Economic Evaluation of Scale Dependent and Irreversible Technology Investments

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Abstract: We study the effect of risk on the economic evaluation of projects where capacity decisions are made in advance and are irreversible. Capacity decisions have an important effect on the projects value through the up-front investment, the associated operating cost, and constraints on output. Increased scale raises up front investment and fixed operating cost in future periods, raising the financial risk. When a firm has the ability to affect the price it charges, the capacity decision also influences the market price. Increased scale allows higher output and sales, but, because of market power, greater output lowers prices. Although there are many sources of risk, we focus on (1) the up front investment, (2) the operating cost of the project, (3) uncertainty in market price, and (4) the constraints imposed by the capacity choice.

Our results include: the riskiness of the project depends both on the technology selected and on the capacity chosen. Project risk, as measured by the required rate of return, is related to the inverse of the expected profit per unit sold. Managers often used a fixed prescribed rate to evaluate the project and choose its scale. With such a fixed rate, a manager will generally choose sub-optimal capacity that reduces project value. This is true even if the prescribed discount rate equals the optimal scale-and-technology-dependent rate. It is also possible to set the prescribed rate to induce the optimal scale decision. However, in this case, the manager will generally greatly under-value the project. Use of the former (a pre-set rate close to the actual rate) is preferable to the latter (a rate that induces the correct quantity decision). A general result is that the assignment of fixed discount rates to capacity decisions (or other decisions that affect project risk) causes errors in project scale and value.

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1 Introduction

In this paper, we study the effect of risk on the economic evaluation of projects where capacity decisions are made in advance and are irreversible. Capacity decisions have an important effect on the project value through the up-front investment, the associated operating cost, and the constraints on output. Increased scale raises up front investment and fixed operating cost in future periods, raising the financial risk. When a firm has the ability to affect the price it charges, the capacity decision also influences the market price. Increased scale allows higher output and sales, but because of market power, greater output lowers price. Although there are many sources of risk, we focus on (1) the up front investment, (2) the operating cost of the project, (3) uncertainty in market price, and (4) the constraints imposed by the capacity choice.

Our results include: the riskiness of the project depends both on the technology selected and on the capacity chosen. Project risk, as measured by the required rate of return, is related to the inverse of the average expected per-unit profit. We compare this project-sensitive assessment of risk with the use of a fixed, prescribed, discount rate. A general result is that the use of such a fixed discount rate for capacity decisions (or other decisions that affect project risk) causes errors in project scale choice and valuation. Even if the prescribed rate induces the optimal capacity decision, the project will generally be greatly undervalued. Alternatively, if the pre-set rate equals the discount rate for the optimally designed project, the project scale will be larger than optimal. Between these two, the latter results in a smaller deviation from the optimal profit, and, consequently, is preferred as the lesser of two evils.

The results and the methodology make this research important because it uses modern finance theory to explicitly model the effect of technology choice and project scale on project risk and economic value. It also highlights an unrecognized problem with Net Present Value Analysis (NPV) as conventionally practiced to value projects. Clearly most operations capital investments involve decisions of capacity. Yet, by establishing an a priori discount rate, firms fail to incorporate project scale in the evaluation of project risk and value.

In this paper, we assume that a firm that wishes to economically evaluate a project is a publicly owned firm with shares that trade on a stock exchange. This assumption allows use of
tools of modern finance theory to value risky cash flows. Thus, risk adjustment need not be
based upon risk aversion of decision makers, but instead on risk adjustment of financial markets.

As discussed in Lederer and Singhal (1989), there are three non mutually exclusive
sources of risk associated with investment in any capital project: implementation risk, industry
risk, and business risk. Implementation risk is the risk associated with the time and cost to
complete the project and uncertainty about operating costs and output rates for the project ex-
post. Industry risk is that risk associated with future market prices and market shares within an
industry. Business risk is the risk associated with the variation of project cash flows with the
general state of the economy. A publicly traded firm incorporates only last type of risk into its
decision making process. This is because an investor can hold a diversified portfolio to eliminate
the risk associated with a specific project or industry. Further, we consider the scenario where
scale can affect both the expected market price and the maximum sales rate. These, in turn,
affect expected revenues and the riskiness of the cash flows with respect to the general economy.
We consider both factors in capacity decisions and project valuation.

Discounted cash flow analysis (sometimes called net present value analysis, or NPV)
values future cash flows by discounting the expected value of each future period's cash flows at a
risk adjusted "hurdle" rate, and then summing over all periods. A necessary condition for
properly using NPV analysis is that the estimates of future cash flows are unbiased, and that the
discount rate used has been properly adjusted for risk. It is not hard to show that the risk of the
project depends on the design of the project itself. Thus, the discount rate ought to be
technology and scale specific. However, there is little literature or modeling on how to do this.
Some examples exist – see Lederer and Singhal (1989)

Problems with NPV analysis are outlined in several papers. Kaplan (1986) points out
that relevant cash flows are sometimes ignored because they are hard to quantify. Similarly,
(1983), Jelinek and Goldhar (1984), and Meredith (1987) provide a discussion of the strategic
benefits of capital projects that are often ignored.

Although projects should be evaluated according to the rate specific to a project, there is
some evidence that firms use only one rate or, at most, a small set of discount rates. Chadwell-

Hatfield et al [1996] report that only 21% of the managers they surveyed used project-specific rates, while others reported that the discount rate varied between 5% and 25%, with 1/3rd of the managers stating they used rates between 10% and 12%. These management-prescribed rates implicitly categorize projects into different risk groups based on firm criteria.

This paper develops a one-period model based upon price uncertainty. The model assumes that the firm knows the industry demand curve and thus the expected price it will receive as a function of supplied volume. However, there is some uncertainty with the demand/supply relationship. Thus, there is uncertainty as to what price it will actually receive. The firm must decide on how much capacity to install. In this paper, we assume that the capacity sets the future production rate. This assumption is reasonable in many large capital process industries (such as paper and petrochemical refining). Van Meigham and Dada (1999) describe this firm strategy as Price Postponement with Clearance.

Another reason we require that capacity choice fixes the future production rate is this assumption allows use of the Capital Asset Pricing Model (CAPM) to adjust for risk. Allowing production rates to vary according to future market conditions requires use of more complex Options Pricing models whose valuation estimates cannot be presented in simple closed form solutions. We present results of this approach in another paper, Mehta and Lederer (2002). That paper shows that the results we find here are general to industries where capacity merely caps output rates and does not mandate output rates. We can also generalize the results to multi-period investments, yielding the same insights.

In Section 2, we introduce the model and problem terminology (2.1), project-specific analysis (2.2), decision-making with a pre-set rate (2.3), comparison of project-specific and preset rates (2.4), errors resulting from seemingly correct pre-set rates (2.5), and analyze the magnitude of errors resulting from such a pre-set rate (2.6). Section 3 contains parametric analysis, Section 4 includes illustrative examples demonstrating some of our results, and Section 5 summarizes the results and discusses possible extensions to the work.
2 The Model

We study the choice of capacity and evaluation of value of a production technology. Here are the assumptions we make in our model. Table 2.1 lists the variables and parameters we use in this paper.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>price $/unit</td>
</tr>
<tr>
<td>( \hat{p} )</td>
<td>uncertain price $/unit</td>
</tr>
<tr>
<td>( d )</td>
<td>production rate (units), also referred to as scale</td>
</tr>
<tr>
<td>( K )</td>
<td>capacity of the project (units per period)</td>
</tr>
<tr>
<td>( R, \hat{R}_{proj} )</td>
<td>required rate of return, required rate of return for (%)</td>
</tr>
<tr>
<td>( \bar{R}, \bar{R}_{proj} )</td>
<td>Uncertain rate of return, for the project (%)</td>
</tr>
<tr>
<td>( \bar{c} )</td>
<td>Uncertain error term in inverse demand function ($).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>Market price for risk (1/%)</td>
<td>( \frac{ER_m - R_f}{\sigma_m^2} ) Estimated to be 2.25</td>
</tr>
<tr>
<td>( \sigma_m^2 )</td>
<td>Variance in market returns</td>
<td>Estimated to be 0.04</td>
</tr>
<tr>
<td>( R_f )</td>
<td>Risk free rate of return</td>
<td>Estimated to be 3%</td>
</tr>
<tr>
<td>( ER_m )</td>
<td>Expected one period stock market return</td>
<td>Estimated to be 12%</td>
</tr>
<tr>
<td>( i_o, h )</td>
<td>Fixed and per unit initial investment ($, $/unit)</td>
<td></td>
</tr>
<tr>
<td>( f_o, f_1 )</td>
<td>fixed and per unit period cost ($, $/unit)</td>
<td></td>
</tr>
<tr>
<td>( a_o, a_i )</td>
<td>Parameters of inverse demand function</td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>Variable cost per unit ($/unit)</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.1 – Variables and parameters in the model

A1. The firm manufactures and sells a single product at a price that is a function of output. The inverse demand function that the firm faces is:
\[ \tilde{p} = a_0 - a_1 d + \tilde{\varepsilon} \]

The uncertainty term \( \tilde{\varepsilon} \) has a normal distribution with mean zero and known variance.

