When Good News About Your Rival Is Good for You: The Effect of Third-Party Information on the Division of Channel Profits

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Abstract
The Internet has led to a large number of third-party sources that offer high-quality information about firms’ products at little or no cost to consumers. As a result, many of these sources have grown in popularity, extending well beyond the usual reach of traditional third parties such as Consumer Reports and Kelly’s Blue Book. For example, the online version of Edmunds offers, at no cost to consumers, information about new products, existing products, long-term tests, and buyers’ guides, all relating to the automotive industry. AvWeb.com delivers weekly aviation news and new product reviews to its readers, and a large number of websites follow developments on computer platforms such as the Apple Macintosh.

In this paper we analyze how the provision of third-party information affects the division of profits in a multiproduct distribution channel. To illustrate, consider the competition between Microsoft and Apple in the operating systems (OS) market and their channel relationship to CompUSA, a retailer that sells both Macs and Windows-based PCs. Consider two pieces of third-party information. First, suppose that CNET, an Internet technology site, reviews the newest upgrade of the MacOS and writes that the new user interface is even easier to use than previously. Second, suppose that an article in the technology section of the Wall Street Journal notes that changes in Apple’s networking support now enable Macs to be better integrated into PC networks. These two pieces of information are similar in the sense that they both express good news about the MacOS and thus they both can be expected to benefit Apple by increasing consumer demand for Macs. One might also expect that in both cases CompUSA will capture some of the gains that come from the increased demand for Macs and that Microsoft will lose because the good news about the MacOS will induce some consumers to switch from Windows-based PCs to Macs. The reason is that the two reviews differ on one crucial dimension: the group of customers for whom they are primarily relevant. The CNET review talks about improvements in the customer interface—precisely what Apple’s core consumers care about. The Wall Street Journal review talks about compatibility with prevailing PC standards—important to consumers who care relatively more about compatibility and who are thus more likely to prefer Windows (Apple’s noncore consumers). We show that good news about the MacOS that is more relevant to Apple’s core consumers (the CNET review) benefits Microsoft but harms CompUSA, while good news about the MacOS that is more relevant to Apple’s noncore consumers (the Wall Street Journal review) has the opposite effect. It harms Microsoft but benefits CompUSA.

Stated more generally, our main result is that when third-party information affects consumers’ product valuations, the type of information that induces the change is critical to understanding which firms gain and which firms lose. In particular, depending on the type of third-party information, we find that (1) a retailer can be harmed by good news about a product that it carries; (2) a manufacturer can gain from good news about a rival’s product; and (3) good news about a product category need not benefit all the manufacturers in that category.

There are three novel features of the analysis. First, we derive the equilibrium division of profit among firms when a retailer sells the products of competing manufacturers, and we have done so while placing few restrictions on the feasible set of contracts. Second, we show how this equilibrium division of profit lends itself to a simple graphical interpretation that depicts which firms gain and which firms lose from third-party information. Third, we provide a taxonomy of information types and identify the key features of each type that cause profit incentives to vary. In particular, we conceptualize information as having three components, namely (1) the products to which the information pertains, (2) whether the information is positive or negative, and (3) the consumers to whom the information is relevant. We show that all three information components play a role in determining the change in each firm’s profit.

Our framework can also be used to analyze a variety of other settings of interest; for example, it can be used to analyze profit incentives when the retailer has bargaining power, when there is downstream competition, and when there are non-information-based changes in consumers’ valuations. In addition, our framework may be used both to analyze the effects on profits of persuasive advertising and to predict advertising content.

(Game Theory; Channel Coordination; Third-Party Information; Advertising; Internet; Distribution Channel)
1. Introduction

The Internet has led to a large number of third-party sources that offer high-quality information about firms’ products at little or no cost to consumers. As a result, many of these sources have grown in popularity, extending well-beyond the usual reach of traditional third parties such as Consumer Reports and Kelly’s Blue Book. For example, the online version of Edmunds offers, at no cost to consumers, information about new products, existing products, long-term tests, and buyers’ guides, all relating to the automotive industry. AvWeb.com delivers weekly aviation news and new product reviews to its readers, and a large number of websites follow developments on computer platforms such as the Apple Macintosh. MacInTouch, MacFitIt, MacCentral, and several other sites offer daily industry news and review hardware and software.

