

HASSLE COSTS: THE ACHILLES' HEEL OF PRICE-MATCHING GUARANTEES*

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We show that price-matching guarantees can facilitate monopoly pricing only if firms automatically match prices. If consumers must instead request refunds (thereby incurring hassle costs), we find that any increase in equilibrium prices due to firms' price-matching policies will be small; often, no price increase can be supported. In symmetric markets price-matching guarantees cannot support any rise in prices, even if hassle costs are arbitrarily small. In asymmetric markets, higher prices can be supported, but the prices fall well short of maximizing joint profits. Our model can explain why some firms adopt price-matching guarantees while others do not.

1. INTRODUCTION

It is common in retail markets for firms to advertise that they will not be undersold. These advertisements often contain promises to match any lower price announced by a competitor. For instance, a typical price-matching guarantee reads: "We'll Match Any Price! Our Prices Are Unbeatable."¹ Price-matching guarantees can be found in such diverse markets as sporting goods, books, housewares, cellular

We thank Eric Rasmusen, Ken Corts, two anonymous referees, and the co-editor for their help in improving this paper. We have also benefited from the comments of workshop participants at Warwick, Queen Mary and Westfield, Dundee, Carnegie Mellon, Birkbeck, Cal-Berkeley, Royal Holloway, Indiana, Rochester, the U.S. Department of Justice, the 1996 EARIE Conference, the 1997 Marketing Science Conference, and the 1998 winter meetings of the Econometric Society. Morten Hviid thanks the University of Warwick's Research and Innovation Fund for financial support.

1. See Washington *Post*, November 5, 1995, insert (Herman's Sporting Goods).

phones, office products, consumer electronics, luggage and travel accessories, furniture, tires, toys, petrol, eyewear, and prescription drugs. Moreover, they have been adopted by numerous department stores, mail-order firms, and even supermarket chains.

To many, price-matching guarantees give an impression of promoting vigorous competition. Newspaper articles, for instance, often equate the introduction of price-matching guarantees with price wars.² The dominant view in the economics literature, however, is that price-matching guarantees facilitate monopoly pricing.³ Using static oligopoly models, the participants in this literature show that price-matching guarantees can support monopoly pricing because they prevent rival firms from gaining market share by cutting price. This theory can also be found in textbooks on industrial organization (Krouse, 1990), management strategy (Oster, 1994; Baye, 1997), game theory (Dixit and Nalebuff, 1991), and antitrust law (Handler et al., 1990; Hovenkamp, 1994). However, it implicitly assumes that firms *automatically* match prices.

In this paper, we show that if consumers must instead request refunds (thereby incurring hassle costs), any increase in equilibrium prices due to firms' price-matching policies will be small; often, no price increase can be supported. For instance, if firms are symmetric, we find that price-matching guarantees cannot support any rise in prices above the Bertrand-Nash level, even if hassle costs are arbitrarily small. Thus, we believe that the ability of price-matching guarantees to facilitate monopoly pricing is overstated. A firm's promise to match the lowest price in the market is not the same as actually having the lowest price. In reality, consumers incur hassle costs when securing a matching lower price from an initially higher-priced firm. Consider Computer City's guarantee:

We'll meet any local competitor's current advertised price for any identical item in stock. Bring us the competitor's actual entire ad—if their price is lower, we'll match the price! . . . Offer excludes coupons, clearance, percent and dollar discounts, special promotions, and going out-of-business sales. (*Washington Post*, September 24, 1995)

2. The day after Esso announced the introduction of its price-matching policy, a headline in the *Financial Times*, January 18, 1996, p. 8, read "Petrol Rivals on Price-War Footing." Similarly, on the even of Tesco's decision to adopt price matching, a headline in the *Financial Times*, September 5, 1996, p. 1, read "Tesco Launches a New Price-War."

3. See, for example, Hay (1982), Salop (1986), Kalai and Satterthwaite (1986), Png and Hirschleifer (1987), Belton (1987), Doyle (1988), Logan and Lutter (1989), Baye and Kovenock (1994), Chen (1995), Zhang (1995), and Edlin and Emch (1999).

To obtain the benefits of Computer City's price-matching guarantee, a consumer must incur several nonpecuniary costs, which collectively constitute the consumer's hassle. First, the consumer must verify that the competitor's price quote is not a coupon, clearance, percent or dollar discount, special promotion, or going-out-of-business sale. Second, the consumer must bring the competitor's actual ad to the store and show it to sales personnel. This imposes a cost on consumers similar to that associated with coupon redemptions. Third, the consumer must incur an additional cost while at the store due to the extra time it takes to obtain a refund, a cost that depends on how easy sales personnel are to find and whether they are initially reluctant to grant the lower price.

Hassle costs have dramatic consequences for equilibrium pricing; their mere existence is enough to make any price above marginal cost unprofitable if firms sell homogeneous products. The reason is that, for any positive hassle costs, consumers would strictly prefer to buy from the firm with the lowest price, whether or not its rival is committed to price-matching. If Computer City were to charge the monopoly price in the mistaken belief that its market share was protected by its price-matching guarantee, its rival's best response would be to charge something slightly less and capture the entire market demand. In this case, price-matching guarantees do not ensure against market-share loss and thus are unable to prevent Bertrand prices from prevailing in equilibrium.

This paper extends previous literature in that it is the first to consider hassle costs when examining the competitive effects of price-matching guarantees. In addition to the case of identical firms and homogeneous products, we consider an asymmetric model of duopoly in which consumers have locational preferences about where to shop. This extension is important not only in adding realism to the model but also because it partly restores the ability of price-matching guarantees to facilitate higher prices. Indeed, we show that if the firms are sufficiently asymmetric, the existence of hassle costs can actually lead to higher prices than would otherwise occur. Nevertheless, we find that any rise in equilibrium prices due to firms' price-matching policies will be small.

Our results are robust to several extensions of the model. For instance, it makes no difference to our results whether we consider a static model in which firms choose price-matching policies and prices simultaneously or a two-stage framework in which price-matching policies are chosen first. We also show how our results extend to situations in which some consumers are uninformed about prices,

and in which firms can adopt price-beating guarantees, making hassle costs negative.

Section 2 describes the model. Section 3 considers the benchmark case of no hassle costs and then examines how our qualitative results differ when hassle costs are added. Section 4 discusses the model's robustness to alternate game forms, informational asymmetries, and price-beating guarantees. Section 5 concludes. The proofs of all propositions are contained in the Appendix.

2. THE MODEL AND NOTATION

Suppose two firms sell a product that is identical in all respects except for the location of sale. Suppose also that consumers have diverse preferences regarding these locations and that each consumer buys at most one unit of the product. We denote firm i 's aggregate demand as $D^i(X_i, X_j)$, where X_k , $k = i, j$, is the effective price (inclusive of hassle costs) to consumers buying from firm k . For all positive values of D^i , we assume demand is differentiable, decreasing in X_i , and weakly increasing in X_j , and that equal increases in both firms' prices decrease firm i 's demand. We further assume that price-matching guarantees are binding commitments and legally enforceable.

Let c_i be firm i 's marginal cost and P_i its posted price. If neither firm adopts a price-matching policy, the posted prices become the actual selling prices and no hassle costs are incurred. Firm i 's profit is then $\Pi^i(P_i, P_j) = (P_i - c_i)D^i(P_i, P_j)$. For all P_i, P_j such that $D^i > 0$, we make four assumptions on this reduced-form profit function. First, we assume $\Pi_{ij}^i \geq 0$. This assumption implies that firm i 's marginal profit is weakly increasing in its rival's price, and is equivalent to assuming that prices are strategic complements (best-response functions are upward sloping). Second, we assume $\Pi_{ii}^i + \Pi_{ij}^i < 0$. This assumption implies that equal price increases by both firms are less profitable for firm i the higher is P_i . Together with the first assumption, it implies that firm i 's profit is concave in its own price and ensures the existence of a unique Bertrand-Nash equilibrium in prices (see Friedman, 1983). Third, we assume $\Pi_{jj}^i + \Pi_{ij}^i \geq 0$. This assumption implies that equal price increases by both firms are weakly more profitable for firm i the higher is P_j . Lastly, we assume $\Pi_{ii}^i + \Pi_{jj}^i + 2\Pi_{ij}^i < 0$. This assumption implies that firm i 's profit is concave along the path of equal price increases by both firms. It ensures that firm i 's profit attains a unique maximum along the line

$P_i = P_j$, and it is satisfied, as are the other assumptions, with linear demands.⁴

Let $BR^i(P_j)$ solve $\max_{P_i} \Pi^i(P_i, P_j)$. Our assumptions guarantee that $BR^i(P_j)$ is single-valued, continuous, and differentiable, and $BR^i(P_j) \in [0, 1)$. Moreover, the intersection of the two best-response functions yields the Bertrand price pair $\mathbf{P}^B \equiv (P_1^B, P_2^B)$, as illustrated in Figure 1. If the two firms were to compete in a simultaneous game in prices only (without price-matching guarantees), this intersection would represent the unique Nash equilibrium. Note that the best-response functions in Figure 1 are drawn linearly for illustrative purposes only and should not be taken to imply any restrictions on demand other than those imposed by the explicit assumptions above.

