

Optimal asymmetric strategies in research joint ventures

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Abstract

This paper identifies an overlooked implication of models of research joint ventures initiated by d'Aspremont and Jacquemin (1988). Even though the aggregate R&D cost of identical firms in a research joint venture would be lowest if they invested equally to reduce subsequent production costs, nonetheless members may often enlarge their overall joint profit by making unequal investments. Such a strategy raises costs in the investment stage but may create more than offsetting benefits in the production stage since industry profits are larger there when the firms are of unequal size. When the consideration leading to asymmetry prevails, we find that, in contrast to previous work, a research joint venture can raise welfare even when there are no spillovers. © 1998 Elsevier Science B.V.

Keywords: Research and development; Cartels; Sidepayments; Asymmetry; Non-convexity

JEL classification: D43; L13; L22

1. Introduction

In their analysis of the consequences of research joint ventures, d'Aspremont and Jacquemin (1988) deduce from the hypothesis of joint-profit maximization the behavior of two firms which are allowed to coordinate investments in cost-reducing R&D but must remain non-cooperative rivals in the product market. Their paper has stimulated further work by De Bondt and Veugelers (1991).

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Kamien et al. (1992), Meister (1992), Suzumura (1992), Motta (1992), Vonortas (1994), Ziss (1994), Poyago-Theotoky (1995), Steurs (1995), and many others.

Since firms are assumed identical and the cost of investing at each firm is assumed to be strictly convex, all contributors to this literature have restricted their search for the investment combination which maximizes joint profits to situations where each member of the research joint venture invests equally. This seemingly innocuous restriction has led to error since joint profits may often be increased if the two firms make *unequal* investments. This error in the positive analysis has in turn contaminated some of the literature's normative conclusions about the desirability of permitting research joint ventures. We use the seminal article of d'Aspremont and Jacquemin to make these points; however, the observations we make apply equally to the subsequent literature.¹

The paper is organized as follows. In Section 2, we explain why asymmetric investments sometimes maximize joint profits. In Section 3, we characterize the solution to the joint-profit maximization problem of a research cartel. In Section 4, we discuss the welfare implications of our analysis. In Section 5, we ask whether the literature's assumption of joint-profit maximization remains appropriate in the absence of side payments. Section 6 concludes the paper.

2. The model of d'Aspremont and Jacquemin

In the model of d'Aspremont and Jacquemin, two firms invest in cost-reducing R&D and then face an inverse demand of $P = a - b(q_1 + q_2)$, where P is the market price and q_i is the quantity produced by firm i , for $i = 1, 2$. Any investment by firm i which lowers its own constant marginal cost of production by \$1 is assumed to spill over to the other firm lowering the rival's constant marginal cost by β . If x_i denotes the amount of investment that firm i undertakes and x_j the amount of investment by its rival, then the production cost function for firm i is given by $(A - x_i - \beta x_j)q_i$, for $i, j = 1, 2$, and $i \neq j$. The cost of investment is also assumed symmetric and is given by $\gamma x_i^2/2$. The exogenous parameters must satisfy the following: $0 < A < a$, $0 < \beta < 1$, and $\gamma > 0$.

As did d'Aspremont and Jacquemin, assume that firms cooperate in their choice

¹In particular, confining attention to equal investments also results in error in, among others, Kamien et al. (1992), Suzumura (1992), and Steurs (1995) since there exists in each instance an admissible region of parameter space where higher joint profits can be achieved when investments are unequal. Meister (1992) reports numerical simulations where two identical firms simultaneously choose one of three levels of R&D and then compete as Cournot duopolists. Some of these simulations suggest that joint profits might be higher if the two firms did not invest identically. Nevertheless, Meister commits the same error as the rest of the literature when extending d'Aspremont and Jacquemin to the n firm case.