In this paper we analyze how the provision of third-party information affects the division of profits in a multiproduct distribution channel. To illustrate, consider the competition between Microsoft and Apple in the operating systems market and their channel relationship to CompUSA, a retailer that sells both Macs and Windows-based PCs. Consider two pieces of third-party information. First, suppose that CNET, an Internet technology site, reviews the newest upgrade of the MacOS and writes that the new user interface is even easier to use than previously. Second, suppose that an article in the technology section of the Wall Street Journal notes that changes in Apple’s networking support now enable Macs to be better integrated into PC networks. These two pieces of information are similar in the sense that they both express good news about the MacOS and thus can be expected to benefit Apple by increasing consumer demand for Macs. One might also expect that in both cases CompUSA will capture some of the gains that come from the increased demand for Macs and that Microsoft will lose because the good news about the MacOS will induce some consumers to choose Macs over Windows-based PCs. However, we will show that this intuition is incorrect. The two reviews can have surprisingly different implications for the profits of Microsoft and CompUSA.

The reason is that the two reviews differ on one crucial dimension: the group of customers for whom they are primarily relevant. The CNET review talks about improvements in the customer interface—precisely what Apple’s core consumers care about. The Wall Street Journal review talks about compatibility with prevailing PC standards—important to consumers who care relatively more about compatibility and who are thus more likely to prefer Windows (Apple’s non-core consumers). We show that good news about the MacOS that is more relevant to Apple’s core consumers (the CNET review) benefits Microsoft but harms CompUSA, while good news about the MacOS that is more relevant to Apple’s noncore consumers (the Wall Street Journal review) has the opposite effect. It harms Microsoft but benefits CompUSA.

Stated more generally, our main result is that when third-party information affects consumers’ product valuations, the type of information that induces the change (whether it is product specific or relates to the whole category and whether it applies equally to all consumers or affects some consumers more than others) is critical in understanding which firms gain and which firms lose. In particular, depending on the type of third-party information, we find that (1) a retailer can be harmed by good news about a product that it carries, (2) a manufacturer can gain from good news about a rival’s product, and (3) good news about a product category need not benefit all the manufacturers in that category.

In addition to distinguishing between information that is more relevant to a firm’s core consumers and information that is more relevant to a firm’s noncore consumers, we also examine the effects of information that applies equally to all consumers and information that pertains to the product category as a whole. (For example: “The SAT scores of children whose families own computers are on average higher than the SAT scores of children without computers.”)

The effect of third-party information on the division of channel profits has not previously been studied. Models that analyze firms’ own incentives to inform consumers or that consider the effects of information on firms’ competitive behavior (Grossman and Shapiro 1984, Zettelmeyer 2000a and b,

The rest of the paper proceeds as follows. Section 2 offers a taxonomy of third-party information. Section 3 derives the equilibrium division of profits in a channel structure with two manufacturers and one retailer. Section 4 looks at how different types of third-party product information affect each firm’s profit. We show that which firms gain and which firms lose depends on whether the information is more relevant to a firm’s core or noncore consumers. Section 5 extends the analysis to information that affects all products in a category. Section 6 discusses extensions of the model to multiple downstream retailers, any distribution of bargaining power, and noninformation-based changes in consumers’ valuations. Section 7 concludes the paper. The proofs of all propositions can be found in the Appendix.

2. Third-Party Information

We conceptualize third-party information of the kind typically found on the Internet and other media as having three essential components, namely (1) the products to which the information pertains, (2) the effect that the information has on consumers’ valuations, and (3) the consumers to whom the information is relevant. The first component captures whether the information affects consumers’ valuations for all products in a category or for just one product. We call the former type category information and the latter type product information. The second component captures whether consumers’ gross utilities for the product(s) to which the information pertains increase or decreases. We say that information is positive if valuations increase and negative if valuations decrease. The third component captures whether the information affects all consumers’ valuations equally or primarily the valuations of a product’s core consumers (this will be made more precise in the next subsection).

We model information in the context of a model of demand in which consumers have a common product-specific utility $v_i$ for product $i$ but differ in their preferences over two product attributes, $a$ and $b$. We assume that there is a negative correlation between these attributes, i.e., those consumers that prefer attribute $a$ the most like attribute $b$ the least, and vice versa.