Define a symmetric market as a market in which $P_1^B = P_2^B$. Define an asymmetric market as a market that is not symmetric. Figure 1 illustrates the equilibrium for a typical asymmetric market. Without loss of generality, we will henceforth assume that firm 1 has

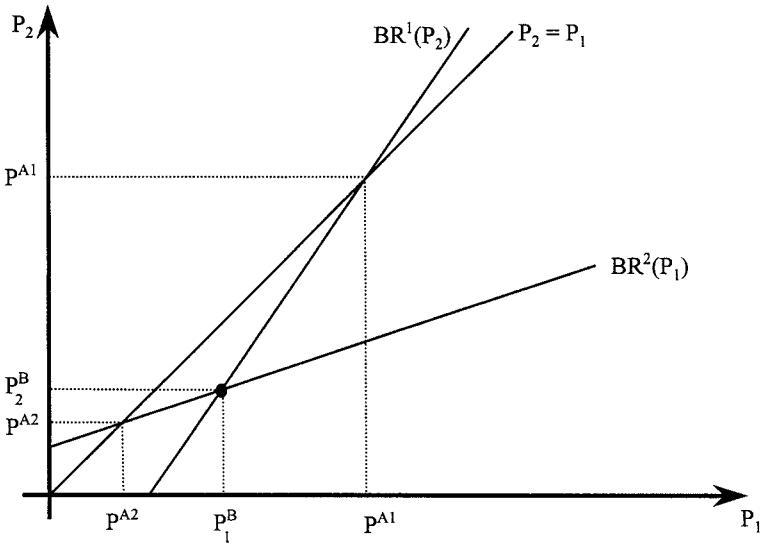


FIGURE 1. BERTRAND-NASH EQUILIBRIUM IN THE ABSENCE OF PRICE-MATCHING GUARANTEES

4. With linear demands, $\Pi_{ij}^i = \Pi_{jj}^i + \Pi_{ij}^i = D_j^i$, which is weakly positive, since firm i 's demand is weakly increasing in P_j . Moreover, $\Pi_{ii}^i + \Pi_{ij}^i = 2D_i^i + D_j^i$ and $\Pi_{ii}^i + \Pi_{jj}^i + 2\Pi_{ij}^i = 2(D_i^i + D_j^i)$, which are strictly negative, given our assumption that equal increases in both firms' prices decrease firm i 's demand.

the weakly higher Bertrand-Nash price. Note that firm 1's higher price can arise because firm 1 has a higher marginal cost than firm 2, a demand advantage relative to firm 2, or some combination of the two.

It is useful at this point to define some additional price pairs that will be used extensively in the next two sections. One such price pair, $\mathbf{P}^{Ai} \equiv (P^{Ai}, P^{Ai})$, occurs at the intersection between $BR^i(P_j)$ and the line $P_2 = P_1$. In a symmetric market, this intersection occurs at the Bertrand price pair, and so $\mathbf{P}^B = \mathbf{P}^{A1} = \mathbf{P}^{A2}$ under symmetry. In an asymmetric market, however, \mathbf{P}^{A1} will not, in general, equal \mathbf{P}^{A2} . Given our assumption that firm 1 has the weakly higher Bertrand price, it must be that $\mathbf{P}^{A1} \gg \mathbf{P}^B \gg \mathbf{P}^{A2}$, as illustrated in Figure 1.⁵

Denote the price pair that maximizes firm i 's profit along the line $P_i = P_j$ as $\mathbf{P}^{Ci} \equiv (P^{Ci}, P^{Ci})$. In an asymmetric market, the two firms will likely prefer different points along this line, and thus, in general, \mathbf{P}^{C1} will not equal \mathbf{P}^{C2} . Define \mathbf{P}^M as the price pair that is closer to the origin in the set $\{\mathbf{P}^{C1}, \mathbf{P}^{C2}\}$. That is, let $\mathbf{P}^M = \mathbf{P}^{C2}$ if $P^{C1} \geq P^{C2}$, and $\mathbf{P}^M = \mathbf{P}^{C1}$ otherwise.

The price pair \mathbf{P}^M is not depicted in Figure 1, because in general there are two possible orderings of \mathbf{P}^M when $\mathbf{P}^{A1} \gg \mathbf{P}^{A2}$. One possibility is $\mathbf{P}^{A1} \gg \mathbf{P}^M > \mathbf{P}^{A2}$. The other possibility is $\mathbf{P}^M > \mathbf{P}^{A1} \gg \mathbf{P}^{A2}$.⁶ The ordering of \mathbf{P}^M will play an important role in the next section, where the existence of equilibria with price-matching guarantees is determined.

3. COMPETITIVE EFFECTS OF PRICE-MATCHING GUARANTEES

Price-matching guarantees are promises by firms to match any lower price announced by a competitor. For instance, if firm i posts a price P_i and adopts price matching, while firm j posts a price $P_j < P_i$, firm i is obligated to sell at price P_j to any consumer who requests it. This situation is typically modeled by assuming that firm i 's price to informed consumers is automatically lowered to P_j . In reality, however, firms tend not to automatically match lower prices. Instead, price matching usually occurs only by request and is not hassle-free.

5. We adopt standard vector notation when ranking price pairs, i.e., $\mathbf{P}^B \gg \mathbf{P}^{A1}$ if and only if $P_k^B > P_k^{A1} \forall k$, and $\mathbf{P}^B > \mathbf{P}^{A1}$ if and only if $P_k^B \geq P_k^{A1} \forall k$.

6. We can rule out $\mathbf{P}^{A1} \gg \mathbf{P}^{A2} \gg \mathbf{P}^M$. Suppose not. Then $\mathbf{P}^{Ai} \gg \mathbf{P}^{Ci}$ for some i . However, an increase in firm j 's price from P^{Ci} to P^{Ai} would increase firm i 's profit, as would an increase in firm i 's own price from P^{Ci} to P^{Ai} , since $P^{Ai} = BR^i(P^{Ai})$. Firm i 's profit is thus strictly higher at the price pair \mathbf{P}^{Ai} than at \mathbf{P}^{Ci} , a contradiction.

Often a consumer must assert the existence of a lower price elsewhere and then a sales person must decide whether to grant it. He may ask the consumer to supply written proof of the lower price, verify the price quote himself by calling the other firm, consult with the manager, or argue that the product is not the same.

In every instance it takes longer to complete a transaction when price matching is requested than when it is not. We can thus quantify a consumers' hassle cost as her opportunity cost of time of waiting while a sales representative (1) confirms the lower price, (2) decides whether to grant the discount, and (3) rebates the difference.⁷ In addition, the consumer may be required to incur other costs, e.g., the cost of having to bring the rival's advertisement to the store. Some consumers may also suffer disutility in having to confront a salesperson. For all of these reasons, we believe it is unlikely that any consumer would bother to request price matching if the difference in posted prices were small enough, e.g., a penny. But unless consumers are willing to incur this hassle, the ability of price-matching guarantees to facilitate higher prices and profits is curtailed.

To model these nonpecuniary costs in the simplest possible fashion, we assume that each consumer must forgo the same monetary amount $z \geq 0$ in order to activate a price-matching guarantee. This assumption implies that no consumer will activate firm i 's price-matching guarantee if firm j 's posted price is within z of firm i 's posted price.⁸ Thus, if hassle costs are \$1.00 and firm i advertises a price of \$294 while its rival advertises a price of \$293.25 for the same item, firm i 's actual selling price to all consumers who buy from it will remain at \$294. On the other hand, if firm j advertises a price that is less than \$293, the informed consumers who buy from firm i will request a refund, and thus sales of firm i 's product will occur at the price set by firm j . As we now show, this consequence of hassle costs has important implications for equilibrium pricing.

Consider a static game of complete information in which firms 1 and 2 simultaneously post prices and decide whether to adopt price matching (\mathcal{PM}). Formally, let $\Sigma_i \equiv \{(P, \delta) \mid P \geq 0, \delta \in \{\text{no } \mathcal{PM}, \mathcal{PM}\}\}$ denote the set of strategies available to firm i . Let $\sigma_i = (P_i, \delta_i)$ denote an element of this set. Since all consumers are fully informed of each

7. Some firms, such as PETsMart, require consumers to fill out a registration card before rebating the difference.

8. This assumption is not as restrictive as it may seem. Our qualitative results are largely unchanged if z is interpreted as the lower support of some arbitrary distribution of consumer hassle costs. See Section 4.

firm's strategy, firm i 's profit can be expressed in terms of P_1 and P_2 for all possible strategy combinations (σ_1, σ_2). See Table I.

If the difference in posted prices is less than or equal to z , firm i 's actual selling price is the same as its posted price. Firm i 's profit is then $\Pi^i(P_i, P_j)$. Price-matching guarantees in this case (column 2) are never activated. If the difference in posted prices is greater than z , consumers who buy from the higher-priced firm will activate its price-matching guarantee (if it has one) and incur the hassle cost z . Thus, if $P_i > P_j + z$ (column 1), firm i 's actual selling price under price matching falls to P_j while its effective price to consumers only falls to $P_j + z$. In this case firm i 's profit is $(P_j - c_i)D^i(P_j + z, P_j)$. If $P_i < P_j - z$ (column 3), firm j 's actual selling price under price matching falls to P_i and its effective price to consumers becomes $P_i + z$. Firm i 's profit is then $\Pi^i(P_i, P_i + z)$.

It is useful to begin by analyzing the case where $z = 0$ so as to compare its predictions with those obtained when hassle costs are present. It is also useful to distinguish between markets for which $\mathbf{P}^M > \mathbf{P}^{A1}$ [each firm's profit-maximizing point on the 45° line equals or exceeds the intersection of this line with $BR^1(P_2)$] and markets for which the opposite is true. The former category encompasses all symmetric markets—because the symmetric Bertrand price is always weakly below the joint profit-maximizing price—as well as markets that are not too asymmetric. Markets for which $\mathbf{P}^{A1} \gg \mathbf{P}^M$, on the other hand, are sufficiently asymmetric to yield qualitatively different results. It should be noted that the magnitude of the cost differences and/or the demand advantage necessary for a market to be characterized as sufficiently asymmetric will depend on, among other things, the size of the market and the degree of substitution between the products of the two firms.

TABLE I.
FIRM i 'S PROFIT FOR ALL POSSIBLE STRATEGY
COMBINATIONS WHEN $z \geq 0$

Strategies	Profit		
	$P_i > P_j + z$	$ P_i - P_j \leq z$	$P_i < P_j - z$
Neither has \mathcal{PM}	$\Pi^i(P_i, P_j)$	$\Pi^i(P_i, P_j)$	$\Pi^i(P_i, P_j)$
Firm i has \mathcal{PM}	$(P_j - c_i)D^i(P_j + z, P_j)$	$\Pi^i(P_i, P_j)$	$\Pi^i(P_i, P_j)$
Firm j has \mathcal{PM}	$\Pi^i(P_i, P_j)$	$\Pi^i(P_i, P_j)$	$\Pi^i(P_i, P_i + z)$
Both have \mathcal{PM}	$(P_j - c_i)D^i(P_j + z, P_j)$	$\Pi^i(P_i, P_j)$	$\Pi^i(P_i, P_i + z)$

3.1 SOLVING THE MODEL IN THE ABSENCE OF HASSLE COSTS

If S_i denotes firm i 's selling price, then a strategy (P_i, δ_i) for firm i yields $S_i = \min(P_i, P_j)$ if $\delta_i = \mathcal{PM}$, and $S_i = P_i$ otherwise. The following proposition characterizes the set of Nash equilibria.

