Spatial retail markets with commuting consumers

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Received 13 June 1994; accepted 22 December 1994

Abstract

In this paper we analyse a model of spatial competition with commuting consumers due to Claycombe (1991, International Journal of Industrial Organization 9, 303-313). We show that results different from Claycombe’s are obtained if a rigorous game-theoretic analysis is applied to the model. Our results provide a theoretical basis for a later study carried out by Claycombe and Mahan (1993, International Journal of Industrial Organization 11, 283-291) and lead to predictions which are in line with the empirical results of that later study. For small commuting distances (relative to the distance between firms), there exists a symmetric equilibrium in which the price is continuous and decreasing in both the commuting distance and the proportion of commuting consumers. For intermediate distances, however, a symmetric price equilibrium in pure strategies in general does not exist. Only if all consumers commute and the commuting distance is large, perfect competition prevails.

Keywords: Commuting; Retail markets; Spatial competition

JEL classification: D43; L81

1. Introduction

In a recent article, Claycombe (1991) introduced commuting into a model of spatial competition, arguing that for many retail markets, consumers’ travel costs are in part attributable to their commuting route and not to the purchase of a good. In a subsequent study, Claycombe and Mahan (1993)
set out a number of predictions which were only loosely related to the formal model of Claycombe (1991), and tested them. In this paper, we apply a game-theoretic analysis to a simplified version of the original Claycombe model. Our results differ considerably from those obtained by Claycombe and at the same time are broadly consistent with the predictions and empirical results of Claycombe and Mahan (1993).

In standard models of spatial competition, the travel cost from a consumer's (home) location to a firm (a retail store) is fully attributed to the net price the consumer has to pay. For a large variety of retail markets, however, the relevant consumers are more appropriately described as living at one location and working at another. Commuting between these locations occurs regardless of any purchases made, and consumers will in general try to combine shopping with commuting in order to save travel costs. Thus the net travelling distance to be attributed to the purchase of a good is the distance between the retailer's location and the closest point on the commuting route, and is zero if the store happens to lie on the route.

Motivated by these considerations, Claycombe (1991) analyses a model in which firms and consumers are located on an infinite line. Firms are spaced at an equal distance, and consumers are uniformly distributed along the line. Each consumer commutes to a location at a constant distance from his home location. Claycombe argues that commuting makes the market inherently more competitive. In particular, he arrives at the result that if the commuting distance exceeds the distance between the firms, Nash price setting behaviour essentially leads to a competitive outcome, whereas for smaller commuting distances, commuting does not matter very much.

Claycombe and Mahan (1993) interpret this theoretical work as implying that if a population consists of both commuting and noncommuting (i.e. standard hotelling) consumers, the equilibrium retail price depends negatively on the commuting distance of the commuters and positively on the fraction of noncommuting consumers. They go on to estimate retail prices for beef in different US cities, using commuting characteristics as explanatory variables in addition to data on concentration levels. The proportion of consumers using mass transit or car pools turns out to have a significant positive effect on prices, whereas the average commuting distance has a marginally significant negative impact. These results seem to offer support to their predictions.

My concern in this paper is not with the econometric specification in Claycombe and Mahan linking data on prices and commuting characteristics to their predictions. Rather, I am concerned with (i) the validity of the original Claycombe (1991) analysis, and (ii) with the basis of the Claycombe–Mahan predictions, i.e. the link between Claycombe (1991) and Claycombe and Mahan (1993).

For a slightly simplified version of Claycombe's (1991) model, we show
that for small commuting distances, prices in a symmetric equilibrium depend continuously and negatively on the commuting distance and positively on the proportion of noncommuting consumers. These results both refute Claycombe's (1991) conjecture that for small distances, commuting does not matter much, and provide a more solid theoretical explanation for the predictions and empirical results of Claycombe and Mahan than their own heuristic arguments. For intermediate commuting distances, we show that a symmetric price equilibrium in pure strategies in general does not exist. Perfect competition, as predicted by Claycombe, does not result since, as the market becomes more competitive and profits decrease, firms have an incentive to cease competing for the marginal consumers and instead increase their price in order to extract profits from consumers over which they have some monopoly power. Thus, while nonexistence results are fairly common in spatial models, the reason for the breakdown of equilibrium in our model is quite different from other models. Only for large commuting distances, viz. at least twice the distance between firms, and if all consumers commute, perfect competition prevails as then every consumer passes at least two firms on her commuting route.

