COMPETITIVE COUPON TARGETING

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With the advent of panel data on household purchase behavior, and the development of statistical procedures to utilize this data, firms can now target coupons to selected households with considerable accuracy and cost effectiveness. In this article, we develop an analytical framework to examine the effect of such targeting on firm profits, prices, and coupon face values. We also derive comparative statics on firms’ optimal mix of offensive and defensive couponing, the number of coupons distributed, redemption rates, face values, and incremental sales per redemption. Among our findings: when rival firms can target their coupon promotions at brand switchers, the outcome will be a prisoner’s dilemma in which the net effect of targeting is simply the cost of distribution plus the discount given to redeemers.

(Competitive Strategy; Promotion)

1. Introduction

Coupons are a widely used promotional tool in firms’ competition for price-sensitive consumers. Having grown at an average rate in excess of 11% during the 1980s, the number of coupons distributed annually by consumer goods manufacturers is now at a near-record level. According to the most recent NCH (1994) report, 298.5 billion coupons, more than 3,000 coupons per household, were distributed in 1993.

One reason why firms compete for price-sensitive consumers by offering coupons instead of simply lowering the price of their product is that coupons can engender market segmentation whereas lower regular prices cannot. Coupons facilitate price discrimination because only those consumers that present a coupon at the point of sale receive a discount; all other consumers pay the full price.\(^1\) As noted by Narasimhan (1984), Levedahl (1984), Sweeney (1984), Varian (1989), and others, such discrimination can be profitable as long as coupon users as a group are more price-sensitive than non-coupon users,\(^2\) an implicit assumption being that firms distribute coupons randomly via the mass-media and rely on consumer self-selection to achieve market segmentation.\(^3\)

\(^1\) Coupons may also motivate retail participation in price promotions (Gerstner and Hess 1991a, b.), create switching costs (Caminal and Matutes 1990), stimulate short term introductory sales, etc. For a complete list of managerial objectives served by coupons, see the excellent survey by Blattberg and Neslin (1990).

\(^2\) Narasimhan (1984) and Babakus et al. (1988) provide empirical evidence to support the claim that coupon users are more price sensitive on average, than noncoupon users.

\(^3\) There are several other instances in the price discrimination literature in which firms structure their pricing to induce consumer self-selection. For example, in Salop (1977), a multistore monopolist charges a distribution of prices to exploit differences in buyer search costs, and, in Jeuland and Narasimhan (1985), differences in buyers’ inventory holding costs motivate firms’ temporary price cuts, etc.
However, firms need no longer rely exclusively on consumer self-selection to discriminate in price. With the advent of panel data on household purchase behavior, and the development of statistical procedures to utilize this data, firms can now target coupons to selected households with considerable accuracy and cost effectiveness.\(^4\) As a consequence, new avenues of competition are beginning to open up in which firms play a much more active role in market segmentation. Some marketing firms and retail chain stores have already coinitiated programs in which plastic identification cards, such as Catalina Marketing’s *Checkout Direct*, are distributed to individuals for use when buying goods. Transactions are then entered into a database each time a customer uses her card,\(^5\) with the intent being to provide targeted coupons based on the customer’s purchasing history.\(^6\) In addition to these nascent point-of-purchase programs, coupons are also being targeted to selected households via direct mail, with firms utilizing self-reported survey data on product preferences, demographics, and lifestyle characteristics.\(^7\) Many analysts predict that these targeted promotions are the wave of the future and will gradually replace mass-media distribution.

The ongoing revolution in coupon targeting capabilities obviously has important implications for firm rivalry and competition. Our purpose in this article is to develop an analytical framework to address several issues. First, what is the relationship between coupon targeting and random mass-media distribution? Will the former replace the latter over time, as some believe, or are the two complementary? Second, how will the ability to target coupons affect regular prices and coupon face values? Does the answer depend on whether firms also distribute coupons via the mass media? Third, do rivalrous firms stand to gain or lose from the increasing cost effectiveness of coupon targeting? Fourth, what types of coupon targeting strategies can be expected to emerge in a competitive environment? What fraction of coupons should be sent to the rival’s customers (offensive targeting) in an effort to increase sales, and what fraction of coupons should be sent to one’s own customers (defensive targeting) in an effort to preempt rivals’ coupon promotions?

Our framework posits a spatial model of product differentiation and assumes that data on past purchasing behavior has given firms information that allows them to discriminate in price according to consumer heterogeneity in brand loyalty.\(^8\) Surprisingly, we find that coupon targeting does not preclude the traditional kinds of price discrimination that arise from consumer self-selection. Whereas targeting coupons to specific individuals exploits differences in brand loyalty, random coupon distribution exploits differences in coupon user/non-user price sensitivity. In many instances, firms will choose to discriminate along both dimensions.

Unlike the traditional kinds of price discrimination, however, coupon targeting intensifies competition without allowing firms profitably to raise their regular prices. This supports the contention of some that the net effect of couponing in a competitive environment is simply the cost of distribution plus the discount given to redeemers (Raphel 1988b, Chiang 1992), and therefore that the outcome of rivalrous coupon targeting is a

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\(^4\) The challenge in using panel data on household purchase behavior is to develop statistical procedures capable of generating household-level estimates of parameters given the relatively small amount of data per household. See the recent approach taken by Rossi and Allenby (1993).

\(^5\) The data may even be stored and updated right on the “smart” card. That is the case, for instance, with Advanced Promotion Technologies’ *Vision Value Club,* as reported by Litwak (1991).

\(^6\) In addition to Catalina Marketing Co., Citicorp P.O.S. Information Services is also in the process of creating a customer database. According to Mayer (1990), the company plans eventually to sell time series data on purchases of approximately 40 million American households.

\(^7\) One of these firms is Computerized Marketing Technologies, Inc., which mails individualized UPC coded coupons to 15 million households three times a year (Business Week 1989). Another firm is Donnelly Marketing, which targets 30 million households through its Carol Wright program (Raphel 1988a).

\(^8\) Heterogeneity in brand loyalty is the sine qua non of sales promotions in Narasimhan (1988).
prisoner’s dilemma in which profits are lower for all firms. Our results also provide some support for the view that coupons should be directed at a rival’s customers for the purpose of increasing brand sales (Neslin and Clarke 1987, Neslin 1990).

This offensive strategy does indeed predominate in equilibrium, if the cost of targeting is sufficiently high. Nevertheless, our analysis suggests that as the marginal cost of targeting declines over time, each firm should adjust its strategy by becoming relatively more defensive.

The rest of the paper is organized as follows. Section 2 specifies the model and notation. Section 3 derives equilibrium coupon targeting strategies. Section 4 considers the impact of competitive coupon targeting on firm profits, prices and coupon face values. Section 5 examines the incidence of offensive and defensive targeting. Section 6 derives comparative statics on the number of coupons distributed, redemption rates, face values, and incremental sales per redemption. Section 7 concludes.

2. The Model and Notation

Consider a market in which two firms sell competing brands of a consumer good that is produced at constant marginal cost c. Since heterogeneity in consumer tastes is essential to study coupon targeting, we adopt a spatial model of product differentiation and assume, à la Hotelling (1929), that consumer tastes differ along a single dimension in product space. For simplicity, we abstract from product design choices by locating firms at opposite ends of the line segment [0, 1].

We consider a two-stage game-theoretic model of pricing and coupon distribution. In the initial stage, firms compete for customers by simultaneously and noncooperatively choosing their regular prices ($R_1, R_2$) and coupon face values. Once pricing and promotion depth decisions have been made, firms proceed in stage two by distributing coupons according to their targeting strategies ($\Omega_1, \Omega_2$), which specify the probability that consumers on any given interval of the line segment [0, 1] will receive a firm’s targeted coupon. Firms may also randomly distribute coupons via the mass media in stage two. If so, these coupons are assumed to reach all consumers with probability one. We use subgame perfection as our solution concept which means that the actions chosen in each stage are required to be Nash given the choices in the preceding stages, and the choices in the early stages are chosen knowing the effects of such actions in the stages to follow.

This two-stage game accentuates the strategic role of firms’ coupon targeting decisions by assuming these decisions are made subsequent to decisions on regular prices and coupon face values. From a game-theoretic point of view, an implicit assumption is that this two-step decision making sequence corresponds to the relative speed with which these choices are typically altered in practice. Hence, firm pricing and promotion depth decisions are thought of as higher level managerial decisions that are relatively less responsive than perturbations in a firm’s tactical choice of coupon targeting strategies. Our set-up is thus analogous to the multistage game employed by Rao (1991) in modeling firms’ price promotion decisions in a competitive environment.

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9 By ascribing a central role to a firm’s incremental sales per redemption, these authors implicitly assume that targeting a rival’s customers is optimal provided the cost of such targeting is not too steep.

10 In our working paper version, Shafer and Zhang (1994a), we show that our qualitative conclusions are robust to any symmetric pair of firm locations on [0, 1] for which a pure strategy Nash pricing equilibrium exists.

11 We agree with Rao (1991, p. 133) that stylized models such as ours are best judged on the usefulness of the insights and the validity of the testable implications they generate. Nevertheless, it is encouraging to note that our analysis is robust to all permutations of play in which regular prices and coupon face values are chosen prior to distribution. Unfortunately, while we don’t believe our results would be sensitive to allowing targeting strategies and coupon face values to be chosen simultaneously, no pure strategy Nash equilibrium exists for such a game (proof available on request), and solving for mixed strategy equilibria in that case is beyond current game-theoretic techniques.
Consumers differ in their willingness to pay for the two brands. The farther away a consumer's tastes are from the product characteristics of a given brand, the less the consumer is willing to pay. Let \( V \) be a common reservation price for each consumer's ideal brand and let \( t_j \) be the transportation cost per distance squared for a consumer of type \( j \). Then a type \( j \) consumer located at \( X \) is willing to pay \( V - t_j X^2 \) for brand \( A \) located at zero, and \( V - t_j (1 - X)^2 \) for brand \( B \) located at one. We assume \( V \) is sufficiently large that all consumers will make a purchase.

