

# On Vertical Restrictions and the Number of Franchises: Comment

## I. Introduction

In a recent issue of this *Journal*, Phillips [3, 423–29] examines the vertical control problem of an upstream monopoly when the number of its retailers is endogenous. He compares a situation in which the upstream firm chooses a two-part tariff contract to maximize its profit with one in which the upstream firm combines a two-part tariff contract with resale price maintenance. Phillips finds that resale price maintenance increases the upstream firm's profit, and in both situations, concludes that the “monopolist will set the wholesale price *below* its marginal cost in order to maximize profits through the franchise fee” [3, 423]. However, his analysis is incorrect.

In this note, I solve Phillips's model and find that the profit maximizing two-part tariff contract *never* exhibits a wholesale price below marginal cost. Furthermore, I show that resale price maintenance does *not* increase the upstream firm's profit. The intuition for this latter finding is that the upstream firm's profit maximizing two-part tariff contract suffices to control perfectly the number of retailers and the retail price, yielding the same profit as would be earned by a vertically integrated firm. Resale price maintenance in this instance is redundant.

## II. Phillips' Model and Notation

Phillips considers a market in which an upstream monopolist sells its product through multiple retailers each with its own distinct geographic territory. The upstream firm is initially restricted to choosing a two-part tariff contract, that is, to setting a franchise fee ( $F$ ) and wholesale price ( $P$ ). Although the upstream firm cannot directly determine the number of retailers that will sell its product or the retail price, it recognizes that both variables are indirectly influenced by its choice of two-part tariff. According to Phillips, “As  $P$  and  $F$  are set, the number of franchises is determined under the conditions that franchisees maximize profits through their choice of [retail price] and that they break even in the downstream market” [3, 425].

The retail equilibrium is summarized in reduced form by  $N(F, P)$ , which is the equilibrium number of retailers as a function of the franchise fee and wholesale price, and  $S(F, P)$ , which is the equilibrium sales made by the upstream firm to each retailer. Several sign restrictions are imposed. First,  $N(F, P)$  is assumed to be decreasing in both the franchise fee and wholesale price, as both decrease the profitability of owning a franchise. Second,  $S(F, P)$  is assumed to be increasing in the franchise fee—because increasing  $F$  reduces the total number of retailers thereby increasing each retailer's market area—and decreasing in the wholesale price—because higher wholesale prices lead to higher retail prices and thus less sales to consumers.

The upstream firm's problem is to choose  $(F, P)$  to maximize

$$\Pi_m = P \cdot N(F, P) \cdot S(F, P) - C(N \cdot S) + N(F, P) \cdot F, \quad (1)$$

where  $C(N \cdot S)$  denotes the cost of producing  $N(F, P) \cdot S(F, P)$  units. Writing the first order conditions with respect to  $F$  and  $P$  in elasticity terms gives

$$1 + \eta_{NF} + (P - C_x) \cdot (S/F) \cdot (\eta_{NF} + \eta_{SF}) = 0, \tag{2}$$

$$(F/P) \cdot \eta_{NP} + S + [(P - C_x)/P] \cdot S \cdot (\eta_{NP} + \eta_{SP}) = 0, \tag{3}$$

where  $C_x$  is production marginal cost and  $\eta_{ij} = (\partial i/\partial j) \cdot (i/j)$ . After some manipulation, these two first order conditions can be rewritten as

$$F/(P \cdot S) = (\eta_{NF} + \eta_{SF})/[\eta_{SP}(1 + \eta_{NF}) + \eta_{NP}(1 - \eta_{SF})], \tag{4}$$

$$(P - C_x)/P = -(1 + \eta_{NF})/[\eta_{SP}(1 + \eta_{NF}) + \eta_{NP}(1 - \eta_{SF})]. \tag{5}$$

From these equations, which are numbers (5) and (6) in [3, 426], Phillips finds that the upstream firm always charges a positive franchise fee and sets its wholesale price below marginal cost. However, his proof follows from an incorrect derivation of the second order sufficient condition  $\partial^2 \Pi_m / \partial F^2 < 0$ . Phillips's mistake was to assume that  $\eta_{NF}$  and  $\eta_{SF}$  were independent of  $F$ . In fact, these elasticities are complicated functions of the franchise fee and can only be calculated from explicit derivation of the retail equilibrium, as done in the appendix. Since these elasticities depend on  $F$ , Phillips's proof is incorrect, and thus his qualitative conclusions are unsubstantiated.

