DUOPOLY COMPETITION IN NETWORKS

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Abstract

This paper presents a new approach to modeling competition between firms in network-based industries, i.e. industries where the firms' technology decisions correspond to choices of networks. Industries having this structure include transportation, telecommunications, and some service industries. Competition is studied between two firms who make both network design decisions and price decisions for services. This situation is modeled as a game, an equilibrium solution corresponding to a Nash equilibrium is defined, and properties of the solution are characterized. Necessary and sufficient conditions are shown for equilibrium solutions and existence of equilibrium solutions is demonstrated. Among the results is that each firm will maximize its own profit by minimizing total industry cost of providing services. An example demonstrating results is presented.

Keywords and phrases

Spatial competition, networks, Nash equilibrium, profit maximization.

1. Introduction

This paper studies competition between two profit-maximizing firms who compete through choices of technologies and prices for the service they provide customers. We examine this problem in a special context where a firm's technology corresponds to a network, and we refer to industries where individual firm technologies correspond to networks as "network-based industries". An example is the railroad industry, the technology decision being the selection of a rail network: what cities to link by rail. It is easy to see that, in general, transportation and communication industries can be viewed as network based. We can use our approach in other service industries when we can think of a service provided by a firm as a link connecting two nodes, the nodes corresponding to the state of a product or process with and
without the service. We view demand in such markets as being generated by customers who have projects composed of a directed set of required services, represented by a series of such connected nodes. In all such industries, we study how two firms will competitively decide what services they will provide to customers and how they will price their services.

For ease of exposition, we will consider competition in the railroad industry where nodes correspond to cities and links to transport services. The reader is advised that for other network-based industries, an identical development with similar wording exists. Our model will assume there are no capacity constraints for service between cities provided by the network and, because of this, our results may not be directly applicable to network-based industries where links have capacity constraints such as in airlines and telecommunications. This assumption on capacity does not seem to be a serious restriction in analyzing the railroad industry, however. Forthcoming work will address the problem of capacity, and results will be seen that are similar to those of this model.

The problem of competitive network design has been little studied. Recently, many authors have studied how firms should locate on an existing network (see Tansel et al. [6]). In this literature, the firm’s problem is to decide where on the network to locate facilities to serve demand at nodes. Our work studies what links the firm should provide between nodes, i.e. what networks to create. Hakimi [1] studies the problem of location on a network in a competitive environment with duopolists seeking to locate a number of facilities. He does not consider price competition. Wendell and McKelvey [7] study competition on a network where the firm that locates on the network first can be assured of serving at least half of the market, but price competition is not considered. The problem of competition between firms in network choices and prices is related to competition in location and pricing in markets where firms may set delivered prices. Markets in this context are studied by Lederer [4], Lederer and Hurter [5], and Hurter and Lederer [2], who study competitive problems with and without production, in markets with and without elastic demand.

Our approach will be to model the problem of two firms’ competition in network choices and prices as a two-stage game, and study the equilibrium of this game and its properties. Section 2 contains our model and properties of equilibrium solutions. Section 3 presents an example demonstrating the results.

2. The model of duopoly competition and results

Let two profit-maximizing firms, denoted $A$ and $B$, desire to compete by providing transport between $K$ cities, which are denoted $\{z_1, \ldots, z_K\}$. To provide transportation service, each desires to create a railroad network connecting some subset of the $K$ cities. To compete, each firm must also price transportation services between cities. We will assume that the firms have not yet created their network, and each anticipates their competitor’s decisions about networks and prices for service.
If there is a demand from cities \( z_i \) to \( z_j \), we will refer to this demand as transport demand \( \tilde{x} \), and write \( \tilde{x} = (z_i, z_j) \). We will measure units of transport demand in some units, say, of weight. The quantity of demand in units associated with transport demand \( \tilde{x} \) will be given by \( \rho(\tilde{x}) \). We will let \( M \) be the number of transport demands where \( \rho(\tilde{x}) > 0 \), and denote the set of non-zero transport demands by \( L = \{\tilde{x}_1, \ldots, \tilde{x}_M\} \).

