the origin. Then, the derivative of firm A's profit with respect to its location is found by calculation:

\[
\frac{\partial \Pi_i(z_A, 0)}{\partial z_A} = \frac{\partial q_i}{\partial z_A} (p_t - C' - z_A - \epsilon),
\]

where \(q_i\) and \(p_t\) are functions of firm A's location at the market. The term within the parentheses is always positive because price is equal to marginal cost. By direct computation,

\[
q_i(z_A) = \frac{a - bC' - b(z_A + \epsilon)}{(z_A + \epsilon) + 2bc(z_A + \epsilon) + 1}
\]

which is differentiable and monotone decreasing in \(z_A\): \(q'(z_A) < 0\). Therefore, firm A's profit is maximized at the origin.

At the location equilibrium, both firms serve the market and earn positive profits. Profits are positive as each firm serves customers and price is equal to marginal cost.

It is interesting to note that firm A's profit is generally not a quasi-concave function of its location. Fig. 8 shows firm A's profit as a function of its location for the case \(z_B = 2\), \(c = 0.75\) and \(C' = 1\). Firm A's profit is clearly not quasi-concave. When \(z_A > 0.201\), firm A shares the market with B and sets delivered mill prices; when \(z_A \leq 0.201\), firm A uses basing point prices. Firm profit is non-differentiable at this breakpoint.

The conclusion of the example is similar to that of Anderson and Engers (1989) which showed that mill pricing firms can agglomerate in equilibrium if the firms are price-takers with non-constant marginal production cost. In our example, of course, firms are not restricted to mill prices.

Although this example assumes a single market, introducing several sufficiently small markets located about the origin will not change the above
argument or the result that a location equilibrium is at the origin and that firms have incentive to locate coincidentally. Therefore, equilibrium locations can be coincident for delivered pricing firms even when there are many markets.

What is different about this example from previous work on competitive location with delivered pricing is that the firms earn positive profit in equilibrium. Earlier work by Lederer and Hurter (1986) showed that only a single firm earns positive profit in equilibrium when costs are quantity independent. Therefore, all firms except one are indifferent as to their location, and thus there are multiple equilibria. The model of this paper can help explain the agglomeration of firms that use mill prices: these can be delivered pricing firms that maximize profits by locating together and, therefore, use mill prices.

6. Conclusions and extensions

This paper has studied spatial price competition of delivered pricing firms with quantity-dependent transportation costs. Firms engage in local Bertrand competition at each individual market inducing complex patterns of delivered pricing. In particular, it is shown that within a single industry mill, monopoly and basing point pricing patterns can occur at different markets. Although these results were derived using the special demand rationing rule (described by Definition 2.1), the results can be generalized to other demand rationing rules such as the parallel and proportional rules described by Tirole (1989). Analysis using these rationing rules is technically complex.
because equilibrium production and prices use mixed strategies. However, the pricing patterns induced will follow the patterns observed here: basing point and monopoly pricing occur at the same markets, and firms use mixed strategies and share service at markets where delivered mill prices are observed here.

The results of this paper also help to explain geographical price patterns in transportation markets. The model is applicable to transportation markets because transportation firms can be viewed as delivered pricing firms that provide transportation services but no good. Transportation tariffs often follow the 'tapering principle' which states that transportation price per quantity shipped is monotone increasing and concave in distance [see, for example, Talley (1983)]. Talley explained the tapering principle as the result of averaging station fixed cost over longer distances. This paper presents an alternate, competitive explanation: prices are tapered because transportation cost is quantity dependent and demand for transportation is price elastic. Tapered prices are readily observed in several examples studied in section 5; see, for example, figs. 2, 4 and 7. An implication is that tapered prices can occur in the absence of perfect competition. For example, in the extreme case of monopoly, fig. 6 shows that monopoly prices are tapered. Thus tapering of prices can occur in competitive, oligopolistic and monopolistic markets without fixed costs. A test to discriminate between the 'demand elasticity—quantity dependent cost' hypothesis and the 'fixed cost averaging induces tapering' hypothesis could be designed by relating a measure of price tapering against measures of demand elasticity, transportation cost convexity and fixed cost.

The model of this paper has already been generalized and extended. Analysis of the model assuming quantity-dependent production cost is found in Lederer (1992). Consumer costs, such as congestion costs, can be introduced into the model with consumers choosing to purchase from the firm with the least price plus consumer cost. Congestion effects associated with the time it takes for firms to produce an order and the cost that this causes customers has been modeled by Lederer and Li (1992).

Appendix

Proof of Theorem 4.1. As the problem of competition in each market is separable from all other markets, we study a single market \( z \).

