

Unequal Treatment of Identical Agents in Cournot Equilibrium

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Oligopoly models where prior actions by firms affect subsequent marginal costs have been useful in illuminating policy debates in areas such as antitrust regulation, environmental protection, and international competition. We discuss properties of such models when a Cournot equilibrium occurs at the second stage. Aggregate production costs strictly decline with no change in gross revenue or gross consumer surplus if the prior actions strictly increase the variance of marginal costs without changing the marginal-cost sum. Therefore, unless the cost of inducing second-stage asymmetry more than offsets this reduction in production costs, the private and social optima are asymmetric. (JEL D43, L13, L40)

Oligopoly models where firms take prior actions which affect subsequent marginal costs have long been useful in illuminating policy debates in areas such as antitrust regulation, environmental protection, and international competition. We discuss an unnoticed and surprising property of such models. Our discovery leads inevitably to a reconsideration of several policy conclusions.

We consider two-stage games where Cournot competition occurs at the second stage. The property we have discovered arises because of the way aggregate production costs change in the Cournot model in response to changes in the cost structure of the industry. We focus on the case where each firm produces at constant marginal cost although, as shown in the concluding section, our results easily gener-

alize.¹ If the vector of constant marginal costs is changed exogenously without altering the sum of its components, it is well known (Theodore C. Bergstrom and Hal R. Varian, 1985b) that industry output will not change provided all firms continue to produce (i.e., the Cournot equilibrium is “interior”). Since industry output is the same, price, industry revenue, and gross consumer surplus are likewise unaffected. Hence, increases in both industry profit and social surplus will result if and only if the change in the vector of marginal costs induces a reduction in aggregate production costs.

Aggregate production costs in turn strictly decrease if and only if the variance of marginal costs (or, equivalently, the sum of their squares) strictly increases (Bergstrom and Varian, 1985a). This implies that aggregate production costs are *maximized* when every firm has the same marginal cost. Contrary to intuition, therefore, when every firm has the same marginal cost, industry profit and social surplus are smaller than when firms have different marginal costs with the same sum. Disregarding equity considerations, we conclude that asymmetry has both social and private advantages.

This conclusion has important implications for two-stage games. Economists have spent more than a decade exploring a broad range of applications where initially identical firms play a Cournot

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¹ In the concluding section, we generalize our results to the case where actions in the first stage alter the intercepts of marginal-cost curves with a fixed common slope.

game after simultaneously taking actions in the first stage that affect second-stage marginal costs. A symmetry restriction is typically imposed at the outset as a simplification. Given our comparative-static result, however, this particular simplification is not innocuous: there is a strong presumption that *nonidentical* actions by identical agents in the first stage are required to maximize social surplus and industry profit. Asymmetric behavior in the first stage is clearly optimal when it is costless to engage in. We provide two examples where it is costless, one involving learning-by-doing and the other involving extraction of a natural resource (whether common or private property) subject to depletion effects.²

But in many cases asymmetric behavior in the first stage does involve additional costs. Because of diminishing returns, for example, it may be cheaper to spread R&D investments evenly across a set of firms than to have unequal investments at the first stage. Nevertheless, the benefits of having unequal marginal costs in the production stage may more than outweigh these increased R&D costs. What leads one's intuition astray is the tendency to focus on changes in the more prominent first-stage costs while ignoring the less visible—but perhaps quantitatively more important—changes in second-stage costs.³ We derive a sufficient condition

for optimal first-stage behavior to be asymmetric. Whenever it holds, the additional benefits of having different marginal costs at the second stage are worth the additional costs of generating these differences in the first stage.

Our discovery about what the various two-stage games actually predict has implications for several policy debates. Recently, for example, there was an international meeting in Kyoto to set emission standards to curb global warming. Individual countries will have to decide whether to meet those standards by curbing production or by spending more on abatement efforts. David M. Newbery (1990) provides a two-stage game which is useful for thinking about such agreements. In his model, choices of emission standards in the first stage affect the abatement cost per unit of output in the second stage. When our sufficient condition holds, it is inefficient to set the same emissions quotas for identical countries.⁴ Moreover, a common Pigouvian tax on emissions is suboptimal in such cases because it inevitably leads to identical behavior, whereas asymmetric behavior generates a higher surplus. For similar reasons, the trading of emission permits is suboptimal since, when identical agents face the same permit price, they take identical actions.

In the international arena, it has long been argued (beginning with Jean Jacques Servan-Schreiber, 1968) that a government should subsidize the research and development of a domestic "national champion" so that the country can better

² The learning-by-doing literature contains contributions by A. Michael Spence (1981), Drew Fudenberg and Jean Tirole (1983), Partha Dasgupta and Joseph E. Stiglitz (1988), Elettra Agliardi (1990), Clement G. Krouse (1994), and many others. Cournot competitors are envisioned as producing a homogeneous good in each of two periods. In most of these models, each firm is assumed to have the same marginal cost in the first stage and the same ability to reduce second-stage marginal costs by learning from the experience of first-stage production.

³ A substantial literature initiated by Claude d'Aspremont and Alexis Jacquemin (1988), and extended by Kamien et al. (1992), Kataro Suzumura (1992), Dermot Leahy and J. Peter Neary (1997), and many others asks how a research joint venture should arrange its members' investments in R&D if it seeks to maximize joint profits but is compelled by law to relinquish control before the production stage. In virtually all of these models there are regions of parameter space where joint-profit maximization requires unequal treatment of identical members of the research cartel. Since it seemed intuitive that members with identical, increasing costs of investing in R&D should invest equally to maximize profits of the joint venture, this entire literature assumed (incorrectly) that it is joint-profit maximizing for the cartel to treat identical members equally. The error arises from failing to recognize that this has

an adverse consequence in the second stage because of the increased production costs it induces.

⁴ To our knowledge, Newbery is the first to recognize the asymmetry phenomenon we investigate although he does not analyze it. Salant and Shaffer (1992, 1998) focus on the R&D literature which follows d'Aspremont and Jacquemin (1988). We discuss the appropriateness of the assumption of maximization of joint profits when a cartel cannot make side payments and show that certain conclusions in the R&D literature no longer follow once optimal R&D investments are recognized to be asymmetric. Ngo Van Long and Antoine Soubeyran (1995), in a paper extending Salant and Shaffer (1992), explain why asymmetric investments maximize the joint profits of an R&D cartel. While they neither discuss the implications of the result for maximization of social welfare nor trace its implications for non-R&D models, they do report a number of interesting results—among them that, "in the case of linear demand, asymmetric cost reduction becomes more likely, the larger the number of participating firms."

compete in the international marketplace. The wisdom of such policies, however, has often been questioned, particularly by domestic firms which resent their government's giving unfair advantages to a rival of no greater competence. The case for subsidization has been regarded as turning on increasing returns to scale in R&D. d'Aspremont and Jacquemin (1988) provide a two-stage game which is useful for thinking about this debate.⁵ In their model, choices of R&D investment in the first stage affect marginal production costs in the second stage. When our sufficiency condition holds, it turns out to be welfare improving for a given country to treat domestic firms unequally even if they are equally competent and even if there are no increasing returns to scale either in the R&D by itself or in the production stage.

In the domestic arena, our results have several implications for antitrust regulation. In general, the perturbation we consider throughout the paper (which raises the variance of marginal costs while maintaining the sum) improves social welfare although it raises the Herfindahl-Hirschman index (HHI). This is an unmistakable reminder that increases in the HHI need not signal a decline in welfare. Our results also have implications for specific antitrust laws. The Robinson-Patman Act, for example, requires an upstream firm to treat downstream firms equally. When our sufficiency condition holds, however, aggregate production costs can be lowered with no adverse effect on consumers if the upstream firm is allowed to price discriminate in violation of the law.

⁵ James A. Brander and Barbara J. Spencer (1983) and Timothy J. Besley and Suzumura (1992) also consider two-stage games where *ex ante* identical Cournot competitors produce homogeneous goods in stage two after investing simultaneously in cost-reducing R&D in stage one. In such cases, government subsidization of research may be called for either to correct a divergence between private and social benefits of R&D (Spence, 1984) or to induce more favorable outcomes in international rivalry (Spencer and Brander, 1983). More recently, Kyle Bagwell and Robert W. Staiger (1994) and Leahy and Neary (1997) calculate the socially optimal subsidy when government intervention is confined to the research stage. None of these authors noticed that asymmetric subsidization of identical firms is sometimes optimal and hence, in the case of international rivalry with multiple competing domestic firms, that it may be national welfare maximizing for the home government to play favorites by creating a "national champion" as a dominant exporting firm. David Collie (1993) does discuss differential subsidization but the firms in his model are unequal at the outset.

The paper proceeds as follows. In the next section, we discuss the comparative-static result. In Section II, we use that result to derive a sufficient condition for asymmetric actions to be socially and/or privately optimal in the first stage of a two-stage game played by *ex ante* identical players. In Section III, we discuss policy implications of our analysis. Section IV concludes the paper.

