SPATIAL DUOPOLY WITH DISCRIMINATORY PRICING

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The concept and existence of an equilibrium is established for profit maximizing competitors whose decisions involve choices of both delivered price schedules and firm locations. Each firm faces a production function; each is allowed to locate in the plane and to set discriminatory prices. Any transport cost function that is continuous in the firm location variable may be used. It is shown that the locations of the two firms are in equilibrium if each firm is minimizing social cost (i.e., the total cost to the firms of supplying the market with the good it demands is minimized) with respect to the opponent's fixed location.

1. Introduction

In this paper, we establish the concept and existence of an equilibrium for profit maximizing competitors whose decisions involve choices of both discriminatory prices and firm locations.

The spatial competition literature, beginning with the work of Hotelling (1929) has been focused on the use of f.o.b. pricing by competing firms. Hotelling considered two identical, single product firms competing in a bounded linear market over which consumers with inelastic demand are uniformly distributed. The firms compete in price and location. Hotelling suggested a Nash equilibrium in prices and locations characterized by back-to-back locations at the center of the market. Subsequent work, most recently by D'Aspremont, Gabszewicz, and Thisse (1979), has demonstrated that, when price is allowed to be used as a strategic variable, no equilibrium in prices or locations exists. If, however, prices are assumed identical and fixed for both firms, Hotelling's central clustering will occur. Work by Smithies (1941), Hartwick and Hartwick (1971), and Eaton (1972) showed that a Nash equilibrium in f.o.b. prices and locations can exist for the two- and three-firm problem in markets with a uniform distribution of consumers each having identical elastic demand functions. In contrast with this litera-

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ture, here firms are allowed to locate in the plane and to set discriminatory prices. Further, the firms are not required to be identical, and they have alternative production possibilities.

This paper develops issues of competitive spatial price discrimination first studied in Lederer (1981) and in Lederer and Hurter (1985), where a more formal treatment of our model is found.

Hoover (1936) analyzed spatial price discrimination for firms with fixed locations. He concluded that a firm serving a particular market point would be constrained in its local price by the marginal cost of service faced by other firms in serving that point. In situations where demand elasticity is 'not too high', price at the market point is equal to the marginal cost of the firm with the next lowest marginal cost of serving the market point. This result was extended and reinforced later by Hurter and Lowe (1976a, b). We find that equilibrium prices in our model, defined and characterized in a careful manner, have a similar property.

Let two firms, denoted A and B, be located on a compact set S, a subset of the plane having a non-empty interior. The locations of the firms are denoted $z_A$ and $z_B$, respectively. The firms are permitted to 'costlessly relocate'. Equivalently, the firms have not yet located in $S$, but desire to, and are considering where to locate in anticipation of their competitor's decisions about price and location. The firms produce a single good. With respect to production, two cases will be considered in separate sections of this paper. In the first case, the marginal cost of production is assumed to be constant and location independent (but not necessarily equal) for each firm. In the second case, a production technology is assumed; and input quantities and the optimal mix problem of the firm, as well as fixed costs, are explicitly considered. In both cases, it will be convenient to denote the marginal cost of production for firms A and B as $C_A$ and $C_B$, respectively.

The customers are assumed to be distributed over the market in a manner describable by the density function $\rho(z)$. We will assume Lebesgue integration will be used, and, as a minor regularity property to insure integrability of the firms' profit functions, assume that $\rho^2(z)$ is a Lebesgue integrable function. Each customer is assumed to wish to purchase a single unit of the good, and will purchase it from the firm offering the lowest delivered price.