Combining commuting with shopping can be interpreted as an example of a multipurpose trip, where there are economies of scale in transport. The literature on trip-chaining (cf. Eaton and Lipsey, 1982; Stahl, 1987; Thill and Thomas, 1987) analyses the consequences of such economies associated with consumers' trips for the spatial distribution of firms, in particular, the emergence of centres. Here, in contrast, the analysis focuses on one market only, while the travel pattern, i.e. the commuting behaviour, is exogenously given and leads to a reduction of travel costs attributable to shopping wherever the commuting and shopping routes overlap.

2. Analysis of a retail market with commuting consumers

In this main section, we analyse a simplified version of Claycombe's (1991) model. First, we analyse the price equilibrium of the basic model in which all consumers commute, for a given number of firms. Then we extend this model to the case in which there is a fraction of consumers who do not commute.

2.1. Analysis of the basic model

Firms and consumers are located on an infinite line. Along this line, firms are evenly spaced at a distance \( d \). We index the firms by integer numbers, where firm \( i \) is located at \( id \).

Consumers are assumed to commute over a constant distance \( c \) from the
location of their home to the east, i.e. in positive direction. In general, a consumer would be characterized by two parameters, i.e. the home and work locations. Given a constant commuting distance, however, these two parameters can be collapsed into one single parameter. For convenience, we denote each consumer by the centre point of her commuting route, i.e. we say a consumer is located at $x$ if she lives at $x - c/2$ and commutes to her workplace at $x + c/2$. Given this notation, let consumers be uniformly distributed along the line, or more precisely, the measure of consumers is given by the Lebesgue measure.

In contrast to Claycombe, we only consider the case of constant marginal costs of production, which for simplicity we set to zero. Costs of entering the market and other setup costs are either sunk or fixed, hence they are irrelevant for the price competition between the firms.

Further simplifying the Claycombe model, we assume that each consumer purchases either exactly one unit of the good or none at all. The net price a consumer has to pay depends not only on the shop price of the good, but also on the travel costs associated with the purchase. If a consumer passes a shop on her commuting route, the net travel distance is zero. If the shop is located to the west of the consumer's home, the net distance is the distance between the firm and the consumer's home, and the distance is analogously defined if the shop is located to the east of the consumer's place of work. Finally, travel costs are assumed to be quadratic in the net travel distance. We discuss the significance of this assumption further below. To summarize, the (indirect) utility for a consumer located at $x$ patronizing firm $i$ to the right (east) of $x$ is given by

$$a - p_i - t(\max\{id - x - c/2, 0\})^2$$

(1)

Here, $p_i$ is firm $i$'s shop price, $t$ is a parameter for travel cost, and $a$ is the utility derived from the good, which we assume to be at least $(1 + c/2)^2$.\(^2\)\(^3\)

In the following, I only consider symmetric price equilibria. In the discussion following Proposition 1 below, however, I will address the question whether asymmetric equilibria are likely to exist.

For the moment, we restrict attention to commuting distances $c < d$, i.e. distances less than the distance between the firms. In this range of values for $c$, each consumer passes at most one firm on her commuting route.

\(^1\) As we will discuss further below, this simplification does not affect our main results.

\(^2\) This assumption merely ensures that neighbouring firms compete for customers rather than just being local monopolists, in order to make the model interesting to analyse.

\(^3\) Compared with a model without commuting and with quadratic travel costs (e.g. d'Aspremont et al., 1979) the specification (2.1) effectively introduces a flat segment in the travel cost function for travelling distances less than $c/2$. This shows more precisely why formally this model has little in common with trip-chaining models, as argued in the introduction.
As a first step in deriving a symmetric Nash equilibrium in prices, we derive the demand of firm 0 charging a price \( p_0 \), assuming that all other firms charge a constant price \( p \). Restricting attention to the positive (east) side of firm 0 (since the analysis is symmetric on the negative side) and to prices \( p_0 \) in a neighbourhood of \( p \), firm 0 can potentially attract three groups of consumers (cf. Fig. 1):

Group \( A_0 \) consists of consumers who freely pass firm 0, but not firm 1. According to the notation introduced above, these are the consumers in the interval \([-c/2, c/2]\). Since firm \((-1)\) charges the same price \( p \) as firm 1, only consumers in \([0, c/2]\) would buy at firm 1 while the consumers in the negative part would rather buy at firm \((-1)\). A consumer located at \( x \) in \([0, c/2]\) decides to purchase at firm 0 as long as

\[
p_0 \leq p + t \left[ d - \left( x + \frac{c}{2} \right) \right]^2.
\]

Defining \( k := \frac{(p_0 - p)}{t} \), this is equivalent to \( x \leq d - c/2 - \sqrt{k} \). For \( k > (d - c/2)^2 \), even consumer 0 will prefer to buy at firm 1 or \(-1\), so firm 0's demand is 0. For \( k \leq (d - c)^2 \), on the other hand, all \( A_0 \)-consumers (with total measure \( c \)) patronize firm 0. For intermediate values \( k \in [(d - c)^2, (d - c/2)^2] \), finally, firm 0's demand from \( A_0 \)-consumers is

\[
2(d - c) - k/(d - c)
\]

for \( k \in [- (d - c)^2, (d - c)^2] \), zero for higher prices (higher values of \( k \)), and \( 2(d - c) \) for lower prices.