### III. Equilibrium Two-part Tariff

Phillips also finds that resale price maintenance increases the upstream firm's profit. If true, this would imply that the profit maximizing wholesale price and franchise fee, in the absence of resale price maintenance, is insufficient to induce the upstream firm's optimal number of retailers and retail price. Yet in the remainder of this section, I demonstrate that the profit maximizing two-part tariff contract does induce the upstream firm's optimal number of retailers and retail price, and therefore yields the same profit as would be earned by a vertically integrated firm. Thus, Phillips's conclusion is incorrect; resale price maintenance does not increase the upstream firm's profit. Moreover, I also show that the profit maximizing wholesale price is never below-cost.

To establish the retail price and number of retailers that would be chosen by a vertically integrated firm, let  $R$  denote the retail price,  $N$  denote the number of retailers,  $\tilde{S}$  denote individual retail sales as a function of  $(R, N)$ , and  $G$  denote the fixed set-up costs of opening a retail outlet. A vertically integrated firm's profit maximizing choice of  $R$  and  $N$  is then determined by

$$\max_{R, N} R \cdot N \cdot \tilde{S}(R, N) - C(N \cdot \tilde{S}) - N \cdot G. \tag{6}$$

The first order conditions with respect to  $R$  and  $N$  are

$$(R - C_x) \cdot \partial \tilde{S}(R, N) / \partial R + \tilde{S}(R, N) = 0, \tag{7}$$

$$(R - C_x) \cdot N \cdot \partial \tilde{S}(R, N) / \partial N + (R - C_x) \cdot \tilde{S}(R, N) - G = 0. \tag{8}$$

Let  $(R^I, N^I)$  be the solution, where the superscript  $I$  stands for the vertically integrated firm's optimal choice. Substituting these values back into (6) gives the vertically integrated profit as

$$\Pi_m^I = R^I \cdot N^I \cdot \tilde{S}(R^I, N^I) - C(N^I \cdot \tilde{S}) - N^I \cdot G. \quad (9)$$

I now compare the vertically integrated profit with the upstream firm's equilibrium profit when it indirectly controls  $R$  and  $N$  through its wholesale price and franchise fee.

Since retailers are identical, and assumed to have their own distinct geographic territory, there are only two conditions that must be simultaneously satisfied in equilibrium.<sup>1</sup> Given  $N$ , the retailer's profit maximizing choice of  $R$  is determined by the first order condition

$$(R - P) \cdot \partial \tilde{S}(R, N) / \partial R + \tilde{S}(R, N) = 0. \quad (10)$$

Given  $R$ , the number of franchises is determined by the break-even constraint

$$(R - P) \cdot \tilde{S}(R, N) - G - F = 0. \quad (11)$$

Solving (10) and (11) simultaneously yields the equilibrium retail price and number of franchises as functions of the upstream firm's franchise fee and wholesale price. Using Phillips's notation, the solution is denoted by  $R = R(F, P)$  and  $N = N(F, P)$  respectively. Substituting into individual retail sales yields  $\tilde{S}(R(F, P), N(F, P)) \equiv S(F, P)$ .

Comparing (7) and (8) with (10) and (11), one finds that  $(R^I, N^I)$  can indeed be induced as a downstream equilibrium. To see this, suppose the upstream firm were to choose its wholesale price equal to production marginal cost, that is,  $P = P^* = C_x$ , and its franchise fee such that  $F = F^* = -(R^I - C_x) \cdot N^I \cdot \partial \tilde{S}(R^I, N^I) / \partial N$ . Substituting  $(P^*, F^*)$  into (10) and (11) and solving for  $R$  and  $N$  gives

$$R^I = R(F^*, P^*) \text{ and } N^I = N(F^*, P^*).$$

This means that the upstream firm can induce with its wholesale price and franchise fee the same retail price and number of retailers that a vertically integrated firm would choose. It remains to show that  $(P^*, F^*)$  is indeed profit maximizing for the upstream firm. Evaluating the upstream firm's maximization problem in (1) at  $(P^*, F^*)$  gives

$$C_x \cdot N^I \cdot \tilde{S}(R^I, N^I) - C(N^I, \tilde{S}) + N^I F^*.$$

But from the break-even constraint given in (11), it is seen that  $F^* = (R^I - C_x) \cdot \tilde{S}(R^I, N^I) - G$ , which implies that the upstream firm's profit evaluated at  $(P^*, F^*)$  can be rewritten as

$$R^I \cdot N^I \cdot \tilde{S}(R^I, N^I) - C(N^I, \tilde{S}) - N^I \cdot G. \quad (12)$$

Since the profit in (12) is the same as the vertically integrated profit in (9), and since the latter is the maximum possible profit, it can be concluded that  $(P^*, F^*)$  is indeed the upstream firm's profit maximizing wholesale price and franchise fee.<sup>2</sup> The intuition is that the two instruments,  $P$  and  $F$ , suffice to control perfectly the two targets,  $R$  and  $N$ . An immediate implication is that the

1. The model can easily generalize to allow each retailer's sales to depend in addition on its rivals' prices. In that case, there would be  $N + 1$  conditions to be satisfied.

