Competitive One-to-One Promotions

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One-to-one promotions are possible when consumers are individually addressable and firms know something about each customer’s preferences. We explore the competitive effects of one-to-one promotions in a model with two competing firms where the firms differ in size and consumers have heterogeneous brand loyalty. We find that one-to-one promotions always lead to an increase in price competition (average prices in the market decrease). However, we also find that one-to-one promotions affect market shares. This market-share effect may outweigh the effect of lower prices, benefiting the firm whose market share increases. Our results suggest that of two firms, the firm with the higher-quality product may gain from one-to-one promotions. Our model also has implications for the phenomenon of customer churn, where consumers switch to a less preferred brand due to targeted promotional incentives. We show that churning can arise optimally from firms pursuing a profit-maximizing strategy. Instead of trying to minimize it, the optimal way to manage customer churn is to engage in both offensive and defensive promotions with the relative mix depending on the marginal cost of targeting.

(Database Marketing; Game Theory; Strategy; Price Discrimination)

1. Introduction

Advances in information technologies and the Internet today allow firms to identify individual consumers with greater accuracy and cost-effectiveness than ever before, which, in turn, allows firms to tailor their promotional prices to consumers on a one-to-one basis. Many firms are already taking advantage of this new-found ability to customize their prices. AT&T, for instance, has successfully lured many MCI customers to switch carriers by offering them personalized checks in the amounts of $25 to $100 depending on each consumer’s long-distance calling history and experience with AT&T (Turco 1993). Other examples include mail-order companies, like LL Bean, which often insert into their catalogs “special offers” that vary across households, and the online data provider Lexis-Nexis, which “sells to virtually every user at a different price” (Shapiro and Varian 1999).

One-to-one promotions are facilitated by many information-intensive marketing approaches such as database marketing, target marketing, micromarketing, and one-to-one marketing. One-to-one promotions are often seen as beneficial to practicing firms because they allow a firm to charge lower prices to new consumers (for the purpose of inducing trial or

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Somewhere, a network has more personal data about you than you probably ever imagined.

—Business Week, April 5, 1999

Practitioners sometimes use these terms interchangeably. Database marketing refers to any marketing activity that is aided by a consumer database. Target marketing refers to the process by which a firm offers promotional incentives tailored to individual consumers or small group of consumers. Micromarketing goes one step further in that a firm also customizes its service, products, or product assortments to satisfy the needs of targeted consumers. One-to-one marketing, a term coined by Peppers and Rogers (1993), focuses on establishing long-term relationships with individual consumers through customized production, individually addressable media, and personalized marketing.

The view that one-to-one promotions (and other forms of customized pricing) ultimately benefit firms by allowing them to generate incremental sales without sacrificing on the profit margins they receive from their loyal customers has been challenged in several recent game-theoretic articles (Shaffer and Zhang 1995, Bester and Petrakis 1996, Chen 1997, Taylor 1999, Fudenberg and Tirole 2000). These articles find that when promotions can be targeted, a prisoner’s dilemma invariably results. Prices decrease as the distinction between marginal and inframarginal consumers becomes blurred, and the alleged gain in incremental sales that the practicing firms are going after never materialize, as the competing firms simply neutralize each other’s promotional efforts.

However, these articles all bias their results in favor of a prisoner’s dilemma because they start with the assumption that firms are symmetric ex ante. Thus, it is not surprising that the firms remain symmetric after all pricing and promotional decisions have been made. In essence, these articles have identified a deleterious price-competition effect from one-to-one promotions, but they have implicitly ruled out the possibility of one of the firms gaining from a market-share effect.

In this article, we consider a game-theoretic model that allows for the possibility of both effects. When competing firms differ in size, and consumers have heterogeneous brand loyalty, we find that one-to-one promotions can affect market shares, even when the targeting technology is available to all firms at the same cost. We also find that this market-share effect may outweigh the adverse effect of lower prices, benefiting the firm whose market share increases. Thus, unlike in the previous literature, we find that one-to-one promotions do not invariably lead to a prisoner’s dilemma.

This has important managerial implications. We incorporate the four main features of one-to-one promotions: individual addressability, personalized incentives, competition, and costs of targeting (Blattberg and Deighton 1991, Schultz 1994). We ask (i) which firms are more likely to benefit from one-to-one promotions, and hence have more incentive to acquire the capability to target; (ii) how might firms best position themselves to take advantage of the new targeting technologies; and (iii) when competing firms are also targeting, which consumers should a firm target, what incentives should these consumers be offered, and how does this depend on firm size and consumer loyalty.

We find that one-to-one promotions tend to favor firms that command stronger brand loyalties and have larger market shares. The smaller market-share firm always loses, but the larger firm may win or lose depending on the magnitude of the market-share effect vis à vis the price competition effect. This is most easily seen when two firms sell vertically differentiated products and compete in prices. In the absence of one-to-one promotions, the higher-quality firm charges a price in excess of its marginal cost, allowing the lower-quality firm to capture some of the market. With one-to-one promotions, the higher-quality firm can outbid the lower-quality firm on each consumer. Although many (but not all) consumers end up paying lower prices, the higher-quality firm can gain because of the resulting increase in its market share. That the lower-quality firm always loses in this case (its market share falls and prices are lower) suggests that positioning one’s brand as the higher-quality product may be even more important in the information age than previously thought.

Our model also provides insights on how to manage customer churn, a phenomenon that arises when consumers are induced to switch to a less preferred product because of having received a targeted promotion. We show that it is not advisable for a firm to eliminate customer churn even if it were possible to do so. Instead, it is optimal for firms to engage in both offensive promotions (targeting consumers who prefer the rival firm’s product) and defensive promotions (targeting consumers who prefer one’s own product) with the relative mix depending on the marginal cost of targeting.

We differ from the above-cited literature on one-to-one promotions in that we are the first to introduce asymmetry in a model in which there are costs of targeting. The asymmetry allows us to see how qualitatively different competitive implications may arise.
from one-to-one promotions when there are market-share effects. The targeting costs give rise to the possibility that firms will randomize their promotions, which allows us to discuss the phenomenon of customer churn.

Shaffer and Zhang (1995) consider a model where firms choose regular prices and coupon face values and then choose how to distribute the coupons. Although they allow for targeting costs, the firms in their model are symmetric and offer at most one promotional price. In this article, we do not restrict the firms’ promotional strategies to just one price (each consumer can potentially be offered a different price) and we allow the firms to be asymmetric. Lederer and Hurter (1986) also allow for asymmetric firms, but they do not incorporate targeting costs nor do they explore the implications of their findings on firm profits and market shares. Corts (1998) and Shaffer and Zhang (2000) were the first to find that targeted promotions need not lead to a prisoner’s dilemma. However, these articles allow for at most one promotional price, and their conclusions arise because of a possible lessening of price competition and not, as in our model, from a market-share effect.

The rest of the paper proceeds as follows. In §2, we set up the model and discuss its properties. In §3, we solve for the second-stage equilibrium of a game in which firms simultaneously choose regular prices in the first stage, and then simultaneously choose which consumers to target and with what promotional incentives in the second stage. In §4, we solve for the mixed-strategy equilibrium and draw conclusions about the competitive effects of one-to-one promotions. In §5, we look at the phenomenon of customer churn and ask whether firms should predominantly focus on offensive or defensive targeting. Section 6 concludes.

2. Model
Suppose firms A and B sell competing brands of a consumer good that is produced at constant marginal cost, \( c \geq 0 \). Each consumer buys at most one unit of the good and all consumers are willing to pay at most \( V \geq c \) for their more preferred brand. Consumers differ in how much they are willing to pay for their less preferred brand, which makes them heterogeneous in brand loyalty.

We define a consumer’s brand loyalty as the minimum price differential necessary to induce her to purchase her less preferred brand. Thus, a consumer who prefers brand A by a loyalty of \( l \) will buy brand B, in the absence of any promotional incentives, if and only if firm A’s regular price, \( P_A \), exceeds firm B’s regular price, \( P_B \), by more than \( l \). That is, the consumer will buy brand B if and only if \( P_A - P_B > l \).

We assume \( l \in [-l_B, l_A] \) follows a uniform distribution, where \( 0 \leq l_i < V \), for \( i \in \{A, B\} \).

Figure 1 illustrates the setup. Consumers with positive loyalty prefer brand A all else equal. Consumers with negative loyalty (toward brand A) prefer brand B all else equal. The consumer located at 0 is just indifferent between purchasing brands A and B at equal prices.

Without loss of generality, we assume \( l_A \geq l_B \). If \( l_A = l_B \), the setup is analogous to the standard model of horizontal differentiation with the loyalty parameter playing the same role as the transportation cost in other spatial models. If \( l_B = 0 \), the setup is analogous to the standard model of vertical differentiation in which all consumers agree that brand A is of higher quality than brand B. If \( l_A > l_B > 0 \), the setup has elements of both horizontal and vertical differentiation.