PROPOSITION 1: *If $z = 0$ and $\mathbf{P}^M > \mathbf{P}^{A1}$, then multiple Nash equilibria exist. The complete set of Nash equilibria is given by all strategy pairs satisfying any of the following conditions:*

(i) *both firms adopt price matching, and posted prices are such that either*

$$P_1 = P_2 \quad \text{and} \quad \mathbf{P}^{A1} < (P_1, P_2) < \mathbf{P}^M$$

or

$$P_j > P_i = P^M \text{ if } P^M > P^{Ci};$$

(ii) *only firm 1 adopts price matching, and posted prices are such that $(P_1, P_2) = \mathbf{P}^{A1}$;*

(iii) *firm 1 does not adopt price matching, and posted prices are such that $(P_1, P_2) = \mathbf{P}^B$.*

Note that it is always an equilibrium for each firm to charge its Bertrand price and for neither to adopt price matching. However, if both firms adopt price-matching guarantees, then there exists a set of Nash equilibria with selling prices ranging from \mathbf{P}^{A1} to \mathbf{P}^M along the 45° line, all of which exceed the Bertrand price pair in asymmetric markets (see Figure 2).⁹ Equilibrium selling prices are bounded above by \mathbf{P}^M because neither firm can be forced to sell at a price that exceeds its profit-maximizing point on the 45° line; unilateral price cutting in this case, even if matched by the rival, is profitable for at least one firm. Equilibrium selling prices are bounded below by \mathbf{P}^{A1} because otherwise firm 1 could increase its profit by dropping its price-matching policy and raising its posted price—and hence selling price—so as to move in the direction of $BR^1(P_2)$.

If only firm 1 adopts price matching, then the unique equilibrium selling prices are \mathbf{P}^{A1} . This exceeds the Bertrand price pair (with asymmetry), but is at the lower bound of what can be supported if both firms adopt price matching. An immediate implication then of

9. Each firm's selling price is the same as its posted price in all but one equilibrium. In that equilibrium, both firms adopt price matching and $P_j > P^M = P_i$. In that case, the selling prices are given by $(S_1, S_2) = \mathbf{P}^M$.

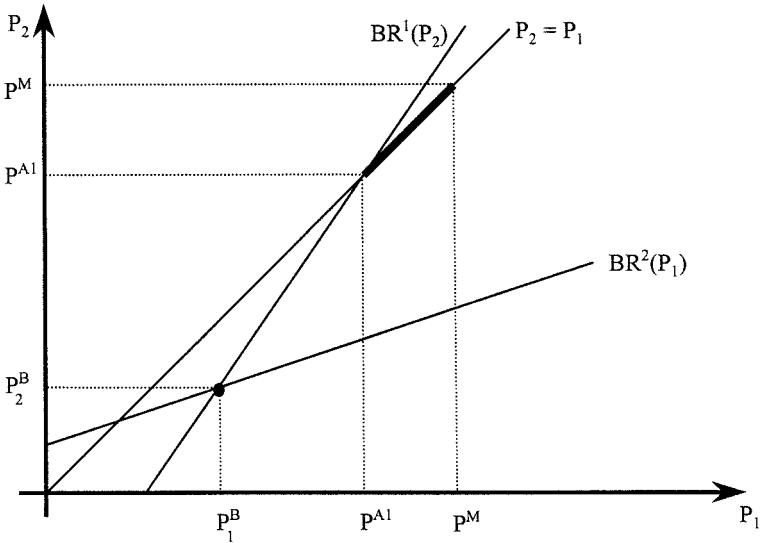


FIGURE 2. EQUILIBRIA WITH PRICE MATCHING IN THE ABSENCE OF HASSLE COSTS

(i) and (ii) in Proposition 1 is that if prices are to rise to P^M , both firms must adopt price matching. For asymmetric markets, firm 1 must adopt price matching if selling prices are to rise above P^B . For symmetric markets, both firms must adopt price matching if selling prices are to rise above P^B .¹⁰

Our framework captures Hay (1982) and Salop's (1986) insight that price-matching guarantees (at least in the absence of hassle costs) can facilitate higher prices and profits. Firm 1's policy of price matching effectively prevents firm 2 from unilaterally undercutting firm 1's selling price (and vice versa): if firm 2 were to advertise a lower price, sales of firm 1's product would automatically be made at the same price. As a consequence, firm 2's incentive to lower its posted price is reduced, which is why higher prices can (sometimes) be supported in equilibrium. We say "sometimes" because there are circumstances, even with no hassle costs, in which supracompetitive

10. Doyle (1988) argues that the set of Nash equilibria can be narrowed if one uses a Pareto dominance criterion to select plausible equilibrium outcomes. Since profits are strictly concave along the 45° line, it is straightforward to show that equilibria in which (1) both firms adopt price-matching guarantees and (2) selling prices are at P^M yield strictly higher profit for each firm than in all other equilibria with price matching.

prices (prices higher than Bertrand) cannot be supported in equilibrium. This is the case if $\mathbf{P}^{A1} \gg \mathbf{P}^M$.

PROPOSITION 2: *If $z = 0$ and $\mathbf{P}^{A1} \gg \mathbf{P}^M$, then multiple Nash equilibria exist. The complete set of Nash equilibria is given by all strategy pairs in which firm 1 does not adapt price matching and posted prices are such that $(P_1, P_2) = \mathbf{P}^B$.*

The multiplicity arises because firm 2 may or may not have a (redundant) price-matching guarantee in equilibrium. Proposition 2 implies that, if $z = 0$ and $\mathbf{P}^{A1} \gg \mathbf{P}^M$, there does not exist a Nash equilibrium in which the selling prices of the two firms are equal. Firm 1 will not acquiesce in identical selling prices below \mathbf{P}^{A1} , because it can earn higher profit by unilaterally dropping any price-matching guarantee it may have and raising its posted price (and hence selling price) to $BR^1(P_2)$. Firm 2, on the other hand, will not accept identical selling prices at \mathbf{P}^{A1} or above, because it can assure itself higher profit by lowering its posted price to \mathbf{P}^M and inducing the activation of firm 1's price-matching guarantee. The ability of price-matching guarantees to support higher prices in this instance fails because of the firms' divergent interests. For sufficiently asymmetric markets, therefore, the Bertrand prices are the only selling prices that can be supported in equilibrium. Note that this case has not previously been discussed in the literature on price matching, since that literature typically assumes homogeneous products and identical firms.

3.2 SOLVING THE MODEL IN THE PRESENCE OF HASSLE COSTS

Suppose hassle costs are strictly positive. Then a strategy of (P_i, δ_i) yields $S_i = P_j$ if $P_j < P_i - z$ and $\delta_i = \mathcal{PM}$, and $S_i = P_i$ otherwise. Can price matching facilitate higher prices in this case? The problem is that consumers will not be willing to activate a firm's guarantee if the hassle costs exceed the savings. For example, if the monopoly price of two equally matched firms is \$75, but hassle costs are \$100, price-matching guarantees will obviously be ineffective in reducing competition because each firm can then undercut the other without consequence. But what if hassle costs are only \$2? The next two lemmas are useful in solving the model in the presence of hassle costs.

LEMMA 1: *If $z > 0$ and firm i adopts a price-matching guarantee, then a necessary condition for (P_1, P_2) to arise in equilibrium is $P_i \leq P_j + z$, $i, j \in [1, 2]$, $j \neq i$.*

Lemma 1 establishes that if hassle costs are strictly positive, then no equilibrium exists in which consumers request price-matching benefits; each firm's selling price is the same as its posted price.

LEMMA 2: *If $z > 0$, then one of the following two conditions must be satisfied for (P_1, P_2) to arise in equilibrium: either $P_i = BR^i(P_j)$ or $P_j - z = P_i > BR^i(P_j)$, $i, j \in [0, 1]$, $j \neq i$.*

Lemma 2 establishes that either firm i will charge $BR^i(P_j)$, or it will be constrained by firm j 's price-matching guarantee to charge $P_i = P_j - z > BR^i(P_j)$. It cannot be an equilibrium for firm i 's posted price to be less than $BR^i(P_j)$, because, given P_j , firm i can increase its profit by increasing its posted price and, if necessary, dropping any price-matching guarantee it may have.

In our example above of two equally matched firms, we can use Lemmas 1 and 2 to show that \$2 is more than enough to render price-matching guarantees completely ineffective. Indeed, the following proposition implies that, in any symmetric market, any positive hassle cost, no matter how small, will suffice to ensure that only the Bertrand-Nash prices can arise in equilibrium.

PROPOSITION 3: *If $z > P_1^B - P_2^B$, then multiple Nash equilibria exist. The complete set of Nash equilibria is given by all strategy pairs in which posted prices are such that $(P_1, P_2) = \mathbf{P}^B$.*

The first thing to notice is that $P_1^B = P_2^B$ under symmetry, and thus Proposition 3 applies to any symmetric market in which hassle costs are positive. More generally, supracompetitive prices cannot be supported in any Nash equilibrium, even in asymmetric markets, if the level of hassle costs exceeds the difference in Bertrand prices. The reason is that, if $z > P_1^B - P_2^B$, strictly positive hassle costs permit price shading on all posted prices above Bertrand. To see this, suppose the market is symmetric. Then, for any posted price pair (on or off the 45° line) above the symmetric Bertrand prices, there exists a firm j and $\varepsilon < z$ such that firm j can increase its profit by lowering its price by that amount. Such a price cut does not induce consumers to activate firm i 's price-matching guarantee (if it has one), and the price cut is profitable, since it is a movement in the direction of $BR^j(P_i)$. In equilibrium, firms may or may not have price-matching guarantees, but if they do have them, they are never activated. The unique equilibrium selling prices are \mathbf{P}^B .

This means that price-matching guarantees fare no better in supporting supracompetitive prices in markets in which $z > P_1^B - P_2^B$, but where products are nonetheless differentiated, than in markets in which the products are homogeneous. This result is surprising, since

there are many differences between the case of imperfect substitutes discussed here and the case of homogeneous products that was discussed in the introduction. In particular, the explanations why, in each case, only the Bertrand-Nash prices can be supported in equilibrium are not the same. When products are homogeneous, *any* price cut will cause *all* consumers to buy from the low-priced firm in the presence of hassle costs, and thus no consumers will request price matching from the high-priced firm. It is not surprising, therefore, that only the Bertrand prices can arise in equilibrium in this case.