Group \( B_{01} \) consists of consumers located between firms 0 and 1 who pass neither firm on their commuting route, i.e. consumers in the interval \([c/2, d - c/2]\). A group-\( B_{01} \) consumer located at \( x \) purchases at firm 0 if

\[
p_0 + t \left( x - \frac{c}{2} \right)^2 \leq p + t \left[ d - \left( x + \frac{c}{2} \right) \right]^2 \quad \text{or} \quad x \leq \frac{1}{2} \left( d - \frac{k}{d - c} \right).
\]

Similarly, we can show that firm 0's demand from \( B_{01} \)-consumers is \( d - c - k/(d - c) \) for \( k \in [- (d - c)^2, (d - c)^2] \), zero for higher prices (higher values of \( k \)), and \( 2(d - c) \) for lower prices.

Group \( A_1 \), finally, comprises consumers freely passing firm 1, but not firm 1.\]
0, i.e. consumers in \([d - c/2, d + c/2]\). A consumer in this group located at \(x\) purchases at firm 0 if

\[ p_0 + t\left(x - \frac{c}{2}\right)^2 \leq p \quad \text{or} \quad x \leq c/2 + \sqrt{-k}. \quad (4) \]

Demand resulting from \(A_1\)-consumers then amounts to \(2[(d - c) - \sqrt{-k}]\) for \(k \in [-d, -(d - c)^2]\). Adding the demand functions for the three different consumer groups gives us firm 0's total demand as a function of \(k\):

\[
D_0(k) =
\begin{cases}
0 & \text{for } k > (d - c/2)^2 \\
2d - c - 2\sqrt{k} & \text{for } k \in [(d - c)^2, (d - c/2)^2] \\
d - k & \text{for } k \in [-(d - c)^2, (d - c)^2] \\
c + 2\sqrt{-k} & \text{for } k \in [-d, -(d - c)^2]
\end{cases}
\quad (5)
\]

This demand curve is shown in Fig. 2 for two different values of \(c\). In contrast to the case of no commuting \((c = 0)\), which would result in a linear demand curve, commuting introduces a region of nonconcavity for higher prices. This is similar to the three-segment demand function obtained in a linear city duopoly in which both firms each have their own hinterland with consumers who would never patronize the other firm because their reserv-

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\(^4\) (i) The lower bound for \(a\) assumed above ensures that this expression is valid, i.e. that in principle firm 0 can cover the market between \([-d - c/2, d + c/2]\] at zero price. (ii) The analysis below shows that \(k < -d\) need not be considered because the requirement of nonnegative prices ensures that the relevant values always exceed \(-d\).
tion price is too low (cf. Gabszewicz and Thisse 1986, p.27). The noncon-
cavity is more pronounced the closer \( c \) approaches to unity.

For \( p \) to be an equilibrium price set by all firms requires that setting \( p_0 = p \) (i.e., \( k = 0 \)) be at least locally optimal for firm 0. With firm 0's profit given by \( \pi_0(k) = p_0D_0(k) \) and \( D_0(0) \) given by (5), the first-order condition is

\[
\frac{\partial \pi_0(0)}{\partial p_0} = D_0(0) + p \frac{\partial D_0(0)}{\partial k} \frac{\partial k}{\partial p_0} = d - \frac{p}{t(d-c)} = 0,
\]

hence the candidate equilibrium price is \( p^* = d(d-c)t \). Our main result
states under which conditions this solution is a Nash equilibrium.

Proposition 1. Let \( \bar{c} := (1/2)(13 - \sqrt{5})d \approx 0.91d \). Then

(a) For commuting distances \( c \in [0, \bar{c}] \), there exists a symmetric Nash
equilibrium in prices in which each firm sets a (unique) price \( p^* = d(d-c)t \),
and the resulting profit is \( \pi^* = d^2(d-c)t \).

(b) For \( c \in (\bar{c}, 2d) \), an equilibrium in symmetric prices does not exist.

(c) For \( c \geq 2d \), there is a unique symmetric Nash equilibrium in which all
firms charge a zero price.