We assume also that firms know the locations of all consumers in the market and thus know their exact brand loyalties. In practice, of course, we would expect firms to be more or less certain about the loyalty of each consumer, depending on the quality and quantity of their data on individuals’ past purchasing behavior. Thus, our results should be interpreted as the solution to an important limiting case—the case of perfect information. Nevertheless, it is an important

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2Our assumption of a uniform distribution is made to obtain explicit solutions. This assumption is not as restrictive as it may seem at first because it turns out that firms’ promotional strategies in equilibrium depend only on \( l \)—and not on the distribution of consumers—for any given pair of regular prices in the market.
case to consider because the effects we identify in this limiting case will also be present in a more general case.

The firms play a two-stage game of pricing and promotions in which regular prices are chosen in the first stage and promotional strategies are chosen in the second stage. This two-stage structure reflects a commonly held view that a firm’s choice of regular price is a higher-level managerial decision and is relatively slower to adjust in practice than a firm’s choice of promotions.\(^3\) Later, in \(\S 6\), we will discuss the implications of one-to-one promotions with no regular prices.

In choosing its promotional strategy, each firm must decide which consumers to target and with what discounts. Targeted promotions are not free; we assume the cost of targeting each consumer is given by \(z \geq 0\) (this cost is incurred whether or not the consumer buys from the firm). Let \(t_i(l)\) be an indicator variable that equals one if firm \(i\) promotes to consumers located at \(l\) and zero otherwise, and let \(d_i(l)\) denote firm \(i\)’s discount to these consumers. For example, if promotions take the form of targeted coupons, then \(t_i(l)\) indicates whether consumers at \(l\) are targeted with a coupon and, if they are, \(d_i(l)\) is the face value of the coupon. More generally, \(d_i(l)\) is the monetary value of any individual-specific promotional incentives, including coupons, premiums, prizes, or alterations in the basic product that firm \(i\) offers to consumers to gain their patronage.

Consumers maximize their surplus given each firm’s regular price and promotional incentives. Thus, a consumer located at \(l\) who receives \(d_A(l)\) from firm \(A\) and \(d_B(l)\) from firm \(B\) will purchase from firm \(A\) if and only if firm \(A\)’s net price does not exceed firm \(B\)’s net price by more than \(l\). If a consumer located at \(l\) only receives promotional incentives from firm \(A\), then she will purchase from firm \(A\) if and only if \(P_A - d_A(l) - P_B \leq l\). If a consumer located at \(l\) only receives promotional incentives from firm \(B\), then she will purchase from firm \(A\) if and only if \(P_A - (P_B - d_B(l)) < l\).

3. One-to-One Promotions
   Given Regular Prices

We assume that the regular prices are observable to both firms when they choose their promotional strategies, and we use subgame perfection as our solution concept. Thus, we solve for the equilibrium strategy of each firm by solving first for the Nash equilibria of the second stage.

Our analysis in this stage is complicated in three important ways by the existence of a targeting cost \((z \geq 0)\). First, it means that the profitability of one-to-one promotions depends on the relationship between \(z\) and \(P_i - c\). This gives rise to four possible subgames. Second, it means that a firm may not want to target all consumers in the market even if it makes use of one-to-one promotions. This gives rise, for each subgame, to a firm’s “targeting zone.” Third, it means that pure-strategy equilibria may not exist. As we shall see, in one subgame, we will need to solve for a mixed-strategy equilibrium, jointly determining each firm’s targeting zone and discount schedule.

Subgame 1: \(P_A \leq c + z\) and \(P_B \leq c + z\).

In this case, the cost of targeting is sufficiently high that neither firm can profitably promote. Thus, the solution is similar to what one gets from a standard spatial model of demand. Let \(\hat{l} = P_A - P_B\) denote the location of the marginal consumer. Then, firm \(A\)’s demand is \((l_A - \hat{l})/(l_A + l_B)\), firm \(B\)’s demand is \((\hat{l} + l_B)/(l_A + l_B)\), and the equilibrium payoffs for the two firms are\(^4\)

\[
\Pi_A^1 = \frac{(P_A - c)(l_A - \hat{l})}{l_A + l_B}, \quad \Pi_B^1 = \frac{(P_B - c)(\hat{l} + l_B)}{l_A + l_B}.
\]

Subgame 2: \(P_A > c + z\) and \(P_B \leq c + z\).

In this case, although firm \(A\) can profitably promote, it will never want to target consumers located at \(l \geq \hat{l}\) because these consumers will buy from firm \(A\) even in the absence of any promotions. As for consumers located at \(l < \hat{l}\), note that the largest discount firm \(A\) can offer any consumer and still earn nonnegative profit is \(d_A \equiv P_A - c\). For all consumers

\(^3\)See, for example, Rao (1991), Shaffer and Zhang (1995), and Banks and Moorthy (1998).

\(^4\)It should be noted that each firm’s demand is bounded below by zero. That is, the demand for firm \(A\) is equal to \(\max(l_A - \hat{l})/(l_A + l_B)\), \(0\). However, for convenience, we suppress writing out the inequalities in the text.
located at $l < \hat{i}_B$, where $\hat{i}_B = c + z - P_B$, however, this discount is not large enough to induce them to switch brands, as they are so loyal to firm $B$ that they will buy from firm $B$ even if they receive $\hat{d}_A$ from firm $A$. Thus, it is not profitable for firm $A$ to target them. The rest of firm $B$'s customers, however, can profitably be induced to switch. Thus, in equilibrium, firm $A$ will target consumers located between $\hat{i}_B$ and $\hat{i}$.

It remains to solve for firm $A$'s optimal discount schedule inside the targeting area. Let $\mathcal{F}_A^3 \equiv \{l \mid \hat{i}_B \leq l < \hat{i}\}$. Then consumers located at $l \in \mathcal{F}_A^3$ will purchase from firm $A$ if and only if $P_A - d_A(l) - P_B \leq l$. It follows that firm $A$ will offer these consumers the smallest discount that satisfies this inequality, i.e., $d_A(l) = P_A - P_B - l$, yielding a profit margin for firm $A$ of $P_A - d_A(l) - c - z = P_B + l - c - z$. Thus, the equilibrium payoffs for the two firms are

$$
\Pi_A^3 = \frac{1}{l_A + l_B} \left( (P_A - c)(l_A - \hat{i}) \right.
+ \int_{l \in \mathcal{I}_A^3} (P_B + l - c - z) \, dl \left.) \right),
$$

$$
\Pi_B^3 = \frac{1}{l_A + l_B} (P_B - c)(\hat{i}_B + l_B).
$$

Firm $A$'s profit is the sum of the profit it earns from sales to consumers who buy at the regular price and the profit it earns from sales to consumers who otherwise would have bought from firm $B$. Its profit margin is lower on the latter consumers due to the cost of targeting and the promotional incentives it must offer them. Nevertheless, firm $A$’s ability to target consumers with individual-specific incentives allows it to create incremental sales at the expense of its nontargeting rival.

Subgame 3: $P_A \leq c + z$ and $P_B > c + z$.

Since this subgame is analogous to Subgame 2, with the role of each firm reversed, we simply list each firm’s equilibrium payoff. Let $\hat{i}_A = P_A - c - z$ and $\mathcal{F}_B^3 \equiv \{l \mid \hat{i}_B < l \leq \hat{i}_A\}$. Then

$$
\Pi_A^3 = \frac{1}{l_A + l_B} \left( (P_B - c)(\hat{i}_B + l_B) + \int_{l \in \mathcal{I}_B^3} (P_A - c - z - l) \, dl \right),
$$

$$
\Pi_B^3 = \frac{1}{l_A + l_B} (P_A - c)(l_A - \hat{i}_B).
$$

Subgame 4: $P_A > c + z$ and $P_B > c + z$.

In this case, both firms can profitably promote and the targeting zone, $\mathcal{F}_A^4$, is simply the union of $\mathcal{F}_A^3$ and $\mathcal{F}_B^3$ as illustrated in Figure 2.5 The targeting zone is the same for both firms because in the region where firm $A$ is trying to induce consumers to switch brands, firm $B$ is trying to defend its customer base, and vice versa. In equilibrium, neither firm targets outside $\mathcal{F}_A^4$, since these consumers are sufficiently loyal that the rival firm cannot profitably attract them.

As in the other subgames, the promotional decisions that each firm makes can be divided conceptually into two components. A firm must decide with what probability to target consumers at each location and, to those it targets, what depth of discount to offer. A major difference from the other subgames, however, is that, in equilibrium, each firm will randomize inside the targeting zone. The randomization will occur not only over whether or not to target consumers at a particular location but also over what discount to offer. To see this, consider the consumers located in region I. Firm $A$ will not want to target consumers in this region if firm $B$ does not, since these consumers are already predisposed to purchasing from firm $A$. If firm $A$ does not target these

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5 Figure 2 illustrates the case where $0 \leq \hat{i} \leq \hat{i}_A$. There is also another case (not shown) where $-\hat{i}_B \leq \hat{i} < 0$. As we will show in Appendix A, however, the second-stage equilibrium does not depend on the relation between $\hat{i}$ and 0.
consumers, however, then firm B will. But if firm B targets these consumers firm A’s optimal strategy is to do likewise, since it would rather outbid its rival and retain its customers than lose them altogether. Finally, if firm A targets these consumers, then firm B would rather not, and so on. Furthermore, given that both firms may target the same consumers, neither firm will use a pure strategy for its discount $d_i(l)$. This is because if the rival firm has provided sufficient incentives to win the patronage of a consumer, then a firm will either increase its own incentives to the consumer to outbid its rival or give up completely on targeting the consumer to save the targeting cost $z$.