There is a cost to the firms associated with the transport of a unit of good from one of the firms to a customer in the market. This cost is a function of the location of the customer, $z$, and the location of the firm, $z_i$ ($i = A, B$). We will write this cost as $f_i(z_i, z)$, and assume it to be proportional to the Euclidean distance from $z_i$ to $z$. Then, for $i \in \{A, B\}$,

$$f_i(z_i, z) = x_i \|z - z_i\|_2.$$  \hspace{1cm} (1.1)
We will model competition of the firms by assuming that the firms choose locations privately and simultaneously; then, with these locations known, each firm chooses discriminatory prices in the market privately and simultaneously. The firms are assumed to understand this game and to understand that the final market prices are conditioned by location choices of the firms. Each firm expresses its price strategy as a function of location choices through a 'price policy'. A 'price policy' is a function which specifies the price at which a firm will offer to sell the good for each market point as a function of the location of each firm. For example, for firm A, a price policy is a function \( p_A(z_A, z_B, z) \), which states that firm A will offer a unit of good to market point \( z \) at \( p_A \) if firm A is located at \( z_A \) and firm B is located at \( z_B \). A price policy for firm B, denoted \( p_B(z_A, z_B, z) \), is similarly defined.

This competitive situation will be analyzed as a non-cooperative game. Because prices chosen are conditioned on locations, the game can be viewed as two-staged: the selection of prices by the firms, assuming fixed location choices can be analyzed for each pair of locations. Then, the problem of location choice can be analyzed, anticipating what prices will be chosen. Our approach will be to find a Nash equilibrium in locations and price policies for the firms: location and price policy choices by the firms that are mutually optimal against each other.

It is assumed that customers purchase from the least costly source. Thus, if \( p_A(z_A, z_B, z) < p_B(z_A, z_B, z) \), the customer located at \( z \) will buy from firm A at price \( p_A(z_A, z_B, z) \). If, at market point \( z \), \( p_A(z_A, z_B, z) = p_B(z_A, z_B, z) \), we will initially assume that the two firms evenly split the market.

At this point in the narrative, it becomes convenient to discuss the two cases mentioned above separately.

2. Constant marginal production costs

If the firms have constant marginal production costs and have given locations and price policies, the profit for firm A can be written

\[
\pi_A(z_A, p_A, z_B, p_B) = \int_{p_A < p_B} [p_A(z_A, z_B, z) - f_A(z_A, z) - C_A] \rho(z) \, dz + \frac{1}{2} \int_{p_A > p_B} [p_A(z_A, z_B, z) - f_A(z_A, z) - C_A] \rho(z) \, dz.
\] (2.1)

\( \pi_B(z_A, p_A, z_B, p_B) \) is similarly defined.

We seek Nash equilibrium pairs \((\bar{z}_A, \bar{p}_A, \bar{z}_B, \bar{p}_B)\), \((\tilde{z}_A, \tilde{p}_A, \tilde{z}_B, \tilde{p}_B)\), such that

\[
\pi_A(\bar{z}_A, \bar{p}_A, \bar{z}_B, \bar{p}_B) \geq \pi_A(\tilde{z}_A, \tilde{p}_A, \tilde{z}_B, \tilde{p}_B)
\] (2.2)
for any $z_A$ in the market region and price policy $p_A$ and

$$
\pi_B(\tilde{z}_A, \tilde{p}_A, \tilde{z}_B, \tilde{p}_B) \geq \pi_B(\tilde{z}_A, \tilde{p}_A, z_B, p_B)
$$

(2.3)

for any $z_B$ in the market and price policy $p_B$.

Examination of the optimal price–location decisions for one firm facing fixed decisions of the other firm will be helpful in obtaining the overall solution. We first focus on the optimal price policy choices. For any fixed locations, $z_A$ and $z_B$ and price policy $p_B(z_A, z_B, z)$, an optimal price policy for firm $A$ is

$$
p_{A^*}(z_A, z_B, z) = \max \{p_B(z_A, z_B, z) - \epsilon, f_A(z_A, z) + C_A'\},
$$

(2.4)

where $\epsilon > 0$ and is arbitrarily small. Notice that $p_{A^*}$, a function of both $z_B$ and $p_B$, maximizes $A$’s profits against $B$’s price policy on the assumption that $z_A$ and $z_B$ are fixed.

The formulation represented by (2.4) assumes that firm $A$ will not price below its marginal cost. This is the expected behavior. However, firm $A$ might price below its marginal cost but above its competitor’s marginal cost if, say, firm $A$ were confident that firm $B$ would undercut its price and thus serve the market point in question. Such behavior is of no direct benefit to firm $A$, but serves to force a lower price on firm $B$. Therefore, this behavior might be strategically important to firm $A$ in attempting to discourage firm $B$ from choosing a particular location. In any case, without the use of explicit threats, a strategy of this type is difficult to communicate in the context of a one-period model. We shall therefore not consider such predatory pricing alternatives: firms are assumed to price above their marginal cost.