**A2** The firm exists for a single period. At the beginning of the period the firm chooses its capacity. The price uncertainty is resolved at the end of the period. Production is at a constant rate equal to the capacity. The firm liquidates itself at the end of the period. The salvage value of the technology is zero.

**A3** The cost structure for technology can be characterized by the parameters \((l_0, i_i, f_0, f_i, c)\).

**A4** The initial investment is incurred at the beginning of the period. The fixed and variable operating cost and the revenue are realized at the end of the period.

**A5** The firm is an all equity firm where the equity holders contribute the initial investment.

**A6** All taxes are zero.

The first assumption states that the firm produces a single product and faces an inverse demand function that has a random additive term. The inverse demand function is given by

\[ p = a_0 - a_1 d, \]

where \( p \) is the expected market clearing price, \( d \) is the quantity brought to the market, and \( a_0 \) and \( a_1 \) are positive constants. Higher production will reduce the expected price the firm receives. While the firm knows the expected price it will receive at the end of the period, the realized price is uncertain and market-driven. We model this relationship with a stochastic term. The firm will earn an uncertain (realized) price, \( \tilde{p} = p + \tilde{\varepsilon} \), where \( \tilde{\varepsilon} \) is the error term. The second assumption specifies the timing of events and resolution of uncertainties. The third assumption characterizes the cost of capacity in terms of the initial investment and period fixed operating costs. The timings of cash flows are done for convenience. Although broader definitions of technology are possible, here we define a technology by the three components of the cost structure. \( \tau = (i_0, i_i, f_0, f_i, c) \). The first two components are the installation cost that the firm incurs at the beginning of the period. The installation cost consists of two elements: a fixed cost, \( i_0 \), that is strictly a function of the selected technology, and a per-installed-unit cost, \( i_i \).

Given the firm's decision to install capacity \( K \), it will incur a total installation cost of \( I = i_0 + i_i * K \). The second component of the cost structure is a 'per-period' cost. This is a cost that the firm incurs each period that it uses the technology and is independent of the production
quantity. In this one-period model, the firm incurs the cost just once – at the end of the period. Like the installation cost, the period cost has two elements. The first is a fixed charge, \( f_0 \), while the second is a constant installed-unit cost, \( f_1 \). Consequently, it will incur a total period cost of \( F = f_0 + f_1 \cdot K \). The third, and last, component of the cost structure is the variable production cost. We assume it is constant, independent of quantity produced. Let it be \( c \). The fourth assumption specifies the timing of cash flows so that they can be valued. The fifth assumption allows the use of the CAPM to adjust for risk and uncertainty. The final assumption frees us from issues of taxes and tax shields due to depreciation.

These assumptions can be generalized. A firm can be allowed to produce multiple products, but the demand functions must be specified. The inverse demand function can be generalized to a non-linear function. The disturbance term can be non-normally distributed, but must remain symmetrical for CAPM to be used to evaluate risk. Under more general distributions, option pricing models could be used to adjust for risk.

Although we assume the firm can exist for a single period, this may be generalized. We discuss how later in this paper. We assume that capacity is chosen first, and it cannot be increased or decreased. Models which allow capacity to be adjusted over time can be constructed, but this is not the aim of this paper. We are concerned with the risk associated with inflexible investments. Production rates could be allowed to be adjusted to maximize profits. This can be done using an options model as has been done in Lederer and Mehta (2001). The cost structure assumed is very general. It is actually far more general than most capital budgeting models.

Issues of debt financing could be introduced and capital structure studied. However, we do not study these issues here. Taxes could be introduced, and the cash flow consequences of depreciation tax shields and tax payments could easily be incorporated into the model. Issues of bankruptcy would complicate the model. See Lederer and Singhal (1994) for models incorporating taxes and bankruptcy considerations.
2.1 Project Risk Adjustment and Discount Rates

For a given technology, \( \tau = (i_o, i_1, f_o, f_1, c) \) and production and scale decisions, \( d \) and \( K \), the expected value of the project is

\[
\pi = E \left[ \frac{(\bar{p} - c)^* d - (f_o + f_1)K}{1 + R} \right] - i_0 - i_1K. \tag{2-1}
\]

where \( R \) is the risk adjusted discount rate which is derived later in this section. The discount rate will be shown to be a function of \( d \) and \( K \).

For fixed production and scale decisions, the expected value of the project can be computed as follows. Initial investments \( i_o, i_1 \) are certain. Future cash flows are uncertain and CAPM can value them\(^1\). According to the certainty equivalent form of CAPM, the present value of uncertain cash flows occurring at the end of period 1 is

\[
P_{V_{proj}} = E \left[ \frac{(\bar{p} - c)^* d - (f_o + f_1)K - \lambda \text{cov}((\bar{p} - c)^* d, R_m)}{1 + R_f} \right] - \lambda \text{cov}(\bar{p}, R_m).
\]

Thus the net present value of the project is

\[
\pi = \frac{(p - c)^* d - \lambda^* d^* \text{cov}(\bar{p}, R_m) - (f_o + f_1)K}{1 + R_f} - i_0 - i_1K. \tag{2-2}
\]

It will be useful to define the certainty equivalent of the unit price:

\[
p_{ce} = p - \lambda^* d^* \text{cov}(\bar{p}, R_m). \tag{2-3}
\]

This is the price adjusted for future risk. Thus,

\[
\pi = \left( p_{ce} - c \right)^* d - (f_o + f_1)K \left( 1 + R_f \right) - i_0 - i_1K.
\]

---

\(^1\) See Brealey and Myers [1991] for discussion of CAPM.
This is important since both (2-1), using the appropriate risk-adjusted discount rate, \( R \), and (2-2), using the risk-free rate, \( R_f \), correctly value the project. Thus, we can derive the discount rate \( R \) by equating these two expressions.

The certainty-equivalent approach is insightful. If the end of period cash flow is denoted \( CF = (\hat{p} - c) * d - (f_0 + f_1 K) \), then the uncertain return on this cash flow is

\[
\hat{R}_{proj} = \frac{CF}{PV_{proj}} = \frac{(\hat{p} - c) * d - (f_0 + f_1 K)}{(1 + R_f)}.
\]

By the Capital Asset Pricing Model (CAPM) the risk-adjusted return on an investment is related to the return on the market portfolio, \( R_m \), and \( R_f \) by

\[
R = R_m + (R_m - R_f) \beta,
\]

where \( \beta \) measures the risk of the uncertain cash flows. Using the definition of \( \beta \)
(Brealey and Myers, 1991), \( \beta = \frac{1}{\sigma_m^2} \text{cov}(\hat{R}_{proj}, \hat{R}_m) \), we find

\[
\beta_{proj} = \frac{(1 + R_f) \text{cov}(\epsilon, \hat{R}_m)}{\sigma_m^2 ((\hat{p} - c) * d - (f_0 + f_1 K))/d}.
\]

Equation (2-5) indicates that project risk (and thus the proper risk adjusted discount rate to use in (2-1) is proportional to the inverse of the operating profit per unit sold. The operating profit per unit sold considers revenues and period costs but excludes the upfront investment.

The firm's problem is to choose production, \( d \), and capacity, \( K \), to maximize the net present value of the project:

\[
\max_{K, d} \pi = E \left[ \frac{(\hat{p} - c) * d - (f_0 + f_1 K)}{(1 + R)} \right] - i_0 - i_1 K.
\]

Since assumption A.2 states that the capacity chosen determines the production rate, we can simplify our analysis and write the firm's problem as

\[
\max_{d} \pi = E \left[ \frac{(\hat{p} - c) * d - (f_0 + f_1 d)}{(1 + R)} \right] - i_0 - i_1 d, \text{ or, using the certainty equivalent form,}
\]
\[
\max_d \pi = \left( p_{ce} - c \right)^* d - \left( f_0 + f_i d \right) \left( 1 + R_f \right) - i_0 - i_1 d.
\]

For the remainder of this paper we assume that the optimal scale equals the optimal production rate. Thus, we can drop \( K \) from the notation, and henceforth refer to \( d \) as the scale of the project. Having developed the model for the firm’s problem in terms of both the risk-adjusted rate and the risk-free rate, we next study its profit maximizing scale decision.