Consumers also differ in their willingness to redeem coupons. A fraction \( \alpha_c \) of consumers incur no costs of coupon usage. Anyone in this group who receives a firm's coupon will redeem it if she purchases from the firm. To simplify the exposition, these consumers will henceforth be known as C-Users. Coupon usage for everyone else is prohibitively costly. These consumers will henceforth be known as Non-Users. Following convention, we assume that C-Users as a group are weakly more price-sensitive than Non-Users. In our spatial framework, this means that \( t_c \), the transportation cost for C-Users, is less than or equal to \( t_n \), the transportation cost for Non-Users.

The marginal consumer among Non-Users is defined as the consumer who is just indifferent between buying from either one of the two firms given \((R_A, R_B)\). Algebraically, the location of such a consumer must satisfy \( R_A + t_n X^2 = R_B + t_n (1 - X)^2 \). Solving yields

\[
\bar{X} = \frac{R_B - R_A}{2t_n} + t_n.
\]

All Non-Users who are located to the left of \( \bar{X} \) will buy from firm \( A \), while all Non-Users located to the right of \( \bar{X} \) will buy from firm \( B \). Note that in the event both firms have equal regular prices, \( \bar{X} = \frac{1}{2} \), and Non-Users simply buy whichever brand is closer to their individual tastes.

Turning to the purchasing behavior of the C-Users, define \( P_i \) as the price C-Users must pay to purchase firm \( i \)'s product if they do not have its targeted coupon. If firm \( i \) does not randomly distribute coupons via the mass media, this price is the same as firm \( i \)'s regular price. Otherwise, \( P_i \) is interpreted as firm \( i \)'s regular price minus the face value of its mass media coupons, which all C-Users receive. Defining \( P_i \) in this way economizes on notation, for under either interpretation, the marginal consumer in the set of C-Users who do not receive a targeted coupon is located at

\[
\bar{X} = \frac{P_B - P_A + t_c}{2t_c}.
\]

Those in the set who are located to the left of \( \bar{X} \) will buy from firm \( A \), while those in the set who are located to the right of \( \bar{X} \) will buy from firm \( B \). At equal prices, consumers in this group buy whichever brand is closer to them in product space.

Now consider the set of C-Users who receive one or both firms' targeted coupons, and define \( d_i \) as the net value of firm \( i \)'s targeted coupon. In the event firm \( i \) does not also randomly distribute coupons, \( d_i \) is interpreted as the actual face value of firm \( i \)'s targeted coupon. Otherwise, \( d_i \) is interpreted as the amount by which firm \( i \)'s targeted coupon face value exceeds firm \( i \)'s mass-media coupon face value. It is now possible to distinguish among an additional four types of C-Users based upon their expected purchasing behavior given \((P_A, P_B, d_A, d_B)\).

Consumers with strong preferences for brand \( A \) will prefer buying from firm \( A \) even if they have \( B \)'s targeted coupon and do not have \( A \)'s targeted coupon. Algebraically, the location of such a consumer satisfies

\[
P_A + t_c X^2 \leq P_B - d_B + t_c (1 - X)^2.
\]
This inequality implies that all C-Users located to the left of
\[
X_A = \frac{P_B - P_A - d_B + t_c}{2t_c}
\]
will buy brand A. Thus, there is no need for firm A ever to target these consumers. In the event \( P_B = P_A \), \( X_A \approx 0 \) requires \( d_B \leq t_c \), which means that the discount offered by firm B falls short of the disutility these consumers would incur if they were to purchase brand B.

Similarly, consumers with strong preferences for brand B will prefer buying from firm B even if they have A’s targeted coupon and do not have B’s targeted coupon. Algebraically, the location of such a consumer must satisfy \( P_A - d_A + t_cX^2 \geq P_B + t_c(1 - X)^2 \). This inequality implies that all C-Users located to the right of
\[
X_B = \frac{P_B - P_A + d_A + t_c}{2t_c}
\]
will buy brand B. Note that with equal prices, \( X_B \leq 1 \) requires \( d_A \leq t_c \).

The remaining C-Users might potentially be induced to switch brands as a consequence of coupon targeting. Define firm A’s potential brand switchers as those C-Users without strong preferences for brand A in the sense that they lie outside the interval \([0, X_A]\), but who nevertheless will buy from firm A conditional on having firm A’s targeted coupon, regardless of whether they have a targeted coupon from firm B. Algebraically, a consumer located at \( X \geq X_A \) is in the set of firm A’s potential brand switchers if and only if
\[ P_A - d_A + t_cX^2 \leq P_B - d_B + t_c(1 - X)^2. \]
Thus, firm A’s potential brand switchers are located at \( X_A \leq X \leq X_S \), where
\[
X_S = \frac{P_B - P_A + d_A - d_B + t_c}{2t_c}.
\]

C-Users located between \( X_A \) and \( X_S \) are consumers whose preferences for brand A are relatively weak, since in the absence of firm A’s targeted coupon, they can be induced to buy brand B if they have B’s targeted coupon. Finally, we define firm B’s potential brand switchers as those C-Users lying outside the interval \([X_B, 1]\), who will buy from firm B conditional on having firm B’s targeted coupon, regardless of whether they have a targeted coupon from firm A. Algebraically, firm B’s potential brand switchers are described by the set of locations \( X_S \leq X \leq X_B \). Note that with equal prices and coupon face values, \( X_S \) is located at \( \frac{1}{2} \).

The relative locations of these four C-User types are well-ordered, although the exact positions are contingent on the regular prices and coupon face values chosen by the firms. When coupled with the location of the marginal consumer among C-Users who do not receive a targeted coupon, there are at most five distinct regions where C-Users exhibit different purchasing behaviors. Figure 1, drawn assuming \( d_A > d_B > 0 \), is illustrative. An analogous figure can be drawn to correspond to \( d_B > d_A > 0 \). If \( d_B \) were to be equal zero, \( \bar{X} - X_A = X_B - X_S = 0 \), and hence regions II and IV would have zero width. If, in addition, \( d_A \) were to equal zero, \( X_S = \bar{X} = 0 \), and region III would also have zero width. If instead, \( d_A = d_B > 0 \), only region III would have zero width.

<table>
<thead>
<tr>
<th>loyal to A</th>
<th>brand switchers</th>
<th>loyal to B</th>
</tr>
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<tbody>
<tr>
<td>I</td>
<td>II</td>
<td>III</td>
</tr>
<tr>
<td>0</td>
<td>( X_A )</td>
<td>( \bar{X} )</td>
</tr>
</tbody>
</table>

**FIGURE 1.** C-User Locations in Product Space.
3. Competitive Coupon Targeting

The targeting information available to firms in practice comes from historical data on household purchasing behavior as well as from information gleaned from market surveys. As an example, Rossi and Allenby (1993) report on a scanner panel dataset which consists of observations on individual household purchases of tuna dating back two and a half years. Using newly developed statistical procedures, they show how the data can be used to rank households according to brand preference and price sensitivity. In our model, we abstract from data estimation problems and simplify by assuming that firms can perfectly distinguish among C-Users with different purchasing behaviors. Thus, if $z > 0$ denotes the marginal cost of distributing targeted coupons, neither firm will ever deliver to the set of Non-Users, since these consumers do not redeem coupons, or to the set of C-Users in regions I and V, since these consumers cannot be induced to switch brands given each firm's discount. The rest of the C-Users, however, are potential brand switchers. These are the consumers over whom rivalry in targeted coupon promotion will occur.

Consider first a representative C-User in region II. This consumer prefers brand $A$, ceteris paribus, and will only buy from firm $B$ if she receives $B$'s targeted coupon and does not receive $A$'s targeted coupon. Whether or not firm $A$ wants to target a coupon to this consumer depends on firm $B$'s coupon targeting strategy and vice versa. For instance, firm $A$ will not want to target its coupon to this consumer if firm $B$ does not target its coupon to her, since all C-Users in region II who do not receive $B$'s coupon will buy from $A$ even without $A$'s coupon. But firm $A$ will want to target its coupon to her otherwise, so as to prevent her from switching brands. Thus, firm $A$ prefers to mimic firm $B$'s strategy. On the other hand, firm $B$ prefers to do the opposite of firm $A$. Given that each firm's targeting strategy is chosen simultaneously, and assuming that firms do not lose money on redemptions, it is clear from the above discussion that no stage-two pure strategy Nash equilibrium exists for C-Users in region II.

We now proceed to derive the unique mixed strategy Nash equilibrium targeting strategies in region II. Define $C^i$ as firm $i$'s pure strategy "target coupons to C-Users at $X \in \{X_A, \hat{X}\}" and let $C^i$ denote firm $i$'s pure strategy "do not target coupons to C-Users at $X \in \{X_A, \hat{X}\}". Then the normal form game between firms $A$ and $B$ for C-Users in region II is given in Figure 2.