of investment levels to maximize joint profits, knowing that they will subsequently compete as Cournot rivals in the product market. If both outputs in the second stage are strictly positive and form a Nash equilibrium given the investments at the first stage, then d’Aspremont and Jacquemin show (p. 1134) that joint profits may be written as the following function of these investments:

$$\hat{\Pi} = \sum_{i=1}^2 \left\{ \frac{1}{9b} [(a - A) - (2 - \beta)x_i + (2\beta - 1)x_j]^2 - \gamma \frac{x_i^2}{2} \right\}, \quad j \neq i. \quad (1)$$

Limiting their search for the joint-profit maximizing investment combination to situations where the firms invest equally, d’Aspremont and Jacquemin propose the following solution

$$\hat{x}_i = \frac{(\beta + 1)(a - A)}{4.5b\gamma - (\beta + 1)^2}$$

and

$$\hat{q}_i = \hat{Q}/2 = \frac{(a - A)}{3b} \left[\frac{4.5b\gamma}{4.5b\gamma - (1 + \beta)^2} \right].$$

The following parameter restriction, reported by them in their footnote 7, ensures that $\hat{\Pi}$ is strictly concave along the path of equal investments: $b\gamma > (1 + \beta)^2/4.5$. We represent this restriction as the set of combinations of β and $b\gamma$ above the upward-sloping curve in Fig. 1. Even when this restriction is satisfied, however, the exogenous parameters may simultaneously satisfy the following condition: $b\gamma < 2(1 - \beta)^2$. We represent this second restriction as the set of combinations of β and $b\gamma$ below the downward-sloping curve in Fig. 1.

When both restrictions hold, i.e., in the shaded region, the symmetric solution

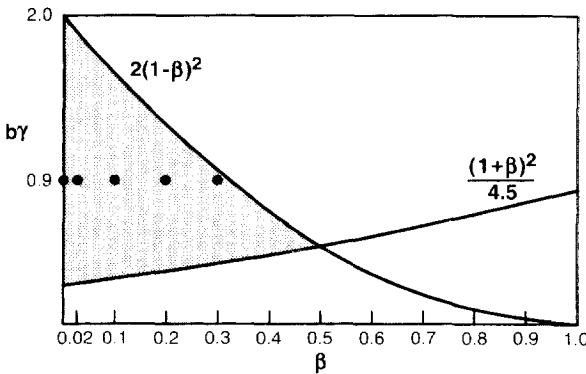


Fig. 1. Region where asymmetric investments are optimal in d’Aspremont and Jacquemin’s model.

that d'Aspremont and Jacquemin identify is never optimal.² Indeed, as we now show, *any* other allocation of the same total investment between the two firms generates higher profits provided it induces duopoly in the second stage. To see this, let k be any positive constant and substitute $x_2 = k - x_1$ into Eq. (1). Joint profits can then be expressed as:

$$\hat{\Pi}(x_1) = F + \frac{x_1(x_1 - k)(2(1 - \beta)^2 - b\gamma)}{b},$$

where F depends only on the exogenous parameters. Differentiating this profit function twice with respect to firm 1's investment, we obtain:

$$\frac{d^2 \hat{\Pi}}{dx_1^2} = \frac{2(2(1 - \beta)^2 - b\gamma)}{b}, \quad (2)$$

which is strictly positive for parameters in the shaded region of Fig. 1. Hence, in this region, joint profits are strictly convex in firm 1's investment if firm 2's investment is altered so as to maintain a fixed sum and the investments induce both firms to produce strictly positive amounts. Since a local change in the neighborhood of d'Aspremont and Jacquemin's proposed solution has no effect on profits and since joint profits are strictly convex when total investment is constant, some non-local changes in the allocation of the same total investment must strictly increase joint profits.