We will assume demand is inelastic with respect to price. However, for each transport demand \( \tilde{x}_j \) there will be a limit price of \( \ell_j \), and we assume that customers will be unwilling to pay more than \( \ell_j \) per unit transported on \( \tilde{x}_j \). Our interpretation of the limit price will be that there is an "alternate carrier" available to customers at the limit price. We assume customers will use the carrier with the lowest price and therefore, each firm will serve those transport demands on which it has the lowest price, if this price is at or below the limit price.

The set of all possible network choices by a firm will be referred to as the set of "feasible network designs" and is a set of possible links connecting cities. Sometimes it may be useful to view the set of links as being directed; sometimes, such as in the railroad industry when traffic can go either way, this distinction will be unimportant. We will denote the set of all directed links between any two cities as \( L \). The set of feasible network designs by firm \( i \) will be \( T_i \subset L \), permitting the possibility that not all designs are allowed for a particular firm and that these sets may be different for the firms, and we will denote an element of \( T_i \) by \( t_i \). We require that \( \Phi \) (null set) belongs to \( T_i \); this allows the firm to decide not to serve any demand. We will allow firms to use the alternate carrier to reduce their variable cost or to extend their network if a transport demand requires service on links that the firm's network has not connected, itself paying the limit price for this part of the service to the alternate carrier. We will also assume that there is no cost to the consumer in switching to or from the alternate carrier. For a firm to provide transport, cities need not be directly connected: service can be performed on the alternate carrier, or on portions of the firm's network. It may be possible that a firm will not use links in its network if service on the alternate carrier at the limit price is more efficient.

For each feasible network design, we may calculate the cost of establishing the networks, this cost having two components: fixed cost (construction, capital financing, and maintenance of the system) and variable cost (cost of actually providing the service). For \( t_i \in T_i \), the fixed cost to firm \( i \) will be \( F_i(t_i) \). We will assume variable cost per unit transported is constant for each \( \tilde{x} \in L \), and is denoted by \( f_i(t, \tilde{x}) \). Because we allow firms to use the alternate carrier if the firm’s network does not provide the complete transport demand, or if it more efficient to do so, we will recognize the variable cost to be the minimum variable cost path using the firm's network supplemented by the best path on the alternate carrier. Thus, \( f_i \) will be defined for all \( \tilde{x} \) for all feasible network designs. Recall that we assume there is no capacity constraint on a firm’s ability to provide transport. This is a realistic assumption for a railroad network.
Each firm expresses its price strategy as a function of network choices through a "price schedule". A price schedule is a function which specifies the price at which the firm will offer to transport a unit of goods for each transport demand as a function of the network of each firm. For example, for firm $A$, a price schedule is a function $p_A(t_A, t_B, \bar{z})$ which states that firm $A$ will offer to transport a unit of good corresponding to transport demand $\bar{z}$ at $p_A$ if firm $A$ chooses network $t_A$ and firm $B$ chooses network $t_B$.

We will model competition for the firms by assuming that the firms choose network designs privately and simultaneously; then, with these networks known, each firm chooses prices privately and simultaneously for all transport demands. The firms are assumed to understand this game and to understand that the final market prices are conditioned by network choices of the firms.

The competitive situation will be analysed as a non-cooperative game. Because prices chosen are conditioned on networks, the game can be viewed as two-staged. The selection of prices by the firms, assuming fixed network designs can be analysed for each pair of locations. Then, the problem of location choice can be analysed, anticipating what prices will be chosen. Our approach will be to find a Nash equilibrium in network designs and price schedules for the firm: network design and price schedule choices are optimal against each other.