Let $a_i$ denote the maximum difference in consumers’ valuations for attribute $a$ in product $i$ and let $b_i$ be defined similarly. Let $\theta \in [0, 1]$ index the unit mass of consumers. Then the utility that a consumer of type $\theta$ derives from consuming product $i$ is

$$U_i(\theta) = v_i + (1 - \theta)a_i + \theta b_i.$$  

(1)

The product-specific utility $v_i$ captures two aspects of consumers’ overall utility. First, $v_i$ is the part of utility for product $i$ that is common to all consumer types; it allows for other attributes of the product that matter for consumers’ utility but whose value to consumers does not vary with $\theta$. Second, $v_i$ subsumes consumers’ minimum utility for attributes $a$ and $b$, allowing consumers to be indexed by $\theta \in [0, 1]$ without implying that consumers at the extremes place no value on attributes $a$ or $b$. For example, while $U_i(0) = v_i + a_i$ is the utility for product $i$ of the consumer type that cares least about attribute $b$, it does not imply that this consumer derives no utility from attribute $b$. The negative correlation in consumers’ preferences over attributes $a$ and $b$ is commonly found in many “location” models of product differentiation in which consumers located closer to attribute $a$ are farther away from attribute $b$ and vice versa.

There are two products, $X$ and $Y$, that are horizontally differentiated. This implies that neither product dominates the other product for all attributes. Without loss of generality, we assume that product $X$ is better on attribute $a$ and that product $Y$ is better on attribute $b$:

$$a_x > a_y, \quad b_x < b_y.$$  

(2)

Vertical differentiation implies that all consumer types’ gross utility is highest for the same product. For example, this would be the case if $v_x > v_y, a_x > a_y$ and $b_x > b_y$. 

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In addition, to ensure that a retailer will want to sell both products, we also assume that each product "specializes" in a different attribute. We assume that product \( X \) is relatively better on attribute \( a \) and that product \( Y \) is relatively better on attribute \( b \):

\[
a_x > b_x, \quad a_y < b_y. \tag{3}
\]

Figure 1 illustrates consumers' valuations for products \( X \) and \( Y \) as a function of their type \( \theta \).

### 2.1. Information Types

We model a piece of third-party information as affecting consumers' valuations of the attributes of one or both products. Information about the category affects both products. Information about only one product affects consumers' valuations of that product only. If the information is positive, valuations increase. If the information is negative, valuations decrease.²

To model the last component of information, namely, the consumers for whom the information is primarily relevant, we must distinguish between core and noncore consumers. From firm \( X \)'s perspective, given that product \( X \) is better than product \( Y \) on attribute \( a \) (\( a_x > a_y \)) and consumers of type \( \theta < 1/2 \) place higher weight on attribute \( a \) than attribute \( b \), we define firm \( X \)'s core consumers as consumers of type \( \theta \in [0, 1/2) \), and we define firm \( X \)'s noncore consumers as consumers of type \( \theta \in [1/2, 1) \). Similarly, given that product \( Y \) is better than product \( X \) on attribute \( b \) (\( b_y < b_x \)), we define firm \( Y \)'s core consumers as consumers of type \( \theta \in [1/2, 1) \) and firm \( Y \)'s noncore consumers as consumers of type \( \theta \in [0, 1/2) \).

Whether the information is primarily relevant to a firm's core or noncore consumers, or whether the information is of equal relevance to all consumers, depends on whether it affects attribute \( a \), \( b \), or the attributes subsumed in \( \bar{v} \). To understand this link, it is useful to consider some examples. In what follows, let Apple computer’s MacOS correspond to product \( X \), Microsoft’s Windows 98 correspond to product \( Y \), attribute \( a \) be the ease-of-use of an operating system, and attribute \( b \) be the operating system’s compatibility with prevailing standards.

Consider positive information about product \( X \) that pertains to attribute \( a \) only. (For example: "The newest upgrade of the MacOS has eliminated the last remaining quirks in the user interface"). Since \( \Delta a_x > 0 \), it will be primarily relevant to firm \( X \)'s core consumers. This is because for \( \Delta a_x > 0 \), consumers’ valuations will rise from \( v_x + (1 - \theta)a_x + \theta b_y \) to \( v_x + (1 - \theta)(a_x + \Delta a_x) + \theta b_y \), which is an increase for all consumers but a bigger increase for consumers with the lowest \( \theta \)s, or firm \( X \)'s core consumers. This corresponds to a rotation of the curve denoting consumers’ valuations of product \( X \) around its intersection with the vertical line at \( 1 \) (see Figure 2).