The first item in each cell corresponds to firm $A$'s per-unit profit, and the second item corresponds to firm $B$'s per-unit profit. If both firms target their coupons in region II, as in the upper-left cell in Figure 2, all C-Users will buy brand $A$. Thus, firm $A$'s per-unit profit in this cell is equal to $(P_A - d_A - c - z)$, whereas firm $B$'s per-unit profit is $-z$. If neither firm targets its coupons in region II, as in the lower right cell in Figure 2, C-Users will buy brand $A$, yielding per-unit profit for firm $A$ of $(P_A - c)$, and per-unit profit for firm $B$ of zero. In the off-diagonal cells in Figure 2, only the targeting firm earns positive profit. Solving for the unique mixed strategy equilibrium profile (see Appendix A) yields $(\sigma_A^*, \sigma_B^*)$, where

$$\sigma_A^*(C^i_A) = \frac{P_B - d_B - c - z}{P_B - d_B - c}, \quad \sigma_B^*(C^i_B) = \frac{d_A + z}{P_A - c},$$

are the respective probabilities that firms $A$ and $B$ target coupons in region II.\(^{13}\)

\(^{12}\) Catalina Marketing, Citicorp P.O.S. Information Services, and Advanced Promotion Technologies have been working with retailers on developing electronic couponing, whereby manufacturers' coupons can be targeted to consumers at the point of sale based on their past purchasing behavior. The long range goal of these firms is to jump from the testing stage of gathering data to the implementation of wide-scale target couponing programs. With the advent of electronic couponing, increasingly complex targeting strategies will become feasible as the technology improves and information on household purchasing behavior accumulates.

\(^{13}\) There are two ways to interpret each firm's mixed strategy. One can think of firm $i$'s mixing in region II as an all or nothing coupon drop that occurs with probability $\sigma_i^*(C^i)$ or does not occur with probability $1 - \sigma_i^*(C^i)$.
<table>
<thead>
<tr>
<th>$C_B^I$ (target)</th>
<th>$C_B^I$ (not target)</th>
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<tbody>
<tr>
<td>$C_A^I$ (target)</td>
<td>$(P_A - d_A - c - z), \ -z$</td>
</tr>
<tr>
<td>$C_A^I$ (not target)</td>
<td>$0, \ (P_B - d_B - c - z)$</td>
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**Figure 2.** Coupon Targeting in Region II.

Firm $A$ prefers not to target coupons to C-Users in this region, since they are already predisposed to buy from $A$. Yet it practices some defensive couponing because otherwise firm $B$ would target coupons to them with probability one. Firm $B$ is aggressive in this region. At a marginal cost $z$ for every targeted coupon, it takes a chance on being able to attract new customers. On balance, however, firm $B$ succeeds in attracting brand switchers only with probability $\sigma_{B}^I(1 - \sigma_{A}^I)$, since its offensive couponing is tempered somewhat by firm $A$'s defensive couponing.

The probability that firm $A$ targets coupons to C-Users in region II is positively related to firm $B$'s net per-unit gain, since the higher is firm $B$'s gain, the more tempting it is for firm $B$ to target coupons, and hence the more defensive couponing firm $A$ must do to retain its customers. The probability that firm $B$ targets coupons in this region is positively related both to the marginal cost of targeting and to the net value of firm $A$'s targeted coupon. The more firm $A$'s cost of defending its customers increases, the more tempting it is for firm $A$ to forego targeting, and hence the more attractive is firm $B$'s offensive couponing. Notice that it is possible for C-Users in this region to have zero, one, or two targeted coupons. An immediate implication when C-Users have both is that the redemption rate for targeted coupons is necessarily less than one.\(^{14}\)

Region IV is symmetric to region II. C-Users in this region prefer brand $B$, ceteris paribus, and will only buy from firm $A$ if they receive $A$'s targeted coupon and do not receive $B$'s targeted coupon. Whether or not firm $B$ wants to target coupons in this region, however, depends on firm $A$'s targeting strategy and vice versa. Not surprisingly, there is no pure strategy Nash equilibrium in this region. Solving for the unique mixed strategy

\(^{(1 - \sigma_{i}^I(C^I))}\). Alternatively, one can think of firm $i$ as randomly selecting a fraction $\sigma_{i}^I(C^I)$ of C-Users in region II to target. Under the former interpretation, couponing emerges endogenously as an occasional price reduction phenomena. Under the latter interpretation, coupons are continuously available, although not always to the same consumers.

\(^{14}\) One might ask how the analysis would change in a dynamic model if, instead of throwing the unused coupon away, a consumer were to retain it until her next purchase occasion. Such a consumer would then prefer buying from the other firm, say firm $B$, in the next period, all else being equal. Assuming firms do not engage in tacit collusion, firm $A$ will no longer be indifferent to sending these consumers a coupon for its brand. The net result is that firm $A$ will target coupons to consumers in this region with probability one and they will once again buy brand $A$ on their next purchase occasion. The original targeted coupon for brand $B$ is saved and the cycle is repeated. Given that firms $A$ and $B$ in the static model are indifferent to sending coupons to C-Users in regions II and IV (property of the mixed strategy equilibrium), firm $A$'s ($B$'s) expected profit in equilibrium from each C-User in region II (IV) is the same as if it sent coupons to them with probability one. It is thus straightforward to show that unless the firms are able to tacitly collude, altering coupon targeting strategies in a dynamic model for period 2 onward, such that firm $A$ ($B$) targets with probability one (zero) in region II and firm $B$ ($A$) targets with probability one (zero) in region IV, does not affect firm profits, prices, or coupon face values as calculated in the next section.
equilibrium profile yields \((\tilde{\sigma}_A, \tilde{\sigma}_B)\), where
\[
\tilde{\sigma}_A(C_A^{IV}) = \frac{d_B + z}{P_B - c}, \quad \tilde{\sigma}_B(C_B^{IV}) = \frac{P_A - d_A - c - z}{P_A - d_A - c},
\]
are the respective probabilities that firms \(A\) and \(B\) target coupons in region \(IV\). Analogous to region \(II\), firm \(B\) would prefer not to target coupons to \(C\)-Users in this region. Yet it does so with positive probability to mitigate the effectiveness of firm \(A\)'s offensive couponing. On balance, firm \(A\) succeeds in attracting brand switchers only with probability \(\tilde{\sigma}_A(1 - \tilde{\sigma}_B)\).

Finally, firm \(A\) always targets (firm \(B\) never targets) coupons to \(C\)-Users in region \(III\), since these \(C\)-Users will buy from firm \(A\) if and only if they have \(A\)'s targeted coupon. To summarize, each firm’s equilibrium coupon targeting strategy for a given region \(r\) in brand space is as follows
\[
\Omega_A(r) = \begin{cases} 
0 & \text{if } r = I, V, \\
\sigma_A(C_A^{II}) & \text{if } r = II, \\
1 & \text{if } r = III, \\
\tilde{\sigma}_A(C_A^{IV}) & \text{if } r = IV,
\end{cases}
\]
\[
\Omega_B(r) = \begin{cases} 
0 & \text{if } r = I, V, \\
\sigma_B(C_B^{II}) & \text{if } r = II, \\
0 & \text{if } r = III, \\
\tilde{\sigma}_B(C_B^{IV}) & \text{if } r = IV.
\end{cases}
\]

We conclude this section by summing each firm's expected profit over all consumers. While this may seem an arduous task because of the induced brand switching among \(C\)-Users in regions \(II, III,\) and \(IV\), the summation is simplified by noting that in any mixed strategy equilibrium, each player is indifferent between mixing or playing one of its pure strategies. This means that firm \(A\)'s expected profit in equilibrium from each \(C\)-User in region \(II\) is equal to the per-unit profit it would receive from playing \(C_A^{II}\) with probability one, that is, \((P_A - d_A - c - z)\), and firm \(B\)'s expected profit in equilibrium from each \(C\)-User in region \(II\) is equal to the per-unit profit it would receive from playing \(C_B^{II}\) with probability one, that is, zero. Similarly, firm \(B\)'s expected profit from each \(C\)-User in region \(IV\) is \((P_B - d_B - c - z)\), while firm \(A\)'s expected profit from \(A\)-Users in region \(IV\) is zero. Firm \(B\)'s profit from \(C\)-Users in regions \(I\) and \(III\) is zero, while firm \(A\)'s profit from each \(C\)-User in regions \(I\) and \(III\) is \((P_A - c)\) and \((P_A - d_A - c - z)\) respectively. Finally, firm \(B\) earns \((P_B - c)\) from each \(C\)-User in region \(V\), while firm \(A\) earns zero in this region. Assuming a uniform distribution of consumers over \([0, 1]\), and summing expected profit over all consumers, yields
\[
\Pi_A^* = (1 - \alpha_c)((R_A - c)\bar{X} + \alpha_c((P_A - c)X_S - (d_A + z)(X_S - \max \{X_A, 0\})),
\]
\[
\Pi_B^* = (1 - \alpha_c)(R_B - c)(1 - \bar{X})
\]
\[+ \alpha_c((P_B - c)(1 - X_S) - (d_B + z)(\min \{X_B, 1\} - X_S)).
\]
Firm \(A\)'s overall profit is equal to its profit from \(Non\)-\(Users\) plus its expected profit from \(C\)-\(Users\) in regions \(I, II,\) and \(III\). Similarly, firm \(B\)'s overall profit is equal to its profit from \(Non\)-\(Users\) plus its expected profit from \(C\)-\(Users\) in regions \(IV\) and \(V\). Note that firm \(B\)'s offensive couponing in region \(II\) yields no expected gain, while firm \(A\)'s expected profit in this region is somewhat dissipated relative to what it would be in the absence of \(B\)'s targeting threat. Similarly, firm \(A\)'s offensive couponing in region \(IV\) yields no expected gain, while firm \(B\)'s expected profit in this region is somewhat dissipated. Thus, whether coupon targeting is profitable in equilibrium turns on whether firms can raise

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15 This assumption allows us to derive explicit solutions for subsequent comparative static analysis. We discuss in appendix D the sense in which our main propositions are robust to nonuniform customer distributions.
prices to the set of all Non-Users and to C-Users in regions \( I \) and \( V \) to offset the expected loss in profit from the discounts given to C-Users in regions \( II, III, \) and \( IV \).