To define the firms' profit functions, we need only resolve the following issue: when the firms price at the same price, how do the firms share transport demand? We do this through a "market share function" $r_i(t_A, p_A, t_B, p_B, \bar{z})$, defined for both firms, which defines the share of transport demand $\bar{z}$ for firm $i$ as a function of the network design and price schedule choices of both firms. We will require that $r_A + r_B = 1$ and $r > 0$. The profit for firm $i$ can now be expressed. The profit for firm $A$ is

$$
\Pi_A(t_A, p_A, t_B, p_B) = \sum_{\{\bar{z} \mid p_A < p_B \text{ and } p_A < \xi_j\}} \left[ p_A(t_A, t_B, \bar{z}) - f_A(t_A, \bar{z}) \right] \rho(\bar{z})
$$

$$
+ \sum_{\{\bar{z} \mid p_A = p_B < \xi_j\}} \left[ p_A(t_A, t_B, \bar{z}) - f_A(t_A, \bar{z}) \right] \rho(\bar{z}) r_A(t_A, p_A, t_B, p_B, \bar{z})
$$

$$
- F_A(t_A),
$$

and $\Pi_B(t_A, p_A, t_B, p_B)$ is similarly defined.

We seek Nash equilibrium pairs $(t^*_A, p^*_A(, ,)),$ $(t^*_B, p^*_B(, ,)),$ such that

$$
\Pi_A(t^*_A, p^*_A, t^*_B, p^*_B) \geq \Pi_A(t_A, p_A, t^*_B, p^*_B)
$$

(2.2)
for any $t_A \in T_A$ and any price schedule $p_A$, and

$$\Pi_B(t_A^*, p_A^*, t_B^*, p_B^*) \geq \Pi_B(t_A^*, p_A^*, t_B, p_B)$$

(2.3)

for any $z_B \in T_B$ and any price schedule $p_B$.

For fixed choices of network designs by firms $A$ and $B$, we now consider properties of price schedules in equilibrium. For a pair of networks and price schedules to be a Nash equilibrium, it is necessary for the price policies to be Nash against each other at the equilibrium network designs. If this is not true, the original pairs are not Nash.

We make the following observations about the Nash price policies. First, in equilibrium each firm is pricing at or above its variable cost for each transport demand that it serves — to do otherwise implies non-optimal behavior, a firm could readjust its price policy to increase profit, and the price policies could not be Nash. In particular, if two firms price identically and share service for a transport demand in equilibrium, both are pricing above their variable cost.

Second, no Nash equilibrium in price policies can exist unless the market share rule has a special property. This property is that if two firms price identically for a transport demand, the firm with the least variable cost will serve all of the demand. For a market share rule not obeying this property, if both firms price identically and serve a transport demand, the low variable cost firm could cut its prices slightly, capture all transport demand traffic and increase its profit. The profit would rise because if the firms are pricing the same and sharing traffic for a transport demand, the price is at least the maximum of the firms’ variable costs. This last statement follows from our first observation. We will require market share rules to obey the above requirement, and refer to the class of market share rules that fall into this class as “cost advantage market share rules”. Our requirement is realistic because no equilibrium in price schedules will exist without it, and equilibrium prices do exist in the real world.

Our last observation is that a firm should never price below its variable cost. If a firm prices below its variable cost for a transport demand, it must be assuming it will not serve any traffic. If it serves customers, it could raise its price and increase its profit. If a firm prices below variable cost, then it must be planning not to serve any traffic. A firm might do this to lower the price, and thereby lower the profit of its competitor: the firm anticipates the competitor will undercut the firm’s price and serve the transport demand alone. This strategy is dangerous though, because if the competitor does not undercut the firm’s price, the firm’s profits will fall. Because the firm has nothing to gain using this strategy, and something to lose, it is not reasonable for a firm to price below its variable cost.
We summarize these comments in the following two assumptions:

(A1) We will assume that the market share rules are cost advantage market share rules, and

(A2) Firms will never price below their variable cost for any transport demand.

These issues are discussed in greater detail in a related work which models competition of firms who locate and set discriminatory prices, and the reader is directed to Lederer [3] or Lederer and Hurter [5] for a fuller development and proofs associated with the above discussion.