The intuition for this result is the following. As long as the *sum* of the investments is constant (with each investment being non-negative), changing the R&D investment at each firm in the joint venture will not alter the sum of marginal production costs. Furthermore, if the sum of marginal production costs is unaltered and all firms continue to produce in the second-stage Cournot equilibrium then, as Bergstrom and Varian (1985) showed, aggregate output, price, and hence industry gross revenue in the second stage will remain unchanged. Since gross revenue is constant, changes in joint profit can only arise because of variations in joint costs: R&D costs and production costs. Making R&D investments differ across firms *does* raise R&D costs but — as Bergstrom and Varian (1985a) first showed in another context — it also *lowers* production costs since more of the unchanged aggregate output would be produced by the lower cost

²In a comment on the d'Aspremont and Jacquemin model, Henriques (1990) considered a game in which firms act non-cooperatively in both output and R&D. She proposed restricting attention to the region of parameter space where the non-cooperative solution is stable in the sense that the reduced form first-stage reaction functions in R&D space cross 'correctly'. See Eq. (6) on p. 639 of her article. Our analysis concerns a different game since we allow cooperation at the first stage. Nonetheless, there exist parameters that satisfy her restriction and also lie within the shaded region in Fig. 1. Hence, her parameter restriction does not eliminate the problem we have identified.

firm. The latter production-cost effect dominates the former R&D-cost effect throughout the region of parameter space shaded in Fig. 1.

As the decreasing curve in Fig. 1 reflects, the considerations leading to asymmetry are more likely to dominate the lower is the spillover parameter (β) and the lower is the R&D cost parameter (γ). Intuitively, the smaller the spillover parameter (β), the larger will be the gap in the marginal costs created by any given reallocation of a fixed aggregate investment, and hence the *larger* will be the second-stage effect in reducing joint production costs. Similarly, the smaller the slope of the marginal cost of R&D investment (γ), the smaller will be the effect on aggregate R&D costs of making the investments unequal.

3. Characterizing the optimal investment combination

To identify what pair of investments is optimal for parameters in the shaded region of Fig. 1, the problem of maximizing joint profits must be reconsidered. The following preliminary observations reduce the number of cases that must be taken into account: (1) it is never optimal for both firms to produce zero output, and (2) it is never optimal for both firms to make zero investment. Denote the firm producing the weakly larger output at the optimum as firm 2. Observation 1 implies $q_2 > 0$. Since firm 2 necessarily makes the weakly larger investment, Observation 2 implies $x_2 > 0$. We incorporate these observations in our formulation of the joint-profit maximization problem:

$$\begin{aligned} \max_{x_1, q_1, x_2, q_2} & (a - bq_1 - bq_2)(q_1 + q_2) - (A - x_1 - \beta x_2)q_1 - (A - x_2 - \beta x_1)q_2 \\ & - 0.5\gamma x_1^2 - 0.5\gamma x_2^2 \end{aligned}$$

subject to:

$$x_2 > 0, \quad x_1 \geq 0, \tag{3}$$

$$q_2 \geq q_1, \quad q_2 > 0, \quad q_1 \geq 0, \tag{4}$$

$$a - 2bq_2 - bq_1 - (A - x_2 - \beta x_1) = 0, \tag{5}$$

$$a - 2bq_1 - bq_2 - (A - x_1 - \beta x_2) \leq 0, \tag{6}$$

$$q_1[a - 2bq_1 - bq_2 - (A - x_1 - \beta x_2)] = 0. \tag{7}$$

Since firm 2's output is set to maximize its profits at the second stage for any pair of investments and for any output of firm 1, no local *increase* or *decrease* in firm 2's output will affect its profits. This is reflected in condition (5). Condition (6) states that firm 1's marginal revenue must be less than or equal to its marginal cost at an optimum. We will henceforth refer to the left hand side of (6) as firm 1's

marginal profit ($M\Pi_1$). Condition (7) is a complementary slackness condition: If firm 1 produces a strictly positive amount at an optimum, then its marginal profit must be zero; if its marginal profit is strictly negative, then it must produce nothing.

Firm 1 may or may not produce at the second stage and may or may not invest at the first stage. Hence, in addition to cases such as the one d'Aspremont and Jacquemin considered in which both firms invest and produce ($q_1 > 0, x_1 > 0$), there are three other possibilities for firm 1: firm 1 (1) produces but does not invest ($q_1 > 0, x_1 = 0$), (2) neither produces nor invests ($q_1 = 0, x_1 = 0$), or (3) invests but does not produce ($q_1 = 0, x_1 > 0$).