2. From the elasticity terms derived in the appendix, it is easily verified that the upstream firm's first order conditions given in (4) and (5) are satisfied at  $(P^*, F^*)$ .

imposition of resale price maintenance cannot further increase the upstream firm's profit, contrary to Phillips' claim.

#### IV. Conclusion

This note has considered the vertical control problem of an upstream firm when the number of its retailers is endogenous. Assuming, as did Phillips, that retailers were granted exclusive territories, I found that the upstream firm should choose its wholesale price equal to production marginal cost and set its franchise fees to extract retailer surplus. Since this yields the same profit as would be earned by a vertically integrated firm, it follows that resale price maintenance cannot do better.<sup>3</sup>

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3. The model can also be readily solved if consumers can substitute among retailers. In that instance, previous literature has shown that two-part tariffs suffice to achieve the vertically integrated profit and that the upstream firm's profit maximizing wholesale price strictly exceeds production marginal cost [1, 497-517; 2, 1293-1311]. Once again, it is never optimal for the upstream firm to set its wholesale price below marginal cost, as claimed by Phillips, nor does resale price maintenance increase profit.

#### Appendix

##### *Determination of Elasticity Terms*

Let  $R$  denote the retail price,  $N$  denote the number of retailers,  $\tilde{S}$  denote individual retail sales as a function of  $(R, N)$ , and  $G$  denote the fixed set-up costs of opening a retail outlet. Then the downstream equilibrium is characterized by the retailer's first order condition for profit maximization

$$(R - P) \cdot \partial \tilde{S}(R, N) / \partial R + \tilde{S}(R, N) = 0,$$

and the break-even constraint

$$(R - P) \cdot \tilde{S}(R, N) - G - F = 0.$$

Totally differentiating this system of equations and setting it up in matrix form gives

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{pmatrix} dR \\ dN \end{pmatrix} = \begin{bmatrix} \partial \tilde{S} / \partial R & 0 \\ \tilde{S} & 1 \end{bmatrix} \begin{pmatrix} dP \\ dF \end{pmatrix},$$

where

$$\begin{aligned} A &= (R - P) \cdot \partial^2 \tilde{S} / \partial R^2 + 2 \cdot \partial \tilde{S} / \partial R < 0, \\ B &= (R - P) \cdot \partial^2 \tilde{S} / \partial R \partial N + \partial \tilde{S} / \partial N < 0, \\ C &= (R - P) \cdot \partial \tilde{S} / \partial R + \tilde{S} = 0, \\ D &= (R - P) \cdot \partial \tilde{S} / \partial N < 0. \end{aligned}$$

The signs of  $B$  and  $D$  follow from the restrictions imposed by Phillips. The sign of  $A$  is negative by concavity

of the retailer's profit function, and the sign of  $C$  is zero from the retailer's first order condition. Using Cramer's rule, the following derivatives can be determined:

$$dR/dP = ((\partial\tilde{S}/\partial R) \cdot D - \tilde{S} \cdot B)/(A \cdot D) > 0, \quad dR/dF = -B/(A \cdot D) > 0,$$

$$dN/dP = \tilde{S}/D < 0, \quad dN/dF = 1/D < 0.$$

Finally, using these derivatives, the following elasticity terms can be calculated

$$\eta_{NF} = (F/N) \cdot (\partial N/\partial F) = F/(N \cdot (R - P) \cdot \partial\tilde{S}/\partial N) < 0,$$

$$\eta_{NP} = (P/N) \cdot (\partial N/\partial P) = (P \cdot \tilde{S})/(N \cdot (R - P) \cdot \partial\tilde{S}/\partial N) < 0,$$

$$\eta_{SF} = (F/S) \cdot (\partial S/\partial F) = (F/S)((\partial\tilde{S}/\partial R) \cdot (\partial R/\partial F) + (\partial\tilde{S}/\partial N) \cdot (\partial N/\partial F)) > 0,$$

$$\eta_{SP} = (P/S) \cdot (\partial S/\partial P) = (P/S)((\partial\tilde{S}/\partial R) \cdot (\partial R/\partial P) + (\partial\tilde{S}/\partial N) \cdot (\partial N/\partial P)) < 0,$$

where  $S(F, P) \equiv \tilde{S}(R(F, P), N(F, P))$  and the signs of the elasticity terms follow from the restrictions imposed by Phillips.

## References

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3. Phillips, Owen R., "Vertical Restrictions and the Number of Franchises." *Southern Economic Journal*, October 1991, 423-29.