I. Equilibrium in the Final Stage

Suppose n (≥ 2) firms with constant marginal costs play a Cournot game. Denote firm i 's marginal cost as c_i (≥ 0) and the inverse demand as $P = P(Q)$, where P is the market price and $Q = \sum_{i=1}^n q_i$ is industry output. Firm i 's profit is $P(Q)q_i - c_i q_i$. Assume $P(Q)$ is twice continuously differentiable, with $P' < 0$, $\forall Q \geq 0$ satisfying $P(Q) > 0$. Assume firm i 's marginal revenue is everywhere strictly decreasing in each rival's output: $P' + P''q_i < 0$, $\forall q_i \in [0, Q]$. These assumptions insure the existence of a unique pure-strategy Nash equilibrium in output (Gerard O. Gaudet and Salant, 1991).

Assume that in this Nash equilibrium each firm produces a strictly positive output ($q_i > 0$ for $i = 1, \dots, n$). We refer to this as an "interior equilibrium." Equilibrium quantities are then determined by the n first-order conditions: $P(Q) + P'(Q)q_i - c_i = 0$. An immediate implication of these conditions is that a firm with a strictly higher marginal cost will have a strictly lower equilibrium output and two firms with the same marginal cost will have the same equilibrium output. Summing the n first-order conditions to obtain $nP(Q) + QP'(Q) - \sum_{i=1}^n c_i = 0$ yields a second implication (Bergstrom and Varian, 1985a): industry output in any interior Cournot equilibrium depends only on the *sum* of the constant marginal costs and not on the distribution of those costs. It follows that if marginal costs change but their sum remains constant, industry output will be unchanged (assuming an interior equilibrium). Moreover, output will contract at each firm experiencing a marginal-cost increase and expand at each firm experiencing a marginal-cost decrease; there will be no change in the output of a firm with unchanged marginal cost.

Although they induce the same industry

output, not all marginal-cost vectors with the same component sum induce the same aggregate production cost in equilibrium. Nonetheless, as we will see, it is possible merely by inspecting these marginal-cost vectors to predict the order of the aggregate production costs they induce. As a preliminary step in understanding this ordering, it is useful to decompose into its various components the induced change in aggregate production costs that would occur if the marginal costs of the n firms were rearranged in a way which preserves their sum. Let Δc_i denote the change in firm i 's marginal cost and Δq_i denote the induced change in firm i 's equilibrium output. Then the change in aggregate production costs is

$$\begin{aligned} (1) \quad & \sum_{i=1}^n (c_i + \Delta c_i)(q_i + \Delta q_i) - \sum_{i=1}^n c_i q_i \\ & = \sum_{i=1}^n \Delta c_i \Delta q_i + \sum_{i=1}^n c_i \Delta q_i + \sum_{i=1}^n q_i \Delta c_i. \end{aligned}$$

Consider the case where marginal costs change only at two firms. Let firm 1 have a weakly larger marginal cost than firm 2 ($c_1 \geq c_2$) and suppose the larger marginal cost is raised while the smaller marginal cost is lowered by offsetting amounts ($\Delta c_1 = -\Delta c_2 > 0$) so that the variance of marginal costs strictly increases. The first term in (1) will be strictly negative in this case ($\sum \Delta c_i \Delta q_i < 0$) since output expands at any firm experiencing a strict marginal-cost reduction and contracts at any firm experiencing a strict marginal-cost increase. We now show that the other two sums in (1) are weakly negative. Recall that the induced changes in output will be offsetting ($\Delta q_1 = -\Delta q_2 < 0$). Then the second sum in (1) is weakly negative ($\sum c_i \Delta q_i \leq 0$) since it is the firm with the weakly larger marginal cost that experiences the strict output reduction. As for the final sum, it is weakly negative as well ($\sum q_i \Delta c_i \leq 0$) since it is the firm with the weakly larger output that experiences the strict cost reduction. Hence, this particular variance-increasing change in marginal costs results in a strict decrease in aggregate production costs.

It turns out that *any* rearrangement in marginal costs which increases their variance while preserving their sum reduces aggregate production costs provided the induced Nash equilibrium remains interior (Bergstrom and Varian, 1985a).

PROPOSITION 1: *Suppose the marginal costs of the n firms in an industry are rearranged in a way which (1) preserves their sum and (2) results in a new Nash equilibrium which is also interior. Then, aggregate production costs ($\sum c_i q_i$) strictly decrease after the rearrangement if and only if the variance of the marginal costs (or, equivalently, the sum of their squares) strictly increases.*

PROOF:

See Bergstrom and Varian's note (1985a) or our generalization in Section IV.⁶

While our formulation of Bergstrom and Varian's comparative-static result concerns aggregate production costs, it has obvious implications for industry profit and social surplus.⁷ Since both industry revenue ($QP(Q)$) and gross consumer surplus ($\int_{u=0}^Q P(u)du$) depend only on industry output, which remains unchanged as long as the marginal-cost sum is fixed, industry profit and social surplus strictly increase if and only if aggregate production costs strictly decline.

⁶ Readers preferring an alternate proof are directed to footnote 23. We prove there a generalization of Proposition 1 in which marginal costs weakly increase at a constant, common slope but the vertical intercepts of these functions are firm-specific.

⁷ These implications do not arise in the specific context considered by Bergstrom and Varian. In particular, they investigate the behavior of firms with identical and unchanging marginal costs of production in response to changes in per-unit tax rates with the same sum. In this context, they show that if the variance of the tax rates increases, the firms' aggregate tax payments decrease in any equilibrium which remains interior. In their context, therefore, aggregate production costs do not change in response to the change in the vector of tax rates since the offsetting output changes occur at firms with identical per-unit costs; for the same reason no change occurs in social surplus. We have reinterpreted their proposition as applying to *any* situation where the constant "marginal payments" of the firms change in ways which increase their variance but preserve their sum. Changes in aggregate costs then occur when the marginal costs of production are changed.

COROLLARY 1: *Suppose the marginal costs of the n firms in an industry are rearranged in a way which preserves their sum but strictly increases (respectively decreases) their variance. Then, if the Nash equilibrium remains interior, industry profit and social surplus must strictly increase (respectively decrease).*

Corollary 1 permits a complete ordering in terms of industry profit and social surplus of all marginal-cost vectors with the same component sum (provided they induce an interior Nash equilibrium). This ordering yields some surprising implications for antitrust regulation. For example, it has implications for the widely-used Herfindahl-Hirschman index (HHI), a measure of market concentration that is constructed by summing the squares of the market shares of the n firms.

COROLLARY 2: *Suppose the marginal costs of the n firms in an industry are rearranged in a way which preserves their sum. Then, if the Nash equilibrium remains interior, the HHI strictly increases if and only if industry profit and social surplus strictly increase.*

PROOF:

From the first-order condition of firm i we have $q_i = (P - c_i)/(-P')$. Dividing both sides by Q , squaring the resulting expression, and then summing over all i yields:

$$\text{HHI} = \frac{\sum_{i=1}^n (P - c_i)^2}{(-QP')^2}$$

$$= \frac{1}{(-QP')^2} \left(\sum_{i=1}^n P^2 - 2P \sum_{i=1}^n c_i + \sum_{i=1}^n c_i^2 \right).$$

As long as the marginal-cost sum is fixed, industry output (and hence market price) are constant, implying that the first two summation terms on the right-hand side are also constant. Any induced change in the HHI can only arise, therefore, because of changes in the third summation term, the sum of the squared marginal

costs. Thus, a strict increase (respectively decrease) in the variance of marginal costs, holding fixed the marginal-cost sum, implies a strict increase (respectively decrease) in the HHI. This implies that the HHI strictly increases if and only if industry profit and social surplus strictly increase.

Measures of market concentration and social surplus are often thought to be inversely related (see Robert E. Dansby and Robert D. Willig, 1979; Willig, 1991) and this notion has formed the basis of many antitrust inquiries. Since the early 1980's, the HHI has received greater prominence than other measures of market concentration in the deliberation of the federal enforcement agencies and the courts.⁸

Despite the notion that an increase in the HHI reflects a decrease in social welfare, Joseph Farrell and Carl Shapiro (1990) show that if aggregate output is unchanged the HHI and social welfare must move in the same direction.⁹ Our Corollary 2 illuminates their observation by establishing the circumstances under which both variables will strictly increase: both variables strictly increase if and only if the variance of marginal costs strictly increases. Ironically, although the courts regard the HHI as superior to other measures of market concentration precisely because "it increases as the disparity in the size between firms increases" (*FTC v. PPG Indus.*, 798 F.2d 1500, 1503, D.C. Cir. 1986), it is precisely this sensitivity to disparities between firms which can undermine

⁸ See *Antitrust Law Developments* (American Bar Association Antitrust Section, 1992 pp. 302-03), the U.S. Department of Justice, *Merger Guidelines* (1992) [reprinted in 4 Trade Reg. Rep. (Commerce Clearing House, 1992 ¶13,103)], and *FTC v. Bass Bros. Enters.*, 1984-1 Trade Cases (Commerce Clearing House, 1994 p. 68,609, ¶66,041) (N. D. Ohio 1984). ("The HHI is premised upon the economic principle that both the distribution of market shares among the leading firms and the composition of the market outside the leading firms are significant in assessing the likelihood of tacit collusion or interdependent, noncompetitive conduct in a market.")