Proof. See Appendix.

For commuting distances sufficiently smaller than the distance between
the firms, prices and profits are continuous and decreasing in the commuting
distance \( c \). The reason for this is that price competition between the firms
is driven by the indifference condition for the marginal consumer, who, for
firms 0 and 1 and equal prices, is the consumer 1/2, a member of group \( B_{01} \).
As \( c \) increases, the net travel distance to both firms decreases. Thus, an
increasing commuting distance has the same effect on price competition as
firms moving closer to each other in a standard hotelling model.

As \( c \) approaches \( d \), however, \( p^* \) ceases to be a Nash equilibrium (part
(b)). This situation is illustrated in Fig. 3, which shows firm 0's profit
function for two different values of \( c \), where all other firms charge the
candidate equilibrium price \( d(d-c)t \). While for values of \( c \) below 0.8d the
profit function is concave, it becomes nonconcave for higher values. Finally,
at \( c \approx 0.91d \), deviating from \( k = 0 \) to a higher price yields a higher profit.

Economically, competition for the marginal consumer drives \( p^* \) and \( \pi^* \)
down to zero. At some point, however, a firm can profitably deviate by
elevating the price to a level where only some group-A consumers close to
the firm are willing to buy.

Similarly, an equilibrium does not exist if \( c \) exceeds \( d \). The reason is the
simultaneous presence of both consumers who pass two firms on their
commuting route and thereby induce the firms to engage in severe price
competition, and consumers who pass only one firm, implying that this firm
can attract these consumers charging a positive price even if the other firms
sell at zero price. This result is in striking contrast to Claycombe's (1991) prediction that for $c > d$, perfect competition prevails.

Finally, if $c > 2d$, every consumer passes at least two firms, resulting in pure Bertrand competition between neighbouring firms.

Thus, while the special case of no commuting ($c = 0$) corresponds to the case considered by Salop (1979), Proposition 1 shows that the existence of equilibrium can be established more generally for both $c \in [0, \bar{c})$ and $c \geq 2d$. The equilibria in these two ranges, however, are of entirely different natures.

We have shown that at most one symmetric equilibrium exists. The question remains whether other asymmetric equilibria exist, particularly in cases where a symmetric equilibrium does not exist.

It can be shown that in any asymmetric equilibrium there must be a pair of firms for which the absolute price differential exceeds $(d - c)^2$, i.e. implying that these two firms operate in the high and low segments of their demand function, respectively.

On the other hand, it can be shown that if two neighbouring firms compete only for consumers located between them, the difference in prices can in equilibrium never exceed $(d - c)^2$ because otherwise their first-order-conditions would be inconsistent. By the same argument, a situation with alternating prices, e.g. where even-numbered firms set a (the same) high

![Fig. 3. Profit $\pi_0(k)$ (for $d = 1$).](image)
price and odd-numbered firms a low price, cannot be an equilibrium if the difference between the high and low price is greater than \((d - c)^2\).

These two arguments combined suggest that for any asymmetric configuration of prices, the first-order conditions for some firms are likely to be inconsistent. Thus, while I have not been able to show global uniqueness of the equilibrium of Proposition 1(a), these arguments lend some support to the conjecture that it is indeed unique.

In the original (1991) model, Claycombe allows for price-elastic individual demand and for increasing marginal costs. If individual demand is elastic, but marginal costs are constant, Proposition 1(b) remains valid, since firms still can avoid being driven into perfect competition and hence zero profits. For rapidly increasing marginal costs, however, (short-run) perfect competition could possibly arise since at a price equal to marginal cost, firms will still be earning positive profits. In this case, it is less likely that a firm can profitably deviate.

The assumption of quadratic travel costs has been introduced into spatial models because this circumvents problems of nonexistence of a price equilibrium commonly encountered if travel costs are linear in distance (cf. Gabszewicz and Thisse, 1986). Here, we have assumed quadratic costs, even though nonexistence does not arise with linear costs if there is no commuting (cf. Salop, 1979). A modified model with linear costs and commuting, however, leads to results quite different from the ones obtained above, as we discuss in the following.

Assume that travel costs are linear in the net travel distance (with \(i = 1\), and that firms \(i \neq 0\) charge some price \(p\). Then it can be shown that for \(k \in [(d - c), (d - c)]\), firm 0's demand is \(d - k\); in particular, it does not depend on \(c\). It follows then that the unique price in a symmetric equilibrium is \(dt\), whatever the value of \(c\).