Fortunately, the equilibrium payoffs of firms A and B in each region are easy to derive (the details can be found in Appendix A) and are illustrated in Figure 3.

<table>
<thead>
<tr>
<th>firm A: l</th>
<th>firm A: $l_B - c$</th>
<th>firm A: $l_A$</th>
<th>firm A: $P_A - c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>firm B: l</td>
<td>firm B: $P_B - c$</td>
<td>firm B: 0</td>
<td>firm B: 0</td>
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Since consumers located at $-l_B \leq l < l_B$ do not receive any promotions and are loyal to firm B, firm A earns zero and firm B earns $P_B - c$ from each consumer in this region. Similarly, since the consumers located at $l_A < l \leq l_B$ do not receive any promotions and are loyal to firm A, firm B earns zero and firm A earns $P_A - c$ from each consumer in this region. The contested consumers are the ones in the middle two regions. For consumers located at $l$ to the left of 0, firm B has a preference advantage of $-l$. For consumers located at $l$ to the right of 0, firm A has a preference advantage of $l$. Equilibrium payoffs in these regions then follow immediately as a direct consequence of mixed-strategy equilibria.

**Proposition 1.** With one-to-one promotions, a firm’s equilibrium payoff from each consumer in the targeting zone depends only on the price premium the consumer is willing to pay for its brand.

Proposition 1 implies that with one-to-one promotions, a firm’s expected payoff from consumers in the targeting zone derives solely from the loyalty these consumers have for its brand. The reason is that although a firm is always able to outbid its competitor for the consumers who prefer its own brand, one-to-one promotions dissipate all potential rents except for the premiums that contested consumers are willing to pay for a brand. This result highlights the vital importance of individual (rather than average) consumer loyalty in the information age and suggests that the increased interest in recent years among marketing practitioners in relationship marketing, customer satisfaction, customer life-time value, and one-to-one marketing can be viewed as a manifestation of firms’ efforts to position themselves for the upcoming information-intensive marketing.

Summing up each firm’s expected payoff in all four regions of Figure 3 yields:

$$
\Pi_A^4 = \frac{(P_A - c)(l_A - \hat{l}_A)}{l_A + l_B} + \frac{\hat{p}_B^2}{2(l_A + l_B)}
$$

$$
\Pi_B^4 = \frac{(P_B - c)(l_B + \hat{l}_B)}{l_A + l_B} + \frac{\hat{p}_A^2}{2(l_A + l_B)}.
$$

### 4. Equilibrium Pricing and Promotions

In this section we address several questions of interest. First, how do one-to-one promotions affect the average prices paid by consumers? Second, which firm has more incentive to initiate one-to-one promotions? Third, who will gain or lose from one-to-one promotions? Intuitively, one might expect that a firm’s decision on whether or not to target will depend on the magnitude of the targeting costs relative to $l_A$ and $l_B$. The next proposition summarizes this relationship.

**Proposition 2.** For all $z \geq 0$, there exists a unique subgame perfect equilibrium.

(a) If $z \geq (2l_A + l_B)/3$, then the equilibrium regular prices are $P_A = (2l_A + l_B)/3 + c$ and $P_B = (l_A + 2l_B)/3 + c$, and neither firm offers one-to-one promotions.

(b) If $l_B \leq z < (2l_A + l_B)/3$ (note $l_A \geq l_B$ by assumption), then the equilibrium regular prices are
P_A = (2l_A + l_B - z)/2 + c and P_B = (l_B + z)/2 + c, and only firm A offers one-to-one promotions.

(c) If z < l_B, then the equilibrium regular prices are P_A = l_A + c and P_B = l_B + c, and both firms offer one-to-one promotions.

Neither Firm Offers One-to-One Promotions

The first thing to notice from Proposition 2 is that there exists a cutoff level of z such that if z is greater than or equal to this cutoff level, then neither firm offers one-to-one promotions. This has both an obvious and a less-than-obvious implication. The obvious implication is that if the cost of targeting is sufficiently high then no firm will promote. The less-than-obvious implication is that, at a given point in time, one-to-one promotions may be optimal in some industries but not others, even if the underlying targeting technology and its associated cost is the same across all industries. To see this, note that the cutoff level of z in this case is equal to firm A’s regular-price markup, which depends on the degree of competition in the market. It follows that firms in less profitable industries (those that have low contribution margins) will be less inclined to offer one-to-one promotions. For example, in an industry where the products are homogeneous (l_A = l_B = 0), we have that P_A = P_B = c and so one-to-one promotions will not occur for any z ≥ 0.

Let \( \tilde{\Pi}_i \) denote firm i’s payoff and \( \tilde{S}_i \) its market share. Then the firms’ payoffs and market shares (the share of total sales accounted for by a firm) when neither firm offers targeted promotions are

\[
\begin{align*}
\tilde{\Pi}_A &= \frac{(2l_A + l_B)^3}{9(l_A + l_B)^2}, & \tilde{\Pi}_B &= \frac{(l_A + 2l_B)^3}{9(l_A + l_B)^2}, \\
\tilde{S}_A &= \frac{2l_A + l_B}{3(l_A + l_B)}, & \tilde{S}_B &= \frac{l_A + 2l_B}{3(l_A + l_B)}. \quad (5)
\end{align*}
\]

If \( l_A = l_B \), then each firm earns the same profit and has the same market share. If \( l_A > l_B \), then firm A earns more profit and has a larger market share than firm B. The dotted lines in Figure 4 illustrate an asymmetric equilibrium; firm B sells to all consumers with loyalties \( -l_B \) to \((l_A - l_B)/3 \) at price \((l_A + 2l_B)/3 + c \), and firm A sells to all consumers with loyalties \((l_A - l_B)/3 \) to \( l_A \) at price \((2l_A + l_B)/3 + c \).

Only Firm A Offers One-to-One Promotions

The second thing to notice from Proposition 2 is that there exist intermediate values of z such that in equilibrium firm A will want to promote but firm B will not (it is never the case that only firm B will promote). The solid lines in Figure 4 illustrate the equilibrium. In Figure 4, firm B chooses a regular price of \((l_B + z)/2 + c \) and sells to all consumers with loyalties \(-l_B \) to \((z - l_B)/2 \). All consumers with loyalties to the right of \((z - l_B)/2 \) buy from firm A. Of these consumers, consumers with loyalties \((z - l_B)/2 \) to \( l_A - z \) receive promotional incentives. These are the consumers in firm A’s targeting zone. The consumers with loyalties \( l_A - z \) to \( l_A \) are sufficiently loyal to firm A that they are not given any discounts. These consumers buy from firm A at firm A’s regular price of \((2l_A + l_B - z)/2 + c \).
This case is interesting for three reasons. First, it is the case implicitly assumed by the non-game-theoretic literature, which views one-to-one promotions as ultimately benefiting firms by allowing them to generate incremental sales without sacrificing on the profit margins they receive from their loyal customers. However, as we shall see, the implications of this case are much richer, and the conventional view that the promoting firm will be better off does not always hold. Second, this case is important because it has been missed by the previous game-theoretic literature, which assumes that firms are ex ante symmetric. Note that at \( l_B = l_A \), the region of \( z \) such that only firm \( A \) promotes, \( l_B \leq z < (2l_A + l_B)/3 \), does not exist, and so our model predicts that in a market with two equally matched firms, we would not expect to see only one firm offering targeted promotions. Third, this case is the only relevant case when the firms’ products are vertically differentiated. To see this, note that if firm \( B \)'s product is of lower quality than firm \( A \)'s product (\( l_B = 0 \)), then the lower bound of \( z \) is never binding, which implies that firm \( B \) never promotes.

Notice that each firm responds differently to a decrease in the marginal cost of targeting; firm \( A \) responds by raising its regular price, while firm \( B \) responds by lowering its regular price. Both intuitions can be seen from the profit expressions in (2). In this case, we see that firm \( A \)'s profit-maximizing regular price balances the marginal profit from selling to a consumer in the “regular-price market” with the marginal profit from selling to a consumer in the “one-to-one market.” When the marginal cost of targeting decreases, the marginal profit of selling to a consumer in the one-to-one market increases, and so to equalize its marginal profits across markets, firm \( A \) will want to raise its regular price.6 In contrast, the effect of a lower cost of targeting on firm \( B \) is much different. As it becomes more profitable for firm \( A \) to promote, firm \( B \) will find it necessary to lower its regular price in an effort to offset firm \( A \)'s encroachment into its regular-price market.