⁹ The idea that the HHI may be positively related to performance has also been noted in Andrew Daughety (1990) and Steven Polasky and Charles F. Mason (1998). Although Corollary 2 reaches a similar conclusion, we investigate a somewhat different phenomenon, namely changes in individual output when market output and the number of active firms is unchanged.

the reliability of this index as an indicator of changes in social welfare.

The policy relevance of Proposition 1 and Corollary 2 extends to the asymmetric treatment of a *subset* of firms. Recall that as long as the marginal-cost sum remains the same, output, revenue, and hence profit, do not change in equilibrium at any firm with an unchanged marginal cost. This observation underlies the following corollary.

COROLLARY 3: *Suppose the marginal costs of firms within one country are rearranged in a way which preserves their sum but strictly increases their variance, while the marginal cost of every firm outside that country is unchanged. Then the aggregate production costs of the firms within the country would strictly decrease while the production cost of each firm outside the country would remain unchanged (assuming an interior Nash equilibrium). Moreover, aggregate profit and social surplus would strictly increase within the country but would remain unchanged outside it.*

Note that by hypothesis the sum of all the marginal costs both inside and outside the country does not change while the sum of their squares strictly increases. It follows from Proposition 1 that overall production costs strictly decrease and from Corollary 1 that overall profit and social surplus strictly increase. But outside the country output does not change at *any* firm. Hence *their* costs and profits do not change. It follows that it is only *within* the country that aggregate production costs strictly decrease and profits strictly increase. Since the net consumer surplus of no individual changes, social surplus likewise strictly increases within the country but is unchanged outside it.

Corollary 3 provides a rationale—even though there are no scale economies—for national policies which favor the R&D investments of those domestic firms designated as “national champions” at the expense of other domestic firms which may be equally capable. It also implies that asymmetries may arise among equally capable members of cartels, research joint ventures, and mergers in the common circumstance where these groups may not contain all firms in the industry and where a firm’s

current actions affect subsequent costs. We clarify these implications in Section III.

While the previous corollaries pertain to situations where initial marginal costs in the industry may differ, the next corollary focuses precisely on the special case where the initial marginal cost of every firm is the same.

COROLLARY 4: *Suppose all firms initially have the same marginal cost (zero variance). Aggregate production costs are then strictly larger while industry profit and social surplus are strictly smaller than they would be under any other configuration of marginal costs with the same sum (provided the induced Nash equilibrium is interior).*

Corollary 4 follows immediately from Proposition 1 and Corollary 2 and is central to the results in the next section. To gain further intuition, we provide a second proof based on (1). Assume initially that all firms have the same marginal cost (c) and, therefore, produce the same output (q). Then the right-hand side of (1) simplifies to

$$(2) \quad q \sum_{i=1}^n \Delta c_i + \sum_{i=1}^n (c + \Delta c_i) \Delta q_i.$$

Since the induced changes in marginal costs sum to zero, no change in aggregate costs would occur if each firm continued to produce the common initial output of q . Thus, the first term in (2) is zero ($q \sum \Delta c_i = 0$). The second term in (2) equals the change in aggregate production costs which would occur once each firm’s output reequilibrated. Since output must strictly decrease (respectively increase) at each firm whose marginal cost strictly increases (respectively decreases), a portion of what was produced by firms experiencing a strict increase in marginal costs is now produced instead by firms experiencing a strict reduction in marginal costs. As a result, the second term in (2) must be strictly negative ($\sum (c + \Delta c_i) \Delta q_i < 0$).¹⁰ Aggregate production costs strictly decrease when

¹⁰ Alternatively, the second term in (2) can be decomposed to $c \sum_{i=1}^n \Delta q_i + \sum_{i=1}^n \Delta c_i \Delta q_i$. We know from Bergstrom and Varian (1985b) that the first term is zero while the fact that marginal cost and output change in

the vector of marginal costs is changed because more of the unchanged aggregate output is produced by the firms whose marginal costs have been lowered. Since the rearrangement of marginal costs affects neither industry revenue nor consumer surplus while it strictly reduces aggregate production costs, industry profit and social surplus must strictly increase. We now examine the implications of these results for two-stage games.

II. Asymmetric Actions by Identical Agents in Two-Stage Games

In a wide variety of two-stage games, *ex ante* identical firms take actions in the first stage which determine the set of constant marginal costs used to play a Cournot game in the second stage. Since the firms are *ex ante* identical, identical first-stage actions by them give rise to identical marginal costs in the second stage. Given Corollary 4, however, there is a presumption that nonidentical actions by identical agents in the first stage will strictly increase social surplus and industry profit—unless the additional cost of introducing asymmetry in the first stage outweighs the net benefit of reduced costs in the second stage. We begin with two examples of games where the net benefit can be achieved without incurring any additional cost at the first stage. It is then *always* optimal, both socially and privately, for the first-stage actions of the identical agents to be asymmetric.

A. Games Where Asymmetries in Marginal Costs Can Be Introduced Costlessly

Learning-by-Doing.—Frequently each firm in an industry can produce at lower cost once experience is gained with a production process. In the absence of any spillovers, each firm “learns-by-doing.” In such circumstances, the government is frequently pressed to expedite this learning process by subsidizing the early production efforts of firms in the industry. But which firms among a set of equally inexperienced domestic firms? If there is in addition international competition, governments are of-

ten pressed to help their own domestic firms “move down the learning curve” ahead of foreign competitors.

Fudenberg and Tirole (1983) provide a model where these issues can be examined. They consider a game in which n Cournot competitors produce a homogeneous good at constant marginal costs in each of two periods. Increased experience in first-period production reduces a firm’s marginal cost of second-period production. In particular, they posit that each firm has the same constant marginal cost \bar{c} in the first period, but firm i ’s marginal cost in the second period is $c^i(\mathbf{x}) = \bar{c} - \lambda x_i$, where x_i ($\leq \bar{c}/\lambda$) denotes firm i ’s first-period output and $\lambda > 0$.

Because each firm’s second-period marginal cost is linear in its first-period output, perturbations which involve an unchanged sum of outputs in the first period preserve the sum of the marginal costs in the second period: $\sum c^i = n\bar{c} - \lambda X$, where $X = \sum x_i$. Since variations in the composition of an unchanged aggregate output produced at the same constant marginal cost do not affect industry profits in the first period ($X(P(X) - \bar{c})$), the perturbation has no cost in the first period.

However, Corollary 4 implies that such perturbations always lower aggregate production costs in the second period. Since there is no cost of introducing asymmetry in the first period and since second-period gains are strictly positive, maximization of industry profits *requires* that firms choose asymmetric actions. Hence, a cartel maximizing joint profits over the two periods but restricted to first-period production quotas would assign identical members unequal quotas.¹¹

In the absence of a cartel, the firms would behave noncooperatively in each period and, as Fudenberg and Tirole (1983 p. 527) show, the subgame-perfect equilibrium would be symmetric. A social planner constrained to intervene in the first period could improve on this market allocation. But to intervene optimally, unequal treatment of identical firms would be required. In international contexts, for instance, it is often

opposite directions ensures that the second term is strictly negative.

¹¹ Costless side payments are sufficient for maximization of joint profits to be the appropriate objective of a cartel; however, as discussed in Salant and Shaffer (1998), they are not necessary.

recommended that a government help equally inexperienced domestic exporters move down their learning curves so they can operate at a cost advantage in the future. However, no matter what the production levels of firms in other countries in the first period, maximizing joint profit or social surplus at home requires—in an example like Fudenberg and Tirole's—unequal production in the first period. This follows since introducing the asymmetry is costless in their example and, given Corollary 3, socially advantageous.

Extraction with Depletion Effects.—A second example where the asymmetry can be introduced costlessly involves extraction with depletion effects. Suppose increasing extraction in the first period makes extraction in the second period more costly on the margin: in the case of oil, increasing current extraction dissipates the pressure driving oil to the surface and makes future extraction more costly; with water, the more withdrawn today, the further distance the water must be lifted tomorrow; in the case of minerals, the more extensive the mining today, the less accessible will be the ore left for tomorrow.