Given our assumptions, the equilibrium prices that will arise in the market for any network designs can be found. Customers for each transport demand will be served by the firm with the lowest variable cost of service and the customer will pay the variable cost of service of the higher variable cost firm. This will occur because the firms will vie for the customers' business, cutting their prices in turn until the variable cost of the higher variable cost firm is reached. Note that this price is at or below the limit price because of the way we defined the variable cost to the firms of service. In equilibrium, the higher variable cost firm will price as low as it can, pricing at its variable cost. This is a necessary condition for an equilibrium to exist and to satisfy (A2). To summarize, a necessary condition on equilibrium price schedules is

\[ p^*_A(t_A, t_B, \bar{z}_j) = \max [f_A(t_A, \bar{z}_j), f_B(t_B, \bar{z}_j)] \]

(2.4)

and similarly for \( p^*_B \). For ease of expression, we will refer to \( p^*_A \) and \( p^*_B \) as "e.p.s." equilibrium price schedules.

Under e.p.s., firm A will serve and earn a positive profit from

\[ L_A(t_A, t_B) = \{ \bar{z}_j \in L | f_A(t_A, \bar{z}_j) < f_B(t_B, \bar{z}_j) \} \]

(2.5)

and B will serve and earn a positive profit from

\[ L_B(t_A, t_B) = \{ \bar{z}_j \in L | f_B(t_B, \bar{z}_j) < f_A(t_A, \bar{z}_j) \} \]

(2.6)

and in the complement set

\[ L_C(t_A, t_B) = L - \{ L_A(t_A, t_B) \cup L_B(t_A, t_B) \}, \]

(2.7)

either the alternate carrier will serve, or the firms will serve and earn zero profit. We note that since the set of transport demands that the firms jointly serve is a subset
of \( L_C \), and the firms earn zero profit on this set, the profit the firms earn will be independent of the particular cost advantage market share rule used.

Then, assuming that (A1) and (A2) hold, we may rewrite the profit function of the firms under e.p.s.:

\[
\Pi_A(t_A^*, p_A^*, t_B^*, p_B^*) = \sum_{\{j \mid j \in L_A\}} \{f_B(t_B^*, \bar{x}_j) - f_A(t_A^*, \bar{x}_j)\} \rho(\bar{x}_j) - F_A(t_A^*)
\]

and similarly for \( \Pi_B \).

Having characterized equilibrium price schedules, we show existence of a network design-price schedule equilibrium by finding network design choices for the firms that are Nash under e.p.s. That is, we seek \( t_A^*, t_B^* \) such that

\[
\Pi_A(t_A^*, p_A^*, t_B^*, p_B^*) \geq \Pi_A(t_A, p_A^*, t_B^*, p_B^*) \quad \text{for all } t_A \in T_A
\]

\[
\Pi_B(t_A^*, p_A^*, t_B^*, p_B^*) \geq \Pi_B(t_A^*, p_A^*, t_B, p_B^*) \quad \text{for all } t_B \in T_B.
\]

We can find such equilibrium designs by using the following definition.

**DEFINITION**

The "social cost" is the cost incurred by the industry to satisfy demand in a cost-minimizing manner.

Then, the social cost is

\[
K(t_A, t_B) = \sum_{\{j \mid j \in L\}} \min \left\{ f_A(t_A^*, \bar{x}_j), f_B(t_B, \bar{x}_j) \right\} \rho(\bar{x}_j) + F_A(t_A^*) + F_B(t_B^*).
\]