4. Prices, Coupon Face Values, and Profit

We address several issues in this section. First, do rival firms stand to gain or lose from the increasing cost effectiveness of coupon targeting programs? Second, how does the ability to target coupons to individual households affect regular prices and coupon face values? Third, what is the relationship between coupon targeting and traditional mass-media distribution? In the process, we hope to shed light on two polar views regarding the effects of couponing in a competitive environment.

One view is that coupons effectively sort consumers into groups with differing elasticities of demand. Relative to a uniform price, firms raise price to the non-coupon users and, by way of the discount, lower price to the coupon users. Intuition from the literature on third-degree price discrimination in oligopoly suggests this type of market segmentation will be profitable even if market demand does not increase.\(^{16}\) An opposing view is that the outcome of couponing in a competitive environment is a prisoner’s dilemma in which all firms lose. According to this view, each firm’s couponing succeeds only in maintaining market share and, as a result, profits fall by an amount equal to the cost of distribution plus the discount given to redeemers. This view implicitly assumes that firms do not recover the cost of their couponing activities with higher regular prices.\(^{17}\)

Our analysis proceeds by examining these alternative views in the context of two scenarios. In the first scenario, we consider the competitive effects of coupon targeting in the absence of mass-media distribution. In the second scenario, we allow both types of coupons to be distributed. The two scenarios are then compared to isolate the effects of targeted couponing.

**Targeting in the Absence of Mass-media Distribution**

For many products, the intended audience is too small for mass-media coupon drops to be cost-effective. To capture this situation, and so to focus exclusively on market segmentation that is induced by targeted couponing, we assume initially that the cost of distributing coupons via the mass media is prohibitive. In this case, \( P_i = R_i \), and \( d_i \) is firm \( i \)'s targeted coupon face value.

In the initial stage, each firm chooses its regular price and targeted coupon face value to maximize its second stage equilibrium profit. Thus, firm \( i \)'s problem is to choose \((R_i, P_i, d_i)\) to maximize \( \Pi_i \) such that \( P_i = R_i \) and \( d_i \geq 0 \). Assuming \( t_n \geq t_c \geq t_n/2 \), we can simultaneously solve the Kuhn-Tucker conditions of both maximization problems and obtain the unique subgame perfect equilibrium regular prices and targeted coupon face values as functions of the exogenous parameters \( z, t_c, \) and \( t_n \).\(^{18}\) The presentation of the solution, given in Figure 3 and derived in Appendix B, is simplified by defining \( t_w = t_d t_c / \)}

\(^{16}\) Borenstein (1985) considers a spatial model in which consumers are located on a circle. Sorting consumers into binary groups by reservation prices, he finds that for any given number of firms, third-degree price discrimination always leads to higher profits. Holmes (1989) considers a symmetric duopoly model with general demand. Exogenously partitioning consumers into two groups, which he calls weak and strong markets, Holmes also finds that profits always rise with third-degree price discrimination when market demand is held constant.

\(^{17}\) Blattberg and Neslin (1990, p. 271, 272) summarize this view as follows: “The strategic problem faced by the manufacturer is that its share is vulnerable to the couponing activities of its competition. However, this view is shared by both manufacturers, so both end up using coupons and succeed in protecting their market share, but have eroded their profits by incurring the costs of couponing.”

\(^{18}\) The upper-bound restriction on \( t_c \), means that C-Users are more price sensitive on average than Non-Users. The lower-bound restriction ensures that both firms will have some loyal C-Users in equilibrium. Otherwise, for \( t_c \leq t_n/2 \), it can be shown that there exist equilibria in which all C-Users are potential brand switchers. Coupon targeting in that case mimics mass-media couponing with little additional insight.
((1 - \alpha_c) t_c + \alpha_d d_c) and interpreting it as a weighted average of the transportation costs of C-Users and Non-Users. It is easily verified that \( t_c \leq t_w \leq t_u \).

**Targeting in the Presence of Mass-media Distribution**

Now suppose that distributing coupons via the mass media is costless, so that market segmentation is jointly induced with targeted coupons. In this case, \( P_i \) is interpreted as firm \( i \)'s regular price minus the face value of its mass-media coupon, \( d_i \) is interpreted as the amount by which firm \( i \)'s targeted coupon face value exceeds firm \( i \)'s mass-media coupon face value, and \( R_i \geq P_i \). Thus, firm \( i \)'s problem is to choose \((R_i, P_i, d_i)\) to maximize \( \Pi^* \) such that \( R_i \geq P_i \) and \( d_i \geq 0 \). The unique subgame perfect equilibrium, given in Figure 4, is derived in Appendix C.

**Comparing across Scenarios**

The left-most column in Figures 3 and 4, entitled *Cost Ratio*, gives the conditions under which coupon targeting will \((d_i > 0)\) or will not occur \((d_i = 0)\). For example, coupon targeting arises in Figure 3 if and only if \( z < t_w \). The analogous condition in Figure 4 is \( z < t_c \). Notice that in both scenarios, firms will not target coupons if the marginal cost of targeting is sufficiently high. Since the transportation cost parameter, which can be thought of as a measure of average consumer brand loyalty,\(^{20}\) equals a firms' markup over production marginal cost in equilibrium, we have the following proposition, which is robust across scenarios.

**PROPOSITION 1.** Coupon targeting will occur in a given market if and only if equilibrium price-cost markups on individual sales to C-Users exceed the marginal cost of distributing targeted coupons.

Coupons will not be targeted in the absence of mass-media distribution if \( z \geq P_i - c = t_u \), since inducing brand switching (or defending market share) under such circumstances is never profitable. Similarly, targeting will not occur in the presence of mass-media distribution if \( z \geq P_i - c = t_c \), where \( P_i \) is now \( t_c + c \) as in Figure 4. Intuitively, profit margins get squeezed when consumer brand loyalty is weak, leaving little allowance for incurring the cost of distributing coupons. This intuition is most transparent when products are perfect substitutes. In that case, the lack of any brand loyalty implies \( t_w = t_c = 0 \), and so the no-targeting conditions are necessarily satisfied \( \forall z > 0 \). Because com-

\(^{19}\) The reader may wonder how it is that “no targeting” can emerge in equilibrium given that the targeting probabilities for both firms over regions \( II \) and \( IV \) were found in the previous section to be strictly positive. The paradox is resolved by recalling that regions \( II, III \), and \( IV \) have zero width when \( d_u = d_r = 0 \).

\(^{20}\) One can also think of the transportation cost parameter as a measure of product differentiation in the market since at \( t = 0 \), the products are perfect substitutes, and as \( t \) increases, the products become less substitutable.
<table>
<thead>
<tr>
<th>Cost Ratio</th>
<th>Regular Price</th>
<th>Promotion Decisions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z/t_c )</td>
<td>( R_A = R_B )</td>
<td>( P_A = P_B )</td>
</tr>
<tr>
<td>No Targeting</td>
<td>( z \geq t_c )</td>
<td>( t_n + c )</td>
</tr>
<tr>
<td>Targeting</td>
<td>( z &lt; t_c )</td>
<td>( t_n + c )</td>
</tr>
</tbody>
</table>


petition in perfect substitutes drives price-cost markups to zero, firms cannot gain by attracting brand switchers even though they could do so at little cost. It is ironic that firms in such markets, and in similar markets with weak loyalty, will compete on price alone, since consumers in these markets are very price-sensitive, and could easily be induced to switch brands.

Surprisingly, a firm’s decision to target does not depend on the size of the discount needed to attract brand switchers. One might think, for instance, that firms in markets where competing brands are strongly differentiated would also not target coupons because it would be too expensive to induce consumers to switch brands. This intuition fails, however, because although it is true that some consumers will be unassailable, there will always be consumers at the margin who are willing to switch brands at zero cost (otherwise, each brand would have a local monopoly, contradicting the assumption that brands are substitutes).21 As long as these marginal consumers can be identified, coupon targeting will be profitable whenever \( P_i - c > z \).22

Notice that the necessary and sufficient condition for targeting to be profitable is more stringent for markets in which mass-media coupons are distributed. When \( t_n \leq z < t_m \), for instance, targeting will not be forthcoming if coupons were being mass distributed, but will be forthcoming if otherwise. Since firms differ a priori in the extent to which their brands are marketed through traditional forms of couponing, our model provides a sharp prediction as to which firms are more likely to be in the vanguard of targeting.

PROPOSITION 2. Marketers of brands for whom mass-media coupon drops are not cost-effective, ceteris paribus, will be more likely to use the new targeting technologies.

The profitability of targeting depends on the price-cost markups to C-Users, which will be smaller if coupons are being mass distributed. Thus, over a range of possible \( z \), a firm which mass distributes coupons is less likely to initiate targeting than a firm that does not use the mass media.23 Proposition 2 should not be construed as implying,

21 The number of potential brand switchers will, of course, depend, inter alia, on the functional relationship between the size of each firm’s discount and average brand loyalty. This issue is discussed more fully in §6.

