Contingent Labor Contracting Under Demand and Supply Uncertainty

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Firms increasingly use contingent labor to flexibly respond to demand in many environments. Labor supply agencies are growing to fill this need. As a result, firms and agencies are engaging in long-term contracts for labor supply. We develop mathematical models of the interaction between firms and labor supply agencies when demand and supply are uncertain. We consider two models of labor supply uncertainty, termed productivity and availability uncertainty, and study how each affects the nature of the contracts formed. These models reflect two major roles played by the labor supply agency. In the case of productivity uncertainty we find that it is possible to construct a contract that coordinates the firm and agency hiring in an optimal way. In contrast, we show that in environments characterized by availability uncertainty, optimal contracts are not possible. However, there is a large range of contract parameters for which both parties would benefit from a contract. We analyze these and discuss the trade-offs that should be considered in contract negotiation.
(*Labor Supply; Contract Analysis; Stochastic; Staffing Strategy*)

1. Introduction

The rapidly growing temporary help industry is indicative of the dramatic changes that the U.S. labor market has been undergoing in the past decade. Throughout the economy, especially in services, firms are relying increasingly on external contract workers to fill key positions that were once considered the exclusive purview of full-time permanent workers (Clinton 1997). At the same time, a variety of flexible staffing arrangements have become prevalent within firms. Some of these arrangements include part-time, comp-time, and on-call work (Christensen 1995). All such flexible labor arrangements are examples of *contingent labor* for which we adopt the definition of "any job in which an individual does not have an explicit or implicit contract for long-term employment or one in which the minimum hours worked can vary in a non-systematic manner" (Polivka 1989, p. 10).

The causes of these changes are quite varied and complex and, being a recent phenomenon, are still under investigation. In fact, the Bureau of Labor Statistics only began collecting data on contingent workers in 1995 (Bureau of Labor Statistics 1999). The need for contingent labor supplied by temporary agencies is commonly attributed to (1) demand volatility, (2) the need for specialized services (a form of supply uncertainty), and (3) potential wage and benefit savings (see Abraham 1996, for example).

In the models we develop, we focus on demand and supply uncertainty as the driving forces behind developments in the supply of contingent labor and, specifically, the creation of long-term contracts. With variable demand, firms need flexibility in their labor force, especially within service or make-to-order environments. The trade-off between the cost of maintaining a full-time staff capable of satisfying peak demand and the cost of frequent adjustment of the size of a

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full-time staff was modeled in the classic work of Holt et al. (1960). The development of the contingent workforce may be seen as a means of reducing the cost of adjusting the workforce. For example, the large temporary labor suppliers such as Manpower have lowered the cost of varying a firm's labor size through the use of long-term service contracts (Staffing Industry Report 1996). Further, by buffering a core group of full-time employees from demand shocks, contingent labor further reduces the need for hiring and firing.

The use of contingent labor has changed in recent years as the diversity of positions filled and the extent of the relationship between the firm and the temporary labor provider has increased. For example, firms that once only provided temporary substitutes for absent secretarial workers now provide extensive training opportunities to workers who want to be placed in a wide array of short-term jobs including IT, software, accounting, finance, and marketing (Manpower 2000). Personnel services companies are entering into exclusive partnerships with firms to provide a wide range of human resource support. For example, these agencies often search for and screen new employees, a job typically considered a core function of human resources departments. Some agencies, such as Initial Staffing Services, have established pools of permanent employees who will perform temporary work as a full-time job (Initial Staffing Services 2000).

The increased use of contingent labor has led to an increase in what is referred to as "market-mediated" labor relations. One can view the acquisition of labor as a stage of production in a firm. As with other inputs into production, a firm can choose to seek labor from an external supplier, in this case an external labor supply agency (or ELSA). ELSAs benefit from economies of scale, often allowing them to specialize in a narrow range of skills. We contrast these ELSAs with the historical role of temporary employment agencies that typically provided workers with broadly required skills, such as clerical. Because of the uncertainty of either the quality or the availability of such specific skill providers, firms and ELSAs may establish contracts to govern and tighten their relationships. Given the importance of supply and demand uncertainty in motivating firms to seek contingent workers, and given the opportunities for managing labor acquisition in a similar fashion as other supply chain activities, we believe it is important to understand how uncertainty affects the relationship between a firm and an ELSA.

In this paper we analyze the growing use of contingent labor supplied by external labor supply agencies (ELSAs). We have modeled the ELSA in two ways to reflect the two major roles an ELSA plays in practice. In the first model, the ELSA screens the labor pool for capable workers. The assumption here is that identifying capable workers with certainty is challenging and time consuming for the jobs in question and the firm has outsourced this activity to the ELSA. In the second model, the prevalence of qualified workers is uncertain, but the ELSA is capable of discerning each worker's qualifications; we call this availability uncertainty. In the second model, the ELSA parlays its more active role in the labor market into a greater ability to find available workers on short notice. We analyze the conditions under which the firm and the agency enter into labor supply contracts in both cases. We show that optimal contracts are possible under productivity uncertainty. However, under availability uncertainty, such contracts are not possible. Rather, using a Stackelberg game, firms and agencies can approach fully optimal contracts but these are contracts that place all of the risk on the agency. However, we do show that there is a wide range of contracts that do provide benefits over traditional use of temporary labor without a contract. By investigating the reasons why a coordinating contract may or may not exist we develop a better understanding of the key factors that determine the nature of the relationship between a firm and an ELSA. In particular we find that the ELSA can serve the role of a supplier or an agent of the firm and that this role is determined by the type of labor supply uncertainty involved and the firm's ability to monitor worker productivity.

Literature Review

While the labor economics literature is vast, there has been little work specifically on the nature of the relationship between ELSAs and firms. Obvious related work includes classical moral hazard problems, where

the firm and workers attempt to provide incentives to work (see Rasmusen 1989), and adverse selection models (e.g., Greenwald l986) where the firm screens candidates, leading to lesser able workers in the market. While in the current paper we do not consider the relationship between the worker and the firm, we are concerned with the principle-agent interaction between the firm and the ELSA. The agency acts on behalf of the firm, and so performs the screening activity of contingent labor for the firm. In particular, we are concerned with the optimal design of a contract that accounts for the demand and supply uncertainty (see Fudenberg and Tirole l991).

Within the operations literature, most related work considers the optimal capacity of a firm. Staffing models in which there is a cost to adjust the size of a workforce to better match demand have appeared as part of aggregate planning models (Holt et al. 1960, Sobel 1970, Hax and Meal 1975). However, these models do not compare differences between external and internal labor, nor the process of finding contingent labor. Abraham (1988) models the use of contingent workers in a single-period model as a response to demand uncertainty, assuming an unlimited supply of such workers. We extend her results to allow for stochastic supply as well as study the relationship between firms and suppliers of contingent labor. Berman and Larson (1994) presented a model that determined the optimal size of a temporary contingent labor force where the contingent workforce is guaranteed a minimum number of hours per month and the demand for labor was based on both random demand from outside sources plus absenteeism within the workforce. Pinker and Larson (1997) describe a similar situation with backlogging of work from day to day and the explicit modeling of demand information when staffing decisions are made each day. These models do not consider the role of the employment agency supplying contingent labor.

The paper is also related to the growing literature on supply contracts (see Tsay et al. 1998 for a review). In particular, papers that consider coordination in stochastic environments, such as Weng (1997), seek to establish cases where coordination is possible and the sensitivity of that coordination to the operating environment. The current paper addresses such coordination in the labor supply context, which is distinguished by the two forms of uncertainty considered. The paper is also related to work on subcontracting models such as Kamien and Li (1990) and Van Mieghem (1999), where two parties establish capacity which ex post demand revelation may be transferred through a contract mechanism. Although we consider a similar operating environment and contract forms, we differentiate our work by considering the role of supply uncertainty ex post demand revelation not present in these papers.

We view the contribution of this paper as threefold: First, we introduce and discuss the growing external labor supply industry in a modeling framework. Second, we add to the growing literature on supply contracts by investigating two forms of supply uncertainty particularly applicable to labor supply that have not been applied in other contexts. Third, we provide some guidance for managers working with contingent labor, identifying the effect various costs have on the resulting contract and showing the tradeoffs that should be considered in contract negotiation.

The remainder of the paper is organized as follows. In the next section we develop a model of contingent work with productivity uncertainty. We determine the conditions under which there a exists a coordinating contract. In §3 we develop a model of contingent labor with what we call availability uncertainty. To do this we introduce a model of uncertain labor supply and a search process conducted by the external labor supply agency. We show that a coordinating contract does not exist, and consider cases in which the firms and agencies both find a contract to be advantageous. We conclude in §4 with a discussion of some of the implications of our analysis.

2. Productivity Uncertainty

In this paper we consider two forms of supply uncertainty of concern in the use of contingent labor: productivity uncertainty and availability uncertainty. The former is of concern when temps may differ in their abilities vis-à-vis full-time workers, and managers may have difficulty in distinguishing between workers who have sufficient skills and those who do not.