In principle, there are two possible subcases associated with each case where firm 1 produces nothing. That is, Cases 2 and 3 can each arise in two ways: a local increase in firm 1's production above zero can strictly decrease firm 1's profit on the one hand ($M\Pi_1 < 0$) or can leave it unchanged on the other ($M\Pi_1 = 0$). Hence, in addition to solutions like the one proposed by d'Aspremont and Jacquemin, there are, in principle, five other candidates where an optimum might occur.

However, one of these possibilities can only arise in the extreme case of no spillovers. Although it remains a logical possibility that firm 1 might invest nothing when subsequent production is anticipated to be strictly unprofitable ($M\Pi_1 < 0$), it is straightforward to show that unless $\beta = 0$, such behavior is jointly suboptimal. For suppose firm 1 did no investment in these circumstances. Then a marginal increase in its R&D would create spillover benefits to firm 2 without bringing firm 1 into competition; and given d'Aspremont and Jacquemin's assumption about the cost of investing in R&D, the marginal cost of doing the first bit of R&D is zero. Since a marginal increase in R&D by firm 1 would have strictly positive marginal benefits and zero marginal cost, it is suboptimal for firm 1 to do no investment in these circumstances (for any strictly positive spillover parameter). Hence, the first potential subcase of Case 2 can only arise when $\beta = 0$.

We have shown, for the shaded region of parameter space, why the joint-profit maximum can never occur at interior points where both firms invest and then produce. There remain four *candidates* for the optimal solution. In three of the four cases, the firms invest at the first stage so as to induce monopoly at the second stage. In the remaining case, a corner occurs in firm 1's investment.

Each of these four candidate solutions can in fact be optimal as we illustrate in Table 1a. Table 1a is constructed for the case where $a = 1.5A$ and $b\gamma = 0.9$. For each of four exogenous specifications of β , we report in Table 1a the aggregate profit from the joint venture as well as individual firm profits, outputs, and R&D investments.³ In addition, we report in the columns of Table 1b (distinguished by

³ Rows 1–3 in Tables 1a and 1b entail $M\Pi_1 = 0$. Row 4 entails $M\Pi_1 < 0$.

Table 1a
Profit in the optimal (asymmetric) solution

β	$\Pi/(A^2\gamma)$	$\Pi_1/(A^2\gamma)$	$\Pi_2/(A^2\gamma)$	x_1/A	x_2/A	$q_1/(A\gamma)$	$q_2/(A\gamma)$
0.3	0.114	0.007	0.107	0	0.650	0.089	0.594
0.2	0.147	0	0.147	0	0.833	0	0.741
0.1	0.157	-0.0	0.157	0.019	0.671	0	0.652
0.02	0.156	-0.0	0.156	0.013	0.625	0	0.625

'hats') the corresponding magnitudes in the symmetric solution proposed by d'Aspremont and Jacquemin. The four exogenous specifications of β used in Tables 1a and 1b are indicated by dots in the shaded region of Fig. 1.

When the symmetric investment strategy characterized in the literature is replaced by the strategy which is actually optimal, the increase in joint profits can be substantial. As Tables 1a and 1b reflect, joint profits nearly double in each of the last two cases.

It is tedious but straightforward to show that throughout the shaded region in Fig. 1, when $a = 1.5A$, aggregate investment and output are higher in the optimal (asymmetric) solution than in the suboptimal (symmetric) solution.⁴ Table 1 illustrates this phenomenon for the four points in parameter space it considers. Since consumer surplus strictly increases with output, a surprising implication of

Table 1b
Profit in the suboptimal (symmetric) solution

β	$\hat{\Pi}/(A^2\gamma)$	\hat{x}_1/A	$\hat{q}_1/(A\gamma)$
0.3	0.106	0.275	0.318
0.2	0.096	0.230	0.287
0.1	0.088	0.194	0.264
0.02	0.083	0.169	0.249

⁴To show this, we first determined the region of parameter space where each of the Kuhn–Tucker cases arises in the cartel's joint-profit maximization problem. For each region we then compared the profit-maximizing choice of investments (and outputs) to their counterparts in the proposed symmetric solution of d'Aspremont and Jacquemin. When the comparisons could not easily be made analytically (essentially the cases where one firm produces zero), we plotted the difference in the investments (and the difference in outputs) numerically for each region as a function of the two parameters (β and $b\gamma$) using *Mathematica*. In making our claim, we have, of course, confined attention to the functional forms used by d'Aspremont and Jacquemin.

the analysis is that, in achieving the additional profits, the research joint venture generates a larger consumer surplus (and hence, *a fortiori*, a larger total surplus) than when it is constrained to equal R&D investment by each member. This raises a question about whether the welfare rankings in the literature require revision.