The equilibrium payoffs and market shares when only firm \( A \) promotes are:

\[
\tilde{\Pi}_A = \frac{(2l_A + l_B)^2 - z(4l_A + 2l_B - 5z)}{8(l_A + l_B)}, \quad \tilde{\Pi}_B = \frac{(l_B + z)^2}{4(l_A + l_B)}.
\]

Differentiating market shares \( \tilde{S}_A \) and \( \tilde{S}_B \) with respect to \( z \), we see that firm \( A \)'s market share increases, and firm \( B \)'s market share decreases, with a lower \( z \). Thus, it follows that when the marginal cost of targeting decreases, firm \( B \) will not be completely successful in stopping firm \( A \)'s encroachment on its regular-price market. As illustrated in Figure 4, firm \( A \)'s sales increase relative to the case of no promotions by the number of additional consumers with loyalties \( (z - l_B)/2 \) to \( (l_A - l_B)/3 \).

All else equal, this market-share effect harms firm \( B \) and benefits firm \( A \). In fact, a smaller market share coupled with a lower regular price implies that firm \( B \) will necessarily be worse off when the cost of targeting falls, as can easily be seen by differentiating \( \tilde{\Pi}_B \) with respect to \( z \). Surprisingly, however, it does not follow that firm \( A \) will necessarily be better off. Although firm \( A \) earns a higher contribution margin from its regular-price market and has a larger market share, the fraction of its consumers that buy on promotion increases when the cost of targeting falls. In fact, contrary to what the non-game-theoretic literature would predict, it is easy to construct examples in which firm \( A \) is worse off when \( z \) falls. For example, in Figure 4, firm \( A \)'s profit when it promotes is given by the vertically shaded area (where we assume \( c = 0 \) for convenience). When \( z \) is sufficiently high that no firm promotes, firm \( A \)'s profit is given by the rectangular region with base \( l_A - (l_A - l_B)/3 \) and height \( (2l_A + l_B)/3 + c \). The difference in this example is a net loss for firm \( A \).

More generally, although firm \( B \)'s profit always decreases when the cost of targeting falls, firm \( A \)'s profit may or may not decrease. Differentiating firm \( A \)'s equilibrium profit in (6) with respect to \( z \), and 6One can think of the two markets as analogous to substitute products in a product line. When the contribution margin of the first product exogenously increases, profit maximization requires that the firm raise the price of the other product, because the consumers who are induced to switch to the first product now bring in more profit.
comparing its equilibrium profit in (5) and (6), yields the following proposition.

**Proposition 3.** There exists $\bar{z} > 0$ such that firm A’s equilibrium profit when only firm A promotes is increasing in the cost of targeting for $z > \bar{z}$, and decreasing in the cost of targeting for $z < \bar{z}$. For $z \geq \bar{z}$, firm A is worse off relative to the case of no promotions. For $z < \bar{z}$, firm A may or may not be better off relative to the case of no promotions. When firm A’s product is of higher quality than firm B’s product, there exists a sufficiently small $z$ such that firm A is always better off.

Figure 5 illustrates the results in Proposition 3 for the case where firm A’s product is of higher quality than firm B’s product. At $z = 0$, firm A’s profit is $l_A/2$ when it promotes, which exceeds $4l_A/9$, its profit when $z$ is sufficiently high that neither firm promotes. As $z$ decreases below the cut-off level for targeting, firm A’s profit initially decreases and then increases. This is the result of two opposing effects. On the one hand, firm A benefits from an increase in its market share when $z$ decreases. On the other hand, firm A loses from an increase in price competition caused by firm B’s lower regular price. For small enough $z$, the beneficial market-share effect outweighs the adverse price-competition effect and firm A’s profit increases as the cost of targeting further decreases.

These results differ from previous literature on targeted promotions. The non-game-theoretic literature fails to identify the adverse effect on firm A’s profit caused by firm B’s lower regular price, and the previous game-theoretic literature, in assuming symmetry, fails to identify the beneficial market-share effect. Figure 5 shows that either of the two effects can dominate for firm A.

**Both Firms Offer One-to-One Promotions**

When both firms offer targeted promotions, each firm’s regular price is increasing only in the loyalty of its own consumers (Part (c) in Proposition 2). Thus, an exogenous change in the maximum loyalty of firm B’s consumers has no effect on firm A’s regular price, and vice versa. This result suggests that one-to-one promotions lead to a much different form of competition than the competition that occurs when each firm is constrained to charge a single price. Intuitively, when both firms offer one-to-one promotions, there is a bandwidth of marginal consumers (where the width is determined by the size of the targeting zone) which buffer each firm’s regular price from the other. As we shall see, this buffering, and what happens in the targeting zone, affects the two firms similarly.

The equilibrium expected payoffs and market shares when both firms promote are:

$$\Pi_A = \frac{l_A^2 + z^2}{2(l_A + l_B)}, \quad \Pi_B = \frac{l_B^2 + z^2}{2(l_A + l_B)},$$

$$\bar{S}_A = \frac{l_A}{l_A + l_B}, \quad \bar{S}_B = \frac{l_B}{l_A + l_B}. \quad (7)$$

In the mixed-strategy equilibrium, firm A and B’s expected market share is independent of the cost of targeting and converges, not surprisingly, to each firm’s base of loyal consumers. This is because with one-to-one promotions each firm can always outbid its rival for the consumers who prefer its product. Thus, when both firms promote, changes in targeting costs operate on each firm’s equilibrium profit only through the price-competition effect, and so it is also not surprising that each firm’s profit is increasing in $z$ (a higher $z$ moderates the price competition effect).

However, it would be incorrect to assume that each firm’s profit in this case is necessarily less than what its profit would be in the case in which no firm offered promotions. It follows by continuity from our results in the previous subsection that there exist parameters...
such that firm $A$ is better off with one-to-one promotions, even when both firms promote. To see this, note that for firm $A$ to be better off (firm $B$ is always worse off) when both firms promote, we require

$$l_A^2 - 8l_Al_B - 2l_B^2 + 9z^2 > 0. \quad (8)$$

When the products are symmetrically differentiated ($l_A = l_B$), the condition in (8) is never satisfied. But when the products are vertically differentiated ($l_A > 0, l_B = 0$), (8) is always satisfied. More generally, the condition in (8) is satisfied when $l_A$ is sufficiently large relative to $l_B$, which is more likely to be the case when the relationship between the firms’ products is closer to vertical differentiation than to symmetric horizontal differentiation. This yields the following proposition.

**Proposition 4.** One-to-one promotions need not lead to a prisoner’s dilemma situation in which all firms are worse off. The firm with the more loyal following and larger market share can be better off when both firms use one-to-one promotions than when neither firm uses one-to-one promotions.

Proposition 4 lends some support to the intuition that one-to-one promotions can favor “large firms with strong brand identities” (Blattberg and Deighton 1991). However, it should be emphasized that in suggesting that one-to-one promotions need not lead to a prisoner’s dilemma, we are comparing a case in which both firms offer one-to-one promotions at a particular marginal cost of targeting to a case in which the cost of targeting is sufficiently high that no firms offer promotions. This comparison is relevant, for example, if the targeting technology and associated consumer data is available for purchase and firms $A$ and $B$ are deciding whether or not to purchase it. If the comparison is instead between two levels of $z$ conditional on both firms offering promotions, then the profits in (7) imply that both firms are worse off when the marginal cost of targeting decreases.

To understand why earlier game-theoretic literature concludes that targeted promotions lead to a prisoner’s dilemma, and to understand why it need not, it is useful to examine through some polar cases how the two effects of one-to-one promotions manifest themselves in a competitive context.

Figure 6 illustrates equilibrium prices and expected market shares under symmetry for the cases in Parts (a) and (c) of Proposition 2 (we assume $z = 0$ for convenience). From Proposition 2, we see that the equilibrium regular prices remain unchanged with one-to-one promotions. From the market-share expressions in (5) and (7), we see that the equilibrium expected market shares also remain unchanged with one-to-one promotions.

Figure 6 illustrates equilibrium prices and expected market shares under symmetry for the cases in Parts (a) and (c) of Proposition 2 (we assume $z = 0$ for convenience). From Proposition 2, we see that the equilibrium regular prices remain unchanged with one-to-one promotions. From the market-share expressions in (5) and (7), we see that the equilibrium expected market shares also remain unchanged with one-to-one promotions.7 Thus, when the loyalties for the two

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7 In fact, when $z = 0$, it can be shown that all consumers with loyalties $-l_B$ to 0 buy from firm $B$ and all consumers with loyalties 0 to $l_A$ buy from firm $A$ in both cases. This should be clear in the case where neither firm promotes. The reason it is also true in the case where both firms promote is that, when $z = 0$, there is no cost of targeting and so each firm’s equilibrium strategy calls for it to distribute defensive promotions with probability one. In other words, the equilibrium mixed strategies in the case where both firms promote become degenerate when $z = 0$. 
firms are identical \((l_A = l_B)\), the only effect of one-to-one promotions is to intensify price competition.