However, when products are differentiated, some consumers (the ones whose disutilities from switching exceed their hassle costs) will prefer shopping at their regular location and getting that firm to match a rival's lower price to actually buying from the rival. To the extent that these inframarginal consumers exist, a high-price firm that has a price-matching guarantee may be able to retain some of the customers that it would otherwise lose when a rival cuts prices. Thus, it would seem that the low-price firm's incentive to cut price in the first place would be dampened, and therefore that at least some supracompetitive prices would be supportable.

However, this intuition is incorrect. While the existence of inframarginal consumers does indeed make the market share of the high-price firm less vulnerable to arbitrary price cuts, the rival firm can still profitably undercut by amounts smaller than these consumers' hassle costs. When that happens, consumers whose disutility from switching is smaller than the difference in prices will switch, while consumers whose disutility from switching is larger than the difference in prices will not switch. Consumers will not base their decision on whether the high-price firm has a price-matching policy (because the difference in prices by assumption is less than their hassle costs). Thus, for small price cuts, it is as if the high-price firm had no price-matching policy. In other words, even though the low-price firm's ability to cut prices is limited, the fact that it can always undercut by a penny without causing any consumers to request price matching from the high-price firm is often enough to unravel *all* supracompetitive equilibria. If $z > P_1^B - P_2^B$, the only selling prices that can be supported in equilibrium are \mathbf{P}^B . In this sense, the case of symmetrically (or near-symmetrically) differentiated products is analogous to the case of homogeneous products.

Suppose now that hassle costs are less than the difference in Bertrand prices and that $\mathbf{P}^M > \mathbf{P}^{A1}$. Then the following proposition implies that price-matching guarantees can once again facilitate higher prices, but any rise in equilibrium prices will be small, i.e., the effectiveness of price matching is sharply diminished. In particular,

the next proposition establishes that selling prices can only increase to the point of intersection between the line $P_2 = P_1 - z$ and $BR^1(P_2)$. We denote this price pair as $\mathbf{P}^Z \equiv (P_1^Z, P_2^Z)$ and illustrate its construction in Figure 3.

PROPOSITION 4: *If $P_1^B - P_2^B \geq z > 0$ and $\mathbf{P}^M > \mathbf{P}^{A1}$, then multiple Nash equilibria exist. The complete set of Nash equilibria is given by all strategy pairs satisfying any of the following conditions:*

- (i) both firms adopt price matching, and posted prices are such that $(P_1, P_2) = \mathbf{P}^Z$;
- (ii) only firm 1 adopts price matching, and posted prices are such that $(P_1, P_2) = \mathbf{P}^Z$;
- (iii) firm 1 does not adopt price matching, and posted prices are such that $(P_1, P_2) = \mathbf{P}^B$.

Because firm 2's posted price is always within z of firm 1's posted price by construction, price-matching guarantees are never activated in equilibrium. Thus, each firm's selling price is the same as its posted price. If the necessary conditions in Proposition 4 are satisfied, but firm 1 does not adopt price matching, then the unique equilibrium selling prices are \mathbf{P}^B . Otherwise, if firm 1 adopts price

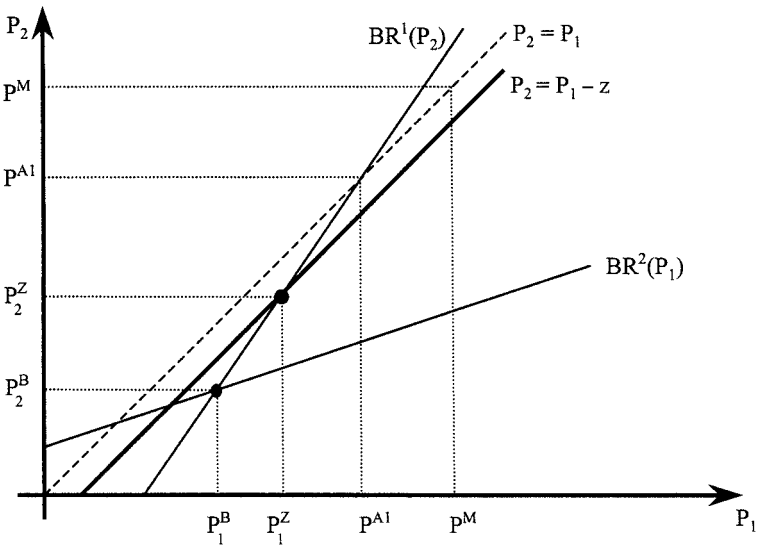


FIGURE 3. EQUILIBRIA WITH PRICE MATCHING IN THE PRESENCE OF HASSLE COSTS

matching, the unique equilibrium selling prices are \mathbf{P}^Z . We know that selling prices are higher in this case because the necessary condition $P_1^B - P_2^B \geq z$ implies $\mathbf{P}^Z > \mathbf{P}^B$.

To understand why selling prices at \mathbf{P}^Z can arise in equilibrium, suppose $(P_1, \delta_1) = (P_1^Z, \mathcal{P}\mathcal{M})$ and $(P_2, \delta_2) = (P_2^Z, \text{no } \mathcal{P}\mathcal{M})$. Given firm 1's strategy, firm 2 does not want to lower its price below $P_1^Z - z$, since that would induce consumers to activate firm 1's price-matching guarantee. But neither does firm 2 want to raise its price above P_1^Z , since that would move selling prices farther away from $\text{BR}^2(P_1^Z)$. Thus, $(P_2^Z, \text{no } \mathcal{P}\mathcal{M})$ is a best response to $(P_1^Z, \mathcal{P}\mathcal{M})$. Given firm 2's strategy, firm 1 cannot improve on its own strategy of adopting a price-matching guarantee and posting a price of P_1^Z , because its selling price when it does so occurs at $S_1 = \text{BR}^1(S_2) = \text{BR}^1(P_2^Z)$.

Note that firm 1's price-matching guarantee supports the equilibrium because it prevents firm 2 from profitably lowering its price below P_2^Z . Any price-matching guarantee by firm 2, however, is superfluous, since firm 2 already has the lower posted price in equilibrium. By contrast, both firms *must* adopt price matching in the absence of hassle costs if the maximum selling prices that can be obtained in equilibrium are to be supported. Thus, whereas previous literature predicts that all firms in a market will adopt price-matching guarantees, we would not be surprised, given the prevalence of hassle costs, to find many markets in which only a subset of firms do so.

While hassle costs do not prevent price-matching guarantees from facilitating higher prices in asymmetric markets, they do limit how much prices can rise. The reason is that $z > 0$ implies $\mathbf{P}^Z \ll \mathbf{P}^{A1}$, and therefore selling prices with hassle costs are bounded above by the lower bound of what can be supported—if firm 1 adopts price matching—in the absence of hassle costs. Indeed, equilibrium prices with hassle costs are often far below what can be supported in the absence of hassle costs. Price-matching guarantees are simply much less effective than conventional wisdom in economics would suggest in increasing prices above \mathbf{P}^B —for many markets.

We say "for many markets" because there are circumstances in which hassle costs lead to higher prices. Suppose $\mathbf{P}^{A1} \gg \mathbf{P}^M$. Then, in the absence of hassle costs, the unique equilibrium selling prices are \mathbf{P}^B . However, if hassle costs are strictly positive, but less than the difference in Bertrand prices, selling prices can be higher than \mathbf{P}^B if, in addition, firm 2's profit-maximizing price along the line $P_1 = P_2 + z$ equals or exceeds P_2^Z . Formally, let P_2^\ominus solve $\max_{P_2} \Pi^2(P_2, P_2 + z)$.

Then higher prices can be supported if and only if $P_2^\theta \geq P_2^Z$. This condition, which appears in the next proposition, ensures that at the price pair \mathbf{P}^Z , firm 2 will not find it profitable to lower its posted price below P_2^Z and induce consumers to activate firm 1's price-matching guarantee. Note that this condition does not appear in Proposition 4, since $P_2^\theta \geq P_2^Z$ is always satisfied when $\mathbf{P}^M > \mathbf{P}^{A1}$.

PROPOSITION 5: *If $P_1^B - P_2^B \geq z > 0$ and $\mathbf{P}^{A1} \gg \mathbf{P}^M$, then multiple Nash equilibria exist. The characterization of these equilibria depends on the relationship between P_2^θ and P_2^Z .*

If $P_2^\theta \geq P_2^Z$, then the complete set of Nash equilibria is given by all strategy pairs satisfying any of the following conditions:

- (i) *both firms adopt price matching, and posted prices are such that $(P_1, P_2) = \mathbf{P}^Z$;*
- (ii) *only firm 1 adopts price matching, and posted prices are such that $(P_1, P_2) = \mathbf{P}^Z$;*
- (iii) *firm 1 does not adopt price matching, and posted prices are such that $(P_1, P_2) = \mathbf{P}^B$.*

If $P_2^\theta < P_2^Z$, then the complete set of Nash equilibria is given by all strategy pairs in which firm 1 does not adopt price matching and posted prices are such that $(P_1, P_2) = \mathbf{P}^B$.

Proposition 5 implies that if firm 1 does not adopt price matching, then the unique equilibrium selling prices are \mathbf{P}^B . Otherwise, if $P_2^\theta \geq P_2^Z$, the unique equilibrium selling prices are \mathbf{P}^Z . If we were to illustrate the case in which $\mathbf{P}^{A1} \gg \mathbf{P}^M$ and $P_2^\theta \geq P_2^Z$, it would be similar to Figure 3, except that P^M would be below P^{A1} on the vertical axis. To satisfy the condition that $P_2^\theta \geq P_2^Z$, it suffices that $P^M \geq P_2^Z$. In other words, $P^M \geq P_2^Z$ implies $P_2^\theta \geq P_2^Z$, for all $z > 0$.¹¹

Note that hassle costs in this case can lead to higher equilibrium prices than would occur in their absence. Recall that selling prices above \mathbf{P}^B cannot be supported in the absence of hassle costs if $\mathbf{P}^{A1} \gg \mathbf{P}^M$ because, for any price pair on the 45° line, either firm 2 would want to deviate by posting a lower price, or firm 1 would want to deviate by posting a higher price. However, if hassle costs are strictly positive and the necessary conditions for the first subcase in Proposition 5 are satisfied, then equilibria can be supported in which firm 2 charges a lower price than firm 1 and both firms' selling

11. To see this, note that P_2^θ is implicitly given by the first-order condition $\Pi_1^2(P_2^\theta, P_2^\theta + z) + \Pi_2^2(P_2^\theta, P_2^\theta + z) = 0$. A straightforward application of the implicit-function theorem yields $dP_2^\theta/dz \geq 0 \forall z$. Since P_2^θ is weakly increasing in z and equal to P^M when $z = 0$, it must be weakly greater than P^M for all $z > 0$.

prices rise above \mathbf{P}^B . Intuitively, hassle costs can facilitate higher equilibrium selling prices if $P_2^\theta \geq P_2^Z$ because they mitigate the asymmetry between firms. Price-matching guarantees then no longer constrain selling prices to lie on the 45° line.