Using this definition, we may rewrite the expression for a firm's profit function:

\[
\Pi_A(t_A, p_A^*, t_B^*, p_B^*) = \sum_{\{j \mid j \in L\}} f_B(t_B^*, \bar{x}_j) \rho(\bar{x}_j) - F_B(t_B^*)
\]

\[
- \sum_{\{j \mid j \in L\}} \left\{ \min \left\{ f_A(t_A, \bar{x}_j), f_B(t_B, \bar{x}_j) \right\} \rho(\bar{x}_j) + F_A(t_A) + F_B(t_B) \right\}
\]

\[
= \sum_{\{j \mid j \in L\}} f_B(t_B^*, \bar{x}_j) \rho(\bar{x}_j) + F_B(t_B) - K(t_A, t_B).
\]
Examination of (2.12) and a similar expression for \( \Pi_B \) reveals that \( t_A^*, t_B^* \) are equilibrium network designs iff

\[
K(t_A^*, t_B^*) \leq K(t_A^*, t_B^*) \quad \text{for all } t_A \in T_A
\]

\[
K(t_A^*, t_B^*) \leq K(t_A^*, t_B^*) \quad \text{for all } t_B \in T_B
\]

(2.13)

and the firms employ e.p.s. Because \( T_A, T_B \) are finite sets, \( K \) has a minimum on \( T_A \times T_B \), and condition (2.12) will hold with \( (t_A^*, t_B^*) \) chosen to globally minimize \( K \). Therefore, an equilibrium in network designs exists.

We note that each firm should minimize social cost – which is the industry's cost, not the firm's cost – to maximize its profit. In an equilibrium of network designs, each firm’s network choice minimizes social cost with respect to the other firm’s fixed network design choice. A network design equilibrium need not minimize social cost globally, only in each firm’s network choice individually. It can be anticipated that there may be multiple equilibria in network designs.

We also see that if \( F_I(\Phi) = 0 \), each firm is assured of non-negative profits, and that entry into the market will occur only if a firm will earn non-negative profit in equilibrium.

Our approach has focused on simultaneous entry, but can also be used to analyse sequential entry of two firms. Our result (2.12), concerning the decomposition of a firm’s profit function, indicates that the first firm entering the market should choose a network design that minimizes social cost, anticipating the network design choice of the second firm, who will seek to minimize social cost with respect to the first entering firm’s network choice. This type of Stackelberg analysis tells us that the first entering firm in a market should not choose a network design to minimize its own cost as the only provider, but instead choose to minimize social cost, anticipating the competitor’s likely network choice.

3. Example

Let two firms \( A \) and \( B \) desire to serve demand for railroad traffic between three cities, 1, 2, and 3 (see fig. 1). The non-zero transport demands for these cities are \( L = \{ \bar{z}_1, \bar{z}_2, \bar{z}_3 \} \):

\[
\bar{z}_1 = (1, 2) \quad \rho(\bar{z}_1) = 10
\]

\[
\bar{z}_2 = (1, 3) \quad \rho(\bar{z}_2) = 10
\]

\[
\bar{z}_3 = (3, 2) \quad \rho(\bar{z}_3) = 10.
\]
The limit prices for service will be taken to be US $3 for both \( \bar{x}_2 \) and \( \bar{x}_3 \) and US $6 for \( \bar{x}_1 \). Because this is a railroad network, we need only be concerned with non-directed links between cities. The set of feasible network designs for the firms will be allowed to be the set of all subsets of \( \{(1, 2), (1, 3), (3, 2)\} \). The fixed costs of the firms and the variable costs of the firms without using the resources of the alternate carrier are found in Table 1 for all choices of network design. The costs indicate that the variable cost to the firms of providing service on two connected links is just the sum of the variable cost of the two separate links. There are also economies of scale, in terms of fixed costs, associated with direct service. The functions \( f_A, f_B \), the variable costs for the firms best utilizing the resources of the firm and the alternate carrier, are found in Table 2 for every choice of network design. We will assume that there is no cost to the customer of switching between two links provided by a firm, if a firm provides a service requiring such switching. This is reasonable in the railroad industry. It might not be in the airline industry because of the cost to passengers of longer travel times.