Figure 7 illustrates the fact that if the firms are asymmetric, there will also be a market-share effect. In the absence of targeted promotions, it is not optimal for the firm with a larger loyal following to sell to all consumers who prefer its product. This is because to capture more consumers with weaker preferences for its product, it must lower its price to all consumers, including those with strong preferences who do not need any additional inducement to purchase from it. Although all firms face this tradeoff between incremental sales and inframarginal losses in their pricing decisions, the fact that firm \(A\) has a larger loyal following tilts the balance of the tradeoff in favor of firm \(B\) setting a lower price than firm \(A\). As a result, firm \(B\) sells to consumers with loyalties \(0 \leq l \leq \frac{(l_A - l_B)}{3}\) even though these consumers would have bought from firm \(A\) at equal prices.

One-to-one promotions allow a firm to generate incremental sales from consumers with weaker preferences without sacrificing margins from consumers with strong preferences. An obvious consequence of this increase in pricing flexibility is more intense price competition (each firm is now free of any inframarginal loss when lowering its price). A more subtle consequence is that the increase in pricing flexibility levels the playing field (the ability to offer one-to-one promotions allows each firm to avoid the aforementioned tradeoff). This accentuates any advantage that a firm may have in customer loyalty. As a result, firm \(A\) can take advantage of its pricing flexibility to increase its market share at the expense of firm \(B\) by capturing those consumers that prefer its product.

It is this tradeoff between market share and price competition that determines whether firm \(A\) will be better or worse off with one-to-one promotions. As we have seen, it can go either way.

5. Customer Churn and Optimal Targeted Promotions

Customer churn arises when consumers who would otherwise prefer a firm’s product switch to a rival firm’s product because of the promotional incentives they receive. So, for example, in Figure 2, those consumers in Region II who buy from firm \(A\) represent positive customer churn for firm \(B\), and those consumers in Regions I and III who buy from firm \(B\) represent positive customer churn for firm \(A\). There are two reasons why a firm might lose customers to its rival even though the consumer prefers the firm’s product given each firm’s regular price. One reason is that the consumer does not receive any promotional incentives from its more preferred firm but does receive promotional incentives from the rival firm, where the incentives are such that \(P_i - P_{-i} -

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Note. Dotted lines illustrate the equilibrium in Part (a) and solid lines illustrate the equilibrium in Part (c).
d_−i(l) > l, with firm i being the consumer’s more preferred firm. Another reason is that the consumer receives promotional incentives from both firms, but the rival firm’s inducement is such that \( P_i - d_i(l) - (P_{−i} - d_{−i}(l)) > l \).

Customer churn has become a source of concern in many industries. For example, it is a concern in long-distance telephone service, wireless communications, and banking, where targeted promotions have induced a large number of consumers to switch. It is thus important for any manager to address the following two questions in an information-intensive marketing environment. First, does customer churn become worse as targeted promotions become increasingly feasible and accurate due to lower costs of targeting? If so, how pervasive will it become? Second, what should a firm do to manage customer churn in an information-intensive marketing environment?

Our analysis can shed light on both questions. Figure 8 illustrates the targeting zone for the case of symmetry when both firms promote and loyalties are \( l_A = l_B = l_S \). In this case, we see that the targeting zone expands when \( z \) decreases until at \( z = 0 \), all consumers in the market receive promotions. One might think, therefore, that as the marginal cost of targeting decreases, the number of consumers that churn will inevitably rise. Surprisingly, this turns out to be incorrect.

**Proposition 5.** When both firms offer one-to-one promotions, the number of consumers that churn first increases and then decreases as the marginal cost of targeting decreases. In the limit, when the marginal cost of targeting is zero, the phenomenon of customer churn disappears.

Intuitively, there are two opposing effects at work that give rise to the inverted-U relationship between customer churn and the cost of targeting. On the one hand, each firm has a greater incentive to offer targeted promotions as the cost of targeting decreases. All else equal, this tends to increase customer churn. On the other hand, each firm also has a greater incentive to retain consumers who are more loyal to its own product as the cost of targeting decreases. All else equal, this tends to reduce customer churn. In the special case where \( z = 0 \), all consumers receive a targeted promotion on the product they most prefer and customer churn is eliminated.\(^9\)

To gain a more complete sense of each firm’s overall strategic orientation, we define an offensive promotion as a promotion that is given to consumers who prefer the rival’s product, and a defensive promotion as a promotion that is given to consumers who prefer one’s own product. Then the number of offensive and defensive promotions that are distributed by each firm is given by:

\[
N_D = \int_{0}^{l_S - z} \tilde{q}_A(l) \frac{dl}{2l_S} = \frac{l_S - z + z \log \frac{z}{l_S}}{2l_S},
\]

\[
N_O = \int_{0}^{l_S - z} \tilde{q}_B(l) \frac{dl}{2l_S} = \frac{l_S^2 - z^2}{4l_S^2},
\]

where \( \tilde{q}_i(l) \) is the equilibrium probability that consumers with loyalty \( l \) receive a promotion from firm \( i \) (see Appendix C). By setting \( z = \beta l_S \) where \( 0 \leq \beta \leq 1 \), we can plot \( N_D \) and \( N_O \) on the vertical axis against \( \beta \) on the horizontal axis in Figure 9. This yields the following proposition.

\(^9\)In long-distance service, one in four residential subscribers switched long-distance carriers in 1997 (Lawyer 1998). Even worse, it has been alleged that some 30% of the 55 million wireless customers in the U.S. churn (Cane 1999).

\(^{10}\)When \( z = 0 \), there is no cost of targeting and so each firm’s equilibrium strategy calls for it to distribute defensive promotions (a promotion that is given to consumers who prefer one’s own product) with probability one. In other words, the equilibrium mixed strategies in the case where both firms promote become degenerate when \( z = 0 \).
Proposition 6. When both firms offer one-to-one promotions, the relative mix of offensive to defensive promotions increases as the cost of targeting increases. A firm’s targeting is predominantly offensive when the targeting cost is high and predominantly defensive when the targeting cost is low.

As the cost of targeting decreases, the targeting zone expands on both sides, encompassing consumers with increasingly stronger loyalties. This implies two things. First, it implies that the total number of offensive and defensive promotions is decreasing in $z$. This can be seen from the fact that both curves decrease from left to right in Figure 9. Second, it implies that offensive promotions must be given with increasingly larger discounts to be effective, while defensive promotions can be effective with increasingly smaller discounts. As a result, the attractiveness of an additional consumer captured with offensive promotions decreases with lower $z$, while the attractiveness of an additional consumer retained with defensive promotions increases with lower $z$. This can be seen from the concavity of the offensive promotion curve, which achieves a maximum of 0.25 at $z = 0$, and the convexity of the defensive promotion curve, which achieves a maximum of 0.5 at $z = 0.11$.

Propositions 5 and 6 imply that customer churn results from firms optimally taking chances with their loyal customers in order to capture as much consumer surplus from them as possible. Consequently, customer churn will occur in equilibrium as long as the marginal cost of targeting is positive. This suggests that the optimal way to manage customer churn is not to eliminate it, but to engage in more defensive promotions (e.g., loyalty programs) as the cost of targeting decreases.

6. Conclusion

The addressability of individual consumers, along with personalized promotional incentives, fundamentally changes a firm’s pricing decision. The parsimonious model we have developed here allows us to investigate this phenomenon. We show that one-to-one promotions allow a firm the flexibility to generate incremental sales without offering any unnecessary discount. But it also tends to intensify competition since all consumers, including loyal consumers, are potentially contestable.

In assuming symmetry, previous studies conclude that one-to-one promotions invariably lead to a prisoner’s dilemma in which all firms are worse off. However, when firms are asymmetric, there will also be a market-share effect. We show that the firms that are best positioned to gain from this additional effect are those that have a larger loyal following. One-to-one promotions gives these firms the flexibility they need to capitalize on their customer loyalty. Thus, one-to-one promotions may be a boon for large firms in today’s market environment, as it allows large companies to compete with small firms for niche markets in an ever more fragmented marketplace (Bessen 1993).

One striking message from our analysis is that building customer brand loyalty is of paramount importance. In an information-intensive marketing environment, all consumers, not just “marginal consumers,” are potentially exposed to competitive bidding, and customer loyalty is the only line of defense for a firm. This is reflected in the fact that a firm’s payoffs from contested consumers can only be as large as the premiums they are willing to pay for the firm’s brand. It is also reflected in the fact that a firm with a large loyal following can best take advantage of one-to-one promotions, while a small firm with a
Our analysis also sheds light on the subject of customer churn, which is increasingly occupying the attention of management in many industries. To our knowledge, it is the first rigorous analysis of this widely publicized phenomenon arising from targeted promotions. We show that customer churn can result from firms optimally taking chances with their loyal customers. Therefore, it is not advisable to eliminate or even to minimize customer churn. While increasing customer loyalty in a cost-effective manner may always be beneficial to a firm, it should not be the focus of any churn reduction program. Indeed, it follows from our results that the optimality of a firm’s pricing strategy may be suspect if customer churn reduces as customer loyalty increases, unless there is already a high level of customer loyalty. We show that the optimal way to manage customer churn is to engage in relatively more and more defensive promotions as the cost of targeting decreases.