Firm $B$ knows that firm $A$ will behave optimally against its price policy and, thus, $B$ will select the price policy

$$
p_{B^*}(z_A, z_B, z) = \max \{f_A(z_A, z) + C_A' - \epsilon', f_B(z_B, z) + C_B'\}.
$$

(2.5)

Firm $A$’s optimal response will be

$$
p_{A^*}(z_A, z_B, z) = \max \{f_B(z_B, z) + C_B, f_A(z_A, z) + C_A'\}.
$$

(2.6)

The firms seek to make $\epsilon, \epsilon'$ arbitrarily small. For small $\epsilon, \epsilon'$, we see that a customer will be served by the firm with the least marginal delivered cost at a price slightly less than the competitor’s marginal delivered cost. In the limit,

$$
p_A^* = \lim_{\epsilon \to 0} p_{A^*} = \max \{f_A(z_A, z) + C_A', f_B(z_B, z) + C_B'\},
$$

(2.7)
\[ p^*_b = \lim_{\varepsilon' \to 0} p^*_{b \varepsilon} = \max [f_A(z_A, z) + C_A, f_B(z_B, z) + C_B]. \]  

(2.8)

We call \( p^*_A, p^*_B \) the equilibrium price policies of the firms.

Under equilibrium price policies, the firms will be pricing identically at each market point for each pair of firm locations. The price paid by a customer is equal to the marginal cost of supplying that customer, including transport cost, experienced by the higher cost supplier.

Under the rule used to help define the profit function, the firms should be evenly splitting demand from each customer in the market, since they charge identical prices to each customer. Note, however, that, for any positive \( \varepsilon \) and \( \varepsilon' \), demand goes to the lower cost firm. This discontinuity in the demand relationship may be eliminated by redefining the equal price rule so that, if two firms price identically to a customer, the firm with the least delivered marginal cost will serve the customer. Such a rule is reasonable and intuitive under equilibrium prices because the firm with the cost advantage can serve the customer alone by cutting his price an arbitrarily small amount. Redefining the profit functions using this rule will result in profits being continuous in \((\varepsilon, \varepsilon')\) at \((0, 0)\) using prices (2.5) and (2.6).

We define a function \( r(z_A, p_A, z_B, p_B, z) \), which describes how the firms share a customer at \( z \), when they price identically at \( z \). Because we require that the firm with the least marginal cost serve the customer, \( r \) is referred to as a cost advantage sharing rule. We specifically require that

(a) \( r(z_A, p_A, z_B, p_B, z) = (r_A(z_A, p_A, z_B, p_B, z), r_B(z_A, p_A, z_B, p_B, z)) \geq 0 \),

(b) \( r_A(z_A, p_A, z_B, p_B, z) + r_B(z_A, p_A, z_B, p_B, z) = 1 \),

(c) \( r_i(z_A, p_A, z_B, p_B, z) = 1 \) if \( f_i(z, z) + C_i < f_{-i}(z_{-i}, z) + C_{-i} \),

for all price policy choices and locations such that \( p_A(z_A, z_B, z) = p_B(z_A, z_B, z) \). \((-i \) is the complement of \( i \), i.e., \( -A = B \), \(-B = A \)).

Then, the profit for firm \( A \) can be written

\[
\pi^*_A(z_A, p_A, z_B, p_B) = \int_{r_A = p_A} \left[ p_A(z_A, z_B, z) - f_A(z_A, z) - C_A \right] \rho(z) \, dz \\
+ \int_{r_B = p_B} \left[ p_A(z_A, z_B, z) - f_A(z_A, z) - C_A \right] r_A(z_A, p_A, z_B, p_B, z) \rho(z) \, dz
\]

(2.9)

and similarly for \( \pi^*_B \). Using equilibrium price policies, \( \pi^*_A \) computes \( A \)'s
profits, assuming that A serves those regions where A has the marginal cost advantage and there prices are at B's marginal cost.