We see that the network designs minimizing social cost are: \( t_A^* = \{(1, 2), (3, 2)\} \) and \( t_B^* = \{(1, 3)\} \), with resulting e.p.s.: \( p_A^*(t_A, t_B, \bar{x}_1) = 5.4 \), \( p_B^*(t_A, t_B, \bar{x}_2) = 3 \), and \( p_A^*(t_A, t_B, \bar{x}_3) = 3 \), for all \( i \in \{A, B\} \), and that these choices form an equilibrium of network designs and price schedules. For this equilibrium, \( A \) carries traffic demands \( \bar{x}_1 \) and \( \bar{x}_3 \), and \( B \) provides \( \bar{x}_2 \). The social cost is 100.5, and the firms’ profits in equilibrium are: \( \Pi_A = 11 \) and \( \Pi_B = 2.5 \).

Another network design equilibrium is \( t_A^* = \{(1, 3), (3, 2)\} \) and \( t_B^* = \{(1, 3)\} \), with \( p_A^*(t_A, t_B, \bar{x}_1) = 5.4 \), \( p_B^*(t_A, t_B, \bar{x}_2) = 2.8 \), and \( p_A^*(t_A, t_B, \bar{x}_3) = 3 \) for \( i \in \{A, B\} \). We note that \( A \) provides \( \bar{x}_1 \) through \( (1, 3) \) and \( (3, 2) \). The firms’ profits are \( \Pi_A = 7.5 \) and \( \Pi_B = 0.5 \), and the social cost is 104.

We note that the only other network design equilibrium is \( t_A^* = \{\Phi\} \), and \( t_B^* = \{(1, 3), (3, 2)\} \) with e.p.s.: \( p_A^*(t_A, t_B, \bar{x}_1) = 6 \), \( p_B^*(t_A, t_B, \bar{x}_2) = 3 \), and \( p_B^*(t_A, t_B, \bar{x}_3) = 3 \), for all \( i \in \{A, B\} \). Firm \( B \) does service for all three transport demands. The social cost is 104.5. Firm \( A \) earns no profit, and \( \Pi_B = 15.5 \).
Table 1

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<thead>
<tr>
<th>Network design</th>
<th>Transport demand</th>
<th>Fixed cost</th>
<th>Variable cost/unit (US $/unit) without using the alternate carrier</th>
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<td>Firm B</td>
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The monopolist who controlled both firm A and B would choose network designs $t_A^* = \{(3, 2)\} \text{ and } t_B^* = \{(1, 3)\}$, and set prices for transport demand at the limit prices. The cost to the monopolist would be 96.5. The monopolist’s profit would be 23.5.

The monopolist’s cost is lower than the social cost. This is because we require customers to buy a service from one firm: we do not allow customers to buy parts of a transport demand from several firms, constructing the transport demand from its components. If such behavior is possible, firms must set prices on networks understanding that a customer may find it cheaper to buy segments of a transport demand from different firms. Customers may also find it cheaper to purchase a transport demand in connected pieces from the same firm, and the firm must worry about this possibility when setting prices.

If we allow customers to "switch" carriers to lower their price for service, we must examine the effect of these changes on equilibrium prices given network design choices, and the resulting equilibrium network design choices. Unfortunately, in this case unique equilibrium price schedules do not exist. Therefore, the network design
Table 2

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<tr>
<th>Network design (q_i)</th>
<th>Transport demand (z)</th>
<th>Fixed cost Firm A</th>
<th>Fixed cost Firm B</th>
<th>(f_A(t_{A,z})^*)</th>
<th>(f_B(t_{B,z})^*)</th>
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*For all unlisted transport demands, the firms will use the alternate carrier: the variable cost is the appropriate limit price.

equilibrium is not well defined. However, if we suppose firms choose network designs anticipating that the price schedule that will occur will be the one giving the firm its lowest profit, then the firms will choose the same network designs as they did in equilibrium when customer switching is not allowed. These results are discussed in Lederer [3], which also analyses network competition for more than two firms, with and without customer "switching".

References