To take account of these cases, modify Fudenberg and Tirole's example so that increased extraction by firm i today raises firm i 's marginal cost in the future: $c^i(\mathbf{x}) = \bar{c} - \lambda x_i$ but now $\lambda < 0$. Since nothing in our discussion of the learning-by-doing example depends on the sign of λ , our previous conclusions still hold: a cartel maximizing the joint profits of its members over two periods but restricted to first-period production quotas would want asymmetric first-period extraction among its members; moreover, a social planner constrained to intervene in only the first period would want to induce unequal extraction in the first period as well.

Sometimes, a firm's extraction in the first period raises not only its own second-period marginal cost but also the second-period marginal costs of the other firms: $c^i(\mathbf{x}) = \bar{c} - \lambda x_i - \lambda \beta \sum_{j \neq i} x_j$ where $\beta \in (0, 1]$ and $\lambda < 0$. Then, as before, the sum of the second-period marginal costs will not change as long as aggregate extraction in the first period does not change: $\sum_{i=1}^n c^i = n\bar{c} - \lambda X(1 + \beta[n - 1])$. Depending on the spillover parameter β , there are a continuum of possibilities, ranging from the standard case of a common property ($\beta = 1$) on the one hand to the previously dis-

cussed case of private property ($\beta = 0$) on the other. In intermediate cases where partial seepage ($\beta \in (0, 1)$) occurs between properties (Dasgupta and Geoffrey M. Heal, 1979 p. 373), one firm's extraction will have a smaller effect on the second-period marginal costs of the other firms than on its own second-period marginal cost.¹² When partial seepage occurs, introducing asymmetry is feasible and—in the modified Fudenberg-Tirole example—optimal. Governments trying to regulate common property extraction via taxes or quotas on the first-period extraction should then treat identical firms unequally by imposing unequal per-unit taxes or quotas. The social optimum cannot be achieved with the same Pigouvian tax on all n firms.

In these two examples, the benefits of generating asymmetry can be achieved without incurring any costs in the first period. The results of the previous section then imply that asymmetry is clearly optimal. Often, however, raising the variance of marginal costs in the second period (without altering the marginal-cost sum) requires that some costs be incurred at the first period. The marginal benefits and costs must then be balanced to determine if the reduction in subsequent production costs outweighs the increase in first-period costs. The analysis is complicated by the fact that a marginal increase in asymmetry in the neighborhood of the best first-period symmetric production (extraction) vector has no first-order effects. Nonetheless, nonmarginal perturbations are beneficial whenever the condition we derive is satisfied. After introducing our notation, we derive our sufficient condition for asymmetric treatment of identical agents to be optimal.

B. Games Where Asymmetries in Marginal Costs Are Costly to Introduce

Consider a two-stage game in which each of n firms chooses an action in the first stage that alters its second-stage constant marginal cost of production. Assume that each firm produces a homogeneous good and competes as a Cournot player in the second stage, given a marginal cost

¹² The seepage aspect of extraction problems involving common properties is analogous to the spillover aspect of problems with cost-reducing R&D discussed in the next section.

that is determined by first-stage actions. Assume the firms are *ex ante* identical. That is, assume the cost of a given action is the same regardless of the identity of the firm undertaking it; if a given set of actions results in a given set of marginal costs in the second stage, rearranging the identities of the firms taking the actions would simply rearrange the identities of the firms with the corresponding marginal costs.

Let $\mathbf{x} = (x_1, \dots, x_n)$ denote the list of prior actions of the n firms. Such actions can be interpreted variously as first-stage production, resource extraction, levels of allowed pollution, and R&D investment, among others. It is convenient to refer to all of these types of first-stage actions as "investments." Denote the cost to firm i of investment \mathbf{x} as $f^i(\mathbf{x})$. Denote industry investment costs as $F(\mathbf{x}) = \sum_{i=1}^n f^i(\mathbf{x})$. Assume that $f^i(\mathbf{x})$ is twice continuously differentiable. Since $F(\cdot)$ is the sum of these individual investment-cost functions, it inherits this property. Denote the constant marginal cost of production at firm i in the second stage as $c^i(\mathbf{x})$. Assume that $c^i(\mathbf{x})$ is twice continuously differentiable. Assume that increased investment by firm i in the first stage *strictly* decreases (respectively *strictly* increases) its own marginal cost of production in the next stage and, because of possible spillovers, at least *weakly* decreases (respectively *weakly* increases) the marginal cost of every other firm. That is, denoting partial differentiation with respect to the independent variable x_i by the subscript i , we assume for $j \neq i$ and $i = 1, \dots, n$, either (1) $c_j^i < 0$ and $c_i^i \leq 0$ or (2) $c_j^i > 0$ and $c_i^i \geq 0$. The first case would arise, for example, if a given firm's increased R&D investment or increased production experience lowers its own second-stage marginal cost of production and, if spillovers are assumed, the marginal costs of its rivals; the second case would arise if increased extraction of a natural resource by one firm *raises* its own marginal cost of extraction in the second stage and, if underground seepage between properties is assumed, the per-unit extraction costs of rival firms on adjacent properties. Our formulation encompasses these cases among others.

Denote the sum of the marginal costs of production in the last stage as $h(\mathbf{x}) = \sum_{i=1}^n c^i(\mathbf{x})$. Given our assumptions about the individual $c^i(\mathbf{x})$ functions, their sum ($h(\mathbf{x})$) is continuously differentiable. Moreover, its partial deriv-

atives are nonzero and of the same sign: $h_i(\mathbf{x}) \neq 0$ and $\text{sgn } h_i(\mathbf{x}) = \text{sgn } h_j(\mathbf{x})$ for all i, j , and any admissible \mathbf{x} .

Let $x_i = x^*$ for $i = 1, \dots, n$ be any investment combination in which all firms invest the same amount x^* and denote the n -tuple of such investments as \mathbf{x}^* . Define $c(x^*)$ as the sum of the second-stage marginal costs of production, evaluated at \mathbf{x}^* : $c(x^*) = h(\mathbf{x}^*)$.

Now fix the investments of $n - 2$ of the firms at x^* but allow the investments of the remaining two firms (labeled, without loss of generality, firm 1 and firm 2) to vary. We denote the resulting n -tuple of investments as $(x_1, x_2, x^*, \dots, x^*)$. Define Ω as the set of investment combinations that result in a marginal-cost sum of $c(x^*)$ in the second stage: $\Omega = \{(x_1, x_2) | h(x_1, x_2, x^*, \dots, x^*) = c(x^*)\}$. Ω is not empty since the point $x_1 = x^*$ and $x_2 = x^*$ belongs to this set. The implicit function theorem then implies that for any x_1 within some neighborhood of x^* , we can express x_2 as a unique function of x_1 : $x_2 = g(x_1; x^*)$ where $g(x_1; x^*)$ solves $h(x_1, g(x_1; x^*), x^*, \dots, x^*) = c(x^*)$.¹³ We refer to $g(x_1; x^*)$ as the "iso-sum" locus since it is the locus of combinations (x_1, x_2) that generate an unchanged marginal-cost sum $c(x^*)$.¹⁴ The x^* after the semicolon indicates that firms 3 through n each invest x^* and that the sum of the n marginal costs in the next period is $c(x^*)$.

With these preliminaries behind us, we can now investigate the optimality of unequal investments by identical firms. Note that if \mathbf{x}^* globally maximizes the objective function (be it industry profit or social surplus), then \mathbf{x}^* must not only be the best equal investment combination but must also be weakly superior to any other point in the domain—including all *unequal* investment combinations on the *same* iso-sum locus. We use this observation

¹³ Since the partials of $h(\mathbf{x})$ are nonzero, the implicit function theorem is applicable. A further implication of this theorem is that $g'(x_1; x^*) = -h_1(\mathbf{x}^*)/h_2(\mathbf{x}^*)$ when $x_1 = x^*$. Since every partial derivative of $h(\mathbf{x})$ is of the same sign, $g'(x_1; x^*) < 0$. Indeed, given the symmetry of the $c^i(\mathbf{x})$ functions, $h_1(\mathbf{x}^*) = h_2(\mathbf{x}^*)$, and hence, $g'(x_1; x^*) = -1$.

¹⁴ Since our analysis involves only local perturbations in the neighborhood of $x_1 = x^*$ and $x_2 = x^*$, we do not have to examine the existence and uniqueness of the iso-sum locus for all $x_1 \geq 0$.

below in deriving a condition sufficient for the social and the private optimum to be asymmetric.