Our conclusions are based on a two-stage game where each firm commits to a regular price. The reader may wonder whether our conclusions would hold if firms do not have regular prices. Could it be the case that the regular price serves no useful function in the age of one-to-one marketing? We can address this question by amending our game to allow each firm to choose whether or not to set a regular price prior to its targeting decisions. Our analysis shows that the regular price plays a strategic role in the game of one-to-one promotions. This strategic role arises from the fact that the regular price places an upper bound on the transaction price that any consumer would have to pay and hence discourages each firm from pursuing its rival’s most loyal customers. Without a regular price to shelter a firm’s loyal consumers from the rival’s targeting, the targeting zone in Figure 3 would expand to cover the entire line and each firm’s payoff would be reduced.

Because of its strategic role, we can show that competing firms all have incentives to choose a regular price in the amended game. This means that the regular price is not passé but instead performs an indispensable function of sheltering its loyal customers from competitive poaching. This also means that our two-stage setup is an innocuous simplification, as it prunes off-equilibrium subgames without affecting any substantive conclusions. In the unlikely case where regular prices cannot arise for institutional reasons (or if firms play a simultaneous game of one-to-one pricing), each firm’s payoff will be the same as its payoff in the two-stage game we consider here when \( z = 0 \), and our main conclusions will not be qualitatively altered.

One intriguing question that our analysis does not answer directly is what role would nontargeted promotions play in the context of targeted promotions? In our model, we implicitly assume that all consumers avail themselves of any promotions offered to them, and so nontargeted promotions if offered would serve the same purpose as the regular price. On the other hand, if not all consumers make use of promotions offered to them, and if these consumers have different aggregate elasticities of demand than consumers who do use promotions, then we conjecture that nontargeted promotions could serve a price-discrimination role and be used by firms even if targeted promotions are also used. In addition, we conjecture that nontargeted promotions could benefit a firm in an environment of one-to-one promotions by discouraging a competing firm from targeting its consumers aggressively. We leave it to future research to confirm these intuitions. We hope that our analysis sparks further interest from both academics and practitioners in this pricing tactic.

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Appendix A
In this appendix, we derive the mixed-strategy equilibrium in each of the regions of Subgame 4. To start, we partition consumers in \( \mathbb{T} \) into three regions, as illustrated in Figure 2. For each region, we

\[\text{[12] We thank an anonymous referee for raising these questions and for suggesting the analysis. The details of this analysis are available from the authors upon request.}\]

\[\text{[13] We thank an anonymous reviewer for raising this question.}\]
show that a mixed-strategy equilibrium exists and we derive for each firm a probability distribution defined over the set of feasible (undominated) pure strategies $\mathcal{A}_i$.

**Region I.** Since all consumers located in Region I will purchase from firm A if they do not receive any promotional incentives from firm B, and any sales that firm B generates through targeted promotions are incremental, the maximum discount that firm B will be willing to distribute to the consumers located at $l$ in Region I must satisfy $P_B - d_A(l) - c - z \geq 0$ (this is the maximum discount that firm B can offer and still earn nonnegative profit). This implies that $d_A(l) \leq P_B - c - z$. Furthermore, firm B's discount in this region must be sufficiently large to overcome consumers' loyalty to brand A. Otherwise, firm B would incur the targeting cost without affecting consumers' purchasing decisions. This means, for the consumers located at $l$ in Region I, $d_A(l)$ is bounded from below such that $P_B - P_A + d_A(l) \geq l$. This implies that $d_A(l) \geq P_B - P_A + l$. Thus, firm B's feasible strategy set is given by $\mathcal{A}_B(l) = \{d_A(l) | d_A(l) \in [P_B - P_A + l, (P_B - c - z)]\}$.

Firm A's maximum discount is determined by insuring that the targeted consumers will buy from firm A and derive minimum surplus even if they get the maximum discount from firm B. Since consumers who receive discounts from both firms will buy from firm A if and only if $P_A - d_A - (P_B - d_B) \leq l$, and since firm B's maximum discount is $d_B(l) = P_B - c - z$, it follows that firm A's maximum discount must satisfy $P_A - d_A - c - z = l$. Firm A's minimum discount is $d_A \geq 0$. Thus, firm A's feasible strategy set is given by $\mathcal{A}_A(l) = \{d_A(l) | d_A(l) \in [0, (P_A - c - z - l)]\}$.

Using the same arguments as in Varian (1985) and Narasimhan (1988), we can show that, in any mixed-strategy equilibrium, there can only be one mass point. This mass point occurs at $d_A(l) = P_B - P_A + l$, and we denote it by $m_B$. We can also show that, conditional on its targeting consumers located at $l \in \mathcal{A}_A(l)$, each firm's distribution function is continuous. We now proceed to show by construction that a unique mixed-strategy equilibrium exists for all $l$ in Region I.

Let $F_B(d; l)$ be the probability that firm A targets consumers located at $l$ with a discount depth less than $d$, conditional on targeting those consumers, and let $m_B + (1 - m_B)F_B(d; l)$ be the probability that firm B targets consumers located at $l$ with a discount depth less than $d$, conditional on targeting those consumers. Then, if $q_i(l) \in (0, 1]$ denotes the probability that firm $i$ will target a consumer located at $l$, the probability that firm A targets these consumers with a discount less than $d$ is given by $q_A(l)F_B(d; l)$, and the probability that firm B targets these consumers with a discount less than $d$ is given by $q_B(l)(m_B + (1 - m_B)F_B(d; l))$.

In any mixed-strategy equilibrium, a firm's strategy must make its rival indifferent among all of its feasible strategy choices. In particular, firm B's strategy must be such that

$$1 - q_B(l)(P_B - c) + q_B(l)0 = 1,$$

where the left side of (A1) is firm A's expected payoff from a consumer located at $l$ if it does not target, and the right side of (A1) is firm A's expected payoff from the consumer if it targets with its maximum discount depth of $(P_A - c - z - l)$, thereby always insuring itself a payoff of $l$.

Furthermore, firm B's strategy must also make firm A indifferent between targeting with any $d_A(l) \in \mathcal{A}_A(l)$, and targeting with its maximum discount depth of $P_A - c - z - l$. We thus have:

$$[1 - q_A(l)][P_A - d_A(l) - c - z]$$

$$+ q_B(l)[m_B + (1 - m_B)F_B(P_B - P_A + d_A(l) + l)](P_B - d_A(l) - c - z)$$

$$+ q_B(l)(1 - m_B)[1 - F_B(P_B - P_A + d_A(l) + l)](-z) = l. \tag{A2}$$

The first term in (A2) is firm A's expected gain from a consumer located at $l$ when firm B happens not to target the same consumer. The second term is firm A's expected gain when it wins the competitive bid for the consumer and the third term is its expected gain when it loses the bid. Using (A1), (A2), and the regularity conditions $0 \leq F_B(d) \leq 1$, we can solve for $m_B$, $q_B(l)$ and $F_B(d; l)$. By setting $P_B - P_A + d_A(l) + l = d$, we have:

$$\tilde{q}_B(l) = \frac{P_A - c - l}{P_A - c}, \quad \tilde{m}_B = \frac{z}{P_A - c - l}, \tag{A3}$$

$$\tilde{F}_B(d; l) = \frac{(l + z)(P_B - P_A + l + d)}{(P_B - l - c - z)(P_B + l - d - c)}, \tag{A4}$$

where tildes denote equilibrium values and $0 \leq \tilde{F}_B(d) \leq 1$ is right continuous and monotonically increasing as are required of a distribution function. Thus, the probability that a consumer located at $l$ will receive a promotional incentive from firm B with a discount value less than $d$ is given by

$$\mathcal{A}(d) = \begin{cases} 0 & \text{if } d < P_B - P_A + l \\ \tilde{q}_B(l)[\tilde{m}_B + (1 - \tilde{m}_B)\tilde{F}_B(d)] & \text{if } d \in \mathcal{A}_B(l) \\ \tilde{q}_A(l) & \text{if } d \geq P_B - c - z \end{cases}. \tag{A5}$$

Similarly, firm A's strategy must make firm B indifferent among all of its feasible strategy choices. In particular, firm A's strategy must be such that

$$[1 - q_A(l)][P_A - l - c - z] + q_A(l)(-z) = 0, \tag{A6}$$

where the left side of (A6) is firm B's expected payoff from a consumer located at $l$ if it targets with its minimum discount depth of $P_B - P_A + l$, and the right side of (A6) is firm B's expected payoff from the consumer if it does not target, thereby insuring itself a payoff of 0. In addition, firm B must also be indifferent between targeting with any $d_A(l) \in \mathcal{A}_A(l)$ and not targeting at all, i.e.,

$$[1 - q_A(l)][P_B - d_A(l) - c - z]$$

$$+ q_A(l)F_A(P_B - P_A + d_A(l) - l)(P_B - d_A(l) - c - z)$$

$$+ q_A(l)[1 - F_A(P_B - P_A + d_A(l) - l)](-z) = 0. \tag{A7}$$

Solving (A6) and (A7), and setting $P_B - P_A + d_A(l) - l = d$, we have:

$$\tilde{q}_A(l) = \frac{P_A - c - l - z}{P_A - c - l}, \tag{A8}$$


\[
F_A(d) = \frac{dz}{(P_A - c - d - l)(P_A - c - z - l)}.
\]  