In this case, a firm seeks to take advantage of an ELSA's capability to screen and train workers. Alternatively, if the capabilities of temporary workers are easy to evaluate but their availability is uncertain, as is often the case where highly trained workers seek short-term work, a firm seeks to take advantage of the ELSA's capability to search for labor. We contrast these two forms of uncertainty, focusing on productivity uncertainty in this section.

We use the following notation in the models developed in §§2 and 3. Any additional notation is introduced in the text.

Q = the number of full-time workers hired;
M_F = the number of contract workers contracted for by the firm;
M_A = the number of contract workers the agency hires;
w = the number of contingent workers requested by the firm;
r = revenue per unit of demand satisfied;
c = cost per full-time worker (hiring, training, and salary included);
p = fee the firm pays to the agency per temporary worker;
F_D = the CDF of demand with density function f_D;
$\pi_F(\pi_A)$ = the firm's (ELSA's) penalty per unit of unfilled demand;
S = the number of temporary workers available in the market (not including those hired under a contract);
$\phi(p)$ = the fraction of available temporary workers hired by the ELSA when paid a fee p;
c_c = the fee per worker hired under the contract;
γ = the fraction of the contracted fee paid to the agency on utilization of a worker ($(1-\gamma)$ is paid to the agency up-front);
c_a = the marginal cost of hiring the first temporary worker by the agency;
c_h = the cost of hiring contracted workers by the agency;
c_s = the revenue the agency receives when contracted workers are not utilized by the firm, and the contracted workers find secondary employment.

We consider a firm that establishes its labor capacity by hiring full-time employees and by utilizing contingent labor supplied by an ELSA. In our discussion, we refer to full-time workers as those hired on by the firm. Contingent labor consists of two groups: contract workers, those hired by the agency in fulfillment of a contract; and temps, those workers found by the agency when additional workers are needed. The contract specifies the number of contract workers the ELSA is to hire prior to demand revelation and the payment terms. Ex post demand revelation, the firm obtains contingent labor from the ELSA, some or all of whom may have been hired by the ELSA as contract workers. That is, contract workers are hired by the ELSA ex ante demand revelation in fulfillment of the contract, and temporary workers are hired by the ELSA ex post and then provided to the firm.

We consider the following single-period scenario modeling the determination of the labor capacity of the firm. Ex ante demand revelation, the firm hires Q full-time workers at a cost of c per worker (this includes hiring costs and wages paid for the period) and contracts with the ELSA to hire M_F contract workers. At the time that the contract is established (ex ante demand revelation), the agency hires any number of contingent workers, M_A, at a cost of c_h per worker hired. If $M_A \geq M_F$, the agency is in compliance with the contract; otherwise, not. The firm pays $(1-\gamma)c_c$ ex ante for each contract worker, and pays γc_c for each contract worker utilized ex post. Therefore, if the firm utilizes the worker, the total cost to the firm per contract worker is c_c. After demand is revealed, the firm asks for w contingent workers. The ELSA then provides all w contingent workers; if $w > M_A$, the ELSA hires the difference at a cost of $c_a > c_h$ for each additional worker. If the firm does not utilize a contract worker, it does not pay the ELSA the additional γc_c. The ELSA supplies unutilized contract workers to a secondary temporary position elsewhere, receiving a payment of $c_s < c_h$ per worker. We assume that the ELSA first provides contingent workers already hired, and then hires additional temps to make up any short fall. (The costs assumed in the model support this assumption.) Let D be the random demand, with known distribution F_D.

We assume the cost to hire a full-time worker is less than the total cost of a contract worker, but greater than the amount paid up front ($(1-\gamma)c_c \le c \le c_c$). We also assume the fee paid for temps is greater than the utilization fee paid for contract workers ($\gamma c_c \le p$).

We assume the full-time and contract workers are uniformly, fully productive (i.e., each is a full-time equivalent). In contrast, in this section we assume that temps (those hired after demand revelation) are not necessarily fully productive. Rather, we assume that temps are homogeneous in their productivity, and this productivity (as measured as a fraction of a full-time equivalent) is a random variable Y on (0, 1] with CDF $H(y)$. The difference between the productivity of the full-time and contract workers and temps reflects the former's hiring prior to demand revelation, which allows screening and/or training to be conducted to assure their productivity. While assuming homogeneity of the temps is a simplification, we show that it is not a restrictive assumption. Similarly, the assumption that full-time and contract workers are also uniformly productive is not restrictive.

In the model, we wish to determine whether the firm can establish a contract with the ELSA that leads to the optimal sizing of the full-time and contract labor. By establishing a contract, the firm would be able to trade-off the uncertainty of the productivity of the temps with the certainty of the productivity of the full-time and contract workers. To do so we propose a contract and show that both the firm and ELSA are willing to participate in it, and that there is no incentive for either to deviate from the contract. That is, we demonstrate that the proposed contract is incentive compatible and individually rational. Further, we show that the proposed contract maximizes the system's profit.

Consider first the firm's optimal labor policy. Letting $T_F = Q + M_F$ and $T_A = Q + M_A$, the firm's expected profit is

$$\begin{aligned} z_F(Q, M_F, M_A) = {} & -cQ - (1-\gamma)c_c M_F + E_D[r \min[D, Q]] \\ & + E_D[(r-\gamma c_c)\min[(D-Q)^+, M_F]] \\ & + E_D[G(D; Q, T_F, T_A)], \end{aligned} \tag{1}$$

where $G(D; Q, T_F, T_A)$ is the expected profit generated by the contingent workers hired in addition to the contracted number, M_F. If $M_F = M_A$, all of the additional workers would be temps. We allow the ELSA to hire any number M_A, but show that under the proposed contract, $M_F = M_A$.

Approaching the firm's profit using dynamic programming, we find

Proposition 1. *If $T_A \ge T_F$,*

$$G(D; Q, T_F, T_A) = \begin{cases} 0 & \text{if } D \le T_F \\ (r-p)(D-T_F) & \text{if } T_F < D \le T_A \\ r(D-T_F) - p(T_A - T_F) & \\ \quad -(r+\pi_F)(D-T_A)H(\bar{a}) & \text{if } T_A < D, \end{cases}$$

where $\int_0^{\bar{a}} y \, dH(y) = p/(r+\pi_F)$.

The value of $\bar{a}$ is the ratio of demand (measured in full-time equivalents (FTEs)) in excess of the number of contract workers to the number of temps hired. Based on Proposition 1, the number of contract workers the firm hires depends on whether the agency prefers to hire $M_A = M_F$ or $M_A > M_F$. Assuming the former holds, we find

Proposition 2. *Let*

$$\begin{aligned} Q' &= F^{-1}((c_c - c)/\gamma c_c); \\ T' &= F^{-1}(((r+\pi_F)H(\bar{a}) - c_c)/((r+\pi_F)H(\bar{a}) - \gamma c_c)); \\ \overline{Q} &= F^{-1}((p-c)/p). \end{aligned}$$

Assuming $M_A = M_F$, if $Q' < T'$, the optimal number of full-time workers is $Q^* = Q'$ and the optimal number of contract workers is $M_F^* = T' - Q'$. Otherwise $Q^* = \overline{Q}$ and $M_F^* = 0$.

We observe that the firm may not wish to hire any contract workers. This is the case when the cost of shortage $(r+\pi_F)$ is small compared with the hiring cost, c, the contract cost c_c, and the exercise cost γc_c.

Consider next the agency's optimal policy. Under the contract that we propose, if the agency accepts the contract, it will always be in compliance. Therefore,

assuming $M_A \geq M_F$, the agency's expected profit as a function of Q, M_F, and M_A is

$$\begin{aligned} z_A(Q, M_F, M_A) &= ((1-\gamma)c_c - c_h)M_F - c_h(M_A - M_F) \\ &\quad + E_D[c_S(M_A - (\min[(D-Q)^+, M_F] + B))^+] \\ &\quad + E_D[\gamma c_c \min[(D-Q)^+, M_F] + pB \\ &\qquad - c_a(B - (M_A - M_F))] \\ &\quad - E_D\left[\pi_A \int_0^1 (D - (Q + M_A + y(B - (M_A - M_F))^+))^+ \right. \\ &\qquad \left. \times dH(y)\right], \end{aligned} \tag{2}$$

where B is the firm's optimal number of temp workers (given in the proof of Proposition 1) and π_A is the shortage penalty the ELSA incurs when the demand exceeds the supply (at the firm). Using this definition,

Proposition 3. *The optimal policy of the ELSA is to choose T_A as*

$$T_A = F^{-1}\left(\frac{p - c_h + (1/\bar{a})(c_a - p) + \pi_A \int_0^{\bar{a}} (1 - y/\bar{a})\, dH(y)}{p - c_s + (1/\bar{a})(c_a - p) + \pi_A \int_0^{\bar{a}} (1 - y/\bar{a})\, dH(y)}\right).$$

Assume that if the agency rejects the contract and only provides temps, there is a prevailing temporary labor rate, p', and a shortage cost, π_A, applicable to the agency that determine the agency's reservation profit. While $M_F = 0$ in this case, the ELSA may choose to hire $M'_A > 0$, determined in part by p' and π_A. We can show that $M'_A = T_A - Q$, where T_A is as given in Proposition 3 using p' throughout. Let $z_A(M'_A)$ be the ELSA's expected profit, which is its reservation profit.