In addition to its inherent reasonableness, we use a cost advantage sharing rule so that the equilibrium price policies are Nash.

Equivalently, we may require that the profit function under equilibrium prices be the limit of the firms' profit functions using prices (2.5) and (2.6) as \( \epsilon \) and \( \epsilon' \) go to zero. It is reasonable to expect such continuity. The profit function under equilibrium prices for \( i \in \{A, B\} \) is

\[
\pi_i^*(z_A, p_A^*, z_B, p_B^*) = \lim_{{\epsilon \to 0}} \pi_i(z_A, p_A^*, z_B, p_B^*).
\]

The definitions of firm profit under equilibrium price policies in (2.9) and (2.10) are identical. We will assume that the firms' profits under equilibrium prices are as described by \( \pi_A^* \) and \( \pi_B^* \).

In general, the regions served by each of the firms under equilibrium prices can now be defined.

\[
S_A(z_A, z_B) = \{ z \in S \mid f_A(z_A, z) + C_A < f_B(z_B, z) + C_B \},
\]

(2.11)

\[
S_B(z_A, z_B) = \{ z \in S \mid f_A(z_A, z) + C_A > f_B(z_B, z) + C_B \},
\]

(2.12)

\[
S_C(z_A, z_B) = \{ z \in S \mid f_A(z_A, z) + C_A = f_B(z_B, z) + C_B \}.
\]

(2.13)

\( S_A \) is the region served by firm A, \( S_B \) by firm B, and \( S_C \) is the region shared by the firms. If \( z_A = z_B \) and \( C_A = C_B \) and \( z_A = z_B \), then \( S_C = S \). However, in any other situation, \( S_C \) will be a line in \( S \) and have Lebesgue measure 0. In any case, the firms earn zero profit in \( S_C \).

Furthermore, the profit functions under equilibrium prices are

\[
\pi_A^*(z_A, p_A^*, z_B, p_B^*) = \int_{S_A(z_A, z_B)} [f_B(z_B, z) + C_B - f_A(z_A, z) - C_A] \rho(z) \, dz,
\]

(2.14)

and

\[
\pi_B^*(z_A, p_A^*, z_B, p_B^*) = \int_{S_A(z_A, z_B)} [f_A(z_A, z) + C_A - f_B(z_B, z) - C_B] \rho(z) \, dz.
\]

(2.15)

Having characterized equilibrium price policies for fixed locations, we further assert that the firms expect equilibrium prices to exist in the market regardless of firm locations. This is equivalent to the requirement that the solution to our competitive game be subgame perfect [Selten (1976)]. Subgame perfectness requires that solutions to the game be Nash on every
proper subgame. In our case, this corresponds to the whole game and to every
subgame corresponding to price policy choices for fixed locations. Our
solution is therefore stricter than that given by the Nash criterion.

Next, we consider the equilibrium locations of the firms under equilibrium
prices. To do this we must define the concept of social cost, the total cost to
the firms of supplying the market with the good it demands in the cost
minimizing manner. The social cost is

\[ K(z_A, z_B) = \int_S \min \left[ f_A(z_A, z) + C_A, f_B(z_B, z) + C_B \right] \rho(z) \, dz. \tag{2.16} \]

We will show that the locations of the two firms are in Nash equilibrium if
each firm is minimizing social cost with respect to the opponent’s fixed
location. Notice that (2.14) gives

\[
\pi_A^*(z_A, p_A^*, z_B, p_B^*) = \int_{z_A} \left[ f_B(z_B, z) + C_B - f_A(z_A, z) - C_A \right] \rho(z) \, dz \\
= \left[ f_B(z_B, z) + C_B \right] \rho(z) \, dz \\
- \int_S \min \left[ f_A(z_A, z) + C_A, f_B(z_B, z) + C_B \right] \rho(z) \, dz \\
= \left[ f_B(z_B, z) + C_B \right] \rho(z) \, dz - K(z_A, z_B).
\tag{2.17} \]

Thus, from (2.17), the profit to firm A, in equilibrium, is equal to the cost
that would be experienced by its rival, firm B, if B were to serve the entire
market, minus the social cost.