### 2.2 Optimum Scale and Related Measures

The firm’s profit maximizing scale choice \( d^* \), satisfies 
\[
\frac{\partial \pi(d)}{\partial d} = 0,
\]
or
\[
d^* = \frac{a_0 - (c + f_i) - i_1(1 + R_f) - \lambda \text{cov}(\xi, \bar{R}_m)}{2a_1}.
\]

The second derivative of profit with respect to \( d \), \( \frac{\partial^2 \pi(d)}{\partial d^2} \), is strictly negative. Thus we are assured that this scale maximizes profit.

The certainty equivalent of price at the optimal scale is 
\[
p_{ce}(d^*) = a_0 - a_1 d^* - \lambda \text{cov}(\xi, \bar{R}_m),
\]
or
\[
p_{ce}(d^*) = \frac{a_0 + (c + f_i) + i_1(1 + R_f) - \lambda \text{cov}(\xi, \bar{R}_m)}{2}.
\]

The profit that corresponds to the optimal scale of \( d^* \) is
\[
\pi(d^*) = \frac{p_{ce}(d^*)}{1 + R_f} \left( \frac{d^*}{1 + R_f} \right) - f_0 + f_i d^* \left( 1 + R_f \right) - i_0 - i_1 d^*. \]

Substituting for \( p_{ce}(d^*) \) and \( d^* \), and simplifying yields
\[
\pi(d^*) = \frac{1}{(1 + R_f)} \left[ \frac{a_0 - (c + f_i) - i_1(1 + R_f) - \lambda \text{cov}(\xi, \bar{R}_m)}{4a_1} \right]^2 - f_0 - i_0(1 + R_f).
\]

This is equivalent to 
\[
\pi(d^*) = \frac{1}{(1 + R_f)} \left( a_1 d^* - f_0(1 + R_f) \right). \]

For the firm to earn a positive profit and undertake the project, a necessary condition is 
\[
d^* \geq \frac{f_0 + i_0(1 + R_f)}{a_1}.
\]

The
inequality simply states that the revenues from producing $d^*$ must exceed the project fixed costs. In turn, it leads to the condition:

$$d^* \geq \sqrt{\frac{f_0}{a_1}}. \quad (2-8)$$

This inequality states that the firm must earn a profit during the production period that exceeds its fixed period cost.

Thus, the optimum profit is a quadratic function of the demand parameters, period cost parameters, and the risk free rate. The discount rate is found by substituting (2-6) into (2-5) and (2-4)

$$R^* = R_f + \frac{(1 + R_f)\lambda \text{cov}(\tilde{c}, \tilde{R}_m)}{a_0 - c - f_1 + i_1(1 + R_f) - \lambda \text{cov}(\tilde{c}, \tilde{R}_m)} - \frac{2a_1f_0}{2a_0 - c - f_1 - i_1(1 + R_f) - \lambda \text{cov}(\tilde{c}, \tilde{R}_m)} \quad (2-9)$$

From (2-9), demand and technology parameters affect project risk. The appropriate risk adjusted discount rate is a decreasing function in $a_o$ and $i_1$; an increasing function of $a_1$, and $f_o$; an increasing function of $f_1$, c and $\text{cov}(\tilde{c}, \tilde{R}_m)$ so long as operating profit per unit is positive; and is a decreasing function of $i_1$ so long as operating profit per unit is positive. Risk is independent of $i_o$. We conclude that technology parameters $(i_1, f_o, f_1, c)$, and demand parameters $(a_o, a_1)$ affect project risk, and must be included in any decision on project scale. Further discussion of how technology and demand affects risk is deferred to Section 3.

### 2.3 NPV Analysis Using A Management Prescribed Discount Rate

Typically in NPV analysis, a firm prescribes the discount rate to be used by a manager. This discount rate, also called a hurdle rate, is set a priori of scale decisions and is used by the manager for the economic evaluation of the project. In practice, we know of no procedure that is actually used to adjust for scale. However, (2-5) clearly indicates that project risk is a function of the project scale! Effectively, when the hurdle rate is established before the project scale is set, the use of such a prescribed discount rate becomes a “second best” procedure. In much of the remainder of section 2 we explore the effect of a fixed prescribed discount rate on scale
choice and profitability estimates. In section 2.3 we show that the loss in profitability is quadratic in the prescribed rate. In section 2.4, we explore the relationship between the actual discount rate and the prescribed rate. In section 2.5, we show the firm faces an unenviable choice. It will either undervalue profitable projects or scale projects suboptimally. Finally, in section 2.6 we compute the magnitude of the errors that result from the use of a prescribed discount rate.

Suppose a manager is told to choose scale and evaluate a project given a prescribed rate $R_p$. The expected project profitability, based upon the prescribed rate, is

$$\pi_p = \frac{(a_0 - a_1 * d - c) * d - f_0 + f_1 * d}{(1 + R_p)} - (i_0 + i_1 * d).$$

If a manager uses this measure of expected profitability, the profit maximizing scale solves $\pi^*_p = \max_{d} \pi_p$. Differentiating and solving for the zero results in

$$d^*_p = \frac{a_0 - c - f_1 - i_1 \ast (1 + R_p)}{2 \ast a_1}. \tag{2-10}$$

If the firm uses a fixed rate prescribed by management, $R_p$, it will fail to account for how scale affects risk. It will choose scale $d^*_p$ rather than the quantity $d^*$ (2-6). The error in scale is

$$\Delta d^* = d^* - d^*_p = \frac{a_0 - (c + f_1) - i_1 (1 + R_p) - \lambda \text{cov}(\bar{\sigma}, \bar{R})}{2a_1} - \frac{a_0 - c - f_1 - i_1 \ast (1 + R_p)}{2 \ast a_1}, \text{ or}$$

$$\Delta d^* = \frac{i_1 \ast (R_p - R_f) - \lambda \text{cov}(\bar{\sigma}, \bar{R})}{2 \ast a_1}. \tag{2-11}$$

If the firm had chosen the optimum scale and produced $d^*$ it would have earned $\pi(d^*)$. Instead, by using a management prescribed discount rate, it produces a quantity $d^*_p$, thereby earning $\pi(d^*_p)$. 
Actual expected profit, given the firm selects a production quantity that maximizes $\pi_p$, is

$$\pi(d_p^*) = -i_0 - \frac{1}{4a_1} \left[ a_0 - (c + f_1) - i_1 (1 + R_p) \right] + \frac{1}{4a_1} \left[ 2i_1 (R_p - R_f) - 2\lambda \text{cov}(\bar{e}, \bar{R}_m) \right] \frac{\left[ a_0 - (c + f_1) - i_1 (1 + R_p) \right]}{(1 + R_f)}.$$

The firm loses an amount given by $\Delta \pi^* = \pi(d^*) - \pi(d_p^*)$, yielding the expression

$$\Delta \pi^* = \frac{1}{4a_1 (1 + R_f)} \left[ i_1 (R_p - R_f) - \lambda \text{cov}(\bar{e}, \bar{R}_m) \right]. \tag{2-12}$$

In summary, use of a prescribed discount rate, $R_p$, leads to a non-optimal scale, $d_p^*$, being chosen which in turn leads to a loss in NPV given by (2-12).

No firm will select a prescribed rate lower than the risk free rate. Figure 2.1 shows the deviation in scale as a function of the prescribed rate. For values of $R_p$ below the zero-deviation value, the firm over-produces, while for higher discount rates, it under-produces. Figure 2.2 plots the loss in NPV with $R_p$. The loss function is a quadratic in $R_p$.

We conclude that use of a prescribed discount rate can cause large errors in the scale decision that reduces firm profit, although there is a prescribed rate that causes a manager to choose the optimal scale! We next study how scale decisions made using a prescribed discount rate affect the project risk. In subsequent sections, we explore the affect of a prescribed rate on project evaluation and project scale from the manager's perspective.
2.4 The Project Risk Induced by a Prescribed Discount Rate.

If a prescribed rate is used to choose scale, then project scale will be set as \(1-5\). By CAPM and the computation of project beta, the actual project risk is

\[
R(R_p) = R_f + \left(1 + R_f\right) \frac{\lambda \text{cov}(\bar{e}, \bar{R}_m)}{p_{ce}(d_p)} - c - \frac{f_1}{d_p}, \quad \text{or}
\]

\[
R(R_p) = R_f + \left(1 + R_f\right) \frac{\lambda \text{cov}(\bar{e}, \bar{R}_m)}{a_0 - c - f_1 + i_0 \left(1 + R_p\right)} - \frac{2a_1 f_0}{a_0 - c - f_1 - i_0 \left(1 + R_p\right)}. \quad (2-13)
\]

Figure 2.3 plots function \((2-13), R(R_p)\), which is the actual risk induced by the prescribed rate, \(R_p\). The figure also reports the identity function \(R(R_p) = R_p\). The two vertical asymptotes are the solutions to the quadratic formed by setting the denominator of \((2-13)\) to zero. The denominator can be interpreted as the risk adjusted operating profit per unit. The asymptotes then correspond to where the project break even ignoring the initial investment.