4. Welfare implications

When spillovers are sufficiently small, d'Aspremont and Jacquemin, and others in the literature,⁵ have concluded that welfare is *reduced* by permitting firms to coordinate their investments through research joint ventures before competing in the product market. But, as we have shown, an error in d'Aspremont and Jacquemin's positive analysis led them to *understate* the aggregate surplus created by the research joint ventures. Once this underestimate is corrected, the conclusion that research joint ventures inevitably lower welfare for sufficiently small spillovers disappears.

Given the functional forms in the d'Aspremont–Jacquemin model, social welfare (the sum of net consumer surplus and profits) can be written as:

$$W = aQ - 0.5bQ^2 - \sum_{i \neq j}^2 (A - x_i - \beta x_j)q_i - 0.5\gamma \sum_{i=1}^2 x_i^2. \quad (8)$$

We will use Eq. (8) to determine net surplus with and without a research joint venture.

We have already discussed the case where firms coordinate their investments in a research joint venture. As for the case where they are not permitted to do so, we utilize the findings of d'Aspremont and Jacquemin (p. 1134). They concluded that the equilibrium in that case is symmetric with investments and outputs as follows:

$$x_i^* = \frac{(2 - \beta)(a - A)}{4.5b\gamma - (2 - \beta)(\beta + 1)}$$

and

$$q_i^* = Q^*/2 = \frac{(a - A)}{3b} \left[\frac{4.5b\gamma}{4.5b\gamma - (2 - \beta)(1 + \beta)} \right].$$

Like Tables 1a and 1b, Table 2 is constructed for the case where $a = 1.5A$ and $b\gamma = 0.9$. We list in the first five columns of Table 2 the net surplus, investments, and outputs of the two firms when a joint venture is permitted.⁶ Substituting the formulas derived by d'Aspremont and Jacquemin for x_i^* and q_i^* into Eq. (8) yields

⁵ See also Vonortas (1994), Ziss (1994), and Steurs (1995).

⁶ To orient the reader, columns 2–5 of Table 2 repeat the information in columns 4–7 of Table 1a.

Table 2
Social surplus with and without cooperation in the first stage

β	$W/(A_2\gamma)$	x_1/A	x_2/A	$q_1/(A\gamma)$	$q_2/(A\gamma)$	$W^*/(A^2\gamma)$	x_1^*/A	$q_1^*/(A\gamma)$
0.3	0.324	0	0.650	0.089	0.594	0.385	0.462	0.408
0.2	0.394	0	0.833	0	0.741	0.340	0.476	0.397
0.1	0.348	0.019	0.671	0	0.652	0.292	0.485	0.383
0.02	0.332	0.013	0.625	0	0.625	0.253	0.488	0.369
0	0.332	0	0.625	0	0.625	0.244	0.488	0.366

social welfare (W^*) in the absence of a research joint venture. We list this information in the last three columns of Table 2 (distinguished by asterisks).

Aggregate investment and output is smaller under a research joint venture in each of the cases examined in Table 2. This implies that consumers are hurt when firms are given the opportunity to coordinate investments in cost-reducing R&D. Nonetheless, the increase in industry profit with a research joint venture *more than offsets* the loss to consumers in four of the five cases reported in Table 2, including the case where there are no spillovers ($\beta=0$). Hence, the conclusion of the literature that research joint ventures inevitably reduce welfare when spillovers are sufficiently small is unwarranted.