Examination of (2.17) and the analogous expression for \( \pi_B^* \) reveals that
\((z_A^*, z_B^*)\) are equilibrium locations iff

\[
K(z_A^*, z_B^*) \leq K(z_A^*, z_B^*) \quad \forall Z_A, \quad \text{and} \\
K(z_A^*, z_B^*) \leq K(z_A^*, z_B^*) \quad \forall Z_B.
\tag{2.18} \]

\( K \) is continuous on \( S \times S \) because \( f_A(\cdot, z) \) and \( f_B(\cdot, z) \) are continuous, and
minimization and integration preserve continuity. \( K \) has a minimum on the
compact set \( S \times S \). Condition (2.18) will hold with \((z_A^*, z_B^*)\) chosen to globally
minimize social cost \( K \), so that existence of a location equilibrium is assured.
Under equilibrium prices, each firm maximizes its profit by minimizing the cost to both firms of serving the market, not by minimizing its own cost.

We now turn to a discussion of the properties of location equilibria. We begin by noting that coincident location of firms that are identical (i.e., have the same marginal cost of production and transport rates) will yield zero profits to both firms. If the density of demand in the market is non-zero on some subset of $S$ having positive measure, the firms will never locate coincidentally in equilibrium. This is because one firm may relocate such that on some set having positive measure and positive demand density the (relocating) firm will have the (total) marginal cost advantage. Then it will earn positive profits on this set. This must be contrasted with Hotelling's result with exogenously fixed, identical prices for the firms, which predicts equilibrium location at a central coincident point.

When firms are not identical and the market region is a disk with uniform demand density, it can be shown that non-coincident locations of firms in equilibrium will also hold. See Lederer and Hurter (1985).

If the market region is convex, the firms in equilibrium will always locate in the interior of the market. This may be seen by considering the gradient of the firm's profit function.

From (2.17), we see

$$\nabla_{z_A} \pi^*_A(z_A, p_A^*, z_B, p_B^*) = -\nabla_{z_A} \mathcal{K}(z_A, z_B)$$

$$= \int_{s_A(z_A, z_B)} \nabla_{z_A} f_A(z_A, z) \rho(z) \, dz. \quad (2.19)$$

Similarly for $\nabla_{z_B} \pi^*_B$.

Thus, the gradient of each firm's profits under equilibrium prices is just the negative of the gradient of the firm's transportation (and production) costs to the market it serves. In other words, a firm locally maximizes profit by relocating (locally) so as to minimize total production and transport cost to the market it serves. If $S$ is convex and firm A is located on the boundary of $S$, A's profits will locally increase by relocating in the interior of $S$. Thus, boundary location of the firms cannot occur in equilibrium if the density of demand in the market is non-zero on a subset of $S$ having positive measure.

Note that (2.19) implies that each firm attempts to locate so as to minimize the total production and transportation costs to the market it serves and must be at such a minimum when at equilibrium. If the firm were not at such a minimum, it could relocate in order to minimize costs with the same prices as before, and its profit would increase.

Now, suppose that a single monopolist controlled both firms. The monopolist would locate the firms to minimize the total production and transportation cost, that is, the social cost. These locations are, therefore, equilibrium
locations for the duopoly problem. It is not true, however, that the social cost minimizing locations are the only equilibrium locations. Instead, the set of social cost minimizing locations is a proper subset of the set of equilibrium locations. Recall that $z^*_A$ must minimize $K$ with $z^*_B$ held fixed, and similarly for $z^*_B$, but this does not imply that $K$ is being globally minimized at $(z^*_A, z^*_B)$ as the following example demonstrates.

Example 1. Consider two identical firms located in market region $S = \{ (x, y) \in \mathbb{R}^2 \mid -1 + \epsilon \leq x \leq 1 - \epsilon, -1 \leq y \leq 1 \}$. Refer to fig. 1 and suppose the distribution of customers is uniform over $S$. The monopolist's cost minimizing locations (which are also the social cost minimizing locations) are $z^*_A = (0, \frac{1}{2})$, $z^*_B = (0, -\frac{1}{2})$, and these are equilibrium locations for the independent firms.