To support equilibrium selling prices at \mathbf{P}^Z , hassle costs must be smaller than the difference in Bertrand prices and yet large enough that $\mathbf{P}^{A1} \gg \mathbf{P}^M$ and $P_2^\theta \geq P_2^Z$ are both satisfied. Note that P_2^Z and P_2^θ are increasing in hassle costs and that $P_2^Z = P^{A1}$ and $P_2^\theta = P^M$ at $z = 0$. Thus, if $z = 0$ and $\mathbf{P}^{A1} \gg \mathbf{P}^M$, then P_2^Z exceeds P_2^θ and the unique equilibrium selling prices are \mathbf{P}^B . As hassle costs increase, the gap between P_2^θ and P_2^Z narrows and eventually changes sign. Proposition 5 then implies that higher equilibrium selling prices are possible. But if hassle costs increase so much as to exceed the difference in Bertrand prices, then we know from Proposition 3 that the unique equilibrium selling prices will once again be \mathbf{P}^B . Thus, higher prices can only be supported when $\mathbf{P}^{A1} \gg \mathbf{P}^M$ if hassle costs are in an intermediate range: less than $P_1^B - P_2^B$ to ensure that $\mathbf{P}^Z \gg \mathbf{P}^B$, but greater than or equal to $\text{BR}^1(P_2^\theta) - P_2^\theta$ to ensure that $P_2^\theta \geq P_2^Z$.

3.3 DISCUSSION

We have considered two additions to the traditional model: asymmetry among firms and positive hassle costs. Table II summarizes our results. In the first full column, the effects of asymmetry among firms in the absence of hassle costs is considered. The first full row examines the effects of adding hassle costs to a market with symmetric firms. The rest of Table II summarizes the results when positive hassle costs and asymmetric firms are jointly considered.

In all circumstances (whether the firms are symmetric or asymmetric and whether hassle costs are zero or positive), we find that it is always a Nash equilibrium for each firm to charge its Bertrand price and for neither firm to adopt a price-matching policy. Thus, it is important to note that price-matching guarantees need not facilitate collusion, even if there are no hassle costs. Nevertheless, our results suggest there is some truth to the existing theory; if firm 1 adopts a price-matching policy, then, in some circumstances, supracompetitive selling prices can be supported in equilibrium.

The existing literature often focuses on symmetric markets, where there exists an equilibrium in which both firms adopt price-matching guarantees and selling prices are \mathbf{P}^M in the absence of hassle costs. However, Table II shows that this result is not robust to hassle costs. In symmetric markets with hassle costs, price-matching

TABLE II.
SUMMARY OF RESULTS

	No Hassle Costs	Positive Hassle Costs	
Symmetric markets: $P_1^B = P_2^B$	$[P^B, P^M]^a$	P^B All price matching is redundant	
Asymmetric markets: $P_1^B > P_2^B$ and $P^M > P^{A1}$	$P^B \cup [P^{A1}, P^M]$	$z > P_1^B - P_2^B$ P^B All price matching is redundant	$z \leq P_1^B - P_2^B$ $P^B \cup P^Z$ Firm 2's price matching is redundant
$P_1^B > P_2^B$ and $P^M \ll P^{A1}$	P^B All price matching is redundant	P^B All price matching is redundant	$P^B \cup P^Z^b$ Firm 2's price matching is redundant

^a As a mnemonic aid, we use the notation $[P^B, P^M]$ to mean equilibrium selling prices are such that $S_1 = S_2$ and $P^B < (S_1, S_2) < P^M$. Note that in symmetric markets, $P^B = P^{A1}$.

^b Selling prices at P^Z can arise in equilibrium in this case if and only if $P_2^\theta \geq P_2^Z$.

guarantees cannot support anything other than the Bertrand-Nash prices, even if hassle costs are arbitrarily small. Although price-matching guarantees may be adopted by one or both firms in equilibrium in the symmetric case with hassle costs, in all instances they are redundant in the sense that they play no role in supporting the equilibrium.

In extending the existing literature (based on no hassle costs) to asymmetric markets (first full column in Table II), we find that if $P^M > P^{A1}$ and firm 1 adopts a price-matching guarantee, then the Bertrand-Nash prices cannot be supported in equilibrium (Proposition 1), and no "equilibrium refinement" is needed to conclude that selling prices must rise. On the other hand, we find that if $P^M \ll P^{A1}$, then no equilibrium exists in which firm 1 adopts price matching (Proposition 2), and the unique equilibrium selling prices in this case are Bertrand (any price-matching guarantee adopted by firm 2 in equilibrium in this case plays no role in supporting the equilibrium).

Surprisingly, if $z \leq P_1^B - P_2^B$, $P^M \ll P^{A1}$, and $P_2^\theta \geq P_2^Z$, then hassle costs may be the *sine qua non* that leads to higher selling prices. Although equilibria exist in which one or both firms adopt price-matching guarantees in this case, any guarantee by firm 2 is redundant.

In general, if $z > P_1^B - P_2^B$, then only the Bertrand-Nash prices can be supported in equilibrium, and all price-matching guarantees

are redundant, which implies that if price-matching guarantees are to facilitate supracompetitive prices in the presence of hassle costs, the market must be asymmetric and the hassle costs must be less than the difference in Bertrand prices ($z \leq P_2^B - P_1^B$). However, any increase in equilibrium prices due to firms' price-matching policies will be small.¹² Whereas existing theory would suggest that selling prices at P^M can often be supported, we have found that selling prices are bounded above by $P^Z \ll P^{A1}$ in any asymmetric market with hassle costs. In this sense, hassle costs are the Achilles' heel of price-matching guarantees. Our results also differ from existing theory in that both firms need not adopt price-matching guarantees for higher prices to be supported. If $z \leq P_1^B - P_2^B$, we would not be surprised to observe only firm 1 adopting a price-matching guarantee; any guarantee by firm 2 in this case is redundant.

3.4 IMPLICATIONS

Our results suggest that firms face a dilemma regarding hassle costs. On the one hand, a minimum level of hassle costs may be necessary if higher selling prices are to be supported in equilibrium: if markets are sufficiently asymmetric, for instance, the existence of hassle costs may actually make firms better off. On the other hand, if the market is not too asymmetric, hassle costs will serve only to dampen equilibrium prices (P^Z is decreasing in z). This suggests that firms should often try to make obtaining price-matching benefits as hassle-free as possible for consumers.

One way to accomplish this is to automatically match competitors' lower prices. For example, Big Star at one time published a weakly guide containing the prices of over 9000 items that overlapped with Food Lion (see Hess and Gerstner, 1991). Matching prices were then automatically given to all consumers. Tweeter etc. advertises an "automatic price protection" program that goes beyond "the standard low-price guarantee in the electronics industry by promising its customers an automatic refund check anytime a competitor advertises a low price."¹³ A company spokesman notes that the purpose of the strategy is to "save customers the trouble of monitoring ads." Presumably this is also the purpose of Esso's new "Pricewatch" initiative in the UK, where the company itself monitors

12. In a previous version of this paper, we worked through a linear example in which P_1^Z was at most 7.1% higher than P_1^B , whereas P^M was as much as 50% higher.

13. See "Tweeter's Customer Told: 'Your Check Is in the Mail': New England Retailer Says Its Computers Shop the Ads of Rivals to Ensure Refunds," *Wall Street Journal*, August 17, 1993, Section B, p. 6.

competitors' petrol prices and automatically matches them if they are lower.¹⁴

So why aren't these strategies more commonly adopted? One reason may be that firms must incur extra costs if they undertake the comparison shopping themselves. In the case of Big Star, employees had to enter each rival's price into a database and then print a weekly guide. Tweeter etc. pays company staffers to monitor one or two newspapers daily in each of its markets, and cost considerations have forced Esso to limit the range of its monitoring to a two-mile radius of its stations. On the benefit side, the gains from reducing hassle costs may not be very large. Our analysis suggests a downward jump in equilibrium prices and profits at $z = 0$. Unless hassle costs can literally be made zero, the gains from reducing them may be small relative to the costs.

A second reason, however, may be that if markets are sufficiently asymmetric ($P^{A1} \gg P^M$), hassle costs must be bounded above zero if higher prices are to be supported. This may explain the strategy of some firms, such as PETSMART, which seemingly go out of their way to increase hassle costs. PETSMART's price guarantee, for instance, contains more than 15 different restrictions.¹⁵ In addition to requiring written proof of a competitor's lower price, among other things, PETSMART also requires consumers to complete a registration card before receiving any refund.

In a market with two firms, we would expect to find price-matching guarantees adopted by either (1) only the firm which has the higher Bertrand-Nash price, or (2) both firms. In a market with more than two firms, we conjecture that the set of firms with price-matching guarantees would include those firms which have the highest Bertrand prices. These predictions are consistent with Hess and Gerstner's (1991) finding that, in Raleigh, North Carolina, Big Star and Winn Dixie adopted price-matching guarantees even though Food Lion, the firm with the lowest Bertrand price, did not. These

14. See "Petrol Rivals on Price-War Footing," *Financial Times*, January 18, 1996, p. 8, and "Phoney War Fuels Forecourt Fury," *The Guardian*, February 1, 1997, p. 9.

15. See PETSMART's low-price guarantee at <http://www.petsmart.com/shop/lowprice.htm>. According to a report from the Buckingham Research group, Inc., dated April 9, 1998, the pet supply industry is a highly fragmented market with two superstores PETCO and PETSMART, several smaller franchise operations (such as Superpetz and Pet Supplies Plus), a few regional chains with less than 100 stores, and over 10,000 mom-and-pop stores. In addition, supermarkets account for about 49% of the market, although this percentage has steadily declined in the last 10 years with the advent of the two superstores. Of the two superstores, PETSMART had double the sales (\$1,500 million to \$750 million) in 1997 but less than $\frac{1}{4}$ the number of stores (340 to 457) of PETCO in the US.

predictions are also consistent with the current situation in the UK petrol market, where Esso is the only firm to have a nationally advertised price-matching policy.