22 Our Proposition 1 contrasts with Raju et al. (1990), who find that firms do not price promote when consumer brand loyalty is sufficiently large. Their result depends crucially, however, on the assumption that price breaks are given to all consumers, loyalists and potential brand switchers. If firms could selectively target coupons, the opportunity cost of promoting in their model would not be increasing in brand loyalty.

23 This finding is not sensitive to our implicit assumption that the decision to mass-distribute coupons depends on factors external to the model, for while it may well be the case that some firms would cease mass-media couponing when targeting became feasible, it is unlikely that all firms would do so, given that mass-distribution can serve other motives in addition to market segmentation. As Jeff McElnea, president-CEO of Emison Freeman puts it, as quoted by Fitzgerald (1994), “FSI coupons are not the most efficient way of couponing, but they’re still a wonderful, inexpensive way to combine advertising with promotion while driving sales volume. FSI’s are not going away.”
however, that "big marketers" will forego targeting altogether.\footnote{According to Tommy Greer, chairman-CEO of Catalina Marketing, as quoted by Whalen (1994), "the marketers most likely to use targeted coupons would be those with relatively small audiences." But, as Greer (1994) cautions, this does not preclude "big marketers" from targeting coupons. "... the vast majority of our clients are big marketers, including Nestle, Coca-Cola, Procter & Gamble, Kraft General Foods and Campbell Soup, Co."} Indeed, there are several reasons why they might target coupons, at least on a subset of their products. First, large firms produce many diverse products, not all of which are promoted equally with mass-media couponing. Second, consumer brand loyalty may be stronger for these firms, allowing them to surpass the higher targeting threshold. Finally, it is evident from Figure 4 that as the marginal cost of targeting falls, all firms can eventually be expected to target coupons, even those that also rely on mass-media distribution.

**Proposition 3.** Targeted couponing need not replace mass-media couponing, as each allows a distinct way to segment the market.

Proposition 3 contrasts sharply with the views of many analysts who predict that coupon targeting spells the demise of mass-media couponing.\footnote{According to Frank Woodard, marketing director for Vons Cos., as quoted by Millstein (1989), "Programs where promotions are tailored to the household are the way to go. I see the mass-media disappearing and the individual marketing becoming almost one on one."} We find that both types of coupons can coexist even as the cost of coupon targeting falls over time. The reason for this is that the two types of coupons segment the market in different ways. Targeting coupons to specific individuals exploits differences in brand loyalty, whereas mass-media coupon distribution coupled with consumer self-selection exploits differences in coupon user/nonuser price sensitivity. These differences lead to some surprising implications for equilibrium prices and profits.

The second column in Figures 3 and 4, entitled Regular Price, gives equilibrium regular prices for the matrix of possible couponing strategies. For example, the regular price in Figure 3 in the absence of targeting is equal to production marginal cost plus a weighted average of the price-sensitivity of \textit{C-Users} and \textit{Non-Users}. Previous literature has noted that from this starting point, firms can increase their profit by mass distributing coupons and relying on consumer self-selection to segment the market. Moreover, as long as \textit{C-Users} are more price-sensitive on average, mass distribution of coupons allows firms profitably to raise regular prices to the \textit{Non-Users} and, by way of the discount, lower prices to the \textit{C-Users}. This intuition can be verified by comparing row 1 across scenarios. After the introduction of mass-media coupons, the equilibrium regular price rises to $t_n + c$ for \textit{Non-Users} and, as seen in the first column under the heading \textit{Promotion Decisions} in Figure 4, the net price to \textit{C-Users} after coupon redemption falls to $t_c + c$.

One might think that the effect on prices would be even more pronounced with targeting, given that firms can choose to direct their discounts only to the most price-sensitive of the \textit{C-Users}. In fact, this is not the case. A comparison of rows 1 and 2 within each scenario yields the following unexpected, and surprisingly strong, conclusion.

**Proposition 4.** Firms will not raise regular prices, or alter mass-media coupon face values, with the advent of coupon targeting. This is so even when coupons can be accurately targeted to potential brand switchers.

With coupon targeting, the division of the market becomes blurred. Each firm lures away a fraction of the rival's brand switchers with the net effect being to increase the area of competition from a single point in the middle of brand space to the entire interval of potential brand switchers. Since the number of these brand switchers is not exogenously determined, the enhanced competition prevents firms from profitably charging higher prices to their more brand-loyal customers. Put differently, if a firm were to raise its
regular price (or alter its mass-media face value) in stage 1, it would expose a fraction of its otherwise loyal consumers to its rival's targeted coupon in stage two. To do so would not be profitable, however, because the gain on inframarginal sales from this higher price turns out always to be more than offset by the loss of those exposed consumers.

Since coupon targeting can be very effective in stealing a rival's potential brand switchers and keeping one's own, each firm avails itself of the new targeting technology regardless of its rival's strategy. But firms are caught in a prisoner's dilemma. Although some consumers are induced to switch brands, expected market shares do not change in equilibrium. And since regular prices do not rise with the introduction of targeting, the net effect of the new targeted forms of couponing in a competitive environment is simply the cost of distribution plus the discount given to redeemers.

**PROPOSITION 5.** Coupon targeting in a competitive environment gives rise to a prisoner's dilemma in which profits are lower for both firms.

The introduction of coupon targeting allows firms to discriminate in price based on heterogeneity in consumer brand loyalty, which, given the targeting strategies, leads to an *endogenous market segmentation*. It is endogenous in the sense that the number of loyal *C-Users* is a function of both firms' regular prices and coupon face values which are chosen in stage one. By contrast, random coupon distribution coupled with consumer self-selection leads to an *exogenous market segmentation*, which is determined by the given number of *Non-Users*. Whether or not market segmentation is endogenously determined as part of the competition between firms can thus be viewed as a critical determinant of whether firms can profitably exploit differences between consumers (as with mass media distribution) or be caught in a prisoner's dilemma (as with targeting). ²⁶

### 5. Offensive and Defensive Targeting

The allure of the new targeted forms of couponing is obvious; they can be used to attract rival firms' potential brand switchers. The idea is that these consumers may be induced to purchase a brand they would otherwise not purchase if they had not received that brand's coupon. Since a firm gains by generating incremental sales in this manner, the advice routinely offered in the literature, not surprisingly, is that firms should target their coupons offensively. For instance, Alsop (1985) recommends that coupons be mailed directly to competitive brand users. Blattberg and Neslin (1990) note that manufacturers can place their coupons in magazines more likely to be read by a rival’s customers. Finally, Rossi and Allenby (1993) suggest that firms may want to target coupons to households "that show loyalty toward other brands and yet are price sensitive."

The advice that coupons should be targeted offensively need not be optimal in a competitive context, however, as coupon targeting can also serve to defend market share by preventing a rival firm's coupon promotion from luring away one's own potential brand switchers. Thus, it comes as a surprise that our analysis strongly concurs with the offensive minded intuition, provided the cost of coupon targeting is relatively high (as is presently the case). Nevertheless, our analysis suggests that as the marginal cost of targeting falls over time, firms should gradually shift emphasis away from attracting brand switchers and more towards defending against the loss of their existing customers.

These managerial prescriptions follow from a comparison of the incidence of offensive and defensive targeting in equilibrium. Given \((R_i, P_i, d_i)\) from Figures 3 and 4, we have

²⁶ This insight applies, for instance, to Narasimhan (1988), who considers a duopoly model in which consumers are either captive brand loyal and not price-sensitive at all, or brand switchers and willing to shop around. He finds that when firms distribute coupons (redeemed only by brand switchers), prices rise to the non-coupon users (captive loyal customers). Market segmentation is not endogenous in his setting, however, because the number of captive brand loyal customers in his model is fixed.
\[ N_O = \alpha c \sigma^* \left( X_S - X_A \right) = \alpha_a \sigma_a \left( X_B - X_S \right) = \frac{\alpha_c (t_k^2 - z^2)}{8 t_c t_k}, \]
\[ N_D = \alpha c \sigma^* \left( X_S - X_A \right) = \alpha_a \sigma_a \left( X_B - X_S \right) = \frac{\alpha_c (t_k - z)^2}{4 t_c (t_k + z)}, \]

where \( k = w \) in the absence of mass-media distribution, and \( k = c \) in the presence of mass-media distribution. To reduce the dimensionality of these expressions, we define \( f = z / t_k \), as the ratio of the marginal cost of targeting to the transportation cost of C-Users, \( 0 \leq f \leq 1 \). The incidence of offensive and defensive targeting in the presence of mass-media coupons can now be depicted in Figure 5 with \( f \) on the horizontal axis and the number of targeted coupons (divided by \( \alpha_c \)) on the vertical axis. An analogous figure corresponding to the number of targeted coupons in the absence of mass-media coupons can also be drawn.\(^{27}\)

**Proposition 6.** Firms should predominantly target offensively when the cost of coupon targeting is relatively high, and adjust their strategy mix by implementing relatively more defensive targeting as this cost falls. Moreover, for any given cost of targeting, firms should implement relatively more defensive targeting the higher is average consumer brand loyalty.