Consider the following contract offered by the firm: The firm offers to set the parameters as $p = c_a$, $c_c = c_h$, $\gamma = c_s/c_h$, and $\pi_A = 0$. In addition, if demand is less than Q or greater than T_F, or if the demand is between Q and T_F and the contingent labor productivity is 100% of a full-time equivalent, the firm pays the agency $z_A(M'_A)$. If, however, the contingent labor productivity in the latter case is not 100% of a full-time equivalent, the firm pays the agency $z_A(M'_A) - c_h M_F/(P(Q < D \leq T_F)(1 - P(Y \neq 1))) - \epsilon$ ($\epsilon > 0$). That is, if the firm detects that the contract labor is not 100% productive as expected, the ELSA receives its reservation value less the maximum savings it may have incurred by not hiring the full number of contract workers divided by the probability that such a savings could be detected.

Using the common definition of an optimal contract as one that ensures that the system profit is maximized, we now show that the proposed contract is optimal.

Proposition 4. *The proposed contract is an optimal contract.*

Because an optimal contract exists, we conclude that under productivity uncertainty firms and agencies should be interested in pursuing contracts. Importantly, the contract is enforceable because the firm is able to distinguish between an ELSA in compliance with the contract and one that is not. Note that, while in our model we assume that the temps are homogeneous and that the full-time and contract workers are uniformly productive, the result does not depend on such restrictive assumptions. As long as it is possible to distinguish a temporary worker from a contract worker, an optimal contract is possible. For example, if the productivity of each contract worker is a random variable with support $S_c \subset [0, 1]$ and that of each temporary worker is a r.v. with support $S_t \subset [0, 1]$, as long as $S_t \nsubseteq S_c$, we can show that there is an enforceable optimal contract. (Note that if $S_t \subseteq S_c$, temps are in some sense indistinguishable from contract workers, and in such a case a contract would provide little value to a firm.) We next contrast the results of this section with an alternative model of contingent labor uncertainty—availability uncertainty.

3. Availability Uncertainty

In this section, we study the potential use of contracts when the availability of temporary workers is uncertain. To do so, we model the process the ELSA uses to obtain temporary workers. As before, the firm hires Q full-time workers and seeks to establish an enforceable contract with the ELSA to hire M_F contract workers. The firm ex post requests a number of contingent workers, some of whom may be contract workers and the balance of whom are hired as temps. In the case that the number of contingent workers needed exceeds the number of contract

workers, the ELSA searches for additional contingent labor, doing so while it is profitable. We assume that the firm cannot observe the ELSA's search process (the ELSA's actions are unobservable), although the firm knows the cost structure of the ELSA. We assume that the ELSA and firm establish a fixed price the firm pays for temporary labor ex ante demand revelation. Otherwise, the ELSA could act opportunistically and derive profits by misinforming the firm of contingent labor availability. In contrast to the productivity model, firms may face a potential shortfall in the available temporary labor. To differentiate between the implications of the previous model and the current one, we assume that all of the workers, full-time and contingent, are uniformly productive.

To clarify our assumption regarding the search by the ELSA for temporary workers, let p be the price per worker paid by the firm as before. We assume that the ELSA will search until its marginal cost exceeds p. We assume that random variable, S, is the number of productive temps available in the labor market and that $\phi(p)$ is the maximum fraction of these that the ELSA wishes to hire for a given p. That is, the maximum number of temps supplied is $\phi(p)S$. We assume that $\phi(p)$ is an increasing, concave function. To justify this assumption, consider the operation of an ELSA. Such agencies search for temps by contacting potential employees that have been identified either through previous employment or through marketing efforts. As the familiarity and association between the agency and the potential employees increases, the cost to the agency of hiring the employee decreases. If we assume all potential employees are homogeneous in their availability, it is natural to assume that the agency would contact potential employees in the order of their familiarity. Thus the marginal cost of the search is a convex increasing function of the number contacted. Because the likelihood of availability of any contacted potential employee is the same, the marginal cost of finding an available temp is also convex and increasing. Therefore, the inverse supply curve is a concave increasing function proportional to $\phi(p)$ for each S. Therefore, $\phi(p)$ is a concave increasing function.

We first investigate the firm's staffing policy and then investigate the ELSA's response to a contract. We then study the possibility of their coordination through a contract.

3.1. The Firm's Staffing Policy

As before, we seek the firm's optimal policy, assuming that the ELSA complies with the contract. We then consider the agency's policy and the potential for a contract.

To determine Q and $M_F = T_F - Q$ (assuming that $T_A = T_F$), observe that the profit to the firm is

$$\begin{aligned} z_F(Q, T_F) = {} & -cQ - (1-\gamma)c_c(T_F - Q) + E_D[r \min[D, Q]] \\ & + E_D[(r - \gamma c_c)\min[(D-Q)^+, T_F - Q]] \\ & + E_D[G(D; Q, T_F)], \end{aligned} \tag{3}$$

where $G(D; Q, T_F)$, the profit generated by the temps given D, Q, and T_F, is now

$$\begin{aligned} G(D; Q, T_F) = {} & E_S[(r-p)\min((D-T_F)^+, \phi S) \\ & - \pi_F((D-T_F)^+ - \phi S)^+] \end{aligned}$$

for $\phi = \phi(p)$ for a fixed value of p. We have the following:

Proposition 5. *If Q and T_F satisfy*

$$F_D(Q) = (c_c - c)/\gamma c_c \tag{4}$$

and

$$\begin{aligned} & -(1-\gamma)c_c + (p - \gamma c_c)(1 - F_D(T_F)) + (r + \pi - p) \\ & \quad \times \int_{T_F}^{\infty} F_S((x - T_F)/\phi) f_D(x)\, dx = 0 \end{aligned} \tag{5}$$

and $T_F > Q$, then $Q^* = Q$ workers are hired full-time and $M_F^* = T_F - Q$ workers are hired under the contract. If $Q > T_F$, no contract workers are hired ($M_F^* = 0$) and the number of full-time workers satisfies

$$\begin{aligned} & -c + p(1 - F_D(Q)) \\ & \quad + \int_Q^{\infty} (R + \pi - p) F_S((x - Q)/\phi) f_D(x)\, dx = 0. \end{aligned} \tag{6}$$

The total number hired (T_F) given by (5) reflects a trade-off between the marginal cost to the firm of temporary workers ($p - \gamma c_c$) and the marginal cost of

contract workers ($(1-\gamma)c_c$) in a newsvendor-like way. Further, when the firm desires a contract ($M_F^* > 0$), comparing Proposition 5 to Proposition 2 we observe that the number of full-time workers is independent of the type of supplier uncertainty.

The next two propositions investigate when full-time and contract workers will be used.

PROPOSITION 6. *Let* $c' = (c-(r+\pi)\theta)/(1-\theta)$, *where* $\theta = E_D[F_S(D/\phi)]$. *The number of full-time workers is positive if and only if* $p > c'$.

The proposition emphasizes the trade-off between the flexibility of temporary workers and their availability, leading to the need for full-time workers. Observe that as the supply of temps increases, the value of θ, the expected likelihood of insufficient temps, decreases, and the expected total cost of a full-time worker, c', increases. Once this cost is greater than the cost of a temp, the proposition implies that no full-time workers are hired. Note that if $p < c$ (so that temps are paid less than full-time workers), it is still possible that $p > c'$. Therefore, even with relatively inexpensive temporary help, the firm may hire full-time workers.

The firm's preference to contract for contingent workers clearly depends strongly on the cost, c_c, as well as how this cost is divided between ex ante and ex post charges. If most of the contracted worker costs are incurred up front, fewer contract workers will be hired, while delaying these charges should increase the preference for a contract.

Formally,

PROPOSITION 7. *If* $\gamma < ((r+\pi_F)/(r+\pi_F-c_c))((c_c-c)/c_c)$, $M_F = 0$. *If* $\gamma > (p/(p-c))((c_c-c)/c_c)$, $M_F > 0$. *In particular, if* $p > c_c$, *there exists a* γ *such that* $M_F > 0$.

Following intuition, the proposition implies that when temporary labor is expensive, the firm wishes to contract for contingent labor, while if the potential lost revenues and shortage costs are small the firm will not seek a contract. Further, when the cost of a contract worker is small, the firm will seek a contract. Note, when the costs of the different types of workers increases in the order: full-time, contract, temp ($c < c_c < p$), corresponding to the notion of costs increasing as the lead time decreases, there is a value of γ which induces the firm to seek a contract. When γ falls between the two values in Proposition 6, the desire for a contract depends on the supply distribution function. As the supply increases (stochastically) to infinity, we can show from (4) and (5) that $M_F = 0$ if $\gamma < (p/(p-c))((c_c-c)/c_c)$.