A Sufficient Condition for the Optimality of Asymmetric Actions by Identical Agents.—When firm 1 invests x_1 , firm 2 invests $g(x_1; x^*)$, and firms 3 through n each invest x^* , so that the marginal-cost sum remains $c(x^*)$, industry profit can be written as a function of a single variable:

$$(3) \quad \Pi(x_1; x^*) = R(x_1; x^*) - I(x_1; x^*) - C(x_1; x^*),$$

where $I(x_1; x^*) = F(x_1, g(x_1; x^*), x^*, \dots, x^*)$ is the aggregate cost of investment in the first stage, $C(x_1; x^*) = \sum_{i=1}^n c^i(x_1, g(x_1; x^*), x^*, \dots, x^*)$ is the aggregate cost of production in the second stage, and $R(x_1; x^*)$ is industry revenue then. Similarly, when firm 1 invests x_1 , firm 2 invests $g(x_1; x^*)$ and the remaining firms invest x^* , social surplus can be written as:

$$(4) \quad W(x_1; x^*) = S(x_1; x^*) - I(x_1; x^*) - C(x_1; x^*),$$

where $S(x_1; x^*)$ is gross consumer surplus. To determine how industry profit and social surplus vary with x_1 along the iso-sum locus, we examine below how each term in (3) and (4) varies with x_1 —holding constant x_3, \dots, x_n at x^* and adjusting x_2 to maintain $x_2 = g(x_1; x^*)$.

By definition, the sum of the marginal costs of production remains unchanged at points along the iso-sum locus. As a consequence, the sum of the outputs associated with these points also remains unchanged as long as all firms produce. Accordingly, neither industry revenue nor gross consumer surplus, the first terms in (3) and (4), respectively, will vary along the iso-sum locus. Industry profit and social surplus will therefore increase whenever the decline in aggregate production costs in the last stage dominates the change in investment costs in the first stage.

Differentiating (3) and (4) with respect to x_1 , and evaluating all functions at $x_1 = x^*$, we obtain

$$(5) \quad \Pi'(x^*; x^*) = W'(x^*; x^*) = -I'(x^*; x^*) - C'(x^*; x^*) = 0.$$

It is straightforward to show that $-I'(x^*; x^*)$ in (5) is zero.¹⁵ As for $-C'(x^*; x^*)$, it is also zero since differentiability has been assumed and we showed in the previous section that aggregate production costs are larger at the equal investment point than at all other points on the iso-sum locus resulting in an interior equilibrium. This also implies that $-C''(x^*; x^*) > 0$.

Evidently x^* is a critical point of (3) and (4). It is a local minimum whenever

$$(6) \quad \Pi''(x^*; x^*) = W''(x^*; x^*) = -I''(x^*; x^*) - C''(x^*; x^*) > 0.$$

Recall that \mathbf{x}^* represents an investment combination in which all firms invest x^* , where x^* can be any nonzero investment. Denote by $\mathbf{x}^{\Pi} = (x^{\Pi}, \dots, x^{\Pi}) \in R^N_{++}$ the investment combination which maximizes *industry profit* subject to the constraint that all firms invest equally. Denote by $\mathbf{x}^W = (x^W, \dots, x^W) \in R^N_{++}$ the investment combination which maximizes *social surplus* subject to the same constraint. Then, by judiciously choosing $x^* = x^{\Pi}$ (respectively $x^* = x^W$), and using $-C''(x^*; x^*) > 0$, we know from (6) that $\Pi''(x^{\Pi}; x^{\Pi}) > 0$ (respectively $W''(x^W; x^W) > 0$) if the aggregate cost of investment ($I(x_1)$) along the iso-sum locus is weakly concave in the neighborhood of x^* . Otherwise, inequality (6) holds if and only if the concavity of the aggregate cost of production along the iso-sum locus predominates. This leads to our key sufficiency condition.

PROPOSITION 2: *Asymmetric investments by identical agents are required at the first stage in*

¹⁵ Differentiating $I(x_1; x^*)$, we obtain: $I'(x_1; x^*) = F_1(\cdot) + F_2(\cdot)g'(x_1; x^*)$. At the equal investment point, symmetry requires that $F_1(x^*) = F_2(x^*)$ and $g'(x^*; x^*) = -1$. Hence, $I'(x^*; x^*) = 0$.

order to maximize joint profits (respectively social surplus) if

$$(7) \quad - \left(2(F_{11} - F_{12}) - 2F_2 \frac{\sum_{i=1}^n (c_{11}^i - c_{12}^i)}{\sum_{i=1}^n c_2^i} \right) - \left(\frac{4(c_1^1 - c_1^2)^2}{P'} \right) > 0.$$

where P' is evaluated at $Q(\mathbf{x}^*) = \sum_{i=1}^n q^i(\mathbf{x}^*)$, and \mathbf{x}^* equals \mathbf{x}^{Π} (respectively \mathbf{x}^W).

PROOF:

Differentiating $I(x_1; \mathbf{x}^*)$ and $C(x_1; \mathbf{x}^*)$ twice, as shown in Appendix A and B, yields, respectively, the first and second terms in parentheses in (7). The inequality follows from (6).

Note that, in general, since $\mathbf{x}^{\Pi} \neq \mathbf{x}^W$, the condition in (7) may hold at one point but not the other. In such cases, it is possible that maximizing one function (be it industry profit or social surplus) requires asymmetric investments while maximizing the other requires symmetric investments.

The second term in parenthesis in (7) is strictly negative since its numerator is a squared real number and its denominator is negative; this confirms our conclusion in Corollary 4 that aggregate production costs are maximized when all firms produce at the same marginal cost. Without some restrictions on the second derivatives of $F(\mathbf{x})$ and $c^i(\mathbf{x})$, however, the first term in parenthesis cannot be signed in general. Whenever that term is nonpositive ($I'' \leq 0$), introducing asymmetry in the second stage is optimal [the inequality in (7) always holds] since the benefit of reduced second-stage costs can be achieved without incurring any additional costs at the prior stage.

When inducing asymmetry in the second stage raises first-stage costs ($I'' > 0$), the two effects work in opposite directions and the optimal policy depends on which effect predomi-

nates. The following proposition provides a sufficient condition for the introduction of asymmetry to be costly.

PROPOSITION 3: Suppose (1) $F(\mathbf{x})$ is strictly increasing in each x_i and is quasi-convex (respectively strictly quasi-convex) while (2) $\sum_{i=1}^n c^i(\mathbf{x})$ is strictly decreasing in each x_i and is strictly convex (respectively convex). Then inducing asymmetry at the second stage while maintaining the same marginal-cost sum must raise costs at the first stage.

PROOF:

Under our assumptions, the set of investments with a marginal-cost sum ($\sum_{i=1}^n c^i(\mathbf{x})$) no larger than a specified level is strictly convex (respectively convex). Pick from this set the investment vector generating the *smallest* investment cost at the first stage. Given our convexity assumptions, this constrained minimum must occur at a unique point. Given our symmetry assumptions, investment costs are minimized when the n firms invest equally. Since unequal investments generating the same second-stage marginal-cost sum are also feasible, they must result in a strictly larger investment cost at the prior stage. Hence, introducing asymmetry raises investment costs.

Each of the applications discussed in the next section satisfy the assumptions of Proposition 3. As a result, the benefits of introducing asymmetry discussed in Section II must be weighed against the additional costs of introducing it. The sufficiency condition in (7) is helpful in such circumstances. Whenever the second term predominates, *overall* costs can be reduced by introducing asymmetry at the first stage even though doing so increases investment costs at the first stage.

III. Applications of the Sufficient Condition and Policy Implications of the Analysis

In this section, we consider applications where inducing marginal-cost asymmetry in the second stage is costly, but nonetheless optimal in some regions of parameter space. In each case, we discuss potential implications of the asymmetry result for government policy.

A. Pollution with Costly Abatement

Recently, an international agreement was reached among countries to reduce emissions contributing to global warming. Carbon dioxide emissions depend not merely on subsequent production levels but also on efforts to abate. The question naturally arises as to whether maximization of social surplus requires that countries with identical costs of abatement be assigned identical emissions quotas. A further question is whether the socially optimal agreement can be decentralized in the usual way either by a common tax on emissions or, equivalently, by a market-generated price which all agents would face for the purchase or sale of tradable emission permits (Varian, 1996 pp. 568–70).

Newbery (1990) provides a model of emissions and abatement where these issues can be examined. Consider a two-stage game played by n firms, one in each country. In the first stage, the maximum pollution level allowed for each firm is chosen. Given this, each firm must decide in the second stage how much to invest in abatement equipment and how much to curtail production so as to comply with its first-stage emissions quota. Firms play a Cournot game and have abatement costs but, for simplicity, no production costs. Whatever production decision a firm makes in the second stage must be matched by a decision to purchase abatement equipment to bring its pollution within the emissions quota. Since equipment is costly, no firm pollutes less than it is permitted.

Let x_i denote the first-stage maximal pollution permitted firm i , z_i denote the amount of abatement equipment purchased by firm i , and r be the exogenous per-unit cost of this equipment. Let the amount of pollution released by firm i be increasing in its output and decreasing in its abatement effort—specifically, $x_i = q_i/z_i$. Then if firm i produces q_i in the second stage it must have abatement equipment $z^i(\mathbf{x}) = q_i/x_i$ which will cost rq_i/x_i . Hence, it is as if firm i has a constant marginal cost of production of $c^i(\mathbf{x}) = r/x_i$. Firm i 's overall profit is $(P(Q) - r/x_i)q_i$. Following Newbery, assume that the inverse demand curve in the second stage is linear: $P(Q) = a - Q$. Suppose social damage depends on the aggregate pollution of the n firms: $F(\mathbf{x}) = (\sum_{i=1}^n x_i)^2$.