As the marginal cost of targeting decreases (\( f \) decreases), stealing a rival’s customers and defending one’s own customers becomes more attractive. In equilibrium, the number of offensive and defensive coupons both increase. Yet the number of defensive coupons

\(^{27}\) In the absence of mass-media coupons, the number of offensive and defensive coupons depends, among other things, on \( t_s \), which in turn depends on \( t_c \). If one assumes that \( t_s = \alpha s \), it becomes straightforward to show that over the feasible range of parameter space, the number of offensive (defensive) coupons is monotonically decreasing and strictly concave (convex) in \( f \). Moreover, the graph of the two curves always intercept at an interior point similar to that shown in Figure 5.
increases faster, as each firm increases the probability of its defensive targeting \( (d\sigma_d / dz > 0) \) and decreases the probability of its offensive targeting \( (d\sigma_o / dz < 0) \). Intuitively, if its rival were to continue to target defensively with the same probability, the increasing attractiveness of stealing a rival’s customers would induce a firm to target offensively with certainty. Firms preempt this anticipated strike by increasing their probability of defensive targeting so as to defend market share. The same relationships also hold when average consumer brand loyalty increases, since stealing a rival’s customers and defending one’s own becomes more attractive the higher are equilibrium regular prices.

6. Comparative Statics

Aside from coupon face values, many other variables are important for managers to assess the profitability of coupon promotions. In their survey chapter on coupons, Blattberg and Neslin (1990) provide a useful framework in this regard. Their method weighs the incremental gains engendered by a firm’s coupon promotion against its associated incremental costs. One key variable is the number of coupons distributed. In addition, two other variables of interest stand out. One is the coupon redemption rate. The other is incremental sales per redemption, defined as the fraction of redemptions that are from consumers who would not have bought the firm’s product had they not received its coupon. In this section, we derive comparative statics concerning how coupon face values and these other important variables change across markets with differing degrees of average consumer brand loyalty and over time as the marginal cost of coupon targeting falls. To facilitate the comparative statics calculations, we assume henceforth that changes in average consumer brand loyalty are well defined in the sense that changes in the transportation costs of C-Users and Non-Users are always proportional, that is, \( t_n = gt_c \), where \( g \) is a constant.

Targeted Coupon Face Values

In the absence of mass-media distribution, each firm’s targeted coupon face value, reported in the second column under the heading of Promotion Decisions in Figure 3, is \( (t_n - z)/2 \). When coupons are, in addition, distributed via the mass media, each firm’s targeted coupon face value is given by \( R_t - P_t + \alpha_c = t_n - (t_c + z)/2 \). In both cases, face values are increasing in average consumer brand loyalty, and decreasing in the marginal cost of distributing targeted coupons. Intuitively, firms adjust their face values to reflect their perceived gain from attracting a brand switcher. The higher this perceived gain \( (P_t - c - z) \), ceteris paribus, the larger the discount firms will offer as an inducement. Not surprisingly, larger discounts are needed as consumers become less price-sensitive.

Number of Targeted Coupons Distributed

The number of targeted coupons distributed by each firm depends on the number of potential brand switchers and on the probability of offensive and defensive targeting. It is given by

\[
N = N_o + N_d = \alpha_c[(X_S - X_A)\sigma_o^* + (X_B - X_S)\sigma_d] = \frac{\alpha_c(t_k - z)(3t_k^2 + z^2)}{8t_c t_k(t_k + z)}.
\]

Since both \( N_o \) and \( N_d \) decrease in \( z \) and increase in \( t_c \), regardless of whether mass-media coupons are distributed, as discussed previously, so does \( N \). Intuitively, the greater is the marginal cost of targeting relative to average consumer brand loyalty, the smaller is the perceived gain from attracting a brand switcher. Hence, firms simply have less incentive to target coupons when \( z \) is high and when \( t_c \) is low.
One might think that the incidence of targeting should be decreasing in average consumer brand loyalty, since inducing additional consumers to switch brands would become increasingly more expensive. Yet this factor is offset by the increasingly attractive per-unit price-cost markup; although the size of the discount needed to induce consumers to switch brands is increasing in average consumer brand loyalty, the regular prices that firms charge are increasing even more.

**Targeted Coupon Redemption Rate**

Since defensive coupons are always redeemed, and offensive coupons are redeemed only if a C-User does not also have a defensive coupon, each firm's targeted coupon redemption rate is given by

\[
R = \frac{\alpha \left[ (X_S - X_A)\sigma_A^* + (X_B - X_S)\tilde{\sigma}_A(1 - \tilde{\sigma}_B) \right]}{N} = \frac{2(t_k^2 + z^2)}{3t_k^2 + z^2}.
\]

Each firm's targeted coupon redemption rate is increasing in \(z\) and decreasing in \(t_k\). Since defensive coupons are uniformly redeemed by C-Users, whereas offensive coupons are not, the change in the redemption rate is largely determined by the change in the fraction of offensive coupons redeemed. Referring to Figure 5, as the marginal cost of distributing targeted coupons decreases (lower \(f\)), or as average brand loyalty increases (lower \(f\)), firms become relatively more defensive in their targeting, and so a relatively lower fraction of offensive coupons will be redeemed. This implies a lower redemption rate.

**Targeted Incremental Sales per Redemption**

Each firm's targeted incremental sales, defined as the decrease in sales that would occur if the firm defected from equilibrium by not targeting coupons, is equal to the number of consumers who have received both targeted coupons plus the number of offensively targeted coupons that are redeemed. Thus, each firm's targeted incremental sales per redemption is given by

\[
I = \frac{\alpha \left[ (X_S - X_A)\sigma_A^* + (X_B - X_S)\tilde{\sigma}_A(1 - \tilde{\sigma}_B) \right]}{N \cdot R} = \frac{(t_k^2 + z^2)^2}{2(t_k^2 + z^2)}.
\]

Each firm's targeted incremental sales per redemption is increasing in \(z\) and decreasing in \(t_k\). Intuitively, the only reason why sales from targeted coupon redeemers would not be incremental is if some of a firm's potential brand switchers received only its (defensively) targeted coupon. Since these consumers would have bought from it even in the absence of its coupon, they cannot be considered incremental. Since these nonincremental redemptions increase as firms become more defensive oriented, it is clear from Figure 6 that targeted incremental sales per redemption will increase with the marginal cost of distributing targeted coupons and decrease with average consumer brand loyalty. This comparative static result, and the preceding ones, are summarized in Figure 6. We use pluses (minuses) signify a positive (negative) relationship.

<table>
<thead>
<tr>
<th>Face Value</th>
<th>Number Distributed</th>
<th>Redemption Rate</th>
<th>Incremental Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brand Loyalty</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Cost of Targeting</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

**Figure 6. Summary of Targeted Coupon Comparative Statistics Results.**
PROPOSITION 7. Firms will distribute more targeted coupons, choose higher targeted coupon face values, experience lower targeted coupon redemption rates and achieve lower incremental sales per redemption the higher is average consumer brand loyalty. The same changes will take place as the cost of targeting coupons falls over time.

Proposition 7 predicts a negative association between targeted coupon face values and targeted coupon redemption rates, and between targeted coupon face values and targeted incremental sales per redemption, over time, and across industries with varying consumer brand loyalty. At first blush, these predictions appear to be at odds with some established empirical literature which suggests the opposite is true. Based on observations from actual coupon drops, Reibstein and Traver (1982) and Ward and Davis (1978) find that higher coupon face values are associated with higher redemption rates. Similarly, Klein (1985) and Shoemaker and Tibrewala (1985) find a positive relationship between coupon face values and incremental sales. There need not be a contradiction between our predictions and this literature, however, since these studies demonstrate a functional relationship between two variables while holding other factors constant, whereas our predictions turn on equilibrium comparisons in which all factors vary simultaneously. Thus, for example, our predictions are fully consistent with the trend during the 1980s in which coupon face values increased in excess of inflation while average coupon redemption rates uniformly declined.  

8. Conclusion

Our primary objective in this article has been to provide an analytical framework to investigate the competitive implications of the new forms of coupon targeting in which discounts can be directed at individual consumers. In the process, we have compared rivalry in price discrimination through random coupon distribution with rivalry through the new forms of coupon targeting. We found that mass media and targeted coupons were complementary in the sense that they exploited different consumer characteristics to achieve market segmentation. However, while the former was associated with higher regular prices and was profitable for the firms, the latter was found to increase competition for the potential brand switchers and was deleterious to firm profits. Thus, our main conclusion is that the outcome of coupon targeting is a prisoner's dilemma in which the net effect of targeting is simply the cost of distribution plus the discount given to redeemers.

We have also derived managerial implications concerning the optimal mix of offensive and defensive targeting, and testable implications concerning how face values, redemption rates, incremental sales per redemption, and the overall number of coupons distributed will change over time and across industries with differing consumer brand loyalties. Although presently available data mixes targeted coupons with mass-media coupons, and so cannot be reliably used to test our implications, it is of some assurance to note that the number of coupons distributed during the 1980s increased dramatically, and that casual observation suggests current targeting strategies are primarily designed to induce brand switching. Both of these observations are consistent with the model's predictions. The decision by NCH to track in-store coupons as a separate category for the first time in 1994 holds promise for empirical testing in the near future.

Our framework consists of a relatively simple, stylized two-parameter model. Yet our main insights are robust to several modeling extensions as has been discussed previously.

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28 A similar apparent paradox and resolution apply to the relationship between redemption rates and incremental sales. Intuition suggests a negative functional relationship between R and I, since pure defensive targeting would achieve a 100% redemption rate but garner relatively few incremental sales, and a pure offensive targeting strategy would achieve a 100% incremental sales per redemption but have a low redemption rate. However, this intuition holds only if all other factors (coupon face values, number of coupons distributed, etc.) are held constant. Comparing across equilibria, as we do in Proposition 7, yields different insights.
They include allowing the firms to locate symmetrically at any pair of locations on [0, 1], adding dynamics to enable consumers to use unredeemed coupons on future purchase occasions, and modifying the linear demand specification by allowing for symmetric nonuniform distributions.