5. Joint-profit maximization in the absence of sidepayments

Since the literature following d'Aspremont and Jacquemin has uniformly assumed joint-profit maximization, we have sought to deduce the positive and normative implications of that assumption. While maximization of joint profits is clearly appropriate when costless sidepayments can be made, how a cartel would behave in the absence of sidepayments requires further investigation. We focus here on the case where the cartel members participate in multiple markets, which are possibly quite dissimilar. We show that in the absence of any sidepayments, investing in the symmetric but suboptimal manner discussed in the literature in more than one of the multiple markets can *always* be Pareto-dominated. Hence, this should never be observed. In addition, we show that selecting the investments that maximize joint profits in each of several markets and then allocating the asymmetric roles in each market to the cartel members so as to address the cartel's distribution problem has effects similar to what can be achieved with costless sidepayments. Each alternative allocation of roles results in a profit combination on the profit possibility frontier.

Suppose two firms participate in a research cartel. Any agreement on investments in R&D induces a Cournot equilibrium and results in a net profit to each firm. Consider the *set* of such profit pairs (referred to as 'the profit possibility set') induced by alternative investment agreements. In a regime of complete in-

formation and costless bargaining and enforcement, economic theory offers one guiding principle to determine which cooperative agreement the cartel will choose: no agreement on investments will be chosen by a cartel if an alternative agreement exists which results in a weakly higher profit for every cartel member and a strictly higher profit for some cartel member.

In a world of *costless* sidepayments, this principle insures that any cartel will invest so as to maximize joint profits regardless of its ranking of the profits of its members.⁷ For, any other investment decision would result in a profit pair which could be Pareto-dominated by a suitable redistribution of maximized joint profits.

Suppose, however, that sidepayments are prohibitively costly.⁸ But assume, as is frequently the case, that the two firms participate in *several* markets. It does not matter whether these markets are identical or dissimilar. We require, however, that (1) actions taken in one market have no effect on costs or demand in any other market; (2) the cartel members agree on investments in all markets and then the members choose outputs in each market non-cooperatively at the second stage; and (3) given the exogenous parameters, the investment combination which maximizes joint-profit in each market is asymmetric. We want to know how much each firm will invest in the various markets under the cartel agreement.

To begin, we show why the symmetric solution discussed in the R&D literature will *never* arise in more than one market. For, suppose the symmetric solution arose in two or more markets. Pick any two of these markets and refer to them as market A and market B. Make no changes in any of the other markets. In market A, marginally increase firm 1's investment while contracting firm 2's investment by the same amount. By assumption, firm 1's profits will increase and so will aggregate profits. In market B, decrease firm 1's investment (while increasing firm 2's investment so the aggregate investment is restored in that market) by enough that firm 1's total profit across the *two* markets is unaltered. Since firm 1's total profit across the two markets is unaltered while aggregate profit in each market has strictly increased, firm 2's profit across the two markets must have strictly increased. But then since we have Pareto-dominated the profit pair associated with

⁷This parallels the theory of the firm in which, when the Separation Theorem holds (Hirshleifer, 1970, p. 14), input and output choices of a firm are 'separated' from the preferences of the consumers (shareholders) owning it.

⁸If sidepayments are not too costly, the equal investment strategy proposed by d'Aspremont and Jacquemin can be Pareto-dominated even in the absence of multi-market contact. To illustrate, assume that for each \$1 transferred from one firm to the other, only $\alpha \in (0,1)$ is received. Then, if joint profits are maximized by investing asymmetrically and sidepayments are used to equalize the payoffs, the most profitable equal investment point can be Pareto-dominated provided α is sufficiently large; in the examples in the four rows of Table 1a, Pareto domination occurs if at least 85, 48, 39, and 36 cents, respectively, of every dollar relinquished by one firm is received by the other.

the symmetric investment pair characterized in the literature, it should never arise in more than one market even in the absence of any sidepayments.⁹

Then what investment choice *will* the cartel make in the absence of sidepayments? To a limited extent, the investment choice of the cartel will depend on how the cartel ranks allocations of profits to its members. That is, the separation of investment behavior from questions of distribution is incomplete in the absence of sidepayments. Nonetheless, the opportunity of the cartel to alter the assignment of roles in the asymmetric joint-profit maximizing solution serves to a considerable extent as a substitute for sidepayments.¹⁰