If our sufficient condition (7) holds, the

second-stage benefits of introducing the asymmetry outweigh the additional costs incurred in the first stage.¹⁶ It is straightforward to show that $I''(x^*) = 16$ and $C''(x^*) = -4r^2/(x^*)^4$. Thus, if emissions quotas are assigned in the first stage taking into account that firms will subsequently compete as Cournot oligopolists in the second stage, condition (7) indicates that whenever $r > 2(x^w)^2$, social optimality requires that *different* pollution standards be set for firms even though they are *ex ante* identical.¹⁷ This condition is satisfied, for instance, if $n = 2$ and $a = 2r$.

Pigouvian Taxes and Tradable Emissions Permits.—Economists often prefer Pigouvian taxes or tradable emissions permits to firm-specific standards because of their flexibility. If the output market is competitive, a Pigouvian tax on pollution or, alternatively, allowing permits to be traded results in a decentralized achievement of the social optimum. Firms that produce more efficiently will buy more permits than firms that produce less efficiently until marginal abatement costs are equalized. However, neither Pigouvian taxes nor a competitive market of tradable emissions permits can achieve an interior *asymmetric* social optimum when Cournot competition occurs under *laissez-faire* at the second stage since each

¹⁶ It is straightforward to verify in this example that the assumptions of Proposition 3 are satisfied; hence, additional costs must be incurred at the prior stage in order to induce unequal marginal costs with the same sum in the final stage. Intuitively, if the emissions quota of firm 1 is increased by $\Delta_1 > 0$ while the emissions quota of firm 2 is reduced by $\Delta_2 > 0$ then the sum of the marginal costs in the second stage will strictly increase for any $\Delta_2 \geq \Delta_1 > 0$: that is, $r/(x^* + \Delta_1) + r/(x^* - \Delta_2) > 2r/x^*$. On the other hand, for any $x^* > 0$ and $\Delta_1 > 0$, reducing firm 2's emissions by $\Delta_2 = \Delta_1/(1 + 2\Delta_1/x^*) < \Delta_1$ results in asymmetric marginal costs in the second period with an *unchanged* marginal-cost sum. By the usual argument, this second-stage asymmetry results in smaller aggregate abatement costs. However, there is a trade-off. Since $\Delta_1 - \Delta_2 > 0$, emissions at firm 1 would increase by more than emissions at firm 2 would decrease. Hence, the reduction in aggregate abatement costs comes at the expense of increased pollution damage. When inequality (7) holds, however, the benefits of introducing asymmetry outweigh these additional costs.

¹⁷ In Newbery (1990), each firm's cost of investment depends on the actions of others. Nonetheless, the expression for I'' in (7) simplifies considerably because although $F_{11} > 0$ and $F_{12} > 0$, $F(\cdot)$ depends only on the sum of the first-stage actions, which implies that $F_{11} - F_{12} = 0$.

policy would induce identical behavior by identical agents.

B. Investment in Research and Development

Firms often invest heavily in research and development (R&D) to lower their production costs. It is sometimes argued that in the absence of government intervention or a privately funded research joint venture, R&D investment would be socially and privately suboptimal because each firm would fail to take account of the beneficial spillovers it provides other firms. Given our results, one might ask whether a research joint venture could increase social welfare even in the absence of spillovers because it could achieve cost reductions by creating asymmetries in R&D investments among participating firms which would not occur under *laissez-faire*. In the absence of a research joint venture, a government constrained to intervene only in the investment stage would have to subsidize domestic firms of equal competence differentially in order to achieve the social optimum. Governmental promotion of the R&D investments of some domestic firms at the expense of others is sometimes referred to as the creation of "national champions." Arguments in favor of creating national champions typically rely on the existence of economies of scale in R&D. Given our results, one might ask whether—even in the absence of such scale economies—promoting national champions is socially optimal because it results in cost reductions which would not be achieved if the government was more evenhanded in its promotion of the R&D of domestic firms in the industry.

d'Aspremont and Jacquemin (1988) are among the first to formulate an oligopoly model in which these issues could be addressed. They envision firms engaged in a two-stage game. In the first stage, firms invest in R&D while in the second stage they produce as Cournot competitors. d'Aspremont and Jacquemin assume an inverse demand of $P(Q) = a - bQ$ in the second stage. Firm i 's constant marginal cost of production is assumed to be a linear function of first-stage investments but with *spillovers* from the other firms: $c^i(\mathbf{x}) = \bar{c} - x_i - \beta \sum_{j \neq i} x_j$.

Hence, $\sum_{i=1}^n c^i(\mathbf{x}) = n\bar{c} - \beta[n - 1]X$ and the sum of the marginal costs of production is affected only by changes in aggregate R&D and not by its composition. The R&D cost of any firm is assumed to depend only on its own investment: $f^i(x_i) = \gamma x_i^2/2$, where the exogenous parameters satisfy $0 < \bar{c} < a$, $0 < \beta < 1$, and $\gamma > 0$. The aggregate cost of first-stage investment \mathbf{x} is thus $F(\mathbf{x}) = 0.5\gamma \sum_{i=1}^n x_i^2$, and it is straightforward to verify that the assumptions of Proposition 3 are satisfied.

Substituting these functional forms into the first term in (7) we conclude that $I''(x^*) = 2\gamma > 0$. Moreover, substitutions into the second term in (7) imply that $C''(x^*) = -4(\beta - 1)^2/b < 0$. Hence, condition (7) indicates that joint-profit maximizing investments are unequal for a research cartel¹⁸ whenever $\gamma < 2(\beta - 1)^2/b$. Since this inequality depends only on exogenous parameters, and not on a particular level of x_1 , the *same* condition holds if instead the objective function is social surplus. Indeed, in this example, asymmetric investments are optimal for a social planner constrained to intervene only in the investment stage if and only if the joint-profit maximum is also asymmetric.

Research Joint Ventures.—If the firms form a research joint venture and maximize joint profits, the cost savings from making the R&D investments of identical firms unequal may be so large that social surplus also increases even if there are no spillovers (Salant and Shaffer, 1998).

U.S. antitrust law has long viewed research joint ventures in a favorable light. Recently, Congress concurred by passing the National Cooperative Research Act (NCRA), which states that a research joint venture will "be judged on the basis of its reasonableness, taking into account all relevant factors affecting competition."¹⁹ Research joint ventures are thought

¹⁸ d'Aspremont and Jacquemin (1988 p. 1134, footnote 7) impose the parameter restriction $\gamma > 2(1 + \beta)^2/9b$ to ensure that the joint profit function in the first stage is strictly concave in x_1 along the path of equal investments. However, as long as $\beta < 0.5$, it is possible to satisfy this restriction but nonetheless also satisfy $\gamma < 2(\beta - 1)^2/b$, the condition in the text requiring profit-maximizing investments to be unequal.

¹⁹ The NCRA also allows parties to the research joint venture to limit their antitrust exposure in private suits to

to confer benefits in sharing risk, achieving economies of scale, pooling complementary assets, and mitigating free-riding when there are research spillovers. Our analysis suggests an additional benefit: members in a research joint venture may be able to achieve production-cost savings by allocating investments within the joint venture asymmetrically. Some members will be asked to cut back on their R&D spending in this scheme, while others will be asked to expand. Ironically, a research cartel that attempted to realize the cost savings we identify would risk antitrust prosecution under the NCRA. For, although government challenges of research joint ventures are infrequent, they are more likely to occur when the effect of the joint venture is to "retard the pace of individual company research."²⁰

R&D Subsidization and the Creation of "National Champions."—In the absence of such a research joint venture, firms would invest non-cooperatively and, as d'Aspremont and Jacquemin show, R&D investments at the first stage would be symmetric in the absence of government intervention.²¹ If the condition in (7) holds, a surplus-maximizing government constrained to intervene only at the R&D stage

would tax or subsidize asymmetrically. By setting the taxes and subsidies so that the sum of marginal costs of production at the second stage is unchanged, the government could induce a reduction in aggregate production costs.

Such a cost reduction could be achieved even if only a subset of the firms in the industry are domestic. Suppose that $m \leq n$ of the firms are domestic and domestic firm i 's second-stage marginal cost is $c^i = \bar{c} - x_i - \beta(X_D - x_i) - \beta_F X_F$, where X_D denotes aggregate domestic R&D, X_F denotes aggregate foreign R&D, $\beta \geq \beta_F$ are the spillover parameters, and $\beta < 1$. No matter what the R&D investments of the foreign firms, it is suboptimal for identical domestic firms to invest equal amounts in R&D. For, suppose initially equal domestic investments were altered while maintaining their sum (X_D). Then since no foreign investment would change and spillovers from the domestic industry in aggregate would be unchanged, no foreign marginal cost of production would change. While the marginal production cost of individual domestic firms would change, the sum of these marginal costs would not since $\sum_{i=1}^m c^i = m\bar{c} - X_D - \beta[m - 1]X_D - m\beta_F X_F$. It follows that domestic profits and social surplus would increase.