To see why demand does not depend on \(c\) for intermediate values of \(k\), assume \(k > 0\) and consider the marginal consumer who is indifferent between patronising firm 0 or firm 1, and who, by assumption on \(k\), is a member of the \(B_{01}\)-group. If her net travel distances to firms 0 and 1 are \(a\) and \(b\), respectively, it follows that \(b - a = k\) (in particular, \(b > a\)). An increase in \(c\) will lead to a decrease in \(a\) and \(b\) by \(\Delta c/2\), leaving \(b - a\) unaffected. This implies that the location of the marginal consumer does not change, and hence that demand for firm 0 remains the same.

With quadratic travel costs, on the other hand, the condition for the marginal consumer is \(b^2 - a^2 = k\), and it is easy to see that the l.h.s. decreases as \(a\) and \(b\) decrease by the same amounts. This consumer then prefers to purchase at firm 1, hence the new marginal consumer must be located to the left of the previous one, implying that demand for firm 0 has decreased, as reflected in the demand function (5).

The linear case, then, illustrates the limits of treating an increase in
commuting distance as equivalent to a decrease in the distance between firms. Here, demand and equilibrium price clearly depend on the distance between the firms, but not on commuting distance. It should be pointed out, however, that this is an extreme case in the sense that any strictly convex travel cost function will lead to a demand function and an equilibrium price which qualitatively depend on the commuting distance in the same way as in the quadratic case discussed above.

A second important difference to the above analysis relates to the breakdown of equilibrium as \( c \) approaches \( d \). It can be shown that \( p^* = dt \) in the linear case ceases to be an equilibrium if \( c \) is, approximately, greater than 0.85\( d \), because firm 0 will have an incentive to undercut its rivals. This situation then, shown in Fig. 4, is rather similar to the reason for nonexistence in a standard Hotelling model with linear travel costs if firms are close together.

2.2. Commuting and noncommuting consumers

We modify the model of the previous subsection by assuming that a proportion \( \alpha \) of the consumers does not commute. The significance of this assumption is the fact that a proportion of consumers uses mass transit (or car pools) to commute rather than their own car. These consumers, then, are less likely to combine purchases with their commuting, as pointed out by Claycombe and Mahan (1993). Thus, if their travel costs for shopping are attributed to the purchases to full extent, this is appropriately modelled by the assumption that these consumers do not commute at all.

Adhering to the notation of Section 2.1, a noncommuting consumer \( x \) located between firms 0 and 1 will patronize firm 0 if

\[
 p_0 + tx^2 \leq p + (d - x)^2 \quad \text{or} \quad k \leq d - 2x. \tag{7}
\]

Fig. 4. Profit \( \pi_0(k) \) for linear transport costs (\( d = 1 \)).
Hence, in the range \( k \in [-d, d] \), firm 0's demand from noncommuting consumers is \( \alpha(d - k) \). Combining this with the demand by commuting consumers given by (5), we obtain the total demand for firm 0 for small absolute values of \( k \):

\[
D_0(k) = (1 - \alpha)\left(d - \frac{k}{d - c}\right) + \alpha(d - k) = d - k\left(\frac{\alpha}{d} + \frac{1 - \alpha}{d - c}\right) \quad \text{for} \quad k \in [-\sqrt{(d - c)^2}, (d - c)^2].
\]  

(8)

Similarly, as in Section 2.1, we can derive the price and profit for the unique candidate for a symmetric equilibrium.

**Proposition 2.** For any given \( \alpha < 1 \) there exists a \( c(\alpha) < d \) such that

(a) For \( c \in [0, c(\alpha)] \), a symmetric price equilibrium is given by

\[
p^* = d^2\frac{(d - c)t}{d - \alpha c} \quad \text{and} \quad \pi^* = d^3\frac{(d - c)t}{d - \alpha c}.
\]

(9)

(b) For any \( c > c(\alpha) \), a symmetric equilibrium does not exist.

**Proof.** See Appendix.

Similarly as before, \( \partial p/\partial c < 0 \), thus prices fall as the commuting distance increases, where the magnitude of the effect varies proportionally with the fraction of commuting consumers. Moreover, \( \partial p/\partial \alpha > 0 \), thus prices decrease continuously as the proportion of commuting consumers becomes larger. This result nicely corresponds to the Claycombe and Mahan (1993) finding that the proportion of consumers using mass transit or car pools has a significant positive effect on retail beef prices.

For commuting distances close to or larger than \( d \), however, the nonexistence problem encountered in the basic model still arises. According to (9), even in the presence of noncommuting consumers, firms still compete for the marginal commuting consumer, which drives prices and profits to zero as \( c \) approaches \( d \). Then, again, for some critical value of \( c \), a firm can profitably deviate from \( p^* \) by extracting a positive price from its local consumers.