By maximizing joint profits in each market and then assigning to the two firms the asymmetric roles in each market in the various ways possible, the cartel can often achieve its distributive goals with little or no sacrifice in joint profits. Suppose two firms participate in three dissimilar markets, denoted A, B, and C. By assumption, the joint-profit maximizing investments in each market are asymmetric. If these investments were carried out, the cartel would have to decide whether firm 1 or firm 2 would make the larger investment in any given market. There are

⁹Our conclusion generalizes under plausible assumptions to the more realistic case where firms outside the research joint venture participate in one or more of the multiple markets in which the two members of the RJV participate. One condition assuring this is that there are no spillovers from the RJV to any of the outside firms.

Since all firms invest simultaneously in the first stage, the perturbation in the investments of RJV members discussed in the text would not be observed by any outside firm prior to its own investment and consequently could not affect its choice. Hence, in the absence of spillovers from the RJV to the outside firms, the marginal cost of each outside firm in the second stage would be unchanged. Since the proposed perturbation also preserves the sum of the marginal costs of the two members of the RJV in market A, the Bergstrom and Varian (1985) result implies that each outside firm will produce the same output as in the original Cournot equilibrium. For the same reasons, no outside firm would change its output in market B – whether or not it also participated in market A.

The rest of our argument needs no modification. The proposed perturbation leaves the cartel's revenue unchanged in each market but, by assumption, lowers the sum of the cartel's production and R&D costs in each market. Hence, the cartel's profit across the markets strictly increases. Since the perturbation lowers firm 1's profit in market B by as much as it is raised in market A, firm 2's profit across the two markets must strictly increase.

¹⁰In the absence of any multi-market contact, a portion (if not all) of the profit possibility *frontier* can also be achieved without sidepayments, assuming risk neutrality, by *randomly* assigning – with a probability ranging between zero and one – the larger role in the joint-profit maximizing solution to firm 1. For a discussion of binding agreements which specify the use of lotteries over alternative strategy combinations of the players, see Binmore (1992, p. 175). Schmalensee (1987, p. 356) and Harrington (1991, p. 77) discuss illegal cartel agreements reached by firms with profit possibility sets displaying similar non-convexities and express skepticism about agreements involving randomization. As they point out, a player with an adverse realization could simply *refuse* to carry out his part of the agreement. This objection does not apply in the case of research joint ventures since they are legal. Members of the joint venture can presumably rely on the courts to enforce such agreements.

eight (2^3) different choices the cartel could make: firm 1 could have the larger role in (1) no market; (2) market A only; (3) market B only; (4) market C only; (5) markets A and B only; (6) market A and C only; (7) market B and C only; or (8) all three markets. To fix ideas, suppose that in the joint-profit maximizing solution, the profit to the firm making the larger investment in R&D were 100, 200, and 300, respectively, in markets A, B, and C. These numbers differ because the markets are assumed to be dissimilar. Suppose the profit to the smaller investor in these three markets were 10, 20, and 30, respectively. Then these eight different assignments of roles would respectively result in joint profits across markets to firm 1 and firm 2 of (60,600), (150,510), (240,420), (330,330), (330,330), (420,240), (510,150), and (600,60). It should be noted that, in this example, giving equal profits to the two firms requires *no* sacrifice of joint profits despite the absence of sidepayments.

Although the assignment of roles in the different markets will affect the aggregate profit earned by firm i (denoted Π_i for $i=1,2$), each of these eight assignments of roles will result in the *same* joint profit ($\Pi_1 + \Pi_2$) to the two firms. This follows since the assignments of alternative roles leave unaltered the pair of investments in each market and hence does not affect the joint profits earned in each market.¹¹ It follows that each of these eight profit possibilities lies on the line $\Pi_1 + \Pi_2 = k$. Moreover, it is straightforward to show that each of these profit pairs lies on the *boundary* of the profit possibility frontier and cannot be Pareto-dominated.¹²

It should be intuitively clear to the reader from what has been said that if the cartel can assign asymmetric roles to the two firms in sufficiently *many* markets, it need sacrifice little or no joint profits to achieve its distributive goals.¹³

¹¹ For example, in our numerical illustration the joint profit of the two firms across the three markets is always 660.