The following proposition formalizes the changes expected in the division of labor between full-time and contract workers and the firm's profit as the value of γ changes.

PROPOSITION 8. *When a firm prefers a contract* ($M_F > 0$), *the following comparative statics hold*: $dQ/d\gamma < 0$, $dM_F/d\gamma > 0$, *and* $dz_F/d\gamma > 0$. *Further*, $d^2z_F/d\gamma^2 > 0$.

Again following intuition, the firm prefers a larger proportion of a contract worker's cost to be paid ex post and will increase the number of contract workers accordingly. Importantly, the profit is convex and increasing in γ. We next consider the agency's policy in considering such contracts.

3.2. The Agency's Policy

In this subsection we consider the ELSA's perspective on a contract. We assume that an ELSA has sufficient lead time to obtain any number of contract workers ex ante and the marginal cost of hiring them, c_h, is constant. Further, as described above we assume that all unutilized contract workers are placed in a secondary temporary position elsewhere, with the agency receiving c_s per worker. We first derive the agency's profit function and then consider when the agency would enter a contract.

The marginal cost of finding temporary workers ex post is not constant. Rather, we let $C(x)$ be the cost of trying to hire x temporary workers (in addition to the M contract workers). Recall that $\phi(p)$ is the fraction of workers found when the price paid by the firm for each temporary worker is p. Because we assumed that the agency would continue to search until its marginal costs equaled its marginal revenues, $\phi^{-1}(\xi)$ is the marginal cost incurred in hiring a fraction ξ of the available workforce. Therefore,

$$C(x) = E_S\left[\int_{y=0}^{\min(x,\,\phi S)} \phi^{-1}(y/S)\,dy\right].$$

Given the contract calls for M workers at a hiring cost of c_h per worker, and the firm requests w workers from the agency, the agency's profit is

$$((1-\gamma)c_c - c_h)M + c_s(M-w)^+ + \gamma c_c \min(w, M) + pE_S[\min((w-M)^+, \phi S)] - C((w-M)^+).$$

As w is determined ex post, the number of contingent workers requested $w = (D-Q)^+$. Let $F_W(w; Q) = F_D(Q+w)$ be the CDF for the number of contingent workers requested. We assume that the agency knows this distribution.

The expected profit of the ELSA for a contract, which specifies that M, γ, and c_c is

$$\begin{aligned} z_A(M) &= ((1-\gamma)c_c - c_h)M \\ &\quad + \int_{w=0}^{M} (c_s(M-w) + \gamma c_c(w))\, dF_W(w) \\ &\quad + \int_{w=M}^{\infty} \left(\begin{array}{l} \gamma c_c M + \int_{S=0}^{\frac{w-M}{\phi}} (p\phi S - S\int_{x=0}^{\phi} \phi^{-1}(x)\,dx) f_S(S)\,dS \\ + \int_{s=\frac{w-M}{\phi}}^{\infty} (p(w-M) - S\int_{x=0}^{\frac{w-M}{S}} \phi^{-1}(x)\,dx) f_S(S) dS \end{array} \right) \\ &\quad \times dF_W(w). \end{aligned} \tag{7}$$

From the ELSA's perspective, a contract for the appropriate number of workers can be beneficial. Intuitively, if the marginal profit of a utilized contract worker is positive and greater than the marginal profit of a temporary worker, the ELSA would agree to some contract. Next we establish that the agency's profit is concave in M.

Proposition 9. *If ex post demand revelation the agency would prefer to provide a contract worker over a temporary worker, $z_A(M)$ is concave in M.*

The condition on the proposition is consistent with logic; if the agency could make a greater profit from placing a temporary worker than from placing a contract worker ex post, the agency would not agree to any contract. The following proposition indicates when the agency would prefer to hire some contract workers as opposed to operating without a contract.

Proposition 10. *If the agency's profit from a placed contract worker exceeds the profit derived from the first temporary worker, the agency would prefer to hire some contract workers.*

In Proposition 7 we observed that if $p \geq c_c$, the firm would be agreeable to a contract if γ is sufficiently large. From Proposition 10 we observe that if $\phi^{-1}(0) - c_h > p - c_c \geq 0$, both the firm and the agency would be willing to contract for some number of workers, given large enough γ. The condition implies that the agency would enter a contract when the additional cost of a temporary worker exceeds the additional revenue provided by not one, while the firm would when the cost of a temporary worker exceeds that of a contract worker. In cases where the condition does not hold, it is unclear whether a contract will exist. Further, even when it does hold, the particular value of M that holds is not clear. While the firm prefers to hire M_F, letting M_A be the optimal number of contract workers for the agency, we have the following:

Proposition 11. *Assuming ex post the agency would prefer to provide a contract worker over a temporary worker, $dM_A/d\gamma < 0$.*

The existence of a contract depends on the proposed parameter values, c_c and γ, as well as the values of the alternatives, p and c. The agency is most receptive to a contract for small values of γ and large values of c_c. Of course, we have shown that this is opposite the case of the firm. As was done in §2, we next investigate whether there are cases in which the firm and the agency can coordinate.

3.3. Joint Firm/ELSA Profits and Coordination

We next investigate whether there is an optimal contract that coordinates the firm and agency hiring, similar to the one found for the case of productivity uncertainty. If we can establish that such a contract exists, one might conclude that the contract would hold in practice. Below, we show that this is not the case, so that any contract that exists will not be coordinating, and therefore optimal outcomes (in the sense of maximizing system profit) cannot be expected. In the following section we then investigate the degree to which potential contracts are optimal.

Considering the firm and agency as a system under centralized control, the system revenue generated (excluding that of temporary workers) is

$$E_D[r\min(D, Q+M) + c_s \min(M, (Q+M-D)^+].$$

The ex ante hiring cost is $-cQ-c_hM$. If x temporary workers are needed (beyond the $Q+M$), the expected profit generated is

$$K(x) = E_S\left[r\min(x,\phi S) - \pi_S(x-\phi S)^+ - \int_{y=0}^{\min(x,\phi S)} \phi^{-1}(y/s)\,ds\right].$$

Therefore, letting $T=Q+M$, the total system profit is

$$z_s = r\left(\int_0^T yf_D(y)\,dy + T(1-F_D(T))\right) + c_s\left((T-Q)F_D(Q) + \int_Q^T (T-y)f_D(y)\,dy\right) - (c-c_h)Q - c_hT + \int_T^\infty K(y-T)f_D(y)\,dy. \quad (8)$$

Proposition 12. *First-order necessary conditions imply*

$$F_D(Q) = (c_h - c)/c_s \quad (9)$$

and

$$\frac{\partial z}{\partial T} = -c_s(1-F_D(T)) + (c_s - c_h) + (r+\pi_S)\int_T^\infty F_S((y-T)/\phi)f_D(y)\,dy + E_S\left[\int_{w=0}^{\phi} S\phi^{-1}(w)f_D(Sw+T)\,dw\right] = 0. \quad (10)$$

The full-time staffing level is determined by the cost difference between hiring a contingent worker and a full-time worker and the secondary revenue a contingent worker can generate. The total number hired is determined by the expected marginal cost of finding temporary workers, the revenue full-time and contingent workers generate, and the value of secondary work for contingent workers.

By viewing the firm and the ELSA as being a part of the same firm, we identify the appropriate level of hiring that maximizes the system profit. However, as we show in the next proposition, given the framework we have assumed for the contingent work, there is no contract that induces the firm to choose the correct employment level.

Proposition 13. *There are no values of c_c, γ, and p that coordinate the labor contract.*

The result follows from the nonlinear cost structure of finding temporary workers and the fact that the actions of the agency are unobservable to the firm. Because the transfer price p is linear and the ELSA searches until the marginal cost of finding an additional temporary worker is p, the firm will pay more for temporary workers than it costs to find them. Therefore, it would choose to hire more contingent workers than in a jointly coordinated solution.