Our model is also consistent with the casual observation that firms that adopt price-matching guarantees often charge higher prices than firms that do not. Note that we would not always expect to find a divergence of prices between two firms in a market with price-matching guarantees. To the extent that firms buy their product from the same upstream supplier *and pay the same price*, it may be that the market is best characterized as symmetric. In that case, our model would predict that selling prices would be identical—albeit competitive.

4. ROBUSTNESS OF THE MODEL

We now consider the robustness of the model to (1) two-stage games in which firms choose their price-matching policies prior to posting prices, (2) information asymmetries in which some consumers are uninformed, (3) the distribution of hassle costs, and (4) price-beating guarantees.

4.1 TWO-STAGE GAME

We have analyzed a game in which each firm simultaneously chooses a price-matching policy and posts a price. In this subsection, we consider a two-stage game in which the firms first choose their price-matching policies and then advertise prices. It turns out that, if we use subgame perfection as our solution concept, this alternative game form makes little difference for our main results. In particular, if $P_1^B - P_2^B > z > 0$, the maximum selling prices that can be supported in any equilibrium are \mathbf{P}^Z , and there exist subgame perfect equilibria in which the selling prices are \mathbf{P}^B . Moreover, if $z > P_1^B - P_2^B$, the unique equilibrium selling prices are Bertrand.

To prove these claims, note that while Lemma 1 continues to hold in the two-stage game, Lemma 2 must be modified to reflect that the fact that, in the last stage of the game, firm i cannot deviate by dropping its price-matching guarantee (if it has one).

LEMMA 3: *If $z > 0$, then one of the following three conditions must be satisfied for (P_1, P_2) to arise in a subgame perfect equilibrium: either $P_i = \text{BR}^i(P_j)$, or $P_j - z = P_i > \text{BR}^i(P_j)$, or $P_j + z = P_i < \text{BR}^i(P_j)$, for $i, j \in [0, 1]$, $j \neq i$.*

Lemmas 1 and 3 can be used to characterize the complete set of Nash equilibria in the second stage. To conserve space, we focus on the case in which $P_1^B - P_2^B \geq z > 0$ and $\mathbf{P}^M > \mathbf{P}^{A1}$. Let $\tilde{\mathbf{P}}^Z$ denote the price pair that solves $P_2 = P_1 - z = \text{BR}^2(P_1)$.

PROPOSITION 6: *If $P_1^B - P_2^B \geq z > 0$ and $\mathbf{P}^M > \mathbf{P}^{A1}$, then Nash equilibria exist in each subgame, and the complete set is characterized as follows:*

- (i) *If both firms adopt price matching in the first stage, then the complete set of Nash equilibria in the second stage is given by all price pairs that satisfy $P_2 = P_1 - z$ and $\tilde{\mathbf{P}}^Z < (P_1, P_2) < \mathbf{P}^Z$.*
- (ii) *If only firm 1 adopts price matching in the first stage, then the complete set of Nash equilibria in the second stage is given by all price pairs that satisfy $(P_1, P_2) \in \{\tilde{\mathbf{P}}^Z, \mathbf{P}^Z\}$.*
- (iii) *If neither firm adopts price matching in the first stage, or if only firm 2 does so, then the unique Nash equilibrium in the second stage is given by $(P_1, P_2) = \mathbf{P}^B$.*

Note that the maximum selling prices that can be supported in any stage-2 equilibrium are \mathbf{P}^Z . Note also that the unique equilibrium selling prices if firm 1 does not adopt price matching are \mathbf{P}^B . We now show that both price pairs can arise in a subgame perfect equilibrium.

PROPOSITION 7: *Subgame perfect equilibria exist in which selling prices are \mathbf{P}^Z , and in which selling prices are \mathbf{P}^B . The maximum selling prices that can be supported in any equilibrium are \mathbf{P}^Z .*

Allowing firms to commit *ex ante* to price-matching policies makes little difference for our qualitative results. Thus, our conclusions are robust to the two-stage game.

4.2 INFORMED AND UNINFORMED CONSUMERS

We have assumed that all consumers are fully informed of each firm's price. In this subsection, we consider whether hassle costs have a dampening effect on prices when some consumers are uninformed. One might conjecture that hassle costs are less important in this case. After all, it has been argued that the full force of price-matching policies is understated in models of complete information because the Bertrand price pair often remains as an equilibrium (as it does in our model). By contrast, Edlin (1990) argues that in models where some consumers are uninformed, a price-matching policy allows a firm profitably to raise its price, even if its rivals do not. The firm does so in order to exploit its uninformed customers, confident that its price-matching policy will ensure against the loss of informed cus-

tomers. Similarly, Png and Hirschleifer (1987) argue that price-matching policies, because they permit firms to price-discriminate, can lead to higher equilibrium prices than would occur with a nondiscriminating monopolist. In both models, the informed consumers always buy from the lowest-priced firm in the absence of price matching.

How would the introduction of hassle costs affect the results in these papers? Quite simply, positive hassle costs would undermine all collusive equilibria. To see this, recall our argument in the introduction of this paper for why price-matching cannot facilitate supra-competitive prices when products are homogeneous. In short, for any positive hassle costs, informed consumers would never buy from the high-priced firm whether or not it had a price-matching policy. Thus, price-matching policies offer no protection against the loss of market share in the presence of hassle costs.

Now suppose that we modify our framework to allow some consumers to be uninformed. In particular, suppose $D^i(X_i, X_j) \equiv D^{im}(X_i) + D^{id}(X_i, X_j)$, where D^{im} is the demand facing firm i from the uninformed (monopoly) consumers and D^{id} is the demand facing firm i from the informed (duopoly) consumers. Assume that the uninformed consumers know the price of only one firm, while the informed consumers know the prices of both firms at the time of purchase. Assume also that, for all positive values of D^i , demand is differentiable, decreasing in X_i , and weakly increasing in X_j , and that equal increases in both firms' prices decrease firm i 's demand.

In this setting, assume parameters are such that Lemma 1 holds.¹⁶ Then Lemma 2 must also hold, and all of our qualitative results follow. The reason is that, as before, a small amount of undercutting will not trigger consumer requests of the high-priced firm to price-match (due to the hassle costs involved). The fact that only some of the consumers are informed does not matter for this argument. Neither the informed nor the uninformed consumers will request price matching. Suppose, for example, that $z > 0$ and the market is symmetric. Then, in any equilibrium in which $P_i \leq P_j + z$, the unique equilibrium selling prices are \mathbf{P}^B . The proof of this assertion is none other than the proof of Proposition 3. Similarly, in any equilibrium in which $P_i \leq P_j + z$, Propositions 4 and 5 imply that the maximum selling prices that can be supported are \mathbf{P}^Z .

16. Let $P_i^* \equiv \arg \max_{P_i} (P_i - c)D^{im}(P_i)$ and $P_j^{**} \equiv \arg \max_{P_j} (P_j - c) [D^{im}(P_j) + D^{id}(P_j, P_j + z)]$. Then Lemma 1 holds if either (1) $P_j^{**} + z > P_i^*$, or (2) $\Pi^i(P_j^{**}, P_j^{**}) > (P_i^* - c)D^{im}(P_i^*) + (P_j^{**} - c)D^{id}(P_j^{**} + z, P_j^{**})$.

We conclude this subsection by noting that Lemma 1 is more likely to hold for larger hassle costs, all else equal. Thus, small hassle costs need not have as dramatic a consequence as they do in our model in models of price matching in which some consumers are uninformed—and for which the informed consumers have locational preferences about where to shop.¹⁷

4.3 DISTRIBUTION OF HASSLE COSTS

One unrealistic implication of our model is that no consumer requests price matching in equilibrium. To account for this feature, suppose that consumers differ in their hassle costs. Then any given price cut would segment consumers into one of two groups, those who request price matching from the high-priced firm and those who do not, where the size of each group is determined endogenously.

How would this extension change our results? We conjecture that our qualitative results would be unchanged if all consumers have hassle costs bounded above zero.¹⁸ In that case, let z be the lower support of the distribution of hassle costs. Then, our proof that supracompetitive prices cannot be supported in symmetric markets goes through unchanged, as do our results that hassle costs limit the ability of price-matching guarantees to facilitate high prices. Indeed, our results can be interpreted as providing an upper bound on the maximum amount that equilibrium prices can rise above P^B . If consumers differ in their hassle costs, and there does not exist a mass of consumers with zero hassle costs, one firm would still be able to undercut the other firm's price by up to z without inducing any consumers to request price matching. But it might want to undercut by even more, knowing that its losses would be limited to those consumers with the lowest hassle costs. The profitability of such a strategy would depend on the shape of the distribution.

4.4 PRICE-BEATING GUARANTEES

The last extension we consider is one in which the strategy space is extended to allow firms to adopt price-beating guarantees. This is an important extension to consider, since one might think that firms can negate the adverse effects of hassle costs if they can compensate consumers with extra incentives. To address this issue, we will

17. We thank an anonymous referee for making this observation.

18. It may be that some consumers actually have negative hassle costs—they take delight in getting a low price, even if it costs them enormous amounts of time. We will ignore such consumers.

consider two types of price-beating guarantees: price beating by a percentage of the difference in prices and price beating by a lump-sum amount.

4.4.1 PRICE BEATING BY A PERCENTAGE OF THE DIFFERENCE IN PRICES In a sample of 515 low-price guarantees, Arbatskaya et al. (1998) found that price beating by a percentage of the difference in prices accounts for approximately 32% of all low-price guarantees (price-matching guarantees are nearly twice as common). A typical example is given in Dixit and Nalebuff (1991, p. 103): "If after your purchase, you find the same model advertised or available for less... we, *Newmark & Lewis* will gladly refund (by check) 100% of the difference, plus an additional 25% of the difference." We now show that this type of price-beating guarantee does not get around the problem of price shading in the presence of hassle costs.