(A9)

Thus, the probability that a consumer located at \(l\) will receive a promotional incentive from firm \(A\) with a discount value less than \(d\) is given by

\[
\mathbb{F}_a(l, d) = \begin{cases} 
0 & \text{if } d \leq 0 \\
\hat{q}_a(l)F_A(d) & \text{if } d \in \mathcal{A}(l) \\
\hat{q}_a(l) & \text{if } d \geq P_A - c - z - l 
\end{cases}.
\]  

(A10)

It follows from these derivations that the expected payoff from any consumer located at \(l\) in Region I is given by \(\pi_a(q_a, \hat{q}_a, \mathbb{F}_a) = l\) for firm \(A\) and \(\pi_\theta(q_\theta, \hat{q}_\theta, \mathbb{F}_\theta) = 0\) for firm \(B\). It also follows that neither firm can do better by deviating unilaterally to some other possible strategy.\(^{14}\)

**Region II.** The equilibrium in Region II is symmetric to that in Region I and can be derived by substituting \(A\) for \(B\), \(B\) for \(A\), and \(l\) for \(-l\) in Equations (A5) and (A10). Thus, in Region II, firm \(A\)'s expected payoff is zero and firm \(B\)'s expected payoff from a consumer located at \(l\) is \(l\).

**Region III.** In Region III, a consumer will purchase from firm \(B\) absent any promotion and the consumer is loyal to firm \(A\). We can show, following the same process as we did for Region I, that in equilibrium, we must have

\[d_A(l) \in \mathcal{A}(l) \in (0, P_A - c - z - l).\]

Furthermore, firm \(A\) always promotes with a mass point, \(m_A\), at the minimum value, and firm \(B\) promotes with probability \(q_B \leq 1\). In equilibrium, the following three conditions must be satisfied:

\[
\begin{align*}
(1-q_B)(P_B + l - c - z) + q_B(l-z) &= l, \quad (A11) \\
(1-q_B)(P_B - d_A - c - z) + q_B(d_A+l) &+ (P_A - d_A - c - z) \\
+ &q_B(1-F_A(P_B - d_A+d_A+l))(l-z) = l, \quad (A12) \\
m_A(P_B - d_A - c - z) + (1-m_A)F_A(P_B - d_A - d_A+l) &+ (P_B - d_A - c - z) \\
+ &+ (1-m_A)(1-F_A(P_B - d_A+d_A+l))(l-z) = 0. \quad (A13)
\end{align*}
\]

Equation (A11) ensures that firm \(A\)’s payoff is indifferent between \(d_A = P_A - P_B - l\) and \(d_A = P_A - c - z - l\) given firm \(B\)'s strategy. Equation (A12) assures that firm \(A\) is indifferent between any \(d_A \in (P_A - P_B - l, P_A - c - z - l)\) and \(d_A = P_A - c - z - l\) given firm \(B\)'s strategy. Equation (A13) assures that firm \(B\) is indifferent between any \(d_B \in (0, P_A - c - z)\) and the maximum value \(P_A - c - z\). Solving A1, A2, and A3, along with the regularity conditions, we have:

\[
\mathbb{T}_A(l, d) = \begin{cases} 
\tilde{m}_A + (1-\tilde{m}_A)F_A(d) & \text{if } d \in [P_A - P_B - l, P_A - c - z - l] \\
1 & \text{if } d \geq P_A - c - z - l 
\end{cases}
\]  

(A14)

\(^{14}\) If firm \(A\) unilaterally deviates to any other strategy \((q_A, F_A(d))\), firm \(A\) can only do worse if \(d \notin \mathcal{A}_A(l)\). If \(d \in \mathcal{A}_A(l)\), then firm \(A\)'s expected payoff is \(l\) given any strategy of firm \(B\). The same logic also applies to firm \(B\). Thus, the singular distributions in (A5) and (A10) are the Nash equilibrium strategies for both firms.

\[
\mathbb{T}_B(d) = \begin{cases} 
\hat{q}_B(l)F_B(d) & \text{if } d \leq 0 \\
\hat{q}_B(l) & \text{if } d \geq P_B - c - z,
\end{cases}
\]  

(A15)

where

\[
\tilde{m}_A = \frac{z}{P_B - c}, \quad \hat{m}_A = \frac{(d+l+P_B-P_A)z}{(P_B-c-z)(P_A-l-d-c)},
\]  

(A16)

\[
\tilde{q}_B(l) = \frac{P_A - c - z}{P_B - c + l}, \quad \hat{q}_B(l) = \frac{d(l+z)}{(P_B + l - c - d)(P_B - c - z)},
\]  

(A17)

In this region, only firm \(A\) has positive profit. Firm \(A\)'s payoff from a consumer is simply equal to the premium the consumer is willing to pay for firm \(A\)'s brand.

Thus, we can conclude that a firm’s payoff from a contested consumer is equal to the premium the consumer is willing to pay for the firm’s brand. Q.E.D.

**Appendix B**

**Proof of Proposition 2.** The steps we take to prove Parts (a), (b), and (c) are the same. In each case, we derive the necessary conditions for an equilibrium. We then show that in the proposed equilibrium, neither firm has an incentive to deviate unilaterally. We will first take up Part (a).

Neither Firm Offers One-to-One Promotions. Note that in any equilibrium where \(P_A \leq c + z\) and \(P_B \leq c + z\) hold, both firms must have positive market shares. Otherwise, a monopolist firm should set its price at \(V > c + z\). This means that in any such equilibrium we must have \(l_A - l > 0\) and \(l_B > 0\). The equilibrium regular prices \((\tilde{P}_A, \tilde{P}_B)\) then necessarily satisfy the following conditions for all \(P_A\) and \(P_B\):

\[
\tilde{P}_A = \arg \max_{r_A} \frac{1}{l_B + l_a} (P_A - c)(l_A - l) \\
s.t. \ P_A \leq c + z
\]

\[
\tilde{P}_B = \arg \max_{r_B} \frac{1}{l_A + l_b} (P_B - c)(l_B + l) \\
s.t. \ P_B \leq c + z
\]

In other words, in equilibrium, \((\tilde{P}_A, \tilde{P}_B)\) must solve the following system of first-order conditions

\[
\frac{1}{l_A + l_B} (l_A - l - P_A - c) - \mu_A = 0
\]

\[
\frac{1}{l_A + l_B} (l_B + l + P_B - c) - \mu_B = 0
\]

\[
\mu_A(P_A - c - z) = 0, \quad \mu_A \geq 0, \quad P_A \leq c + z
\]

\[
\mu_B(P_B - c - z) = 0, \quad \mu_B \geq 0, \quad P_B \leq c + z,
\]

where \(\mu_A\) and \(\mu_B\) are Lagrange multipliers.

If \(\mu_A = \mu_B = 0\), i.e., none of the constraints bind, a unique solution to the above first-order conditions exists if \(z \geq (2l_A + l_B)/3\), and we have the following candidate equilibrium

\[
\tilde{P} = \frac{2l_A + l_B}{3} + c, \quad \tilde{P} = \frac{l_A + 2l_B}{3} + c
\]

\[
\tilde{P}_A = \frac{(2l_A + l_B)^2}{9(l_A + l_B)}, \quad \tilde{P}_B = \frac{(l_A + 2l_B)^2}{9(l_A + l_B)}.
\]
We now show that the strategies in this candidate equilibrium indeed constitute a Nash equilibrium since none of the firms can do better by deviating unilaterally from its strategy above. Given \( \tilde{P}_B \), the first-order conditions already ensure that firm A never wants to deviate from \( \tilde{P}_B \) by choosing some \( P_A > c + z \). However, firm A may want to deviate by choosing some \( P_A > c + z \) so that targeted promotions then become profitable. This gives a deviation profit of

\[
\Pi'_A = \frac{1}{l_A + l_B} \left\{(P_A - c) \max\{l_A - \tilde{l}, 0\} + \int_{l_A/\theta}^{l_A} (\tilde{P}_B - c - z + l)dl\right\}.
\]

We now show that any deviation \( P_A > c + z \) by firm A can only make firm A worse off. First, firm A can deviate by choosing some \( P_A > l_A + \tilde{P}_B \). In that case, firm A’s profit is nonzero only if \( l_A - c - z + \tilde{P}_B > 0 \) holds, or equivalently the condition holds

\[
l_A + l_B > \frac{3}{2} z. \tag{B1}\]

Given Condition (B1), we have

\[
\Pi'_A = \frac{1}{l_A + l_B} \int_{l_A/\theta}^{l_A} (\tilde{P}_B - c - z + l)dl.
\]

However, in that case, we have

\[
\tilde{\Pi}_A - \Pi'_A = \frac{-2(l_A + l_B)^2 + 3(c(8l_A + 4l_B - 3z))}{18(l_A + l_B)}
\]

\[
\geq \frac{-2(l_A + l_B)^2 + (2l_A + l_B)(8l_A + 4l_B - 3z)}{18(l_A + l_B)}
\]

\[
= \frac{(2l_A + l_B)(2l_A + l_B) - 3z}{18(l_A + l_B)}
\]

\[> 0. \]

The first inequality follows from the fact that we have \( z \geq (2l_A + l_B)/3 \), and the second inequality follows from Condition (B1). Thus, firm A never deviates by setting some \( P_A > l_A + \tilde{P}_B \).