![Figure 1](image-url)
However, for \( \varepsilon > 0 \) sufficiently small, locations \( \hat{z}_A = (\frac{1}{2} - \varepsilon/2, 0) \) and \( \hat{z}_B = (\frac{1}{2} + \varepsilon/2, 0) \) are also equilibrium locations for the firms, but \( \hat{K}(z_A^*, z_B^*) < K(\hat{z}_A, \hat{z}_B) \).

3. Production technology

In this section the model developed in section 2 is generalized by assuming that the firms face a production technology that can be represented by a linear homogeneous production function and that they have location dependent fixed costs.

The notation employed in section 2 will be used here. In addition, we assume that the firms produce their output using \( r \) input factors. The level of input \( j \) used by firm \( i \) \( \{i = A, B; j = 1, \ldots, r\} \) is denoted \( q_{ij} \) and the vector of inputs used by firm \( i \) is denoted \( q_i \). The input factors are assumed to be supplied from \( r \) point sources. The location of the source of the \( j \)th input factor is denoted \( z_j \). The cost to firm \( i \) of acquiring a unit of factor \( j \) is the sum of a purchase price \( p_{ij} \) paid at the source, plus the transportation cost of moving the factor from \( z_j \) to the firm’s location at \( z_i \). The transport cost is \( \beta_{ij} z_i - z_j \) where \( \beta_{ij} > 0 \).

Each firm is assumed to have a continuous linear homogeneous production function, \( H_i(q_i) \), which yields the output of firm \( i \) when using input vector \( q_i \). Further, we assume for any \( q_i \geq 0 \), \( H_i(q_i) \geq 0 \).

We will need the location dependent function representing the cost to firm \( i \) \( \{i = A, B\} \) of producing a single unit of output, using the most efficient input mix. This function, denoted \( C_i(z_i) \), for \( i \in \{A, B\} \) may be found by solving

\[
C_i(z_i) = \min_{q_i \in \mathbb{R}^r} \sum_{j=1}^r \frac{q_{ij}(p_{ij} + \beta_{ij} z_i - z_j)}{q_i}, \text{ subject to } H_i(q_i) = 1. \tag{3.1}
\]

Because \( H_i(q_i) \) is linear homogeneous, \( C_i(z_i) \) is the marginal cost of production for firm \( i \) at location \( z_i \), and it is a continuous function of \( z_i \).

Let each firm experience location dependent fixed costs with \( F_i(z_i) \) denoting the fixed costs to firm \( i \) at \( z_i \). We assume \( F_i(z_i) \) is continuous on \( \mathcal{S} \).

We will assume that, where the firms offer equal prices, a cost advantage sharing rule, \( r \), decides how the firms will share the market. The profit of firm \( A \) then will be

\[
\pi_A(z_A, p_A, z_B, p_B) = \int_{p_A = p_B} \left( p_A(z_A, z_B, z) - f_A(z_A, z) - C_A(z_A) \right) \rho(z) \, dz \\
+ \int_{p_A = p_B} \left[ p_A(z_A, z_B, z) - f_A(z_A, z) - C_A(z_A) \right] 
\]
with \( \pi_b(z_A, z_B, p_A, p_B) \) similarly defined.

The competition between the firms takes the form described in section 2. In a manner similar to the development of expressions (2.4) through (2.8) of section 2, for given locations, the price policy equilibrium is

\[
p_A(z_A, z_B, z) = \pi_b(z_A, z_B, z) = \max \{ [f_b(z_B, z) + C_b(z_B)], [f_A(z_A, z) + C_A(z_A)] \}. \tag{3.4}
\]

Under equilibrium price policies, the market regions controlled by the firms can be characterized as

\[
S_A(z_A, z_B) = \{ z \in S | f_A(z_A, z) + C_A(z_A) < f_b(z_B, z) + C_b(z_B) \}, \tag{3.5}
\]

\[
S_b(z_A, z_B) = \{ z \in S | f_A(z_A, z) + C_A(z_A) > f_b(z_B, z) + C_b(z_B) \}. \tag{3.6}
\]

These are parallel to (2.11) and (2.12). Now the profit functions become

\[
\pi_A(z_A, p_A^*, z_B, p_B^*) = \int_{S_A} \{ f_b(z_B, z) + C_b(z_B) - f_A(z_A, z) - C_A(z_A) \} \rho(z) \, dz - F_A(z_A), \tag{3.7}
\]

\[
\pi_b(z_A, p_A^*, z_B, p_B^*) = \int_{S_b} \{ f_A(z_A, z) + C_A(z_A) - f_b(z_B, z) - C_b(z_B) \} \rho(z) \, dz - F_B(z_B), \tag{3.8}
\]

which are analogous to (2.14) and (2.15).