A simple example illustrates this point. Suppose hassle costs are \$2 and *Newmark & Lewis* advertises a price of \$294 while its rival advertises a price of \$293 for the same item. Then, even with its price-beating policy, *Newmark & Lewis's* selling price to all consumers will remain at \$294. The reason is that the \$1 difference in posted prices, plus the extra 25% (\$.25) that consumers would receive in addition were they to activate the price-beating guarantee, is not enough to compensate them for their hassle costs. If *Newmark & Lewis* had instead promised to refund double any price difference, a competitor might indeed be dissuaded from pricing at \$293—but it could still set a lower price without repercussion, e.g., by advertising a price of \$293.50. It should be clear that whatever additional percentage refund *Newmark & Lewis* might offer to consumers, its rival can always find a profitable deviation, however small, that allows it to successfully undercut *Newmark & Lewis's* selling price of \$294. It should thus be clear that the most commonly observed kinds of price-beating guarantees cannot prevent a firm from losing market share, and therefore we would expect hassle costs to continue to have a dampening effect on equilibrium prices.

4.4.2 PRICE BEATING BY A LUMP-SUM DOLLAR AMOUNT We now consider guarantees in which firms promise to match any competitor's lowest price plus pay consumers a lump-sum fee for their hassle. This type of price-beating guarantee is rare. Arbatskaya et al. (1998) found only two such instances. In one instance, an optician promised a lump sum of \$10. In the other case, a computer store promised a lump sum of \$50.

As one might expect, compensating consumers directly for their hassle, with lump-sum payments, can prevent undercutting. To see

this, consider a symmetric market in which both firms advertise the monopoly price P^M and offer to beat a rival's lower price by a lump-sum dollar amount greater than z . In this case, neither firm has an incentive to undercut the other's advertised price, since, regardless of the initial difference in prices, consumers will be induced to activate the high-priced firm's price-beating guarantees in order to collect the lump-sum payment.

However, even price-beating guarantees of this type will not, in general, restore monopoly pricing. The reason is that the above strategies may not form a Nash equilibrium. Suppose firm i advertises the monopoly price, say \$500, and promises to match any lower advertised price by firm j plus give consumers a lump-sum amount greater than z . Let $z = \$5$. In this case, if firm j wants to undercut firm i 's price by some amount, say \$50, the way to do it is not to advertise a price of \$450, which would cause consumers to activate firm i 's price guarantee, but instead to advertise a *higher* price, say \$510, and to adopt a price-beating guarantee of its own with a lump-sum offer of \$55. In this case, consumers who buy from firm j will activate firm j 's price-beating guarantee and receive the benefits of price matching plus pocket \$55, making these consumers better off by \$50.

Is this strategy profitable for firm j ? Firm j faces the same demand as it would have if it could have lowered its price to \$450 without affecting its rival's price. However, its strategy of "overcutting" costs the firm an amount equal to z for each consumer. Whether firm j 's "overcutting" strategy is profitable thus depends on whether the additional marginal cost of serving each consumer, in this case \$5, is worth the gains from unilaterally deviating from the monopoly price. If hassle costs are small enough, unilaterally deviating from the monopoly price is always profitable.¹⁹

5. CONCLUSION

It is widely believed that price-matching guarantees facilitate monopoly pricing by preventing rival firms from increasing their sales if they attempt to cut prices. As we have seen, however, an implicit assumption in the literature is that firms automatically match prices, thus ignoring the role of hassle costs. Such an omission might be justified if hassle costs are typically nonexistent, or if their existence does not matter. It is our contention in this paper that (1) hassle

19. For a more complete analysis of the effects of price-beating guarantees, see Hviid and Shaffer (1994) and Corts (1995).

costs do exist for almost all markets and (2) even arbitrarily small hassle costs render price-matching guarantees much less effective than existing theory would suggest in raising prices above the competitive level.

When there are positive hassle costs, a small amount of undercutting (say a penny) will not trigger consumer requests for a refund from the high-priced firm (due to the hassle cost involved). This implies, contrary to what is widely believed, that price cutting *can* be effective in increasing sales, and therefore that each firm will often have an incentive to slightly undercut its rival's price if the rival's price is supracompetitive. In other words, the existence of hassle costs can promote healthy competition; each firm attempts to undercut the other until either the Bertrand equilibrium is reached or a pair of prices is reached in which one firm prefers to have the higher price and the other cannot lower its price any further without inducing consumers to request price matching from the higher priced firm. Our findings suggest that the welfare gains to be had from prohibiting these guarantees are thus much smaller than previous literature has suggested. Because of this, and because there may be more benign reasons for adopting price-matching guarantees than the "facilitating practice" motive considered in this paper, we believe that recent calls (Simons, 1989; Sargent, 1993; Edlin, 1997) for outright prohibition by antitrust authorities of the use of price-matching guarantees are premature.

We also believe that firms should be wary of basing important nonprice strategic decisions on the presumption that price-matching guarantees will facilitate monopoly pricing. Consider, for example, two firms deciding where to locate in product space. Zhang (1995), who examines the role of price-matching guarantees in a two-stage Hotelling model in which firms first choose product locations and then choose prices, argues that firms should minimally differentiate their products, confident that subsequent price competition will be eliminated with price-matching guarantees. Our findings suggest that this strategy is unlikely to be profitable. If both firms locate at the same point in product space, then Proposition 3 implies that both firms will end up pricing at marginal cost and earning zero profit. Thus, in our view, it would be a mistake for the firms to adopt such a strategy.

We conclude this paper by commenting on the apparent widespread popularity of price-matching guarantees even in symmetric markets. Although our analysis finds that Nash equilibria in which one or both firms adopt price-matching guarantees exist in symmetric markets, they are redundant in the sense that they play no

role in supporting the equilibrium. This suggests that price-matching guarantees may arise for reasons other than the collusive rationale that we have examined here.

One possibility is that price-matching guarantees may arise for price-discrimination purposes (see Corts, 1997). Another possibility is that price-matching guarantees may alter consumer search behavior to the benefit of the price matcher. For instance, it may be that consumers would prefer to check prices first at a firm that does not have a price-matching guarantee before visiting *and patronizing* a firm with a price-matching guarantee. In doing so, the consumer gains if the firm with price matching happens to have the low price (since it would save on transportation costs vis-à-vis the alternative order of search), and loses little (except the hassle cost) if it turns out that the first firm had the lower price. While a complete analysis of these cases is beyond the scope of this paper, it is likely that the existence of hassle costs will be important there as well.

APPENDIX

Proof of Proposition 1. There are three cases to consider: Nash equilibria in which (1) both firms adopt price matching, (2) only firm 1 adopts price matching, and (3) firm 1 does not adopt price matching.

Case 1: Both firms adopt price-matching guarantees. If an equilibrium exists in which both firms adopt price matching, the selling prices must equal the minimum of the posted prices: $S_1 = S_2 = \min\{P_1, P_2\}$. This allows us to restrict attention to posted prices that induce selling prices along the 45° line.

Consider first the set of Nash equilibria that can be supported with equal posted prices, $P_1 = P_2 = P^*$. Then, by the reasoning in the text, it must be that $P^* \in [P^{A1}, P^M]$. To show that Nash equilibria do, in fact, exist in which $P_1 = P_2 = P^*$, suppose $\sigma_1 = (P^*, \mathcal{P}\mathcal{M})$ and $\sigma_2 = (P^*, \mathcal{P}\mathcal{M})$. Given firm j 's strategy, firm i does not want to raise its posted price and drop its price-matching guarantee, as that would result in selling prices that are farther from $BR^i(P^*)$. Nor does firm i want to lower its posted price, since that would automatically be matched by its rival, moving selling prices down the 45° line away from P^M . Thus, these strategies form a Nash equilibrium.

Now consider the set of Nash equilibria that can be supported with unequal posted prices. Without loss of generality, assume $P^M = P^{Ci}$. Then no Nash equilibrium exists in which $P_i > P_j$. To see this, suppose not. Then we know that $P_j \in [P^{A1}, P^M]$. However, by raising its posted price to $\min\{P^{Cj}, P_i\}$, firm j can strictly increase its profit, a contradiction.

If an equilibrium exists in which $P_j > P_i$, then $P_i = P^M$, for if $P_i > P^M$, firm i can increase its profit by lowering its price, and if $P_i < P^M$, firm i can increase its profit by raising its price. Now consider the pair of strategies $\sigma_i = (P^M, \mathcal{PM})$ and $\sigma_j = (\tilde{P}_j, \mathcal{PM})$, where $\tilde{P}_j > P^M$. We have shown that firm i does not want to deviate in this situation. To see that firm j does not want to deviate, note that at $P_i = P^M$ firm j wants to undercut firm i 's price but is prevented from doing so by firm i 's price-matching guarantee. Thus, these strategies form a Nash equilibrium.

Case 2: Only firm 1 adopts price matching. If an equilibrium exists in which only firm 1 adopts price matching, it must be that $P_2 = S_2$ and $S_1 = \min(P_1, P_2)$, implying that $S_1 \leq S_2$. It must also be the case that $S_1 = \text{BR}^1(P_2)$, for otherwise, firm 1 could increase its profit by dropping its price-matching guarantee and setting $P_1 = \text{BR}^1(P_2)$. This would increase firm 1's profit by forcing selling prices to be on BR^1 .

Thus, there are three possibilities to consider: selling prices are (1) below \mathbf{P}^{A1} on BR^1 , (2) above \mathbf{P}^{A1} on BR^1 , or (3) equal to \mathbf{P}^{A1} . Selling prices below \mathbf{P}^{A1} on BR^1 cannot arise in equilibrium, since that would entail $S_1 > S_2$, a contradiction. Selling prices above \mathbf{P}^{A1} on BR^1 cannot arise in equilibrium, since that would imply $S_1 = P_1 < P_2 = S_2$, and firm 2 would be better off lowering its posted price to P_1 . Such a deviation would move selling prices in the direction of $\text{BR}^2(P_1)$, implying an increase in its profit. The only possibility left is that the selling prices occur at the price pair \mathbf{P}^{A1} . This implies that $P_1 = P_2 = P^{A1}$, for $P_1 < P_2$ would induce selling prices other than \mathbf{P}^{A1} , and $P_1 > P_2$ would allow firm 2 to increase its profit by raising its posted price.