Firm A may, however, deviate by choosing some \( P_A \in (c + z, l_A + \tilde{P}_B] \). In that case, we have

\[
\Pi'_A = \frac{1}{l_A + l_B} \left\{(P_A - c)(l_A - \tilde{l}) + \int_{l_A/\theta}^{l_A} (\tilde{P}_B - c - z + l)dl\right\}. \tag{B2}
\]

By differentiating Equation (B2), we have

\[
\frac{\partial \Pi'_A}{\partial P_A} = \frac{1}{l_A + l_B} \left\{l_A - P_A + 3 + z\right\}
\]

\[
< \frac{2}{l_A + l_B} \left[\frac{2l_A + l_B}{3} > z\right]
\]

\[\leq 0. \]

The first inequality follows from \( P_A > c + z \), and the second follows from \( z \geq (2l_A + l_B)/3 \). The above derivation implies that firm A’s optimal deviation is to set \( P_A \) arbitrarily close to \( c + z \), and not to use targeted promotions. Thus, the optimal deviation will make the second part of Equation (B2) drop out. Obviously, \( \tilde{\Pi}_A > \Pi'_A \) in this case since \( \tilde{P}_B \) maximizes the first part of Equation (B2).

Thus, we have shown that firm A will not unilaterally deviate from \( \tilde{P}_B \). Using the same steps, we can also show that firm B will not unilaterally deviate from \( \tilde{P}_B \) so that \( (\tilde{P}_A, \tilde{P}_B) \) is an equilibrium given \( z \geq (2l_A + l_B)/3 \). Furthermore, we can show, by going through the same steps, that no other equilibrium exists where \( P_A \leq c + z \) and \( P_B \leq c + z \). Q.E.D.

**Only Firm A Offers One-to-One Promotions.** To prove Part (b) of Proposition 2, we first show that it is impossible for an equilibrium to exist where \( \tilde{P}_A \leq c + z \) and \( \tilde{P}_B > c + z \) such that firm B targets while firm A does not. In this were not true, we must have in this equilibrium either \( \tilde{P}_A = c + z \) or \( \tilde{P}_A < c + z \). In the former case, the payoffs for each firm are given by

\[
\tilde{\Pi}_A = \frac{2l_A}{l_A + l_B},
\]

\[
\tilde{\Pi}_B = \frac{1}{l_A + l_B} \left[(\tilde{P}_B - c)(c + z - \tilde{P}_B + l_B) + \int_{l_A/\theta}^{l_A - \tilde{P}_A - z} (\tilde{P}_A - c - l)dl\right].
\]

As \( \tilde{P}_B \) must maximize firm B’s payoff, we necessarily have \( \tilde{P}_B = l_B + c \) in this equilibrium.

We now consider firm A’s deviation by setting \( P_A = l_A + c > c + z \) so that we are in the subgame of competitive targeting. In that case, firm A’s payoff becomes \( \Pi'_A = (\tilde{P}_A + z^2)/(2l_A + l_B) > \tilde{\Pi}_A \). Therefore, if an equilibrium exists where \( \tilde{P}_A \leq c + z \) and \( \tilde{P}_B > c + z \), we must have \( \tilde{P}_A < c + z \). In this case, both firms’ payoffs are given by

\[
\tilde{\Pi}_A = \frac{2l_A}{l_A + l_B},
\]

\[
\tilde{\Pi}_B = \frac{1}{l_A + l_B} \left[(\tilde{P}_B - c)(\tilde{P}_A - \tilde{P}_B + l_B) + \int_{l_A/\theta}^{l_A - \tilde{P}_A - z} (\tilde{P}_A - c - l)dl\right].
\]

By examining the first-order necessary conditions for an equilibrium with these payoff functions, we must have \( \tilde{P}_A = (l_A + z)/2 + c \) and \( \tilde{P}_B = (l_A + 2l_B - z)/2 + c \). However, \( \tilde{P}_A < c + z \) and \( \tilde{P}_B > c + z \) must imply \( l_A < z < (l_A + 2l_B)/3 \), which is not possible given \( l_A \geq l_B \).

We can then go through the same steps to show that the equilibrium exists where firm A targets unilaterally under the conditions specified in Proposition 2. Q.E.D.

**Both Firms Offer One-to-One Promotions.** We now prove Part (c) of Proposition 2. In any equilibrium where \( \tilde{P}_A > c + z \) and \( \tilde{P}_B > c + z \) such that competitive targeting ensues, the firms’ payoff functions are given by

\[
\tilde{\Pi}_A = \frac{1}{2l_A + l_B} \left[\tilde{P}_A - c\right] + \frac{\tilde{P}_A - c}{l_A + l_B},
\]

\[
\tilde{\Pi}_B = \frac{1}{2l_A + l_B} \left[\tilde{P}_B - c\right] + \frac{\tilde{P}_B - c}{l_A + l_B},
\]

where, as we recall, \( l_A = \tilde{P}_A - c - z \) and \( l_B = c + z - \tilde{P}_B \). Through first-order conditions, we necessarily have \( \tilde{P}_A = l_A + c \) and \( \tilde{P}_B = l_B + c \). Then, to insure \( \tilde{P}_A > c + z \) and \( \tilde{P}_B > c + z \), we need to have \( z < l_B \). We now check if any firm has any incentive to deviate from this candidate equilibrium.

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Part (c) of Proposition 2. In any equilibrium where \( \tilde{P}_A > c + z \) and \( \tilde{P}_B > c + z \) such that competitive targeting ensues, the firms’ payoff functions are given by

\[
\tilde{\Pi}_A = \frac{1}{2l_A + l_B} \left[\tilde{P}_A - c\right] + \frac{\tilde{P}_A - c}{l_A + l_B},
\]

\[
\tilde{\Pi}_B = \frac{1}{2l_A + l_B} \left[\tilde{P}_B - c\right] + \frac{\tilde{P}_B - c}{l_A + l_B},
\]

where, as we recall, \( l_A = \tilde{P}_A - c - z \) and \( l_B = c + z - \tilde{P}_B \). Through first-order conditions, we necessarily have \( \tilde{P}_A = l_A + c \) and \( \tilde{P}_B = l_B + c \). Then, to insure \( \tilde{P}_A > c + z \) and \( \tilde{P}_B > c + z \), we need to have \( z < l_B \). We now check if any firm has any incentive to deviate from this candidate equilibrium.
Given \( \tilde{P}_A = l_A + c > c + z \), firm A may deviate by setting \( P'_A < c + z \) and thus giving up on targeting. In that case, firm A’s payoff is given by

\[
\Pi'_A = \frac{1}{l_A + l_B} (P'_A - c)(l_A - P'_A + c + z).
\]

The best firm A can do through this deviation is to set \( P'_A = c + z \) and obtain the payoff \( \Pi'_A = zl_A/(l_A + l_B) \). However, we have

\[
\tilde{\Pi}_A - \Pi'_A = \frac{(z-l_A)^2}{2(l_A + l_B)} > 0.
\]

Thus, firm A has no incentive to deviate.

Similarly, the optimal deviation for firm B is to set \( P'_B = c + z \), which yields \( \Pi'_B = zl_B/(l_A + l_B) \). However, we have

\[
\tilde{\Pi}_B - \Pi'_B = \frac{(z-l_B)^2}{2(l_A + l_B)} > 0.
\]

Therefore, firm B has no incentive to deviate. Q.E.D.

**Appendix C**

**Proof of Proposition 5.** Note that from Proposition 2 and Equations (A5) through (A10), the expected number of consumers who, all else being equal, prefer firm A’s product but purchase from firm B due to targeted promotions is

\[
\mathcal{E} = \frac{1}{2l_A} \int_{l_B}^{l_A - z} \{\tilde{\phi}(l)\tilde{\phi}_l(l)\delta(l) + \int_{d_B < d_A(l)} \tilde{\phi}_l(d_A(l)) - l d\tilde{\phi}_l(d_A(l))\} dl,
\]

where \( \mathcal{E} \) is concave in \( z \) and approaches 0 as \( z \) approaches \( l_A \) or \( 0 \). A similar expression holds for the expected number of consumers who are, all else being equal, loyal to firm B but purchase from firm A due to targeted promotions. Proposition 5 then follows after substituting the equilibrium values into (C1) and conducting comparative statics with respect to \( z \). Q.E.D.

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