Given price schedules, sales take place at the lowest price at each market, \( p \in \mathbb{R}_+^n \). Consider the problem:

\[
\max_{q_i(z); \quad d(p(z))p(z) \geq q_i(z) \geq 0} \left[ p(z) - C'_i(z_i) \right] q_i(z) - F_i(z_i, z, q_i(z)).
\]  

(A.1)

Let \( S_i(p(z)) \) be the solution to this problem; this is firm \( i \)'s profit-maximizing
supply to z. Lemma 4.1 ensures that firm i's production decision for z is equal to $S_i(p(z))$. This follows because equilibrium prices and production decisions must solve problem $P(i)$. Problem (A.1) has a strictly concave objective function on the convex constraint set and is continuous in $p$. This implies that $S_i(p(z))$ is unique and is continuous in $p(z)$. Let $S(p(z)) = \sum_{i \in N} S_i(p(z))$. $S(p(z))$ is the sum of all firms' supply vectors to the markets.

$E(p(z)) = S(p(z)) - p(z)d(p(z))$ is the excess demand function for this market: if $p(z)$ is a price equilibrium, then $E(p(z)) = 0$, since Property C holds in equilibrium. Uniqueness of equilibrium price $p(z)$ and the production equilibrium can be ensured by studying the solutions to $E(p(z)) = 0$ on $p(z) \in [0, \alpha]$.

One of the assumptions in section 2 was that there exists an $\alpha > 0$ such that $d(\alpha) = 0$. This implies that a solution to $E(p(z)) = 0$ exists on $[0, \alpha]$: $E(p(z))$ is continuous and $E(0) < 0$ and $E(\alpha) \geq 0$.

Consider the relaxed problem:

$$\max_{q_{i}(z) \geq 0} \left[ p(z) - C_i(z_i) \right] q_i(z) - F_i(z_i, z, q_i(z)).$$

Call the solution $T_i(p(z))$. The solution to this problem is a monotonic increasing function of $p(z)$. Define

$$p'(z) = \arg \min_{p(z) \geq 0} \left\{ \min_{i \in N} \{ p(z) \mid T_i(p(z)) = p(z)d(p(z)) \} \right\}.$$ 

For all $p(z) \geq p'(z)$ there is some firm $i$ such that $T_i(p(z)) \geq S_i(p(z)) = p(z)d(p(z))$. Also, for all $p(z) > p'(z)$, $E(p(z)) > 0$.

Consider the set of solutions $R = \{ p(z) \mid E(p(z)) = 0 \}$. If $p''(z) \in R$, and $S_i(p''(z)) > 0$ and $S_j(p''(z)) > 0$, for some $i \neq j$, then $R$ is unique and equal to $\{ p''(z) \}$. Why? It must be true that $p''(z) < p'(z)$, else demand will exceed supply. For all $p(z) < p'(z)$, $\partial E(p(z))/\partial p(z) > 0$, so that $p''(z)$ is an isolated solution on the interval $[0, p']$. For $p(z) > p'(z)$, $E(p(z)) > 0$, since more than one firm serves at price $p'(z)$. So, $R = \{ p''(z) \}$ and the solution is unique. This market price generates equilibrium price schedules and productions as follows: for each firm $i$, define firm $i$'s price schedule for $z$ as the maximum of its marginal cost, or $p''(z)$, and define its production decision as $S_i(p''(z))$. At these prices—productions, Properties A, B and C hold, therefore, is an equilibrium, and is unique, since property C will not hold at any market price.

If $p''(z) \in R$, and $S_i(p''(z)) > 0$ and $S_j(p''(z)) = 0$ for all $j \neq i$, then $p''(z) \geq p'(z)$, else $p''(z)$ would not solve $E(p) = 0$. All $p(z)$ in the interval from $[p'(z), p''(z)]$ solve $E(p(z)) = 0$, by the definition of $p'$. Define $p''(z) = \inf \{ p(z) \mid S_i(p(z)) > 0 \}$ for all $j \neq i$. Any $p(z)$ in the interval from $[p''(z), p''(z)]$, must also solve $E(p(z)) = 0$. Any $p(z) > p''(z)$ does not solve $E(p(z)) = 0$, nor does any $p(z) < p'(z)$. Therefore solutions to $E(p(z)) = 0$ are all values in the interval
[\(p'(z), p''(z)\)]. Firm \(i\)'s equilibrium price schedule for \(z\) lies in the interval and is the \(p(z)\) that generates maximum profits. If \(p_M(z)\) (\(i\)'s monopoly price) lies in this interval, this is the equilibrium price. Otherwise, \(p''(z)\) is the equilibrium price at \(z\), because \(i\)'s profits are greatest there. The other firm's price for \(z\) is just the minimum of its marginal cost, or \(p''\). Candidate equilibrium production decisions for market \(z\) correspond to \(S_i(p(z))\) for firm \(i\) and zero for all other firms. These price schedules and production decisions satisfy Properties A, B and C, and define a price-production equilibrium.

This proof can be viewed as constructive: all of the points in \(R\) can be found by increasing \(p(z)\) and finding all solutions to \(E(p(z)) = 0\). Then the above analysis can find the equilibrium market prices, price schedules and production decisions. Q.E.D.

References