Finally, even for large commuting distances \( (c > 2d) \), perfect competition never prevails since positive profits can be earned from the fraction \( \alpha \) of noncommuting consumers.

How does the critical value \( \tilde{c} \) depend on \( \alpha \)? Unfortunately, a closed expression for \( \tilde{c}(\alpha) \), which is a nonmonotonic function, cannot be obtained for this generalized model. Based on numerical experiments, we make the following observation:

**Observation 1.** (a) A lower bound to \( \tilde{c} \) is at around 0.85d, i.e. whatever the
value of \( \alpha \), \( p^* \) as given in (9) will always be an equilibrium price as long as \( c < 0.85d \). The lowest values of \( \bar{c} \) are obtained for values of \( \alpha \) of around 0.7.

(b) As \( \alpha \) approaches unity, \( \bar{c} \to d \).

Therefore, whatever the value of \( \alpha \), \( c \) has to be fairly large for nonexistence to obtain. Moreover, since for \( \alpha \to 1 \) the model converges to a standard model without commuting, there is no upper bound to \( c \) less than \( d \). Nevertheless, any positive fraction of consumers who do commute leads to the breakdown of equilibrium for every \( c > \bar{c} \) (Proposition 2).

3. A review of Claycombe’s analysis and the predictions of Claycombe and Mahan

In this section we address the concern mentioned in the introduction, viz. the validity of Claycombe’s (1991) analysis and the link between this work and Claycombe and Mahan (1993).

As to the latter, we observe that the Claycombe–Mahan predictions cannot be derived from Claycombe (1991). According to that analysis, perfect competition prevails whenever \( c > d \), whether there is a fraction of noncommuting consumers or not. In a free-entry equilibrium, therefore, this situation can never arise, since firms would be making losses. However, the paper remains silent on the properties of the price equilibrium for the case \( c < d \).

On the other hand, the theoretical explanations Claycombe and Mahan provide for their predictions are only heuristic and not directly related to the Claycombe (1991) model: average commuting distance determines the number of shops consumers pass on average, which in turn partially determines the breadth and hence competitiveness of (local) markets, and therefore, prices. Moreover, consumers using mass transit or car pools “are unlikely to use the commute to stop at a store... Hence, a high proportion of these consumers is expected to generate high prices”.

At this point it is appropriate to return to Claycombe’s (1991) analysis in order to see where the difficulties arise.

On p. 308, Claycombe correctly argues that for \( c > d \) and a given number of firms, price shading will occur as long as price exceeds marginal cost. On the other hand, the “prediction of competitive pricing” which seems to follow from this is valid only if competitive pricing can actually be sustained as a Nash equilibrium. Our Proposition 1 shows that this is not the case.

Claycombe then discusses the free-entry equilibrium for a situation with some fixed cost and a constant marginal cost. With this cost structure, competitive pricing implies losses for the firms, therefore they exit (or more appropriately, they do not enter in the first place). Assuming that distance between the firms is inversely related to the number of firms, as is the case
for example if firms are spaced evenly on a circle rather than a line, this reasoning implies that under the free-entry assumption there can never be an equilibrium for which \( c > d \). However, this does not tell us anything about the actual equilibrium outcome. There is no reason to assume that “when exit drives \( d \) up to the level of \( c \) an equilibrium is reached” (p.309), in which prices can be held above marginal cost, even if an equilibrium existed (contrary to Proposition 1(b)). On the contrary, by continuity Proposition 1(a) would imply that for \( c = d \), competitive pricing would still prevail, leading to losses for the firms. If, on the other hand, supramarginal prices are merely due to an integer number of firms and the required nonnegativity of profits, this result is quite obvious and has nothing to do with commuting.

Claycombe’s above result that for \( c > d \), perfect competition prevails, carries over to the case where there is a fraction of noncommuting consumers (mixed demand, p.310). According to our analysis (Section 2.2), however, in this case a symmetric pure-strategy equilibrium does not exist even if \( c > 2d \).

For small commuting distances (\( c < d \)), finally, Claycombe is concerned with the derivation of market boundaries for given prices, but does not carry this analysis further to determine equilibrium prices. The results of our analysis, as given by Proposition 1, certainly are at odds with Claycombe’s assertion that for this parameter range “the model not differ substantially from the model where there is no commuting at all” (p. 310). At the same time, they provide a more rigorous theoretical basis for the predictions of Claycombe and Mahan, which appear to be supported by their data.