3.4. Comparison of Solutions

In this section we consider numerically the performance of the firm, the ELSA, and the system as a whole to evaluate the potential for contingent labor contracts. We consider several potential contract outcomes: (1) a contract maximizing the firm's profit assuming the agency complies, (2) a contract determined by the solution to a Stackelberg game where the firm is the leader, and (3) a contract determined by the solution to a Stackelberg game where the ELSA is the leader. We refer to these as the firm-maximizing, Stackelberg, and ELSA-maximizing solutions, respectively. We compare these solutions to the system-profit-maximizing solution and the firm and agency actions under no contract. The firm-maximizing solution is given by optimizing (1) using (4) and (5). The following equations describe the firm and ELSA profits when M_F and M_A differ:

$$\begin{aligned} z_F(Q,M_F,M_A) &= -cQ-(1-\gamma)c_cM_F \\ &\quad + E_D[r\min(D,Q)+(r-\gamma c_c)\min((D-Q)^+,M_F)] \\ &\quad + E_{D,S}[(r-p)\min((D-T_F)^+,(\phi S-(M_F-M_A))^+)] \\ &\quad - E_{D,S}[\pi_F((D-T_F)^+-(\phi S-(M_F-M_A))^+)^+] \end{aligned} \quad (11)$$

and

$$\begin{aligned} z_A(Q,M_F,M_A) &= (1-\gamma)c_cM_F - c_hM_A \\ &\quad + E_W[c_s(M_A-W)^+ + \gamma c_c\min(W,M_F)] \\ &\quad + E_{W,S}[p\min((W-M_F)^+,\phi S-(M_F-M_A))] \\ &\quad - E_{W,S}\left[\int_0^{(\min(W,M_F)-M_A)^+/S} S\phi^{-1}(\xi)\,d\xi + \int_{(M_F-M_A)/S}^{\min(\phi,(W-M_A)/S)} S\phi^{-1}(\xi)\,d\xi\right]. \end{aligned} \quad (12)$$

The Stackelberg solution (with the firm as leader) is given by solving (11) and (12), allowing the firm to choose Q and M_F first, assuming an optimal response by the agency for a value of M_A. The ELSA-maximizing solution is similarly solved, with the ELSA first determining M_A and the firm subsequently determining Q and M_F. The system-profit-maximizing solution is given through Proposition 12, while the "no contract" solution is found by maximizing the firm profit (11) over Q with $M_F = M_A = 0$.

We make several assumptions in conducting the numerical experiments. Recall that $\phi(p)$ is the fraction of the available labor supply that the agency will hire ex post (unless contractually obligated to hire more). In the tests we let $\phi(p) = 1 - (c_a - c_h)/(p - c_h)$ so that the marginal cost of hiring a fraction y of the available supply is $\phi^{-1}(y) = c_h + (c_a - c_h)/(1 - y)$. We assume common knowledge of costs. Within the Stackelberg game, we assume that the ELSA's actions are unobservable by the firm.

We consider several parameter values in our studies. In our base case we let $c = 1.0$, $c_c = 1.05$, and $p = 1.2$, representing an increase in the costs based on the immediacy of the need, i.e., full time, contract, temporary. We then vary γ, the fraction of the contract fee paid ex post. In addition, we use the following cost parameters: $r = 1.5$, $\pi_F = 0.25$, $c_s = 0.25$, $c_h = 0.9$, and $c_a = 1.1$. We consider a single demand case and two supply cases. We let demand be normally distributed with mean 30 and standard deviation 10. In the case of low supply, we assume supply follows an Erlang order-2 distribution with mean 10, shifted so that the smallest value of supply, $\widetilde{S}$, has $\widetilde{S} = 10$ (the supply mean is therefore 20 with a c.v. of $\sqrt{2}/4$). We include the shift so that a minimum supply is available for the ELSA to hire ex post if needed. We acknowledge that this assumption becomes restrictive as the value of γ approaches 1 (as in Figure 4). However, for moderate values of γ it is not restrictive and represents the notion that the ELSA has access to some minimum labor supply. Similarly, for the high-demand case we let supply be an Erlang order-2 r.v. with mean 30 again shifted with $\widetilde{S} = 10$ (the supply mean is 40 with c.v. $\sqrt{2}/2$).

In our numerical results shown in Figures 1–5, we demonstrate the following:

(1) There is a large range of γ where both the firm and ELSA prefer a contract.

(2) While the firm prefers a higher γ in all cases, the ELSA prefers a moderate γ.

(3) The Stackelberg solution (with the firm as leader) provides lower profits for both the firm and the agency (vis-à-vis each one's maximizing solution) but for higher values of γ, the total profit approaches 85% to 90% of the system optimal profit.

(4) The Stackelberg solution provides greater profits than the "no contract" solution over a large range of γ.

(5) Assume the Stackelberg solution holds with the ELSA subject to individual rationality and incentive compatibility (IR and IC) constraints.[1] If the firm can choose the temporary worker wage, p, it would choose higher values when (a) supply is lower and (b) γ is higher.

(6) In this case, we observe that under both high- and low-supply cases, the Stackelberg solution approaches 100% of the system optimal profit.

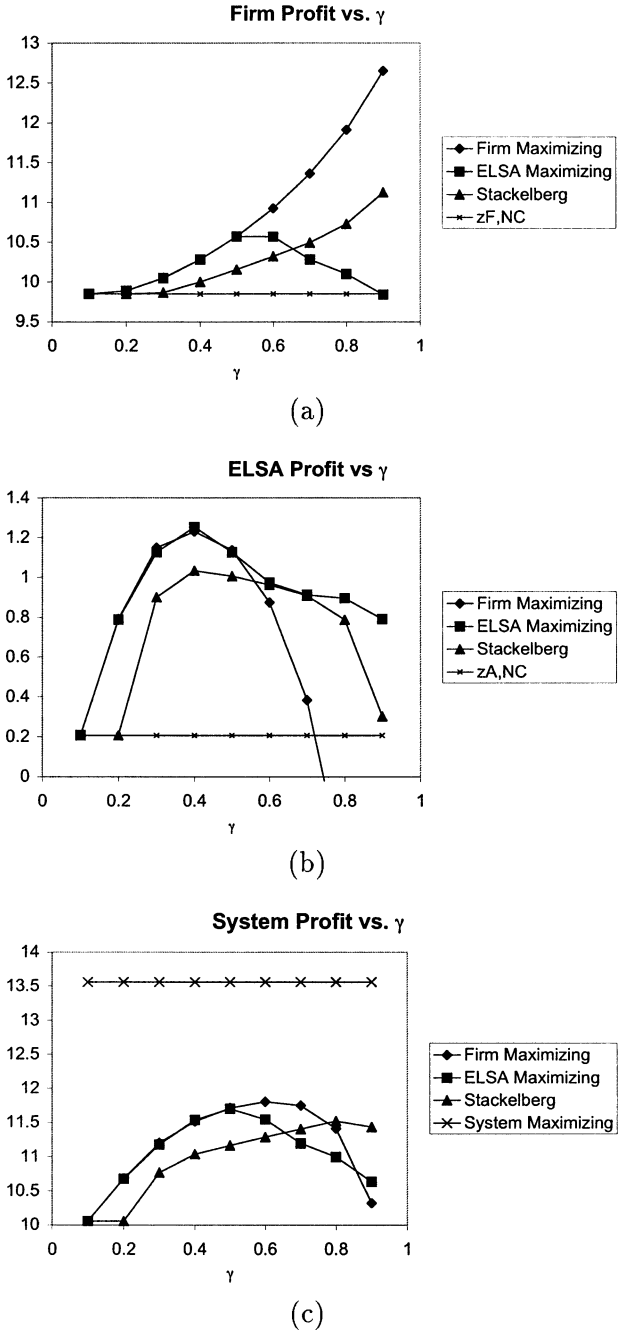
(7) As the total wage for contract workers c_c increases, the minimum value of γ acceptable to both the firm and agency increases; however, the combined effect of greater revenue from contracts with a greater amount of it delayed payment ex post results in an increasing firm profit and decreasing ELSA profit.

To illustrate these points, we begin by presenting in Figure 1 the firm, ELSA, and system profits for the base case with $\mu_S = 20$. We observe that for small values of γ, the firm-maximizing and ELSA-maximizing strategies are identical and both prefer the common solution to that of the Stackelberg game or no contract. Note that in concert with Proposition 7 and 10, because $\phi^{-1}(0) - c_h > p - c_c$, there is a large range of γ where both the firm and ELSA would prefer a contract. However, because the firm- and ELSA-maximizing solutions presuppose that the ELSA hires the contracted number, neither of these solutions may hold.

Considering the Stackelberg solution, we observe that the firm would prefer γ as large as possible, while the ELSA would prefer a lower value of

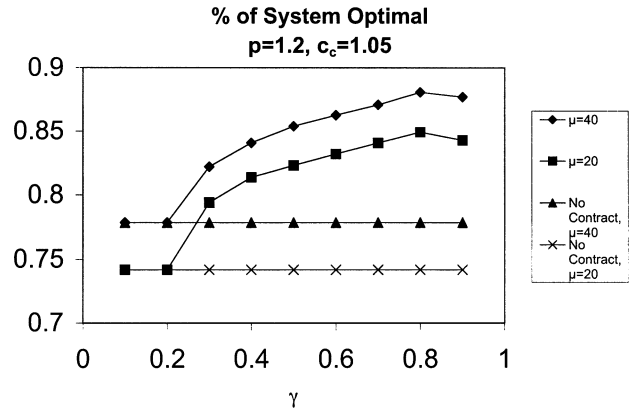
[1] The ELSA IR and IC constraints imply that the ELSA maximizes (12) given Q and M_F, and agrees to a contract only if its profit exceeds its "no contract" profit.