¹² Let j index the n markets in which both firms participate, let $\pi_{i,j}$ denote the profit of firm i in market j and distinguish with an asterisk (*) the profits associated with the joint-profit maximizing investments in a market. Then, by definition $\pi_{1,j}^* + \pi_{2,j}^* \geq \pi_{1,j} + \pi_{2,j}$ for $j = 1, \dots, n$. Summing, we obtain: $\sum_{j=1}^n \pi_{1,j}^* + \sum_{j=1}^n \pi_{2,j}^* \geq \sum_{j=1}^n \pi_{1,j} + \sum_{j=1}^n \pi_{2,j}$. Letting Π_i denote the profit of firm i aggregated across markets ($\sum_{j=1}^n \pi_{i,j}$) we conclude: $\Pi_1^* + \Pi_2^* \geq \Pi_1 + \Pi_2$. Hence, no point on the profit-possibility frontier can lie outside the line passing through the profit pairs resulting from joint profit-maximization in each market. It follows that these pairs lie on the boundary of the profit-possibility set.

¹³ The argument above is reminiscent of that made by Farrell (1959) regarding non-convexities and competitive equilibrium. One would conjecture, by analogy with the extensive subsequent literature formalizing Farrell's insight, that the loss in joint profits could be made arbitrarily small if the markets in which the two firms participated were replicated appropriately. Moreover, joint-profit maximization in every market would arise if there were a *continuum* of markets. Suppose for example that there were a continuum of markets and that the larger firm in any market earns \$100 in the joint-profit maximizing solution while the smaller firm earns \$0. Then by assigning firm 1 the lead role in a fraction δ of the markets ($\delta \in [0,1]$) one can achieve any point on the profit possibility frontier. In such a case, a cartel would maximize joint profits in each market regardless of its ranking of the profits of its members.

6. Conclusion

In the d'Aspremont–Jacquemin model and the literature it has spawned, firms coordinate research investments in a first stage prior to competing as Cournot rivals in a second stage. We have shown that, for a particular region in parameter space, the joint-profit maximizing solution in the d'Aspremont–Jacquemin model is not symmetric as the previous literature assumed. Rather, joint profits can be increased by reallocating the same total investment between the two firms. As a result, the joint-profit maximizing solution for a research cartel is to choose asymmetric investments at the R&D stage. This in turn generates increased concentration at the production stage.

Since the R&D cost for each firm is regarded as the same strictly convex function of own investment, the least cost way of achieving any aggregate investment level is to invest equally at the two firms. However, joint profits in the subsequent production stage are maximized when one firm is given a sufficiently large advantage in marginal cost (through greater R&D investment). Whether or not an asymmetric solution maximizes joint profits depends on the relative trade-off of these two considerations.

For the shaded region of parameter space in Fig. 1, the consideration leading to asymmetry prevails. Increased concentration in the product market then inevitably results when firms are permitted to coordinate their R&D investments through a research joint venture. No collusion in the product market is necessary. No matter how vigilant antitrust authorities are in policing product-market collusion, increased concentration will occur.

That permitting cooperation through research joint ventures may subsequently lead to increased concentration in the product market is a possibility policy makers may want to take into account. This possibility has not previously been addressed in the literature since attention has erroneously been restricted to cases of equal investment. The resulting increase in concentration is at least in some circumstances accompanied by a deterioration in welfare in the model proposed by d'Aspremont and Jacquemin. The welfare changes associated with increased concentration remain to be determined in more general formulations.

Acknowledgements

We thank Jim Adams, Bruno Cassiman, John Hartwick, Alexis Jacquemin, Tracy Lewis, Patrick Meister, David Simpson, Hal Varian, Desire Vencatathellum, and especially Joseph Harrington for helpful comments.

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