4. Concluding remarks

In this paper we have shown that introducing commuting consumers into an otherwise standard spatial model has a significant impact on the price competition among retail firms. For commuting distances which are small compared to the distance between firms, prices are decreasing in both the commuting distance and the proportion of commuting consumers. For larger commuting distances, however, a price equilibrium in general does not exist. Only in the extreme case where each consumer freely passes at least two shops on her commuting route (i.e. in particular, all consumers commute), perfect competition is the equilibrium outcome. Our results thus strikingly differ from those obtained by Claycombe (1991).

The results of our model appear to be, at first glance, similar to results frequently encountered in spatial models: (i) As firms move closer to each other (directly or indirectly), equilibrium prices decrease. (ii) On the other
nonexistence is a common result particularly if firms are close to each other (cf. Gabszewicz and Thisse, 1986).

A closer look, however, reveals some deviations from this pattern. First, the analogy between an increase in commuting distance and a decrease in the distance between firms has its limits, as the discussion of the case of linear travel costs has shown. Second, and more importantly, the reason why a price equilibrium ceases to exist for higher commuting distances is very different in nature from the breakdown of equilibrium in the standard hotelling model with linear travel costs. There, if firms are close together, a firm will be tempted to shade its price in order to capture a large share of the market of the neighbouring firm; here, in contrast, a firm will give up competing for the marginal consumer located between the firms and instead increase the price in order to extract profits from its local consumers.

The nonexistence problem is a quite general phenomenon which can be shown to arise in other spatial models with commuting consumers as well, e.g. models in which the commuting distance is not the same for every consumer. It arises because of the presence of both consumers who freely pass two shops on their commuting route and thus are only concerned about the shop price, and other consumers who pass only one shop and hence have to incur travel costs if they purchase at another shop. Prices above marginal costs cannot be sustained as firms compete for consumers in the former group, whereas on the other hand firms can always secure positive profits by charging supramarginal prices from consumers in the latter group.

Thus, while the assumption of a constant commuting distance in this paper is quite restrictive, relaxing it only makes nonexistence problems more likely to occur: consumers commuting over long distances make the market more competitive, whereas the existence of local consumers who commute only over short distances (or not at all, cf. Section 2.2) gives firms some monopoly power, disrupting competition for the marginal consumer.

Theoretical and empirical work both suggest that commuting plays an important role for the price determination in retail markets. Apart from the price level, commuting is also likely to affect firms' location choices; for example, petrol stations tend to be located on radial routes of cities, where commuters pass by, rather than in suburbs or in the centre. The nonexistence of pure-strategy price equilibria in apparently plausible models, however, poses a serious obstacle to the analysis of models involving location choice. Further research will hopefully lead to models in which these issues can be analysed.

Acknowledgements

I would like to thank John Sutton for his encouragement and advice, and Michael Hardt, Alison Hole, and two anonymous referees for very helpful
Appendix

Proof of Proposition 1.

1. First consider the case \( c < d \). Obviously, since (6) is a necessary condition for a symmetric price equilibrium, \( p^* = d(d-c)t \) is the only candidate for an equilibrium price. Given the concavity of \( \pi_0 \) for \( k \in [-(d-c)^2, (d-c)^2] \), \( p^* \) is locally optimal in this price range. It remains to be shown under which conditions deviating to a price outside this range will not be profitable.

1.1. Consider a deviation by firm 0 from \( p^* \) such that \( k > (d-c)^2 \). Since \( p_0 = kt + p^* \), by (5) firm 0's profit is

\[
2[t(k + d(d-c)t)(d-c/2 - \sqrt{k})].
\]

Therefore, deviating is profitable if

\[
2[k + d(d-c)t](d-c/2 - \sqrt{k}) - d^2(d-c) > 0. \tag{A.2}
\]

In order to obtain the maximum of this expression in \( k \), we take a look at the first-order condition

\[
d - c/2 - \sqrt{k} = \frac{1}{2\sqrt{k}}[k + d(d-c)t]. \tag{A.3}
\]

Unless \( c \) is relatively close to \( d \), this equation has no real root in \( k \), in which case the l.h.s. of (A.2) is decreasing in the entire relevant range. For larger \( c \) the solution to (A.3) is

\[
\sqrt{k} = \frac{1}{6} \left(2d - c + \sqrt{c^2 + 8cd - 8d^2}\right), \tag{A.4}
\]

where indeed a maximum is obtained. Substituting this expression in (A.2) and furthermore expressing \( c \) as a fraction of \( d \), \( c = \lambda d \), the l.h.s. of (A.2) becomes

\[
2 \left(1 - \frac{\lambda}{2} - \frac{2-\lambda}{6} - \frac{1}{6} \sqrt{\lambda^2 + 8\lambda - 8}\right)
\times \left[1 - \lambda + \left(\frac{2-\lambda}{6} + \frac{1}{6} \sqrt{\lambda^2 + 8\lambda - 8}\right)^2\right] - (1 - \lambda). \tag{A.5}
\]