Figure 1 Firm, ELSA, and System Profits vs. γ Under the Firm-Maximizing, ELSA-Maximizing, and Stackelberg Solutions ($\mu_S = 20$)



γ, although not the smallest possible. The system profit is maximized for an intermediate value. In Figure 2, we present the percentage of the optimal system profit achieved by the Stackelberg game solution.

Figure 2 System Profit of the Stackelberg Game as a Percentage of the System Optimal z_S Given in (8)



The relatively large value of 84% and 88% at $\gamma = 0.8$ for the two supply cases indicates that the contract formed could be generally considered efficient.

If p rather than γ is the variable under control of the firm, its preferred value depends on the value of γ and the supply. In Figure 3, we present the firm's profit as a function of p for $\gamma = 0.4$ and 0.8 and $\mu_S = 20$ and 40 using the Stackelberg solution with the ELSA subject to the IR and IC constraints. Recall that p is the incentive given to the agency to search for temporary labor. Therefore, we observe that as the supply decreases, the firm must increase p. Similarly as γ increases, the ELSA is less likely to hire the contracted number, so the firm must increase p to provide an incentive for the ELSA to search for more than just $M_F - M_A$. Together, these imply that the firm should

Figure 3 Firm Profit in the Stackelberg Game vs. p

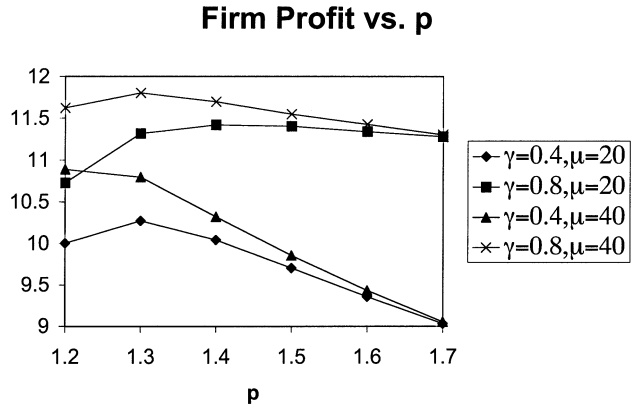
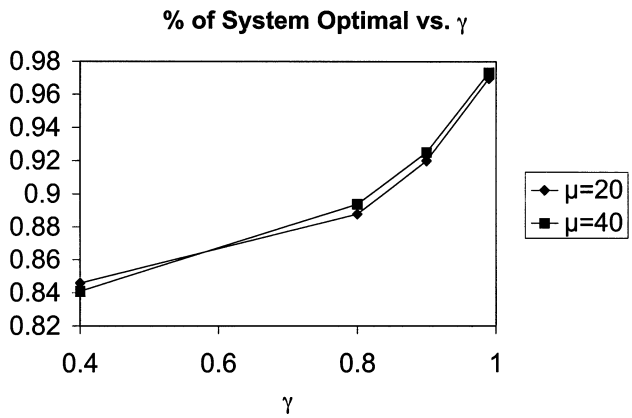


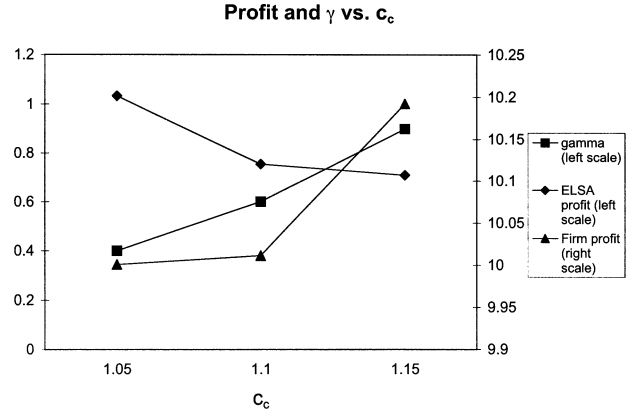
Figure 4 System Profit in the Stackelberg Game As a Percentage of the System Optimal vs. γ for Low and High Supply ($\mu_S = 20, 40$)



not unvaryingly press for lower temporary worker costs.

If we consider (as in Figure 4) the system profit for the Stackelberg game subject to the ELSA's IR and IC constraints, we see that the efficiency of the contract approaches 100% as γ approaches 1 for both high- and low-supply cases. This shows that if the firm can increase γ, it can effectively achieve an efficient contract while maintaining the ELSA's agreement to it. However, as noted above, as γ approaches 1, M_A approaches $M_F - \widetilde{S}$ so that the ELSA hires the minimum number of contract workers necessary to avoid the risk of being detected as not complying with the contract. Because this number is unlikely to be known with certainty by the firm, in reality such a constraint is less likely to be binding. The figure demonstrates that if the ELSA can be pressed to a position where deviations are detectable, the firm can force an efficient contract and increase its profit.

Figure 5 Minimum γ, Girm Profit, and ELSA Profit As a Function of c_c



We note that as γ increases, the agency increasingly bears the risk. As we have chosen to make all decisions on a risk-neutral basis, discussion of risk is beyond the scope of the model (as for all similar expected profit-maximizing models, such as the newsvendor model). Nevertheless, we recognize that very high levels of γ are unlikely in practice. The minimum value of γ we would expect would be that which maximizes the ELSA's profit under the Stackelberg game. In Figure 5 we plot this value as a function of c_c. We also plot the firm and ELSA profit. We observe that as c_c increases, the minimum value of γ increases, the firm profit increases, and the ELSA profit decreases. This implies that the ELSA should seek contracts with more payment assured up front in exchange for smaller total contract revenue.

4. Conclusions

In this paper we have investigated the interaction between a firm and an external labor supply agency, or ELSA. We have modeled the ELSA in two ways to reflect the two major roles an ELSA plays in practice. In the first model the ELSA screens the labor pool for capable workers. The assumption here is that identifying capable workers with certainty is challenging and time consuming for the jobs in question, and the firm has outsourced this activity to the ELSA. In the second model the ELSA parlays its more active role in the labor market into a greater ability to find available workers on short notice. In both models the ELSA helps provide the firm with labor supply flexibility in an environment in which there is uncertainty in both labor supply and demand. In both cases we model the formation of labor supply contracts between a firm and an ELSA. Clearly, both types of uncertainty are typically present in environments where there is a need for contingent labor. We have considered the two forms separately to gain insight into the nature and requirements needed to establish contracts.

At first glance the differences between the two models are quite subtle. In both, the firm is establishing a contract with a supplier that provides an

uncertain quantity of labor. Interestingly, we find that for the model with capability uncertainty there exists an optimal coordinating contract, while for the model with availability uncertainty such a contract does not exist. This result raises two questions of managerial significance. The first question is: What is the difference between the two roles played by the ELSA that causes coordination to be possible in one case but not the other? The second question is: Will contracts between firms and ELSAs be more prevalent in practice for one form of uncertainty versus another?

From the mathematical analysis in §2 we see that the reason coordination is possible in the capability uncertainty model is that the firm can monitor the ELSA's compliance with the contract. By observing the productivity of the workers supplied by the ELSA, both contract and temporary, the firm can identify cases in which the ELSA has shirked its responsibility to find the contracted number of capable workers ex ante demand. The existence of the coordinating contract is therefore predicated on the assumption that the firm can monitor worker productivity. In the case of a supplier of components to a manufacturer, this assumption is relatively straightforward. In the case of labor, productivity monitoring may not always be possible, implying that contracts between a firm and ELSA will be harder to establish. In contrast, we show in §3 that because of the nature of the search process for labor conducted ex post, the firm cannot monitor the agency's compliance with the contract, and therefore no coordinating contract can be established. We recognize that in practice, where this form of uncertainty dominates (e.g., IT workers), contracts do exist. As shown in §3, there are a wide range of contracts in which both parties are better off and contracts that reflect that the Stackelberg solution can be quite efficient.

The existence or nonexistence of coordinating contracts can have broad implications for the relationship between a firm and an ELSA. This issue highlights the dual nature of an external labor supply agency, that of a supplier and that of an agency working on behalf of the firm. If the firm has the ability to monitor the performance of the ELSA—in this case the capability of the provided labor—the ELSA's role becomes one of a supplier in the sense usually applied to firms providing raw materials and subassemblies to other manufacturers. In this role, the interaction between the firm and the ELSA can be strengthened by actions usually recommended in the context of supply chain management. Such actions include closer coordination between the parties through joint forecasting, cooperative improvement of the quality of supply (for example, in the current context through better training of workers by the ELSA), and shared planning of development (which in the context of labor supply may mean disclosure of the firm of future labor needs, including the capabilities its workers will need to possess). This contrasts with the limited role that an ELSA takes on when the firm cannot monitor the actions of the supplier. In such a case, the ELSA is reduced to the role of an agency acting on the request of the firm. Because coordination of the number of workers hired cannot be achieved, other types of coordination, such as in training and planning, are also unlikely.