One way of introducing asymmetry among the domestic firms would be to choose arbitrarily from the set of domestic firms one to be designated as the "national champion." Suppose the increase in the per-unit subsidy to that firm's R&D is offset by a reduction in the per-unit subsidy to the R&D of the other domestic firms sufficient to maintain the same final output. Then, the market price will not change and no domestic (or foreign) consumers will be affected. However, the sum of domestic R&D costs and subsequent domestic production costs will decline; hence, the policy will improve national welfare. Arguments in favor of designating a "national champion" usually rely on scale economies in production. Objections to such policies are in turn based on concerns about the impact on foreign industries and the possibility of retaliation. As our discussion above reflects, however, neither of these arguments is complete. Designating one firm a national champion in the example above benefits the home country even in the absence of any scale

actual damages, with provisions to recover costs and attorney's fees in the case of frivolous suits, providing proper notification of the joint venture is filed with the federal enforcement authorities.

²⁰ In *United States v. Automobile Mfrs. Ass'n*, 1969 Trade Cases (Commerce Clearing House, 1969 ¶72,907) (C. D. Cal. 1969), the Department of Justice challenged a research joint venture of automobile manufacturers alleging that it had the effect of delaying the installation of pollution control equipment "by at least some of the manufacturers."

²¹ In discussing taxes/subsidies imposed by governments, it is convenient to transform d'Aspremont and Jacquemin's formulation by letting $y_i = 0.5\gamma x_i^2$ so that $c^i = \bar{c} - (2y_i/\gamma)^{1/2} - \beta \sum_{j \neq i} (2y_j/\gamma)^{1/2}$. While x_i represents the amount of investment (denominated in "efficiency units") needed to reduce marginal costs of production by \$1, the new variable y_i denotes R&D investment denominated in dollars. As investment (y_i) increases, the induced marginal cost of production in the second stage declines but at a decreasing rate (that is, c^i is strictly decreasing but strictly convex in the y 's). In our new notation, if the inequality condition holds, it is joint-profit maximizing and socially optimal to have $y_i \neq y_j$. If the government taxes (respectively subsidizes) firm i , its R&D expenses would become $y_i(1 + t_i)$, where $t_i > 0$ (respectively $t_i < 0$).

economies in production and these benefits need not come at the expense of consumers or foreign firms.

C. Intermediate Goods Markets

The Robinson-Patman Act.—Our results also have implications for domestic antitrust policy. In situations where an upstream supplier sells an input to competing downstream firms, the Robinson-Patman Act prohibits price discrimination: it requires that the upstream firm offer the same contract terms to each downstream buyer.²² The law presumes that social welfare will be higher when the downstream firms are treated equally. But suppose the upstream firm produces machines at a constant marginal cost (normalized to 1) and sells them to two identical downstream purchasers which then use the machinery to produce a final product for consumers. Assume the inverse demand of these final consumers is linear: $P = a - bQ$. Assume that the more machinery downstream firm i purchases, the lower will be its per-unit cost of production although the per-unit cost reductions are subject to diminishing returns: $c^i = \bar{c} - (2y_i/\gamma)^{1/2}$ where y_i denotes the number of machines purchased by downstream firm i . Then, whenever condition (7) holds ($b\gamma < 2$), the requirement that each downstream purchaser pay the same price per machine mandates inefficiency.

To see this, note that by forbidding price discrimination, the law requires that the upstream firm charge the downstream firms the same per-unit price, inducing them to purchase the same amount of machinery and, therefore, to compete as equals in the product market. If the upstream firm instead were permitted to charge different per-unit prices, it could generate greater profit in the vertical

chain and would have an incentive to do so if it could then extract the additional profit with firm-specific access charges. Specifically, by charging one of the downstream firms a lower price for each machine, it would induce that firm to purchase more machinery and, with its lower marginal costs of production, to produce a larger final output than the other downstream firm. If the upstream firm simultaneously raised the machine price to the other downstream firm so that, between them, the two downstream firms produced the same aggregate output as before, then both the gross benefit and the outlay of every final consumer would be unchanged. But this unchanged surplus would have been provided at a strictly smaller production cost. Moreover, since condition (7) holds by assumption, the reduction in the aggregate production costs at the two downstream firms exceeds the cost increase of the upstream firm resulting from having to provide a strictly larger number of machines than before. If the upstream firm is to operate so as to maximize social surplus or, alternatively, its own profit in such circumstances, it will want to charge the downstream purchasers different prices—precisely what the Robinson-Patman law forbids.

IV. Conclusion

In this paper, we have examined a phenomenon which arises when firms take actions (“investments”) that subsequently alter their final-stage marginal costs of production. Given Cournot behavior in the final stage, we show that it is sometimes both socially and privately optimal—if coordination is possible at the prior stage—to invest asymmetrically then.

The same phenomenon arises even when the assumption of constant marginal costs of production is relaxed. Suppose, for example, that each firm’s marginal cost is instead an increasing linear function of its output with a fixed common slope and a firm-specific intercept which can be reduced by prior investment. Then, it is easily shown that: (1) aggregate output, price, industry revenue, and gross consumer surplus are the same in every interior equilibrium where the sum of the

²² The Robinson-Patman Act makes it unlawful for a seller to “discriminate in price between different purchasers of commodities of like grade and quality” where substantial injury to competition may result. In practice, the courts have often found substantial injury to competition, and thus, illegal price discrimination, on a mere showing of a persistent difference in input price. See Antitrust Law Developments (American Bar Association Antitrust Section, 1992 pp. 401–52).

firm-specific intercepts is unchanged; (2) industry profit and social welfare strictly increase whenever the variance of the firm-specific intercepts increases; and consequently (3) there is a presumption that a cartel coordinating actions in the first stage or a social planner able to intervene directly only in the first stage would each prefer asymmetric investments.²³

Our asymmetry result is surprising because as a profession we have become accustomed to thinking about problems where coordination occurs at *neither* stage. In such *fully decentralized* problems, symmetric investments at the first stage by *ex ante* identical firms typically occur in equilibrium. But if the point of departure of analysis were instead the *fully centralized* problem, where a single decision maker—whether a cartel or social planner—controlled production in the second stage as well as investments in the first, asymmetric investments would be the norm. For simplicity, assume no spillovers. Suppose the planner (or cartel manager) invested equally at any two identical firms in the first stage and, as a result, they had the same constant marginal cost of production in the second stage. If the planner used only one of these firms to produce in the second stage, the cost of providing the given aggregate output could be strictly reduced by eliminating investment at the unutilized firm. If, on the other hand,

²³ The first-order condition for firm i is $P(Q) + q_i P'(Q) = a_i + m q_i$, where $m \geq 0$ is the common slope and a_i is the firm-specific intercept. Summing, we obtain $nP(Q) + Q(P' - m) = \sum_{i=1}^n a_i$ which implies that the aggregate output depends only on the sum of the firm-specific intercepts. The first-order condition can be solved to obtain $q_i = (P - a_i)/(m - P')$. Substitute this into the following expression for industry cost: $\sum_{i=1}^n a_i q_i + m \sum_{i=1}^n q_i^2/2$. This substitution yields an expression which depends only on $\sum_{i=1}^n a_i$ and $\sum_{i=1}^n a_i^2$. Since it is strictly decreasing in the latter, industry costs strictly decrease if $\sum_{i=1}^n a_i$ is unchanged but the sum of squares of the firm-specific intercepts strictly increases. Among all sets of marginal costs curves with the same intercept sum, the set with equal intercepts minimizes social surplus and profit. Our Proposition 1, first proved by Bergstrom and Varian (1985a), follows as a special case ($m = 0$).

the planner utilized *both* firms at the second stage, the cost of providing the given aggregate output could be strictly reduced by eliminating investment at one of the two firms and replacing all the output it previously produced with increased output at the other firm.²⁴ Thus, in the fully centralized problem, the planner (or cartel manager) would never invest equally at two identical firms.

Once it is appreciated that asymmetric investments may be optimal when production decisions are fully centralized, it should not be surprising that this phenomenon carries over to the situation where outputs are chosen noncooperatively in the final stage.