In the relevant range for \( \lambda \), i.e. between \( \sqrt{24} - 4 \) and one, (A.5) is monotonically increasing and has a unique root at \((13 - 5\sqrt{5})/2\), which gives

\footnote{For smaller \( \lambda \), (13) is not real.}
us the critical value \( \hat{c} \) of the Proposition.\(^6\) We have thus shown that a deviation from \( p^* \) to a higher price such that \( k > (d - c)^2 \) is not profitable if and only if \( c \leq \hat{c} \).

1.2. To complete part (a), we have to show that deviations below \( p^* \) such that \( k < -(d - c)^2 \) are not profitable either. First of all, since \( p_0 \) cannot fall below 0, the relevant range for \( k \) is \( -(d - c), -(d - c)^2 \). Similarly as above, using (5) the condition for a profitable deviation is

\[
[k + d(d - c)](c + 2\sqrt{-k}) - d^2(d - c) > 0.
\]

(A.6)

This expression has its maximum at \( k = -(d - c)^2 \), where the value is \( -(d - c)^2 \). Therefore, such a deviation is never profitable.

2. Now let \( c \in (d, 2d) \).

(a) Then there exists a group of consumers located between \( [d - c/2, c/2] \) who can reach both firms 0 and 1 at zero cost. A price \( p > 0 \) charged uniformly by all firms is not sustainable as an equilibrium since a firm could, by shading its price by an infinitesimal amount, increase its demand by \( 2(c - d) \) and thus increase profits.

(b) On the other hand, \( p = 0 \) is not sustainable either because consumers located in \( (-d + c/2, d - c/2) \) reach firm 0 at zero cost, but neither of the neighbouring firms. By charging an arbitrarily small price, firm 0 can thus attract a subset of this group and thereby attain positive profits. Therefore, a symmetric price equilibrium does not exit. By continuity, arguments 1 and 2 extend to the case \( c = d \).

3. Finally, for \( c \geq 2d \), argument 2(a) still applies, but not argument 2(b), as every consumer can reach at least two firms. Therefore, \( p^* = 0 \) is the unique symmetric equilibrium. \( \Box \)

Proof of Proposition 2.

1. First of all, for \( p \) to be a symmetric equilibrium price (implying \( k = 0 \)) requires

\[
\frac{\partial \pi_0(0)}{\partial p_0} = d - \frac{p}{t} \left( \frac{\alpha}{d} + \frac{1 - \alpha}{d - c} \right) = 0,
\]

(A.7)

which leads to the candidate equilibrium (9).

2. To show the existence of a critical value \( \hat{c}(\alpha) \), we first have to show that for any \( \alpha < 1 \), there exists a \( c \) such that a deviation from \( p^* \) into the range \( k \in [(d - c)^2, (d - c/2)^2] \) is profitable, which (proceeding as in the proof of Proposition 1, cf. (A.2)) will be the case if

\(^6\) This result was obtained with the help of Mathematica.
\[ r(\alpha, c, k) := \left[ (1 - \alpha)(d - c - 2\sqrt{k}) + \alpha(d - k) \right] \]
\[ \times \left( k + \frac{d^2(d - c)t}{d - \alpha c} \right) - \frac{d^3(d - c)t}{d - \alpha c} > 0. \tag{A.8} \]

Specifically, consider \( \bar{k} := (d - 0.75c)^2 \). Then \( r(\alpha, c, \bar{k}(c)) > 0 \) is equivalent to
\[ s(\alpha, c) := 256(1 - \alpha c)r(\alpha, c, \bar{k}(c)) > 0. \tag{A.9} \]

where \( s \), a polynomial of fifth order in \( c \), is continuous in both parameters and strictly increasing in \( c \) for all values of \( \alpha \). Since \( s(\alpha, 1) \) is unambiguously positive, continuity and monotonicity of \( s \) in \( c \) imply the existence of a \( \bar{c} \) such that \( s(\alpha, c) > 0 \) for all \( c \in [\bar{c}, 1] \) and \( s(\alpha, c) > 0 \) for all \( c < \bar{c} \). By choosing \( k \) to maximize the l.h.s. of (A.8) one then obtains the smallest value \( \bar{c} \) such that this holds. \( \square \)

References


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