Appendix

Proof of Proposition 1. Let B be the number of temporary workers sought by the firm ex post demand.

$$G(D; Q, T_F, T_A) = \max_B r(D-T_F)^+ - pB - (r+\pi_F) \times \int_0^1 (D-(T_A+y(B-(T_A-T_F))^+))^+ \, dH(y).$$

It is easy to see that if $D \le T_F$, $B=0$, and if $T_F < D \le T_A$, $B = D - T_F$. If $D > T_A$,

$$G(D; Q, T_F, T_A) = r(D-T_F) - pB - (r+\pi_F) \times \int_0^{\bar{a}} (D-(T_A+y(B-(T_A-T_F)))) \, dH(y),$$

where $\bar{a} = (D-T_A)/(B-(T_A-T_F))$. In this case, $\partial G/\partial B = -p + (r+\pi_F)\int_0^{\bar{a}} y\, dH(y)$ so that at optimality, $\int_0^{\bar{a}} y\, dH(y) = p/(r+\pi_F)$. Therefore,

$$B = \begin{cases} 0 & \text{if } D \le T_F \\ D - T_F & \text{if } T_F < D \le T_A \\ T_A - T_F + (D-T_A)/\bar{a} & \text{if } T_A < D. \end{cases}$$

Substituting into $G(D; Q, T_F, T_A)$ provides the result. □

Proof of Proposition 2. The first-order necessary conditions of maximizing (1) using Proposition 1 together with standard convexity arguments imply the result.

Proof of Proposition 3. Implied by the first-order necessary and second-order sufficiency conditions of maximizing (2). □

Proof of Proposition 4. To prove that the contract is optimal, we need to show that (i) the firm's orders under the contract are

the same as in a centralized system with a single decision maker, (ii) that the agency will comply with the contract ($T_A = T_F$), (iii) that the firm's profit under the contract is at least as good as if there were no contract, and (iv) that the agency will agree to the contract. Therefore we show the following lemmas.

Lemma 1. *Let Q_S and T_S be the number of full-time and contract workers hired in a centralized system. Assume the system shortage cost, π_S, is incurred only when demand exceeds capacity. Then the system profit is maximized when $F(Q_S) = (c_h - c)/c_s$ and $F(T_S) = ((r+\pi_S)H(\bar{a}) - c_h)/((r+\pi_S)H(\bar{a}) - c_S)$, where π_S is the shortage cost of the system.*

Proof. Under a centralized system the expected profit as a function Q and T is

$$\begin{aligned} z_S(Q,T) &= -cQ - c_h(T-Q) + E_D[r\min(D,T) \\ &\quad + c_s(T-Q) - \min((D-Q)^+, T-Q)] \\ &\quad + E_D\Big[\max_B -c_aB + r(D-T)^+ - (r+\pi_S) \\ &\qquad \times \int_0^{(D-T)^+/B} (D-T-yB)\,dH(y)\Big] \\ &= -cQ - c_h(T-Q) + E_D[r\min(D,T) \\ &\quad + c_s(T-Q) - \min((D-Q)^+, T-Q)] \\ &\quad + E_D[(r-(r+\pi_S)H(\bar{a}))(D-T)^+], \end{aligned}$$

where $\int_0^{\bar{a}} y\,dH(y) = c_a/(r+\pi_S)$. First-order sufficient conditions imply the result. □

Therefore, under the contract, if $\pi_F = \pi_S$, then $Q = Q_S$ and $T_F = T_S$.

Lemma 2. *Under the contract, $T_A = T_F$.*

Proof. Using the contract cost parameters, by Proposition 2, $T_F^* = F^{-1}(((r+\pi_F)H(\bar{a}) - c_h)/((r+\pi_F)H(\bar{a}) - c_s))$ and by Proposition 3, $T_A^* = F^{-1}((p-c_h)/(p-c_s))$. Because $H(\bar{a}) = \int_0^{\bar{a}} dH(y) > \int_0^{\bar{a}} y\,dH(y) = p/(r+\pi_F)$, $(r+\pi_F)H(\bar{a}) > p$ so that $T_F^* > T_A^*$. Because the agency receives $z(M_A^*)$ only if $T_A > T_F$, convexity arguments imply $T_A = T_F$ if the agency is in compliance with the contract. That is, under the contract the agency orders no more than that which is required. Therefore, if the agency is in compliance with the contract, it incurs an initial cost of c_hM_A and receives ex post demand revelation $z_A(M_A') + c_hM_A$ so that it achieves its reservation profit.

If the agency agrees to comply with the contract but does not and chooses $T_A < T_F$, its expected revenue is $z_A(M_A') - C_hM_F - \epsilon$. By not hiring any contract workers, its maximum savings is c_hM_F, so that in expectation it does not achieve its reservation profit, and therefore will comply. □

Because the ELSA complies with the contract with $T_A = T_F$, the total system profit is as in the centralized system. Because the ELSA only receives its reservation utility, the firm cannot expect a higher profit, and so prefers the contract to no contract being formed. Finally, as noted, since the ELSA's expected profit equals its reservation profit, the ELSA will agree to the contract. □

Proof of Proposition 5. Equations (4) and (5) are implied by the first-order sufficient conditions that hold if $T_F \geq Q$. Therefore, if T_F is a root of (5) and solving for Q implies $T_F \geq Q$, then $M_F = T_F - Q$. However, if we find $Q > T_F$, the constraint, $T_F \geq Q$, is tight, and solving (6) for Q provides the solution. The optimality of the results is implied by the concavity of the cost $z_F(Q, T_F)$, which may be shown through standard methods. □

Proof of Proposition 6. If $p > c'$, and if $T_F > Q$, then by (4), $Q > 0$. If $T_F \leq Q$, then $-c + p + (r+\pi-p)\int_0^\infty F(x/\phi)f_D(x)\,dx > 0$ so that by (6), $\partial z_F(0,0)/\partial Q > 0$. By the concavity of $z_F(Q, T_F)$, $Q^* > 0$.

If $p \leq c'$, then $p \leq (c_c - (r+\pi)\theta)/(1-\theta)$, so that by (5), $T = 0$. Therefore (6) holds and $Q = 0$. □

Proof of Proposition 7. Let Q' be the solution to (4) and Q'' be the solution to (6). If $Q' \leq Q''$, assume $T_F \leq Q'$, then since $p > \gamma c_c$,

$$\begin{aligned} &\gamma c_c(F_D(Q') - F_D(T_F)) + \int_{Q''}^\infty (r+\pi-p)F_S\left(\frac{x-Q''}{\phi}\right)f_D(x)\,dx \\ &\quad < p(F_D(Q'') - F_D(T_F)) + \int_{T_F}^\infty (r+\pi-p)F_S\left(\frac{x-T_F}{\phi}\right)f_D(x)\,dx, \end{aligned}$$

which by (5) implies

$$\begin{aligned} 0 &= -(1-\gamma)c_c + (p-\gamma c_c)(1-F_D(T_F)) \\ &\quad + \int_{T_F}^\infty (r+\pi-p)F_S\left(\frac{x-T_F}{\phi}\right)f_D(x)\,dx \\ &> -c_c + p(1-F_D(Q'')) + \gamma c_cF_D(Q') \\ &\quad + \int_{Q''}^\infty (r+\pi-p)F_S\left(\frac{x-Q''}{\phi}\right)f_D(x)\,dx = 0 \end{aligned}$$

by (4) and (6), which is a contradiction. So $T_F > Q'$ and $M_F > 0$. Similarly, if $Q' > Q''$, assuming $T_F > Q'$ leads to a contradiction. So $T_F \leq Q'$ and $M_F = 0$.

If $\gamma < ((r+\pi)/(r+\pi-c))((c_c-c)/c_c)$, then $F_D(Q') = (c_c-c)/\gamma c_c > (r+\pi-c)/(r+\pi)$. But by (6),

$$\begin{aligned} 0 &= -c + p(1-F_D(Q'') + \int_{Q''}^\infty (r+\pi-p)F_S\left(\frac{x-Q''}{\phi}\right)f_D(x)\,dx \\ &< -c + (r+\pi)(1-F_D(Q'')), \end{aligned}$$

so $F_D(Q'') < (r+\pi-c)/(r+\pi) < F_D(Q')$ so $Q'' < Q'$ and $M_F = 0$.