There are several extensions that we do not consider in the paper, but which are nonetheless important. First, it would be useful to allow for asymmetric customer distributions, in order to investigate the relationship between targeted coupon promotions and firm size, particularly as it relates to market share. For instance, is there a market share effect that counteracts what would otherwise be a prisoner's dilemma? A second interesting extension would be to allow for multiple coupon face values so as to explore the possibility that firms would choose a different face value for offensive coupons and for defensive coupons. For example, firms might want to use smaller face values to defend and larger face values to attack.39 Finally, a third useful extension would be to examine the effects of weakening firms' household-specific targeting information. Such an extension would help to clarify the relationship between the two polar types of market segmentation considered in this article, and would, in addition, increase the scope of the analysis to include less accurate forms of targeting such as placing coupons in magazines more likely to be read by rivals' customers.30,31

Acknowledgements. We thank John Hauser, two anonymous referees, and especially Scott Neslin for helpful suggestions that have improved this article.

39 We thank Scott Neslin for pointing out this possibility.
30 See Shaffer and Zhang (1994b) for a start on this third extension.
31 This paper was received September 14, 1993, and has been with the authors 8 months for 2 revisions. This paper was processed by Scott Neslin, former Area Editor.

Appendix A

Let \( (\sigma_d(C'_d), \sigma_d(C''_d)) \) be a mixed-strategy equilibrium profile of the normal game given in Figure 2, where \( \sigma_d(C'_d) \) and \( \sigma_d(C''_d) \) are the respective probabilities that firms \( A \) and \( B \) send coupons to \( C \)-\textit{Users} in region II. Since in any such equilibrium a firm's mixed strategy makes its rival indifferent between its two pure strategies, \((\sigma_d(C'_d), \sigma_d(C''_d)) \) necessarily satisfy the following equations:

\[
-(\sigma_d(C'_d))z + (1-\sigma_d(C'_d))(P_B - d_B - c - z) = 0 \tag{1}
\]

\[
(P_A - d_A - c - z) = (1-\sigma_d(C''_d))(P_A - c), \tag{2}
\]

where the left hand side of Equation (1) (Equation (2)) is firm \( B \)'s (\( A \)'s) expected payoff from targeting coupons in region \( II \) and the right hand side of Equation (1) (Equation (2)) is firm \( B \)'s (\( A \)'s) expected payoff from not targeting coupons in region \( II \). The unique solution is given in the text.

Appendix B

In this appendix, we derive the unique subgame perfect equilibrium in the absence of mass-media coupons assuming \( \tau > \mu/2 \). Our derivation consists of four parts. In Part 1, we characterize the necessary conditions for existence of an equilibrium in which \( \lambda_x > 0 \) and \( \lambda_y < 1 \). In Part 2, we solve these necessary conditions and thereby identify a candidate equilibrium. In Part 3, we show that neither firm can profitably deviate and hence establish that the solution identified in Part 2 is indeed a subgame perfect equilibrium. In Part 4, we demonstrate uniqueness by proving there exists no other subgame perfect equilibrium.

Part 1. In the absence of mass-media coupons, firm \( i \) chooses \( R_i, \Pi_i, \) and \( d_i \) by maximizing \( \Pi_i^* \) as defined in \S 3 such that \( \Pi_i = R_i \) and \( d_i = 0 \), taking its rival's choices as given. Substituting \( R_i \) in for \( \Pi_i \) firm \( i \)'s Lagrange function is given by \( \Pi_i = \Pi_i^* + \lambda_d d_i \), Any subgame perfect equilibrium in which \( \lambda_x > 0 \) and \( \lambda_y < 1 \) can now be characterized by the following necessary first order conditions derived from each firm's constrained optimization:

\[
\frac{\partial L_i}{\partial R_i} = \frac{1}{2\mu} (R_i - 2R_d + c) + \frac{\alpha}{2\mu} (d_i - d_B) + \frac{1}{2} = 0, \tag{3}\]

\[
\frac{\partial L_i}{\partial d_i} = \frac{\alpha}{2\mu} (R_i - 2d_i - c - z) + \lambda_d = 0, \tag{4}\]
It is easily verified that the second order conditions for constrained optimization are satisfied.

**Part 2.** Let \((\bar{R}_A, \bar{P}_A, \bar{d}_A, \bar{\lambda}_A)\) for \(i = A, B\) satisfy Conditions (3) to (8). The solution is derived by solving the following four Kuhn-Tucker cases:

**Case 1.** \(\bar{\lambda}_A > 0\) and \(\bar{\lambda}_B > 0\) (No Targeting).

In this case, \(\bar{d}_A = \bar{d}_B = 0\) and, as can be verified, \(\bar{R}_A - \bar{R}_B = \bar{P}_A = \bar{P}_B = t_u + c\). The \(\bar{\lambda}_A > 0\) and \(\bar{\lambda}_B > 0\) imply \(t_u < z\).

**Case 2.** \(\bar{\lambda}_A = \bar{\lambda}_B = 0\) (Targeting).

Solving Equations (3), (4), (6), and (7) by setting \(\lambda_A = \lambda_B = 0\), we have \(\bar{R}_A = \bar{R}_B = \bar{P}_A = \bar{P}_B = t_u + c\) and  \(\bar{d}_A = \bar{d}_B = (t_u - z)/2\). Equations (5) and (8) imply \(t_u \geq z\). Given \(t_u > t_u/2\), \(X_A > 0\) and \(X_B < 1\) are indeed satisfied.

**Case 3.** \(\bar{\lambda}_A > 0\) and \(\bar{\lambda}_B = 0\): In this case, \(\bar{d}_B = 0\). Solving Equations (3), (6) and (7) gives:

\[
\bar{R}_A = \bar{P}_A = t_u + c + \frac{6t_c(t_u - z)}{6t_c - \alpha d_u},
\]
\[
\bar{R}_B = \bar{P}_B = z + c + \frac{6t_c(t_u - z)}{6t_c - \alpha d_u},
\]
\[
\bar{d}_B = \frac{3t_c(t_u - z)}{6t_c - \alpha d_u}.
\]

However, \(\bar{d}_B \geq 0\) implies \(t_u \geq z\) and \(\bar{\lambda}_A > 0\) implies, by Equation (4), \(t_u < z\). A contradiction.

**Case 4.** \(\bar{\lambda}_A = 0\) and \(\bar{\lambda}_B > 0\): This case is symmetric to case 3.

Thus, if a subgame perfect equilibrium exists in which \(X_A > 0\) and \(X_B < 1\), it is uniquely defined by cases 1 and 2 for the given parameter values therein.

**Part 3.** We now establish that the solution identified above is indeed a subgame perfect equilibrium. This is accomplished by showing that neither firm can profitably deviate. In particular, it must be that firm \(A(B)\) cannot profitably deviate such that \(X_A \leq 0\) (\(X_B \geq 1\)).

For \(t_u \geq z\) and given \(\bar{R}_A = t_u + c\) and \(\bar{d}_B = (t_u - z)/2\), firm \(A\)'s optimal deviation such that \(X_A \leq 0\) is given by:

\[
(\bar{R}_A, \bar{d}_A) = \arg \max_{R_A, d_A} \Pi_A^*(R_A, d_A, \bar{R}_B, \bar{d}_B) \text{ such that } d_A \geq 0 \text{ and } X_A \leq 0.
\]

It is straightforward to show that \(\bar{R}_B = (t_u + 2t_c + z)/2 + c\), and \(\bar{d}_A = (t_u + 2t_c - z)/4\), which implies \(X_A = 0\). Relaxing the constraint to allow \(X_A > 0\) as in Case 2 above yields strictly higher profit. Hence, firm \(A\)'s deviation is unprofitable. By symmetry, it is also never optimal for firm \(B\) to deviate. Hence, the solution defined by Case 2 is indeed a subgame perfect equilibrium for \(t_u \geq z\). In the same way, it is straightforward to show that the solution given in Case 1 defines a subgame perfect equilibrium for \(t_u < z\).

**Part 4.** To establish uniqueness, we consider whether other subgame perfect equilibria exist. For instance, can there be an asymmetric subgame perfect equilibrium in which \(X_A \leq 0\) and \(X_B \leq 1\)? If so, it is necessarily characterized by:

\[
(R_A', d_A') = \arg \max_{R_A, d_A} \Pi_A^*(R_A, d_A, R_B, d_B) \text{ such that } d_A \geq 0 \text{ and } X_A \leq 0,
\]
\[
(R_B', d_B') = \arg \max_{R_B, d_B} \Pi_B^*(R_A, d_A, R_B, d_B) \text{ such that } d_B \geq 0 \text{ and } X_B \leq 1.
\]

It is straightforward, albeit arduous, to show that, for \(t_u > t_u/2\), the unique solution requires \(X_A = 0\) and no other constraints bind. However, at the candidate equilibrium, firm \(A\) can profitably deviate so that \(X_A > 0\). The case where \(X_A \geq 0\) and \(X_B \geq 1\) is symmetric. Thus, it can be concluded that no asymmetric subgame perfect equilibrium exists.

We can similarly show that no subgame perfect equilibrium exists where \(X_A \leq 0\) and \(X_B \geq 1\). Since the proof is analogous, we spare readers the details. This completes our proof that, for \(t_u > t_u/2\), the unique subgame perfect equilibrium is defined in Cases 1 and 2.
Appendix C

In this appendix, we derive the unique subgame perfect equilibrium in the presence of mass-media coupons. Our derivation consists of four parts. In Part 1, we characterize the necessary conditions for existence of an equilibrium in which \( X_A > 0 \) and \( X_B < 1 \). In Part 2, we solve these necessary conditions and thereby identify a candidate equilibrium. In Part 3, we show that neither firm can profitably deviate and hence establish that the solution identified in Part 2 is indeed a subgame perfect equilibrium. In Part 4, we demonstrate uniqueness by proving there exists no other subgame perfect equilibrium.