To show that Nash equilibria exist in which $P_1 = P_2 = P^{A1}$, suppose $\sigma_1 = (P^{A1}, \mathcal{PM})$ and $\sigma_2 = (P^{A1}, \text{no } \mathcal{PM})$. Given firm 2's strategy, firm 1 has no incentive to deviate, since $S_1 = \text{BR}^1(S_2)$. Given firm 1's strategy, firm 2 has no incentive to increase its posted price above P^{A1} , since this would result in selling prices that are farther away from $\text{BR}^2(P_1)$. Nor does it have an incentive to decrease its posted price below P^{A1} , since that would result in selling prices on the 45° line below \mathbf{P}^{A1} (farther away from \mathbf{P}^M). Hence, these strategies form a Nash equilibrium.

Case 3: Firm 1 does not adopt price matching. It is trivial to show that if neither firm adopts price matching, then the unique equilibrium in posted prices is $(P_1, P_2) = \mathbf{P}^B$.

If an equilibrium exists in which only firm 2 adopts price matching, then it must be that $S_2 \leq S_1$, $P_1 = S_1$, and $S_2 = \text{BR}^2(S_1)$. Thus, in this case, we can restrict attention to equilibrium selling

prices on BR^2 . Selling prices below \mathbf{P}^{A2} on BR^2 cannot arise in equilibrium, since that would entail $S_2 > S_1$, a contradiction. Selling prices weakly higher than \mathbf{P}^{A2} but below \mathbf{P}^B also cannot arise in equilibrium, since firm 1 can increase its profit by raising its posted price to $BR^1(P_2)$. Selling prices above \mathbf{P}^B on BR^2 likewise cannot arise in equilibrium, since firm 1 can increase its profit by lowering its posted price. Thus, if an equilibrium exists in which only firm 2 adopts price matching, selling prices must be \mathbf{P}^B , implying that $P_1 = P_1^B$ and $P_2 = P_2^B$. It is straightforward to verify that $\sigma_1 = (P_1^B, \text{no } \mathcal{P}\mathcal{M})$ and $\sigma_2 = (P_2^B, \text{no } \mathcal{P}\mathcal{M})$ form a Nash equilibrium. \square

Proof of Proposition 2. If an equilibrium exists in which both firms adopt price-matching guarantees, we can use the reasoning in the text to establish that $S_1 = S_2$ and $\mathbf{P}^{A1} < (S_1, S_2) < \mathbf{P}^M$. But since $\mathbf{P}^{A1} \gg \mathbf{P}^M$ is now the maintained assumption, this interval is empty. Therefore, there does not exist a Nash equilibrium in which both firms adopt price matching.

If an equilibrium exists in which only firm 1 adopts a price-matching guarantee, we can use the reasoning from case 2 in the proof of Proposition 1 to establish that $(S_1, S_2) = \mathbf{P}^{A1}$ and $S_2 = P_2$. But these selling prices cannot arise in equilibrium, since, given the maintained assumption $\mathbf{P}^{A1} \gg \mathbf{P}^M$, firm 2 can increase its profit by lowering its posted price to P^M . Therefore, there does not exist a Nash equilibrium in which only firm 1 adopts a price-matching guarantee.

If an equilibrium exists in which firm 1 does not adopt a price-matching guarantee, we can use the reasoning from case 3 in the proof of Proposition 1 to establish that $(P_1, P_2) = \mathbf{P}^B$. We leave it to the reader to verify that $\sigma_1 = (P_1^B, \text{no } \mathcal{P}\mathcal{M})$ and $\sigma_2 = (P_2^B, \text{no } \mathcal{P}\mathcal{M})$ form a Nash equilibrium.

Thus, we have shown that if $\mathbf{P}^{A1} \gg \mathbf{P}^M$, a Nash equilibrium in the absence of hassle costs exists if and only if firm 1 does not adopt price matching and $(P_1, P_2) = \mathbf{P}^B$. \square

Proof of Lemma 1. Let $z > 0$ and $\delta_i = \mathcal{P}\mathcal{M}$, and suppose, to the contrary, that a Nash equilibrium exists in which firm i chooses $P_i > P_j + z$. Then, since all consumers who buy from firm i will activate its price-matching guarantee, firm i 's profit is $(P_j - c_i)D^i(P_j + z, P_j)$. However, by choosing instead $P_i = P_j$, firm i 's profit strictly increases to $(P_j - c_i)D^i(P_j, P_j)$, a contradiction. \square

Proof of Lemma 2. It cannot be an equilibrium for firm i to post $P_i < BR^i(P_j)$, since firm i can strictly increase its profit by dropping

any price-matching guarantee it may have and increasing its posted price. It also cannot be an equilibrium for $P_j - z > P_i > \text{BR}^i(P_j)$, since that would violate Lemma 1 if firm j has a price-matching guarantee, and if firm j does not have a price-matching guarantee, firm i can increase its profit by decreasing its posted price. Finally, it cannot be an equilibrium for $P_i > P_j - z$ and $P_i > \text{BR}^i(P_j)$, since firm i can increase its profit by decreasing its posted price. \square

Proof of Proposition 3. Using Lemma 2, there are four possible pairings of posted prices. If $z > P_1^B - P_2^B$, however, only one of these pairings is feasible: $P_1 = \text{BR}^1(P_2)$ and $P_2 = \text{BR}^2(P_1)$. Thus, it must be that $(P_1, P_2) = \mathbf{P}^B$ in any Nash equilibrium. Since hassle costs exceed the difference in Bertrand prices, \mathbf{P}^B can be supported by any combination of price-matching policies. That is, it is straightforward to show that the strategy pairs (1) $(P_1^B, \text{no } \mathcal{PM})$ and $(P_2^B, \text{no } \mathcal{PM})$, (2) $(P_1^B, \text{no } \mathcal{PM})$ and (P_2^B, \mathcal{PM}) , (3) (P_1^B, \mathcal{PM}) and $(P_2^B, \text{no } \mathcal{PM})$, and (4) (P_1^B, \mathcal{PM}) and (P_2^B, \mathcal{PM}) all form Nash equilibria. \square

Proof of Propositions 4 and 5. Using Lemma 2, there are four possible pairings of posted prices that may arise in equilibrium. We consider each in turn.

Case 1: $P_1 = \text{BR}^1(P_2)$ and $P_2 = \text{BR}^2(P_1)$. This implies $(P_1, P_2) = \mathbf{P}^B$. But, using Lemma 1, this price pair cannot arise in equilibrium if $z < P_1^B - P_2^B$ and firm 1 adopts a price-matching guarantee. It is easy to verify, however, that $(P_1, P_2) = \mathbf{P}^B$ can arise in equilibrium if firm 1 does not adopt price matching.

Case 2: $P_1 = \text{BR}^1(P_2)$ and $P_1 - z = P_2 > \text{BR}^2(P_1)$. In this case, it must be that $(P_1, P_2) = \mathbf{P}^Z$. If firm 1 does not adopt price matching, then firm 2 can increase its profit by decreasing P_2 . Hence, $(P_1, P_2) = \mathbf{P}^Z$ cannot arise in equilibrium if firm 1 does not adopt price matching. We now check whether $(P_1, P_2) = \mathbf{P}^Z$ can arise in equilibrium if firm 1 does adopt price matching. It is easy to verify that (1) (P_1^Z, \mathcal{PM}) and (P_2^Z, \mathcal{PM}) and (2) (P_1^Z, \mathcal{PM}) and $(P_2^Z, \text{no } \mathcal{PM})$ form a Nash equilibrium if and only if $P_2^0 \geq P_2^Z$.

Case 3: $P_2 - z = P_1 > \text{BR}^1(P_2)$ and $P_2 = \text{BR}^2(P_1)$. This combination of inequalities is not satisfied by any price pair. Hence, no equilibrium can arise with these characteristics.

Case 4: $P_2 - z = P_1 > \text{BR}^1(P_2)$ and $P_1 - z = P_2 > \text{BR}^2(P_1)$. It is not possible for firm 2's posted price to exceed firm 1's posted price and vice versa. Hence, no equilibrium can arise with these characteristics.

In summary, the combined analysis in the four cases yields Propositions 4 and 5. \square

Proof of Lemma 3. The third condition arises because firm i cannot drop its price-matching guarantee (if it has one). It cannot be an equilibrium for $P_j + z > P_i < BR^i(P_j)$, since firm i could then strictly increase its profit by increasing its posted price. It also cannot be an equilibrium for $P_j + z < P_i < BR^i(P_j)$, since that would violate Lemma 1 if firm i has a price-matching guarantee, and if firm i does not have a price-matching guarantee, then firm i can strictly increase its profit by setting $P_i = BR^i(P_j)$. \square

Proof of Proposition 6. There are five cases to consider in addition to those in the proof of Propositions 4 and 5. Of these, only two cases yield a feasible combination of price pairs.

Case 1: $P_2 + z = P_1 < BR^1(P_2)$ and $P_2 = BR^2(P_1)$. In this case, it must be that $(P_1, P_2) = \tilde{\mathbf{P}}^Z$. If firm 1 has not adopted price matching, then firm 1 can strictly increase its profit by increasing P_1 . Hence, $(P_1, P_2) = \tilde{\mathbf{P}}^Z$ cannot arise in equilibrium if firm 1 has not adopted price matching. It is easy to verify that $(P_1, P_2) = \tilde{\mathbf{P}}^Z$ form a Nash equilibrium in the two subgames in which firm 1 has adopted a Nash equilibrium.

Case 2: $P_2 + z = P_1 < BR^1(P_2)$ and $P_1 - z = P_2 > BR^2(P_1)$. In this case, it must be that $(P_1, P_2) \in \{(P_1, P_2) \mid P_2 = P_1 - z \text{ and } \tilde{\mathbf{P}}^Z \ll (P_1, P_2) \ll \mathbf{P}^Z\}$. It is easy to verify that all price pairs that satisfy these criteria can arise in equilibrium if and only if both firms adopt price matching in stage 1.

In summary, the combined analysis in the nine cases yields Proposition 6. \square

Proof of Proposition 7. A subgame perfect equilibrium that supports selling prices at \mathbf{P}^Z is: firm 1 adopts price matching and chooses P_1^Z if firm 2 adopts price matching, and P_1^B if otherwise; firm 2 adopts price matching and chooses P_2^Z if firm 1 adopts price matching, and P_2^B if otherwise.

A subgame perfect equilibrium that supports selling prices at \mathbf{P}^B is: firm 1 does not adopt price matching and chooses \tilde{P}_1^Z if firm 2 adopts price matching, and P_1^B if otherwise; firm 2 does not adopt price matching and chooses \tilde{P}_2^Z if firm 1 adopts price matching, and P_2^B if otherwise. \square

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