Thus, within the asymptotes, the firm's risk adjusted period operating profit is positive. The risk
function and the identity function intersect at two points, \( R_p^i, R_p^p \). Within the interval \([R_p^i, R_p^p]\)
the project's actual risk is lower than the prescribed risk implies.
Figure 2.3 - Scale-dependent risk, $R$, as a function of management prescribed rate, $R_p$, as defined by (2.13). The risk-adjusted operating profit is positive between the bolded vertical asymptotes. Between $R_p$ and $R_{p}^{*}$, the scale-dependent discount rate is less than the prescribed rate; $R_{p}$ is the profit maximizing scale-adjusted discount rate; $R_{p}^{*}$ is the rate that induces the correct production decision.
2.5 Prescribed Discount Rates Cause Managers to Undervalue Profitable Projects, or To Suboptimally Scale Them.

In section 2.3, we showed that the error in the project valuation is quadratic in the firm’s prescribed rate. Further, there is a rate such that the error induced by the use of a prescribed rate is zero. This might lead one to conclude that it is possible for the firm to prescribe a rate such that the project is scaled and valued optimally. In this section, we show that to be untrue. The use of a prescribed rate causes a manager to either undervalue a profitable project or to scale it in a suboptimal fashion. A lose-lose proposition for a firm that prescribes a discount rate before the project scale is set.

Suppose that the prescribed rate that induces the optimal scale decision is $R_p^*$. By examining (2-7), this prescribed rate is

$$R_p^* = R_f^* + \frac{\lambda \text{cov}(\tilde{e}, \tilde{R}_m)}{i_1}. \quad (2-14)$$

We show that $R(R_p^*) < R_p^*$. From (2-9) and (2-14),

$$R(R_p) - R_p = -\lambda \text{cov}(\tilde{e}, \tilde{R}_m) \begin{bmatrix} p_{ce} - c - f_1 - i_1 \left( + R_f \right) - \frac{f_0}{d_p} \\ i_1 \left( p_{ce} - c - f_1 - \frac{f_0}{d_p} \right) \end{bmatrix},$$

The numerator is the NPV divided by the scale of the optimally designed project, and the denominator $p_{ce} - c - f_1 - \frac{f_0}{d_p}$ is the average operating profit. Both will be positive if the project at a scale of $d_p$ has a positive NPV. Consequently, $R(R_p) < R_p$. This immediately implies that if a prescribed rate induces the manager to choose the optimal scale, the manager will strictly undervalue the project! Clearly if a manager uses another prescribed rate, it will suboptimally scale the project.
This result can be extended to include all prescribed rates in the interval $R_p \in [R'_p, R''_p]$. For such a $R_p$, a manager will overestimate risk and undervalue the project. Outside this interval the opposite conclusion holds. We conclude with a stronger characterization of where $R_p^*$ lies on the $R_p$ domain. We show that it is to the left of the minimum point of $R(R_p)$ on the interval $[R'_p, R''_p]$. This property is useful in understanding the effect of using the discount rate of the optimally designed project as the prescribed rate.

**Property 1.1:** Suppose the project is profitable at the optimal scale. Consider Figure 2.3. 

\[ R_p^* \in [R', R''] \] where \[ B_p = \underset{R_p \in [R', R'']}{\text{Arg min}} R(R_p). \]

Proof: $R_p^*$ lies in the interval $[R', R'']$ if the project is profitable. We next show that the risk minimizing prescribed rate (denoted by $B_p$) is larger than $R_p^*$. The minimum of the project risk occurs at a value of $B_p$ which induces scale decision $d_p$, that is the solution to

\[
\frac{\partial \beta_{\text{proj}}}{\partial d_p} = \frac{1}{\sigma_m^2} \text{cov}(\epsilon, \bar{R}_m) \left[ \left( p_{ce} * d_p - c * d_p - F \right) - d_p \frac{\partial}{\partial d_p} \left( \frac{p_{ce} * d_p - c * d_p - F}{p_{ce} * d_p - c * d_p - F} \right) \right] = 0.
\]

As long as the firm earns a positive income during the production period (so the denominator is non zero), this first order condition becomes

\[
(p_{ce} - c - f_1)^* d_p + d_p \frac{\partial p_{ce}}{\partial d_p} = (p_{ce} - c - f_1) - \frac{f_0}{d_p}.
\]

(2-15)

Similarly, the production volume that maximizes true profit is given by $d_p^*$, which is the solution to

\[
\frac{\partial \pi}{\partial d_p} = \frac{\partial}{\partial d_p} \left[ \left( p_{ce} - c - f_1 \right)^* d_p - f_0 \left( 1 + R_f \right) \right] = 0. \text{ This yields}
\]

\[
(p_{ce} - c - f_1)^* d_p^* + d_p^* \frac{\partial p_{ce}}{\partial d_p} = i_l \left( 1 + R_f \right) \]

(2-16)

Note that the LHS of both (2-15) and (2-16) are equal and are monotone decreasing in $d_p$. The firm will earn a positive profit (excluding its initial investment, $i_0$) when the RHS of
(2-15) is greater than the RHS of (2-16). This requires $d_p < d_p^*$, which implies by (2-6),

$$R_p > R_p^*.$$ 

The next section studies the magnitude and sign of errors made when prescribed discount rates are used.

2.6 Magnitude of Errors When Using Prescribed Discount Rate to Make Decisions

We have established that the use of a prescribed discount rate leaves a firm with an unenviable choice. It will either undervalue a project, or choose a suboptimal scale, or both. This is the inevitable result of choosing a discount rate before setting the project scale. Given that an error of one sort or another is unavoidable, we next ask: when are errors using prescribed discount rates large? We explore two situations. They are:

i. The project manager is told to use the prescribed rate that will induce the optimal scale decision.

ii. The manager is told to use the prescribed rate equal to the risk adjusted rate for the optimally scaled project.

We restrict the analysis to only these two instances since any other choice of a discount rate will lead to a decision where the firm will scale the project suboptimally, earn a suboptimal profit, and estimate the project risk incorrectly. A reasonable reader might well ask the question of how the firm decides on the discount rate that corresponds to cases i and ii above. The question for case i is answered in Section 2.3 above. The rate that corresponds to case ii might be the result of an earlier project that had been scaled correctly.

In the first case, although the optimal production decision is made, the manager’s NPV analysis will value the project below its true economic worth. In this case an important question is: when is the difference between the actual and estimated NPVs large?
In the second case, a manager makes a suboptimal production quantity decision and also evaluates the project at a suboptimal discount rate. Two questions arise: does the suboptimal decision cost the firm much? Does the manager undervalue the project as designed by much?

We can show that in general, the losses of the first case are large, and the losses due to suboptimal decisions of the second case are small, and NPV misestimation is also small. Thus, it is generally better to give a manager a prescribed rate set equal to the real project risk (and accept a suboptimal decision) rather than give the manager a rate set induce optimal production.

In the first case, \( R_p = R^*_p \) and the difference between real and estimated NPV is

\[
\Pi^* - \Pi_p(d^*_p) = -\frac{\lambda \text{Cov}(\tilde{\epsilon}, \tilde{R}_M)}{i_1} \left( a_i d^{*2} - f_o \right) \frac{1}{(1 + R_f)(1 + R_f + \frac{\lambda \text{Cov}(\tilde{\epsilon}, \tilde{R}_M)}{i_1})}.
\]

As a fraction of actual profit, the manager underestimates project value by

\[
\frac{\Pi^* - \Pi_p(d^*_p)}{\Pi^*} = -\frac{\lambda \text{Cov}(\tilde{\epsilon}, \tilde{R}_M)}{i_1} \left( a_i d^{*2} - f_o \right) \frac{1}{(1 + R_f + \frac{\lambda \text{Cov}(\tilde{\epsilon}, \tilde{R}_M)}{i_1})} \left( a_i d^{*2} - f_o - i_o(1 + R_f) \right).
\]

Note that this fraction is a large negative one, often close to -1. Projects are undervalued and economic losses arise because managers reject profitable projects.