Similarly if $\gamma > (p/(p-c))((c_c-c)/c_c)$, then $F_D(Q') = (c_c-c)/\gamma c_c < (p-c)/p$. But (6) implies $F_D(Q'') - \int_{Q''}^\infty ((r+\pi-p)/p)F_S((x-Q'')/\phi)f_D(x)\,dx = (p-c)/p$ so $F(Q') < (p-c)/p < F_D(Q'')$ so $Q' < Q''$ and $M_F > 0$. □

Proof of Proposition 8. From (4), $dQ/d\gamma < 0$. Let $\psi(T,\gamma)$ be the left-hand side of (5). By the Implicit Function Theorem, $dT/d\gamma = -(\partial\psi/\partial\gamma)/(\partial\psi/\partial T)$. But $\partial\psi/\partial\gamma = c_cF_D(T) > 0$ and $\partial\psi/\partial T = (p-\gamma c_c)(-f_D(T)) - (r+\pi_F-p)\int_T^\infty f_S((x-T)/\phi)f_D(x)\,dx < 0$ since $p > \gamma c_c$. Therefore $dT/d\gamma > 0$ and $dM_F/d\gamma > 0$. Also,

$$\begin{aligned} \frac{dz}{d\gamma} &= c_c(T-Q) - c_c\left(\int_Q^T (x-Q)f_D(x)\,dx + (T-Q)(1-F_D(T))\right) \\ &\quad + (-c + r(1-F_D(Q)) - (r-\gamma c_c)(F(T)-F(Q)) \\ &\quad + c_c(1-\gamma) - (r-\gamma c_c)(1-F_D(T)))\frac{dQ}{d\gamma} \end{aligned}$$

$$+\left(-c_c(1-\gamma)+(r-\gamma c_c)(1-F_D(T))+(r+\pi_F-p)\right.$$
$$\left.\times\int_T^\infty F_S\left(\frac{x-T}{\phi}\right)f_D(x)\,dx-(r-p)(1-F_D(T))\right)\frac{dT}{dz}$$
$$=c_c\left((T-Q)F_D(T)-\int_Q^T(x-Q)f(x)\,dx\right)>0,$$

where the equality follows using (4) and (5) and the inequality follows since $T>Q$. Also $d^2z/d\gamma^2=c_c(F_D(T)\,dT/d\gamma-F_D(Q)\times dQ/d\gamma)>0$. □

Proof of Proposition 9. Taking the first derivative with respect to M, we find

$$dz_a/dM=((1-\gamma)c_c-c_h)+c_sF_W(M)+\gamma c_c(1-F_W(M))$$
$$+\int_{w=M}^\infty\left(-p\left(1-F_S\left(\frac{w-M}{\phi}\right)\right)\right.$$
$$\left.+\int_{\frac{w-M}{\phi}}^\infty\phi^{-1}\left(\frac{w-M}{S}\right)f_S(S)\,dS\right)dF_W(w),$$

and so

$$d^2z_a/dM^2=(p-\gamma c_c+c_s-\phi^{-1}(0))f_W(M)$$
$$-\int_M^\infty\int_{\frac{w-M}{\phi}}^\infty\frac{1}{S}(\phi^{-1})'\left(\frac{w-M}{S}\right)f_S(S)\,dS\,dF_W(w).$$

Since $\phi(p)$ is increasing, ϕ^{-1} is increasing so that the second term is negative. By the assumption of the proposition, $\gamma c_c-c_s>p-\phi^{-1}(0)$, so the first term is negative by assumption. □

Proof of Proposition 10. The first derivative of the profit function is positive when $M=0$ since

$$dz_a(0)/dM=(c_c-p)+(p-\gamma c_c)F_W(0)+c_sF_W(0)$$
$$-c_h+p\int_0^\infty F_S(w/\phi)\,dF_W(w)$$
$$+\int_0^\infty\int_{w/\phi}^\infty\phi^{-1}(w/S)f_S(S)\,dS\,dF_W(w)$$
$$>c_c-p-c_h+p\int_0^\infty F_S(w/\phi)\,dF_W(w)$$
$$+\int_0^\infty\int_{w/\phi}^\infty\phi^{-1}(0)f_S(S)\,dS\,dF_W(w)$$
$$\text{(since } \phi^{-1}(\xi) \text{ is increasing and } p\geq\gamma c_c \text{ by assumption)}$$
$$=c_c-p-c_h+\phi^{-1}(0)(p-\phi^{-1}(0))\int_0^\infty F_S(w/\phi)\,dF_W(w)>0$$

since we assume $c_c-c_h>p-\phi^{-1}(0)$ and $p>\phi^{-1}(0)$. Observe that $c_c-c_h>p-\phi^{-1}(0)$ implies $p-\phi^{-1}(0)-\gamma c_c+c_s<(1-\gamma)c_c+c_s-c_h<0$ so Proposition 9 implies the profit function is concave. Therefore, the optimal number of contract workers for the agency is positive. □

Proof of Proposition 11. Let $\psi_A(M_A,\gamma)=dz_A(M_A)/dM_A$. Then $\partial\psi_A/\partial\gamma=-c_cF_W(M_A)<0$. From Proposition 9 we know $\partial\psi_A/\partial M_A<0$, so that by implicit diffentiation we have $dM_A/d\gamma=-(\partial\psi_A/\partial\gamma)/(\partial\psi_A/\partial M_A)<0$. □

Proof of Proposition 12. First we have

$$\frac{\partial z_s}{\partial Q}=c_h-c-c_s(F_D(Q)),$$

implying (9). Second, we have

$$\frac{\partial z_s}{\partial T}=R(1-F_D(T))+c_s(F_D(T))-c_h-\int_T^\infty\frac{\partial K}{\partial T}(y-T)f_D(y)\,dy.$$

We find

$$K(x)=E_S\left[R\min(x,\phi S)-\pi_S(x-\phi S)^+-\int_{z=0}^{\min(x,\phi S)}\phi^{-1}(z/s)\,dz\right]$$
$$=\int_{s=0}^{x/\phi}\left(R\phi s-\pi_S(x-\phi s)-\int_{z=0}^{\phi s}\phi^{-1}(z/s)\,dz\right)f_s(s)\,ds$$
$$+\int_{s=x/\phi}^\infty\left(Rx-\int_{z=0}^x\phi^{-1}(z/s)\,dz\right)f_s(s)\,ds.$$

So,

$$\frac{\partial K}{\partial x}=\int_{s=0}^{x/\phi}(-\pi_Sf_S(s)\,ds)+\int_{x/\phi}^\infty(R-\phi^{-1}(x/s))f_S(s)\,ds$$
$$=-\pi_SF_S(x/\phi)+R(1-F_S(x/\phi))-\int_{x/\phi}^\infty\phi^{-1}(x/s)f_S(s)\,ds.$$

So,

$$\frac{\partial z}{\partial T}=(r-c_s)(1-F_D(T))+(c_s-c_h)-\int_T^\infty\frac{\partial K}{\partial T}(y-T)f_D(y)\,dy$$
$$=(r-c_s)(1-F_D(T))+(c_s-c_h)$$
$$-\int_T^\infty\left(-\pi_SF_S((y-T)/\phi)+r(1-F_S((y-T)/\phi))\right.$$
$$\left.-\int_{(y-T)/\phi}^\infty\phi^{-1}((y-T)/s)f_S(s)\,ds\right)f_D(y)\,dy$$
$$=-c_s(1-F_D(T))+(c_s-c_h)+(r+\pi_S)\int_T^\infty F_S((y-T)/\phi)f_D(y)\,dy$$
$$+\int_{y=T}^\infty\int_{s=(y-T)/\phi}^\infty\phi^{-1}((y-T)/s)f_S(s)f_D(y)\,ds\,dy$$
$$=-c_s(1-F_D(T))+(c_s-c_h)+(r+\pi_S)\int_T^\infty F_S((y-T)/\phi)f_D(y)\,dy$$
$$+\int_{s=0}^\infty\int_{y=T}^{\phi S+T}\phi^{-1}((y-T)/s)f_D(y)f_S(s)\,dy\,ds$$
$$=-c_s(1-F_D(T))+(c_s-c_h)+(r+\pi_S)\int_T^\infty F_S((y-T)/\phi)f_D(y)\,dy$$
$$+E_S\left[\int_{w=0}^\phi S\phi^{-1}(w)f_D(Sw+T)\,dw\right].$$ □

Proof of Proposition 13. Letting $c_c=c_h$ and $\gamma=c_s/c_h$ we can equate the value of Q in (4) and (9). However, substituting these values in (5) and (10), the value of T in the decentralized and

centralized system is the same only if $p((1-F_D(T)) - p\int_T^\infty F_S((y-T)/\phi)f_D(y)\,dy) = E_S[\int_0^\phi S\phi^{-1}(w)f_D(Sw+T)\,dw]$. But

$$\begin{aligned} E_S\left[\int_0^\phi S\phi^{-1}(w)f_D(Sw+T)\,dw\right] &= \int_{y=0}^\infty \int_{x=T}^{\phi y+T} \phi^{-1}((x-T)/y)f_D(x)f_S(y)\,dx\,dy \\ &< p\int_{y=0}^\infty \int_{x=T}^{\phi y+T} f_D(x)f_S(y)\,dx\,dy \\ &= p(1-F_D(T)) - p\int_T^\infty F_S((x-T)/\phi)f_D(x)\,dx, \end{aligned}$$

which is a contradiction. □

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