As we have seen, many policy conclusions must be reconsidered when the social optimum entails asymmetric investments at identical firms. In the realm of antitrust policy, for example, the Robinson-Patman injunction against price discrimination by upstream firms can be shown sometimes to reduce social welfare, and research joint ventures which retard the pace of R&D at *some* firms (while increasing it at others) need not be cause for social concern. In the international sphere, it is now possible to justify on efficiency grounds the designation of one domestic firm as a national champion even though it enjoys no scale economies and outperforms no other domestic firm. Finally, if each firm sets production taking full account of its effect on price rather than ignoring this effect as perfectly competitive firms are assumed to do, the work horses of environmental policy—Pigouvian emissions taxes and tradable emissions permits—fail since they induce identical behavior in identical firms whereas asymmetric behavior may be needed.

²⁴ Constant marginal costs are not necessary for the optimum in the centralized problem of a planner or a cartel to be asymmetric. A similar proposition holds if every firm has a linear marginal-cost curve with a common positive slope and an intercept linearly affected at a common rate by the firm's own prior investment provided aggregate investment costs depend on the sum of prior investments.

APPENDIX A

In this Appendix we compute the first and second derivatives of the aggregate investment function $I(x_1; x^*)$ and the aggregate cost function $C(x_1; x^*)$ as investment in firm 1 is increased along the iso-sum locus in the neighborhood of the equal investment point. The analysis proceeds in three steps.

Step 1: To begin, we calculate the first and second derivatives of $g(x_1; x^*)$ and then evaluate them at x^* . Since the iso-sum locus $x_2 = g(x_1; x^*)$ satisfies $\sum_{i=1}^n c^i(x_1, x_2, x^*, \dots, x^*) = c^*(x^*)$, we apply the implicit function theorem to obtain

$$(A1) \quad g'(x_1; x^*) = - \frac{\sum_{i=1}^n c_1^i(x_1, x_2, x^*, \dots, x^*)}{\sum_{i=1}^n c_2^i(x_1, x_2, x^*, \dots, x^*)}$$

Evaluating (A1) at $x_1 = x_2 = x^*$ yields $g'(x^*; x^*) = -1$, since $c_1^1 = c_2^2, c_1^2 = c_2^1$, and $c_1^k = c_2^k, k = 3, \dots, n$, for any equal investment combination. Differentiating $g(x_1; x^*)$ twice with respect to x_1 gives

$$(A2) \quad g''(x_1; x^*) = \frac{-\sum_{j=1}^n c_2^j \sum_{i=1}^n (c_{i1}^i + c_{i2}^i g') + \sum_{j=1}^n c_1^j \sum_{i=1}^n (c_{i1}^i + c_{i2}^i g')}{\left(\sum_{j=1}^n c_2^j\right)^2}$$

Substituting $g'(x^*; x^*) = -1$ into equation (A2) and observing that $\sum_{j=1}^n c_2^j = \sum_{j=1}^n c_1^j$ when evaluated at $x_1 = x_2 = x^*$ yields

$$(A3) \quad g''(x^*; x^*) = \frac{-\sum_{i=1}^n (c_{i1}^i - c_{i2}^i) + \sum_{i=1}^n (c_{i1}^i - c_{i2}^i)}{\sum_{j=1}^n c_2^j}$$

Since $c_{11}^1 = c_{22}^2, c_{22}^1 = c_{11}^2, c_{12}^1 = c_{21}^2, c_{21}^1 = c_{12}^2$, and $c_{12}^k = c_{21}^k, c_{11}^k = c_{22}^k, k = 3, \dots, n$, for any equal investment combination, (A3) can be rewritten as

$$(A4) \quad g''(x^*; x^*) = \frac{-2 \sum_{i=1}^n (c_{i1}^i - c_{i2}^i)}{\sum_{j=1}^n c_2^j}$$

Step 2: Recall that $I(x_1; x^*) = F(x_1, g(x_1; x^*), x^*, \dots, x^*)$. Differentiating $I(x_1; x^*)$ twice yields

$$(A5) \quad I''(x_1; x^*) = F_{11} + F_{12}g' + F_{21}g' + F_{22}(g')^2 + F_2g''.$$

Substituting $g'(x^*; x^*) = -1$ into (A5) and using condition (A4) gives the desired result:

$$(A6) \quad I''(x^*; x^*) = 2 \left(F_{11} - F_{12} - F_2 \frac{\sum_{i=1}^n (c'_{11} - c'_{12})}{\sum_{i=1}^n c'_2} \right).$$

This is the first term in parentheses in (7) in the text.

Step 3: Next we show that aggregate production costs are locally maximized at x^* . Recall that $C(x_1; x^*) = \sum_{i=1}^n c^i(x_1, g(x_1; x^*), x^*, \dots, x^*)q^i(x_1, g(x_1; x^*), x^*, \dots, x^*)$. Differentiating $C(x_1; x^*)$ and noting that the sum of outputs and sum of marginal costs are fixed along the iso-sum locus gives

$$(A7) \quad C'(x^*; x^*) = c \sum_{i=1}^n \frac{dq^i}{dx_1} + q \sum_{i=1}^n \frac{dc^i}{dx_1} = 0,$$

where c and q are the common marginal cost and output of each firm respectively. This confirms our finding in the text that the aggregate production cost function is flat at x^* . Finally, differentiating $C(x_1; x^*)$ twice with respect to x_1 and evaluating the result at $x_1 = x^*$ yields

$$(A8) \quad C''(x^*; x^*) = c \sum_{i=1}^n \frac{d^2q^i}{dx_1^2} + q \sum_{i=1}^n \frac{d^2c^i}{dx_1^2} + 2 \sum_{i=1}^n \frac{dq^i}{dx_1} \frac{dc^i}{dx_1} = 2 \sum_{i=1}^n \frac{dq^i}{dx_1} \frac{dc^i}{dx_1}.$$

Since $dq^i/dx_1 = q'_1 + g'q'_2 = q'_1 - q'_2$ and $dc^i/dx_1 = c'_1 + g'c'_2 = c'_1 - c'_2$, (A8) can be rewritten as

$$(A9) \quad C''(x^*; x^*) = 2 \sum_{i=1}^n (q'_1 - q'_2)(c'_1 - c'_2).$$

Using (B3) from Appendix B gives

$$(A10) \quad C''(x^*; x^*) = 2 \sum_{i=1}^n \frac{(c'_1 - c'_2)^2}{P'}.$$

Since $c'_1 - c'_2 = c'_2 - c'_2 \neq 0$ and $c'_1 - c'_2 = 0, \forall i \geq 3$, (A10) simplifies to

$$(A11) \quad C''(x^*; x^*) = \frac{4(c'_1 - c'_2)^2}{P'}.$$

This is the second term in parentheses in (7) in the text.

APPENDIX B

In this Appendix we derive the result used in condition (A9) of Appendix A. Let $\pi^i(\cdot)$ denote firm i 's second-period profit as a function of the second-period output profile (given marginal costs determined by the first-period action profile). Assuming an interior optimum, firm i 's quantity as a function of x_1, x_2 is implicitly defined by the n first-order conditions:

$$P(Q) + P'(Q)q^i - c^i(\mathbf{x}) = 0, \quad i = 1, \dots, n.$$

Totally differentiating these conditions with respect to x_i and then imposing symmetry yields a two-by-two system of equations which can be expressed in matrix form as

$$\begin{pmatrix} \pi_{ii}^i & (n-1)\pi_{ij}^i \\ \pi_{ij}^i & \pi_{ii}^i + (n-2)\pi_{ij}^i \end{pmatrix} \begin{pmatrix} dq^i \\ dq^j \end{pmatrix} = \begin{pmatrix} c_i^i dx_i \\ c_j^i dx_i \end{pmatrix},$$

where $j \neq i$ and dq^j is the common response of the firms other than firm i to an increase in x_i . Using Cramer's rule, and evaluating at the point of equal investments, we obtain:

$$(B1) \quad q_i^i|_{x_i=x_j} = \frac{c_i^i(\pi_{ii}^i + (n-2)\pi_{ij}^i) - (n-1)c_j^i\pi_{ij}^i}{\pi_{ii}^i(\pi_{ii}^i + (n-2)\pi_{ij}^i) - (\pi_{ij}^i)^2(n-1)} > 0,$$

which gives the effect on firm i 's equilibrium quantity of a marginal increase in own investment, and

$$(B2) \quad q_j^i|_{x_i=x_j} = \frac{c_i^i\pi_{ij}^i - c_j^i\pi_{ij}^i}{\pi_{ii}^i(\pi_{ii}^i + (n-2)\pi_{ij}^i) - (\pi_{ij}^i)^2(n-1)},$$

which gives the effect of a marginal increase in firm j 's investment on firm i 's equilibrium quantity.

Subtracting (B2) from (B1) and simplifying by canceling common fractions yields

$$(B3) \quad (q_i^i - q_j^i)|_{x_i=x_j} = \frac{c_i^i - c_j^i}{\pi_{ii}^i - \pi_{ij}^i} = \frac{c_i^i - c_j^i}{P'} > 0,$$

the result used in (A11) of Appendix A.

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