On the other hand, using \( R_p = R^*_p \) as the prescribed rate induces a fall in real profit:

\[
\Pi^* - \Pi(R_p) = -\frac{\lambda \text{Cov}(\tilde{\epsilon}, \tilde{R}_M)}{4a_1} \left[ \frac{i_1 (1 + R_f) d^*}{a_i d^{*2} + i_1 (1 + R_f) d^* - f_o} \right]^2,
\]

and in fractional terms:

\[
\frac{\Pi^* - \Pi(R_p)}{\Pi^*} =
\]

\[
-\frac{(1 + R_f) \left[ \lambda \text{Cov}(\tilde{\epsilon}, \tilde{R}_M) \right]}{4a_1} \left[ \frac{i_1 (1 + R_f) d^*}{a_i d^{*2} + i_1 (1 + R_f) d^* - f_o} \right]^2 \left[ \frac{1}{a_i d^{*2} - f_o + i_o(1 + R_f)} \right].
\]
This real loss is small when \( 1/\left[ a_1 d^* - f_o + i_o (1 + R_f) \right] \) is small. When \( a_1 d^* - f_o > 0 \), per period operating profit is positive. Therefore, real fractional errors can be large only when optimum profits and the initial fixed investment are both small. Managers do not make economically significant errors when using the optimal project risk.

The manager undervalues the project (compared to the optimal scaled one) when using the optimal project risk as a prescribed rate:

\[
\Pi^* - \Pi_p(R^*) = \frac{i_i^2 \left( \lambda \text{Cov}(\bar{\epsilon}, \bar{R}_M) \right)^2 \left( 1 + R_f \right) d^*}{4a_1^3 (d^* - \frac{f_o}{a_1})^2}.
\]

As a fraction of optimal profit the error is:

\[
\frac{\Pi^* - \Pi_p(R^*)}{\Pi^*} = \frac{i_i \left( \lambda \text{Cov}(\bar{\epsilon}, \bar{R}_M) \right) \left( 1 + R_f \right)^2 d^*}{(d^* - \frac{i_o (1 + R_f) + f_o}{a_1})(d^* + \frac{(1 + R_f) d^* - f_o}{a_1})}.
\]

Thus, the fractional error behaves approximately as \( 1/\left( d^* - \frac{f_o}{a_1} \right)^3 \), which is small so long as \( d^* - \frac{f_o}{a_1} \) is sufficiently large.

Finally, we can deduce that if \( R^* \) is set as the prescribed rate then the firm will choose too large a scale and overvalue the project at the chosen scale. The argument is as follows. Property 1.1 showed that function \( R \) rises to the left of \( R_p^* \) and this implies that \( R(R_p^*) > R^* \).

However, \( R_p^* = R(R_p^*) \) implies \( R_p^* > R^* \) so that by (2.6) \( d_p(R^*) > d_p(R_p^*) = d^* \). At this scale, the manager will overvalue the project because to the left of \( R_p^* \), \( R(R_p^*) > R_p^* \). See Figure 2.3.
3 Effect of Technology and Demand Parameters on Project Value

In previous sections we found mathematical relationships between demand parameters/cost parameters and the project scale, project risk and project valuation. In this section, we more carefully study the effect of technology and demand parameters on firm scale decisions and profit. Knowledge of these relationships allows insights about how these parameters affect project risk, scale decisions and project evaluation. In some contexts knowledge of the partial derivatives of scale, risk and valuation with respect to parameters allows qualitative ranking of different technologies.

For example, Lederer and Singhal (1989) presented empirical evidence that flexible manufacturing system (FMS) technology has lower variable production cost and fixed period cost than conventional labor-intensive technologies. The initial investments to obtain this technology are considerable. In this section, we show that this cost and investment pattern implies that scale dependent investments in FMS are a priori less risky than scale dependent investments in conventional technology. Thus, FMS should be evaluated at a lower discount rate than conventional technology!

Further, the following analysis allows prediction of when small changes in cost/demand parameters will greatly affect scale decisions and project evaluation

3.1 Impact of Technology Choice on Project Risk and NPV

The technology used by the firm is described by the vector \( \tau = (i_o, l, f_o, f_1, c) \). We study how changes in these parameters affect key performance measures: project value, scale choice, project risk and error in project value created when scale decisions are made using a prescribed rate. We answer these questions by finding the derivatives of these performance measures with respect to the parameters and *signing* them. Table 3.1 summarizes the results of the analysis.

Intuitively, one would expect the relation between a cost component and risk would be defined by a non-negative first derivative, i.e., an increase in cost would, at best, be risk-neutral but, more typically, increase the risk. The initial scale-independent investment, \( i_0 \), falls in the first category, and the project risk is independent it – as one would expect. The per-period costs, \( f_o \) and \( f_1 \), fall in the latter category, and an increase in either leads to higher risk. However, the
per-unit cost of initial installation, \( i_0 \), falls into neither of these categories. An increase in \( i_1 \) leads to a lower project risk!

Fixed period cost, \( f_0 \), impacts the firm’s decisions and performance measures. Increasing fixed period cost reduces the profitability of the project by the net present value of the change and a sufficient increase would lead to the cancellation of the project. Consistent with equation (2-5), an increase in \( f_0 \) increases project risk, and this change is inversely proportional to the scale of the project. However, the optimum scale is independent of this fixed cost, since at the optimal production level, marginal cost equal to marginal revenue (and fixed costs play no role in marginal analysis). Further, since fixed period cost has no effect on the optimum production volume, it does not affect the consequence to the firm if it adopts a wrong pre-determined discount rate.

Fixed initial investment, \( i_0 \), has a similar effect on performance: an increase in \( i_0 \) decreases project value by the present value of the change, leaves scale constant, and has no affect on the absolute error made when using a prescribed rate. Unlike \( f_0 \), however, \( i_0 \) does not affect project risk.

Other cost parameters, \( c \), \( f_1 \), and \( i_1 \), affect the project risk either by affecting the per-unit average profit or because of their affect on the scale of the project.

An increase in the variable cost, \( c \), or the per-unit fixed period cost, \( f_1 \), lowers profitability, and operating profit per unit. Thus such changes increase risk. Increases in these two parameters do not affect the error from use of a prescribed discount rate because each parameter has an identical effect on the firm’s decisions under the two regimes. Both \( d^* \) and \( d'^* \) are affected identically by changes in either \( c \) or \( f_1 \) (\( \frac{\partial d^*}{\partial c} = \frac{\partial d'^*}{\partial c} = \frac{1}{2a_1} \frac{\partial d^*}{\partial f_1} = \frac{\partial d'^*}{\partial f_1} \)).

Consequently, the difference between them remains unchanged. Since the error in profit is a function of the difference in production volumes, that, too, remains unaffected.

While at first blush the effect of \( i_1 \) on risk might seem non-intuitive, its effect on the project risk is only indirect – through its effect on the optimal production quantity, \( d^* \), i.e.,
\[
\frac{dR^*}{di_1} = \frac{\partial R^*}{\partial d^*} \frac{\partial d^*}{di_1}. \]

Since \(d^*\) is monotonically decreasing in \(i_1\), \(\frac{\partial d^*}{di_1} < 0\). Also, from (2-9)

\[
R^* = R_f + \frac{\left(1 + R_f\right)\lambda \text{cov}(\tilde{e}, \tilde{R}_m)}{a_0 - a_1 d^* - c - f_1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m) - \frac{f_0}{d^*}}, \text{ and}
\]

\[
\frac{\partial R^*}{\partial d^*} = \frac{\left(1 + R_f\right)\lambda \text{cov}(\tilde{e}, \tilde{R}_m)}{\left(a_0 - a_1 d^* - c - f_1 - \lambda \text{cov}(\tilde{e}, \tilde{R}_m) - \frac{f_0}{d^*}\right)^2} \left[-a_1 + \frac{f_0}{d^*} \right]. \text{ As long as the firm earns a positive profit (2-8),} \frac{\partial R^*}{\partial d^*} > 0. \text{ The combined effect is that the project risk is decreasing in the initial per-unit investment, } i_1, \text{ or } \frac{dR^*}{di_1} = \frac{\partial R^*}{\partial d^*} \frac{\partial d^*}{di_1} < 0.\]

Thus if we compare two technologies where one has lower fixed period cost, lower unit variable cost, and higher scale dependent investment than the other, we know that the former has a lower financial risk than the other and ought to be evaluated at a lower discount rate!