Part 1: In the presence of mass-media coupons, firm \( i \) chooses \( R_i \), \( P_i \) and \( d_i \) to maximize \( \Pi_i \), such that \( R_i \geq P_i \), and \( d_i \geq 0 \), taking its rival's choices as given. We can simplify the analysis considerably by observing that for \( t' \geq t_c \), the first constraint never binds for firm \( i \). Incorporating this observation, the Lagrange function is again given by \( \mathcal{L} = \Pi_i^* + \lambda_i d_i \). Any subgame perfect equilibrium in which \( X_A > 0 \) and \( X_B < 1 \) can now be characterized by the following necessary first order conditions derived from each firm’s constrained optimization:

\[
-2R_i + R_{-i} + t_E + c = 0, \quad (9) \\
-2P_i + P_{-i} + d_i - d_{-i} + t_E + c = 0, \quad (10) \\
\frac{\alpha_i (P_i - 2d_i - c - z)}{2t_E} + \lambda_i = 0, \quad (11) \\
\lambda_i d_i = 0, \quad \lambda_i \geq 0, \quad d_i \geq 0. \quad (12)
\]

It is easily verified that the second order conditions for constrained optimization are satisfied.

Part 2: Let \((R_i, P_i, d_i, \lambda_i)\) for \( i = A, B \) satisfy Conditions (9) to (12). The solution is derived in the same way as in Appendix B and is given in Figure 4.

Part 3: We now establish that the solution identified in Figure 4 is indeed a subgame perfect equilibrium. This is accomplished by showing that neither firm can profitably deviate. In particular, it must be that firm \( A \) (\( B \)) cannot profitably deviate such that \( X_A \leq 0 \) (\( X_B \geq 1 \)). But this is trivial to show. Suppose \( P_a \) and \( d_a \) is firm \( A \)'s optimal deviation in the \( C-\text{Users} \) market such that \( X_A \leq 0 \). Then it must be the case that \( d_a = 0 \), for otherwise, if \( d_a > 0 \), firm \( A \) could increase its profit by reducing its discounted price by this amount and not targeting coupons. However, \( d_a = 0 \) implies \( X_A = X_y \) and therefore firm \( A \)'s deviation profit from \( C-\text{Users} \) equals zero. Hence, it is not profitable for firm \( A \) to deviate. A similar analysis shows that firm \( B \) will not deviate.

Part 4: To establish uniqueness, one must show that no subgame perfect equilibria in which \( X_A \leq 0 \) or \( X_B \geq 1 \) exists. This is straightforward and utilizes the same logic in Part 3 that proved there could be no profitable deviation by either firm.

Appendix D

In this appendix, we consider the robustness of the model to symmetric nonuniform customer distributions. Unfortunately, general conclusions are hard to reach since it is impossible to solve analytically for equilibrium \( R_i, P_i, \) and \( d_i \). Nonetheless, we are able to show that for any symmetric nonuniform customer distribution, equilibrium firm profits necessarily decrease, and equilibrium regular prices do not change, in the neighborhood of \( z \) for which coupon targeting just emerges in equilibrium. This provides support for the proposition that competitive coupon targeting does not allow firms profitably to raise price (Proposition 4) and for the proposition that the outcome of targeting is a prisoner’s dilemma (Proposition 5).

Let \( f(x) \) denote the distribution density function and \( F(x) \) the corresponding cumulative distribution function. We assume that \( f(x) \) is continuous, differentiable, and symmetric over \([0, 1]\). With these assumptions, it can be verified that \( f(\frac{1}{2}) = 0 \) and \( F(\frac{1}{2}) = \frac{1}{2} \). For simplicity, we consider only the case where mass-media couponing is absent and restrict attention to equilibria for which \( X_A > 0 \) and \( X_B < 1 \). All other assumptions in the paper remain unchanged.

Since a firm’s targeting strategy is unaffected by the distribution density of consumers (see derivation in §3), the second stage equilibrium targeting strategies are given by \((\Omega_A, \Omega_B)\) as in the text. What differs with nonuniform customer distribution is the summation of consumers in each region, which yields nonlinear demand functions for each firm. Thus, profits for the respective firms are modified as follows:

\[
\Pi_A = (1 - \alpha_c)(R_A - c)F(X_A) + \alpha_c[(R_A - c)F(X_B) - (d_A + z)(F(X_B) - F(X_A))], \\
\Pi_B = (1 - \alpha_c)(R_B - c)(1 - F(X_A)) + \alpha_c[(R_B - c)(1 - F(X_B)) - (d_B + z)(F(X_B) - F(X_A))],
\]

where the first (second) term in each profit function is the net profit from \textit{Non-Users} (\textit{C-Users}). Assuming both firms target coupons, the subgame perfect equilibrium is then characterized by the following first-order conditions:
\[
\frac{\partial \Pi_A}{\partial R_A} = \alpha \left[ \mathcal{I}(X_A) + (R_A - c) f(X_A) \frac{\partial X_A}{\partial R_A} - (d_A + z) \left( f(X_A) \frac{\partial X_A}{\partial d_A} - f(X_A) \frac{\partial X_A}{\partial R_A} \right) \right] \\
+ (1 - \alpha_c) \mathcal{I} \left( \hat{X}_A \right) + (1 - \alpha_c) (R_A - c) f(\hat{X}) \frac{\partial \hat{X}}{\partial R_A} = 0,
\]
\[
\frac{\partial \Pi_B}{\partial d_A} = \alpha \left[ (R_A - c) f(X_A) \frac{\partial X_A}{\partial d_A} - (d_B + z) \left( f(X_B) \frac{\partial X_B}{\partial R_B} - f(X_B) \frac{\partial X_A}{\partial R_B} \right) \right] \\
+ (1 - \alpha_c) (1 - \mathcal{I}(\hat{X})) = 0,
\]
\[
\frac{\partial \Pi_B}{\partial R_B} = -\alpha_c \left[ (R_A - c) f(X_A) \frac{\partial X_A}{\partial R_B} + (\mathcal{I}(X_B) - \mathcal{I}(X_A)) + (d_B + z) \left( f(X_B) \frac{\partial X_B}{\partial d_B} - f(X_A) \frac{\partial X_A}{\partial d_B} \right) \right] \\
- (1 - \alpha_c) (R_A - c) f(\hat{X}) \frac{\partial \hat{X}}{\partial R_B} = 0.
\]

Let the symmetric solution be given by \( R_A = \hat{R}_A = \bar{R} \), and \( d_A = \hat{d}_B = \bar{d} \). This means that \( X_A = \bar{X} = \frac{1}{2} \) and \( X_A = 1 - X_B \). Since the first order conditions for firms \( A \) and \( B \) are identical, the above system of equations can be reduced to the following two identities:
\[
\frac{1}{2} \frac{f(1/2)}{2t_c} (\hat{R} - c) - \frac{\alpha_c}{2t_c} \left( f \left( \frac{t_c - \bar{d}}{2t_c} \right) - f \left( \frac{1}{2} \right) \right) (d + z) = 0,
\]
\[
\frac{f(1/2)}{2t_c} (\bar{R} - \bar{d} - c - z) - \frac{1}{2} + \mathcal{I} \left( \frac{t_c - \bar{d}}{2t_c} \right) = 0.
\]

Totally differentiating the above two identities with respect to \( z \) and evaluating the resulting two identities at \( \hat{z} \) such that the firms are just indifferent between targeting and not targeting (\( d = 0 \)), we have:
\[
-\frac{f(1/2)}{2t_c} \frac{d\hat{R}}{dz} = 0, \quad \frac{f(1/2)}{2t_c} \frac{d\hat{d}}{dz} = 0.
\]

Solving yields \( d\hat{R} / dz = 0 \) and \( d\hat{d} / dz = -\frac{1}{2} \). This means that equilibrium regular prices do not change in the neighborhood of \( z \) for which coupon targeting just emerges in equilibrium.

To verify that coupon targeting leads to a prisoner’s dilemma, substitute \( (\hat{R}_A, \hat{R}_B, \bar{d}_A, \bar{d}_B) \) into \( \Pi_A \) and \( \Pi_B \) and differentiate with respect to \( z \) to give
\[
\frac{d\Pi_A}{dz} = \frac{d\Pi_A}{dR_A} \frac{dR_A}{dz} + \frac{d\Pi_A}{dd_A} \frac{dd_A}{dz} + \frac{d\Pi_A}{d\bar{d}_A} \frac{d\bar{d}_A}{dz} + \frac{d\Pi_A}{d\bar{d}_A} \frac{d\bar{d}_A}{dz} + \frac{d\Pi_A}{dz}.
\]

Substituting in \( d\hat{R}_A / dz = d\hat{R}_B / dz = 0 \) and \( d\hat{d}_A / dz = d\hat{d}_B / dz = -\frac{1}{2} \), and noting that \( d\Pi_A / dd_A = 0 \) by the envelope theorem, we have
\[
\frac{d\Pi_A}{dz} = -\frac{\alpha_c (\hat{R} - c) / f(1/2)}{4t_c} > 0.
\]

This means that equilibrium firm profits necessarily decrease in the neighborhood of \( z \) for which coupon targeting just emerges in equilibrium.

References


NCH Promotional Services (1994), *Coupon Distribution and Redemption Patterns*, Chicago, IL: NCH.