In the present case, the social cost is given by

\[
\hat{K}(z_A, z_B) = \{ \min_{S} \{ [f_A(z_A, z) + C_A(z_A)], [f_b(z_B, z) + C_b(z_B)] \} \} \rho(z) \, dz + F_A(z_A) + F_B(z_B), \tag{3.9}
\]

and we observe

\[
\pi_A(z_A, p_A^*, z_B, p_B^*) = \int_{S} [f_b(z_B, z) + C_b(z_B)] \rho(z) \, dz + F_B(z_B) - \hat{K}(z_A, z_B). \tag{3.10}
\]
Our earlier results characterizing equilibrium locations again apply. Locations are equilibrium locations iff each firm's location minimizes social cost with respect to the other's fixed location. By globally maximizing $K$, one such pair of equilibrium locations can be found. Again, under equilibrium prices, each firm maximizes its own profit by minimizing social, rather than its own, costs.

We now state and discuss some properties of the location equilibria for our model with production. The properties are stated here without proof. The proofs generally follow those presented in section 2, and details are supplied by Lederer (1981).

If the firms are identical and locate coincidentally, both firms are assured zero profits. Thus, coincident location can occur in equilibrium only when neither firm can relocate and earn positive profits. Likewise, when non-coincident location is the only equilibrium, both firms must earn positive profits in equilibrium.

If $S$ is assumed convex, the firms must locate in the interior of $S$ under general conditions. Such conditions include requiring that all input factor source sites be located in the interior of $S$ and that each firm's fixed costs be non-decreasing in a sufficiently small neighborhood of each boundary point on any path approaching this point from the interior. This last condition may be expressed as 'fixed costs do not decrease near the boundary'.

At equilibrium locations, each firm's location minimizes the cost of serving its current set of customers. If this is not true, social cost is not minimized in the firm's location. If both firm A and firm B are owned by a monopolist, the monopolist locates the firms so as to minimize total production and distribution cost, which is given by $K$ and is a location equilibrium. Again, the globally social cost minimizing locations may not be the only equilibrium locations.

4. Conclusions

A model of competition between firms who set locations and delivered price schedules has been presented. We have shown the existence and properties of an equilibrium for this competitive situation, and we note that the model is readily generalizable. Transport cost need not be based upon Euclidean distance. Any transport function $f_i(z_i, z)$ that is continuous in $z_i$ for all $z$ and such that $f_i^2$ is Lebesque integrable in $z$ for each $z_i$ will result in a model having an equilibrium with the social cost minimizing property. More general measure spaces may be used, thereby allowing atomic markets. The only requirement is that $f_i$ be continuous in $z_i$ for all $z$, $f_i^2$ is integrable in $z$ for all $z_i$, and $\rho^2$ is integrable. With these requirements met, the generalized model has an equilibrium in markets with point demands and can be used to study competition of firms locating in existing networks. Our approach can
also study competition in the transport and telecommunications industries where firms compete by providing networks; see Lederer (1984, 1985).

Our model is not limited to location choice in \( \mathbb{R}^2 \). We can model location choice in any finite dimensional space. The model also readily generalizes to many firm competition. See Lederer (1981) for details.

We can also introduce elastic demand. If demand is not 'too elastic', it may be shown that an equilibrium will exist. Unfortunately, our current existence result, which relies upon the decomposition of the firm's profit function, will not be helpful in demonstrating general existence with elastic demand. A discussion of the model with elastic demand, including examples, is found in Lederer (1984).

References


Lederer, Philip J., 1984, Competition of firms: Network design and pricing, Graduate School of Management working paper QM8430 (University of Rochester, Rochester, NY).