The elasticity of the performance measures with respect to the technology parameters is related to the inverse of the performance measure. For example, the elasticity of project profitability with respect to the parameters is proportional to the inverse of project profitability. A similar result holds for the scale decision and project risk. Consequently, when project profitability is low, the performance metrics exhibit much greater sensitivity to a change in the technology vector, \(\tau\). Table 3.2 summarizes the elasticities of the key performance measures with respect to the parameters.
<table>
<thead>
<tr>
<th></th>
<th>( \pi )</th>
<th>( d^* )</th>
<th>( \beta_{proj} )</th>
<th>( \Delta \pi^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed period cost, ( f_0 )</td>
<td>(- \frac{1}{1 + R_f} ); strictly negative could lead to project cancellation</td>
<td>No impact, given that the project is undertaken</td>
<td>( \frac{\beta^2_{proj}}{d^* \text{cov}(e, \tilde{R}_m)(1 + R_f) / \sigma_m^2} ); strictly positive</td>
<td>No effect</td>
</tr>
<tr>
<td>Variable period cost, ( f_1 )</td>
<td>(- \frac{d^*}{1 + R_f} ); strictly negative could lead to project cancellation</td>
<td>(- \frac{1}{2a} ); strictly negative</td>
<td>( \frac{\beta^2_{proj}}{\text{cov}(e, \tilde{R}_m)(1 + R_f) / \sigma_m^2} \left[ \frac{1}{2} + \frac{f_0}{2a, d^*^2} \right] ); strictly positive</td>
<td>No effect</td>
</tr>
<tr>
<td>Fixed initial investment, ( i_0 )</td>
<td>-1; could lead to project cancellation strictly negative</td>
<td>No impact, given that the project is undertaken</td>
<td>No effect</td>
<td>No effect</td>
</tr>
<tr>
<td>Variable initial investment, ( i_1 )</td>
<td>(- \frac{d^*}{1 + R_f} ); could lead to project cancellation strictly negative</td>
<td>(- \frac{1 + R_f}{2a} ); strictly negative</td>
<td>( \frac{\beta^2_{proj}}{2 \text{cov}(e, \tilde{R}_m) / \sigma_m^2} \left[ \frac{f_0}{a, d^<em>^2} - 1 \right] / \left( \text{cov}(e, \tilde{R}_m)(1 + R_f) / \sigma_m^2 \right) ); strictly negative if the firm earns a positive profit, i.e., ( d^</em> &gt; \sqrt{\frac{f_0}{a}} )</td>
<td>( \frac{(R_p - R_f)(R_f - R_t) - \lambda \text{cov}(e, \tilde{R}_m)}{2a, (1 + R_f)} )</td>
</tr>
</tbody>
</table>

| Variable production cost, \( c \) | \(- \frac{d^*}{1 + R_f} \); could lead to project cancellation strictly negative | \(- \frac{1}{2a} \); strictly negative         | \( \frac{\beta^2_{proj}}{\text{cov}(e, \tilde{R}_m)(1 + R_f) / \sigma_m^2} \left[ \frac{1}{2} + \frac{f_0}{2a, d^*^2} \right] \); strictly positive | No effect                                   |

**Table 3.1** - Derivative of \( \pi, d^*, \beta_{proj}, \) and \( \Delta \pi^* \), respectively, with respect to the row parameters \( f_0, f_1, i_0, i_1, \) and \( c \)

The effect of the row parameters, with one exception, on the various performance metrics and decision variables matches intuitive expectations. The exception is \( i_1 \), which has a non-intuitive relation with project risk because its effect is only indirect – through its effect on optimal project scale, \( d^* \).
<table>
<thead>
<tr>
<th></th>
<th>( \pi )</th>
<th>( d^* )</th>
<th>( \beta_{\text{proj}} )</th>
<th>( \Delta \pi^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed period cost,</strong>( f_0 )</td>
<td>(- \frac{1}{1 + R_f \pi^<em>} \frac{f_0}{\pi^</em>} ); strictly negative</td>
<td>No effect</td>
<td>( \frac{\beta_{\text{proj}} f_0}{d^* \text{cov}(\bar{\varepsilon}, \bar{R}_m)(1 + R_f)/\sigma^2_m} ); strictly positive</td>
<td>No effect</td>
</tr>
<tr>
<td><strong>Variable period cost,</strong>( f_1 )</td>
<td>(- \frac{d^<em>}{1 + R_f \pi^</em>} \frac{f_1}{\pi^*} ); strictly negative</td>
<td>(- \frac{1}{2a_i} \frac{f_1}{d^*} ); strictly negative</td>
<td>( \frac{\beta_{\text{proj}} f_1}{\text{cov}(\bar{\varepsilon}, \bar{R}_m)(1 + R_f)/\sigma^2_m} \left[ \frac{1}{2} + \frac{f_0^2}{2a_q d^*} \right] ); strictly positive</td>
<td>No effect</td>
</tr>
<tr>
<td><strong>Fixed initial investment,</strong>( i_0 )</td>
<td>(- \frac{i_0}{\pi^*} ); strictly negative</td>
<td>No effect</td>
<td>No effect</td>
<td>No effect</td>
</tr>
</tbody>
</table>
| **Variable initial investment,**\( i_1 \) | \(- \frac{d^* i_1}{\pi^*} \); strictly negative | \(- \frac{(1 + R_f) i_1}{2a_i} \frac{d^*}{\pi^*} \); strictly negative | \( \frac{\beta_{\text{proj}} i_1}{2 \text{cov}(\bar{\varepsilon}, \bar{R}_m)/\sigma^2_m} \left[ \frac{f_0}{2a_q d^*} - 1 \right] \); strictly negative if the firm earns a positive period profit (i.e., if \( d^* > \sqrt[1+R_f]{\frac{f_0}{a_i}} \)) | \( \frac{2i_1(R_p - R_f)}{[i_1(R_p - R_f) - \lambda \text{cov}(\bar{\varepsilon}, \bar{R}_m)]} \)
| **Variable production cost,**\( c \) | \(- \frac{d^* c}{1 + R_f \pi^*} \); strictly negative | \(- \frac{1}{2a_i} \frac{c}{d^*} \); strictly negative | \( \frac{\beta_{\text{proj}} c}{\text{cov}(\bar{\varepsilon}, \bar{R}_m)(1 + R_f)/\sigma^2_m} \left[ \frac{1}{2} + \frac{f_0^2}{2a_q d^*} \right] \); strictly positive | No effect |

Table 3.2 – Elasticity of \( \pi^*, d^*, \beta_{\text{proj}} \) and \( \Delta \pi^* \), respectively, with respect to row parameters \( f_0, f_1, i_0, i_1, \) and \( c \)

When the profit is low, its sensitivity to changes in the cost parameters is high; similarly, when the operating margin is low, project risk is high and is more sensitivity to the cost parameters.
3.2 How Changes in the Demand Function Parameters Affect Performance Metrics

In this section, we look at the impact of demand related parameters on performance, specifically, changes in $a_0$, $a_1$, and the covariance, $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$. Adoption of a new production technology can have demand as well as cost effects due to increased quality and customer service. An increase in $a_0$ reflects a favorable shift in the demand curve. One possible cause for this would be a new use for the firm's product, something that would expand its overall potential market. Or, adoption of new technology may increase customer service or product quality also causing demand to favorably shift. A change in $a_1$ could result from a change in the set of substitute products. For example, if another firm introduces a new substitute product, demand would become more elastic, resulting in an increase in $a_1$. Another demand related parameter, $\text{cov}(\tilde{\varepsilon}, \tilde{R}_m)$ – equivalent to $\text{cov} \left( \tilde{p} + \tilde{\varepsilon}, \tilde{R}_m \right)$, or $\text{cov} \left( \tilde{p}, \tilde{R}_m \right)$ – is a significant term since it defines how sensitive the product price is to changes in overall market conditions.

Table 3.3 summarizes the effects on the DCF, optimal production quantity, the project risk, and the error from a prescribed discount rate of a unit increase in $a_0$, $a_1$, and $\text{cov} \left( \tilde{p}, \tilde{R}_m \right)$. 
<table>
<thead>
<tr>
<th>Demand curve y-intercept, $a_0$</th>
<th>$\pi$</th>
<th>$d^*$</th>
<th>$\beta_{proj}$</th>
<th>$\Delta \pi^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d^*}{1 + R_f}$</td>
<td>$\frac{1}{2a_1}$</td>
<td>$-\frac{\beta_{proj}^2}{(1 + R_f) \text{cov}(\bar{\epsilon}, \bar{R}_m) / \sigma_m^2}$</td>
<td>$\frac{f_0}{2a_1 d^* + 2}$</td>
<td>No effect</td>
</tr>
<tr>
<td>Demand curve slope, $a_1$</td>
<td>$-\frac{d^*}{a_1}$</td>
<td>$-\frac{\beta_{proj}^2}{(1 + R_f) \text{cov}(\bar{\epsilon}, \bar{R}_m) / \sigma_m^2}$</td>
<td>$\frac{f_0}{a_1 d^*}$</td>
<td>$-\frac{\lambda \left( \text{cov}(\bar{\epsilon}, \bar{R}_m) \right)}{4a_1^2 (1 + R_f)}$</td>
</tr>
<tr>
<td>Covariance, $\text{cov}(\bar{\epsilon}, \bar{R}_m)$</td>
<td>$-\frac{\lambda d^*}{1 + R_f}$</td>
<td>$-\frac{\lambda}{2a_1}$</td>
<td>$-\frac{\beta_{proj}^2}{(1 + R_f) \text{cov}(\bar{\epsilon}, \bar{R}_m) / \sigma_m^2}$</td>
<td>$\frac{(1 + R_f) \sigma_m^2}{2a_1 d^* + 2}$</td>
</tr>
</tbody>
</table>

Table 3.3 – Rate of change of $\pi$, $d^*$, $\beta_{proj}$ and $\Delta \pi^*$ w.r.t. the row parameters $a_0$, $a_1$, and $\text{cov}(\bar{\epsilon}, \bar{R}_m)$

Project value increases and project risk decreases with an increase in the demand function intercept, $a_0$, an increase in demand inelasticity, $a_1$, or a decrease in demand uncertainty, $\text{cov}(\bar{\epsilon}, \bar{R}_m)$. 
4 Examples and an Extension

Having studied the relationship between the performance metrics, the decision variables and the model parameters, we present some examples based upon the estimates of financial parameters recorded in . We show first that project risk varies according to relationships derived in Table 3.1 and 3.2 and that differences in project risk can be large. We then demonstrate the errors in scale choice and evaluation that occur when prescribed discount rates are used.

The values of the parameters of the demand function are based upon an empirical study of a local manufacturer of prototype lenses. In what we call the base case, the intercept of the inverse demand function, \( a_0 \), is 200. The slope of the function, \( a_1 \), is 0.85. The covariance of the error term and the market return is 5, i.e., \( \text{cov}(\tilde{\epsilon}, \tilde{R}_m) = 5 \). The cost structure of the base case is \( \tau = (t_0 = 0, t_1 = 15, f_0 = 1,000, f_1 = 10, c = 1) \).

First, we compare the risk of the base case with the risk of three different technologies each of which is incrementally different from the base case (for a total of four technologies). Each of the three new technologies differs from the base case by a single parameter, \( i = 5, f_o = 100 \), or \( f_1 = 1 \), respectively. We study the effect of these four technologies for three different demand functions – characterized by \( a_0 \) values of 100, 200, and 500, respectively.

Finally, we study the effect of the correlation between the product price and market return for each of these 12 combinations. We do so through the covariance term, \( \text{cov}(\tilde{\epsilon}, \tilde{R}_m) \), with values of 1, 2, 5, and 10.

Table 4.1 presents the risk-adjusted discount rate for the 48 possible combinations. Compared to the base case for each demand function and covariance (the three columns labeled ‘base’ in Table 4.1), as the fixed period costs \( f_0 \) and \( f_1 \) decrease (the columns labeled \( f_1 = 1 \) and \( f_0 = 100 \)), both the expected and the uncertain future operating profits increase. Consequently, the risk associated with the project decreases, and the project discount rate decreases. These cost parameters can have a large effect: if \( \text{cov}(\tilde{\epsilon}, \tilde{R}_m) = 2 \) and \( a_0 = 100 \), then a shift of \( f_1 \) from 1 to 10 causes the discount rate to fall from 21.3% to 12.3%.
If the firm earns a positive profit, a decrease in $i_1$ leads to an *increase* in risk, a result that is consistent with the parametric analysis of Section 3. This is shown in the result table below. The one exception is when the covariance is 10 and the demand function intercept is 100. In this case, the firm earns a negative profit in the base case itself, and a decrease in $i_1$ leads, as expected, to a decrease in project risk.

As the demand function intercept increases from $a_0=100$ to $a_0=500$, the influence of the error term ($\varepsilon$) on the price decreases. As this uncertainty decreases, the risk of the project decreases. Going from left to right in the table below, the risk of the same technology decreases. For example, if $\text{cov}(\varepsilon, \tilde{R}_m)$, the risk of the base case falls as $a_0$ increases from 21.3% ($a_0=100$), to 14.7% ($a_0=200$) to 4.9% ($a_0=500$).

As the covariance term increases, the volatility increases and the firm compensates with a lower certainty-equivalent of price ($p_{ce} = p - \lambda * d * \text{cov}(\varepsilon, \tilde{R}_m)$). As this risk-adjusted expected price decreases, expected future revenues decline, as do the expected unit margin and the expected average unit profit. The result is increased risk. For the case, $a_0=100$ using base cost parameters, the risk rises with $\text{cov}(\varepsilon, \tilde{R}_m)$: 11.5% (for $\text{cov}(\varepsilon, \tilde{R}_m)=1$), 21.3% (for $\text{cov}(\varepsilon, \tilde{R}_m)=2$), 63.0% (for $\text{cov}(\varepsilon, \tilde{R}_m)=5$) and 305.0% (for $\text{cov}(\varepsilon, \tilde{R}_m)=10$).

A final effect worth noting is the increasing rate-of-change relative to the covariance term. The risk of a project is inversely proportional to the risk-adjusted operating margin. A key adjustment is the reduction in price by the $-\lambda * d * \text{cov}(\varepsilon, \tilde{R}_m)$ term. Consequently, as the variability increases through the covariance term, the operating margin approaches zero and the risk increases sharply.
<table>
<thead>
<tr>
<th>$\text{cov}(\bar{c}, R_m)$</th>
<th>$a_0=100$</th>
<th>$a_0=200$</th>
<th>$a_0=500$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base</strong></td>
<td>$f_1=1$</td>
<td>$f_2=100$</td>
<td>$i_1=20$</td>
</tr>
<tr>
<td>1</td>
<td>11.5%</td>
<td>5.5%</td>
<td>3.94%</td>
</tr>
<tr>
<td></td>
<td>9.7%</td>
<td>5.4%</td>
<td>3.92%</td>
</tr>
<tr>
<td></td>
<td>7.8%</td>
<td>5.3%</td>
<td>3.92%</td>
</tr>
<tr>
<td></td>
<td>11.3%</td>
<td>5.5%</td>
<td>3.93%</td>
</tr>
<tr>
<td>2</td>
<td>21.3%</td>
<td>8.2%</td>
<td>4.88%</td>
</tr>
<tr>
<td></td>
<td>17.2%</td>
<td>7.9%</td>
<td>4.85%</td>
</tr>
<tr>
<td></td>
<td>12.8%</td>
<td>7.7%</td>
<td>4.86%</td>
</tr>
<tr>
<td></td>
<td>20.9%</td>
<td>8.0%</td>
<td>4.86%</td>
</tr>
<tr>
<td>5</td>
<td>63.0%</td>
<td>16.5%</td>
<td>7.77%</td>
</tr>
<tr>
<td></td>
<td>45.5%</td>
<td>15.7%</td>
<td>7.68%</td>
</tr>
<tr>
<td></td>
<td>29.4%</td>
<td>15.1%</td>
<td>7.71%</td>
</tr>
<tr>
<td></td>
<td>62.6%</td>
<td>16.1%</td>
<td>7.71%</td>
</tr>
<tr>
<td>10</td>
<td>305.0%</td>
<td>32.1%</td>
<td>12.77%</td>
</tr>
<tr>
<td></td>
<td>138.0%</td>
<td>30.3%</td>
<td>12.59%</td>
</tr>
<tr>
<td></td>
<td>64.6%</td>
<td>28.8%</td>
<td>12.63%</td>
</tr>
<tr>
<td></td>
<td>358.8%</td>
<td>31.3%</td>
<td>12.67%</td>
</tr>
</tbody>
</table>

Base case definition is $f_1=100$, $f_0=1,000$, and $i_1=15$

Table 4.1—Project risk in percent as a function of technology parameters under different demand parameters. Note that project risk varies greatly in the cases: from 3.93% to 385.8%. For any particular combination of $a_0$ and demand uncertainty, $\text{cov}(\bar{c}, R_m)$, a decrease in the fixed operating cost, $f_0$, or in the per-unit operating cost $f_1$, decreases project risk. A non-intuitive result is that for a profitable project an increase in the per-unit installation cost $i_1$, decreases project risk! An increase in product demand (an increase in $a_0$) decreases project risk and an increase in demand uncertainty, $\text{cov}(\bar{c}, R_m)$, increases project risk.
We now turn our attention to two cases in which a manager uses a prescribed rate for the scale decision and valuation. In the first case (Table 4.2), the prescribed rate is set equal to the optimal scale-dependent discount rate. When a manager treats this rate as a fixed, scale-independent, rate, the result is an increase in the project scale. Consequently, the firm chooses a scale larger than optimal and overvalues the project. In addition, the firm ignores the volatility of the demand function in its analysis, and both effects are exaggerated as the covariance increases. For example, when \( \text{cov}(\bar{e}, \bar{R}_m) = 5 \), the manager will choose a scale 5.69% too large, estimate profits as being 3.56% too high, and actually reduce project value by 1.84% compared to optimal scale decisions. All of these effects increase in value to 12.01%, 83.78% and -32.04%, respectively, when \( \text{cov}(\bar{e}, \bar{R}_m) = 10 \).

In the second case (see Table 4.3), the prescribed rate is set to induce a manager to choose the optimal scale. Asked to use such a prescribed rate, a manager greatly undervalues the project although the scale decision is optimal. In this example, even if demand variability is its smallest value (\( \text{cov}(\bar{e}, \bar{R}_m) = 1 \)), the undervaluation error exceeds 43%! This error only increases with increasing demand variability.
<table>
<thead>
<tr>
<th>$\text{cov}(\bar{c},R_m)$</th>
<th>Scale dependent</th>
<th>Analysis with pre-set rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td></td>
<td>Optimal</td>
<td>Benefit</td>
</tr>
<tr>
<td></td>
<td>quantity</td>
<td>Profit</td>
</tr>
<tr>
<td>1</td>
<td>100.76</td>
<td>2,158</td>
</tr>
<tr>
<td>2</td>
<td>99.44</td>
<td>1,940</td>
</tr>
<tr>
<td>5</td>
<td>95.47</td>
<td>1,301</td>
</tr>
<tr>
<td>10</td>
<td>88.85</td>
<td>294</td>
</tr>
</tbody>
</table>

Table 4.2 – If the prescribed discount rate is equal to the actual risk of the project at the profit maximizing scale, the manager will undervalue the project and choose scale larger than optimal. This example shows that generally, the errors in project valuation are small, as is the loss due to suboptimal scale decisions unless \(\text{cov}(\bar{c}, R_M)\) is large. In this example, \(a_0=200, f_0=1,000, f_1=10, i_0=5,250, i_1=15\). The table records the optimal scale (column a), profit (column b) and the risk adjusted discount rate at these decisions (column c). It also reports the quantity decision if the discount rate in column c is used to set scale (column d) and the actual project value given the suboptimal scale decision (column e). Column g reports the size of errors in production, column h reports overestimation of profit, and column i reports the loss due to suboptimal scale choice, all expressed in percentage of the respective optimal decisions. Errors made in the last three columns rise as \(\text{cov}(\bar{c}, R_M)\) increases.

<table>
<thead>
<tr>
<th>$\text{cov}(\bar{c},R_m)$</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimal</td>
<td>Benefit</td>
<td>Quantity at optimality</td>
<td>Discount rate</td>
<td>Rate to ensure optimal decision</td>
<td>Profit given pre-set rate</td>
</tr>
<tr>
<td></td>
<td>quantity</td>
<td>Profit</td>
<td>((d_p^*))</td>
<td>((R^*))</td>
<td>(R_p(d_p^<em>-\pi^</em>))</td>
<td>((\pi(d_p^*)))</td>
</tr>
<tr>
<td>1</td>
<td>100.76</td>
<td>2,158</td>
<td>5.5%</td>
<td>18.0%</td>
<td>1,217</td>
<td>43.6%</td>
</tr>
<tr>
<td>2</td>
<td>99.44</td>
<td>1,940</td>
<td>8.2%</td>
<td>33.0%</td>
<td>318</td>
<td>83.6%</td>
</tr>
<tr>
<td>5</td>
<td>95.47</td>
<td>1,301</td>
<td>16.5%</td>
<td>78.0%</td>
<td>1,459</td>
<td>212.2%</td>
</tr>
<tr>
<td>10</td>
<td>88.85</td>
<td>294</td>
<td>32.1%</td>
<td>153.0%</td>
<td>-2,993</td>
<td>-1,117.0%</td>
</tr>
</tbody>
</table>

Table 4.3 – If the prescribed rate is set to induce optimal scale decisions, a manager will greatly undervalue the project. The degree of undervaluation is generally large and increases with the \(\text{cov}(\bar{c}, R_M)\). In this example \(a_0=200, f_0=1,000, f_1=10, i_0=5,250, i_1=15\). Column a reports the optimal scale, column b reports the profit and column c reports the risk adjusted discount rate. Column d reports the prescribed discount rate that induces a manager to make optimal decisions. Column d reports the manager's estimate of profit, and column e reports the size of the manager's estimating error as a percentage of actual optimal profits.
4.1 Extensions

In this paper, we assumed the firm makes an \textit{a priori} commitment to its capacity and production quantity choices. While this is true in many instances, there are other operating strategies, in which the firm can adjust the production quantity and maybe even the available capacity after it observes the realization in demand uncertainty. Mehta and Lederer (2002) uses the Options Pricing methodology to analyze risk and profitability of these business scenarios.

We also assumed in this paper that the firm delivers product to its customers as and when it can. In Mehta and Lederer (2002), we analyze the case where the speed of delivery affects the price charged by the firm.

In this section, we show how to extend the model to an infinite horizon or to a multi-period finite horizon. Denote the price disturbance term in period \( t \) by \( e_t \), and the market return in period \( t \) by \( R_{m,t} \). The easiest case occurs when \( \text{cov}(e_t, e_{m,t}) = c \) for all \( t \), and similarly, the risk free rate is constant for all future periods.

Then, the \( n \)-period profit function is

\[
\pi = \sum_{i=1}^{n} \left( \frac{(p_{ce} - c) * d - (f_0 + f_1 K)}{1 + R_f} \right) - i_0 - i_1 K,
\]

or

\[
\pi = (p_{ce} - c) * d - (f_0 + f_1 K) \left[ \sum_{i=1}^{n} \frac{1}{1 + R_f} - \sum_{i=n+1}^{\infty} \frac{1}{1 + R_f} \right] - i_0 - i_1 K.
\]

Thus,

\[
\pi = (p_{ce} - c) * d - (f_0 + f_1 K) \left[ \frac{1}{R_f} - \frac{1}{(1 + R_f)^{n+1} R_f} \right] - i_0 - i_1 K.
\]

We may write

\[
\pi = \frac{(p_{ce} - c) * d - (f_0 + f_1 K)}{(1 + R_f^{\text{multiperiod}})} - i_0 - i_1 K
\]

with

\[
R_f^{\text{multiperiod}} = \left[ \frac{1}{R_f} - \frac{1}{(1 + R_f)^{n+1} R_f} \right]^{-1}.
\]
For example if $R_f = .03$, and $n = 10$, $R_{multi\text{period}} = .0807$. All our results for the one period problem extend to multiple period analysis with this new risk free rate. Thus, the effect of multiple period analysis is to increase the absolute and relative errors made when using prescribed discount rates.

5 Summary

In this paper, we looked at the effect of risk on the economic evaluation of projects in which capacity decisions are made in advance and are irreversible. We measure systemic risk through the discount rate (or, equivalently, $\beta$), a measure from modern finance theory. Using the Capital Asset Pricing Model and its extensions, we show that the risk and the profitability of a project are dependent on both the technology that the firm selects and the chosen scale of the project. The former plays a role through the cost structure (initial investment, periodic operating costs, and production cost). The latter becomes important when the firm may influence the price it charges since increasing project scale requires the firm to lower the price, thereby reducing its operating profit margin.

We show that the project risk is inversely proportional to the average operating profit. This important result implies that any parameter that affects operating profits affects risk. Examples of parameters internal to the firm would be the cost structure resulting from its choice of technology. Examples of parameters external to the firm would be demand for its product, and the degree of demand uncertainty.

A non-intuitive result that stands out is that an increase in the up-front unit cost of capacity actually decreases project risk.

Project scale also affects the risk of the project in a direct fashion, and firms should adjust the discount rate used to evaluate a project according to project scale. This leads to another important conclusion. A firm that sets a discount rate to value the project prior to deciding the project scale faces a set of unenviable choices. It will choose suboptimal scale and incur a loss that is quadratic in its deviation from the optimal discount rate. Even if it chooses a rate that induces the correct capacity decision, the firm will undervalue the project. Finally, if it prescribes a rate equal to the scale-appropriate rate, the firm will, yet again, choose suboptimal
capacity and overvalue the project. If the firm intends to use a fixed discount rate, we show that it is better off using a rate closer to the optimal rate rather than one that induces the correct scale decision. The error in valuation in the former case is significantly lower than in the latter case. Irrespective of how it chooses the prescribed rate, the key result is that the firm cannot correctly assess both the risk and the profitability of the project.

6 References


Mehta and Lederer, 2002, Options Pricing work


Markowitz, H. M., 1952, “Portfolio Selection,” Journal of Finance, 7:77-91 (March 1952)