If the positive information about product \( X \) pertains to attribute \( b \) only (for example: "Apple’s networking support now allows a seamless integration of Macs into existing PC networks"), it will be primarily relevant to firm \( X \)'s noncore consumers.

²Note that the core versus noncore distinction is a function only of a consumer’s type, whereas the same consumer’s actual purchase decision is a function of both its type and the prices of each product.
This is because $\Delta h_x > 0$ implies that consumers' valuations will rise from $v_x + (1 - \theta)a_x + \theta b_x$ to $v_x + (1 - \theta)a_x + \theta(b_x + \Delta h_x)$, which is an increase for all consumers but a bigger increase for consumers with the highest $h_x$, or firm $X$'s non-core consumers. This corresponds to a rotation of the curve denoting consumers' valuations of product $X$ around its intersection with the vertical line at 0 (see Figure 3).

Finally, if the positive information about product $X$ affects only $v_x$ (for example: “The SAT scores of children whose families own a Macintosh are on average higher than the SAT scores of children without a Macintosh”), it will lead to the same change in valuation for all consumer types. This is because for $\Delta v_x > 0$, consumers' expected utilities will rise from $v_x + (1 - \theta)a_x + \theta b_x$ to $v_x + \Delta h_x + (1 - \theta)a_x + \theta b_x$, which is an equal increase for all consumers (see Figure 4).

### 3. Equilibrium Profits

We are ultimately interested in determining how manufacturers and retailers are affected by information that is made available to consumers by third parties. In this section, we solve for firm $X$, firm $Y$, and the retailer's profit given consumer valuations $U_x(\theta)$ and $U_y(\theta)$, $\theta \in [0, 1]$.

The study pertains to the MacOS and the associated utility increase is uncorrelated with ease-of-use or compatibility and is thus subsumed in $v_x$.  

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**Figure 2** Example of Positive Information of Relevance to $X$'s Core Consumers

**Figure 3** Example of Positive Information of Relevance to $X$'s Noncore Consumers

**Figure 4** Example of Positive Information of Equal Relevance to All Consumers

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4The study pertains to the MacOS and the associated utility increase is uncorrelated with ease-of-use or compatibility and is thus subsumed in $v_x$.  

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**3.1. The Incremental Contribution of Products to Total Channel Profits**

Assume that consumer types $\theta$ are distributed uniformly on $[0, 1]$. Then the downward sloping line in Figure 1 represents the maximum price that can be charged for product $X$ if a quantity $x$ of product $X$ is to be sold and product $Y$ is not sold. We denote this as $p_x(x) = v_x + (1 - x)a_x + xb_x$, $x \in [0, 1]$. Similarly, the upward sloping line in Figure 1 represents the maximum price that can be charged for product $Y$ if a quantity $y$ of product $Y$ is to be sold and product $X$ is not sold. We denote this as $p_y(y) = v_y + (1 - y)a_y + yb_y$, $y \in [0, 1]$. Note that $p_x(x)$ and $p_y(y)$ are the inverse demands for products $X$ and $Y$ when each product is the only product sold.

If a monopolist were to sell only product $X$, the monopolist's profit would be $p_x(x)$ (assume, for simplicity, that production costs are zero). Let $p^m_x$ and $x^m$ denote the price and quantity that maximize this profit and let $\Pi_x$ denote its maximized value.
Then $p^m_x = (v_x + a_x)/2$ and $x^m = (v_x + a_x)/(2(a_x - h_x))$ if the market is not covered (i.e., $a_x - v_x > 2h_x$), and $p^m_x = b_x + v_x$ and $x^m = 1$ if the market is covered (i.e., $a_x - v_x \leq 2h_x$), yielding profit $\Pi_x = p^m_x x^m$. In what follows, we refer to $\Pi_x$ as the monopoly profit of the retailer and firm $X$. That is, if firm $X$ produced product $X$ and the retailer distributed it to consumers, then $\Pi_x$ would represent the maximum profit to be allocated between the retailer and firm $X$ (in the absence of product $Y$).

Similarly, a monopolist's maximized profit if it sells only product $Y$ is $\Pi_y = p^m_y y^m$, where $p^m_y$ and $y^m$ are the profit-maximizing price and quantity. In what follows, we refer to $\Pi_y$ as the monopoly profit of the retailer and firm $Y$. Its interpretation is analogous to that of $\Pi_x$.

These profits are illustrated in Figure 5 for the symmetric case in which $a_x = b_y$ and $a_y = b_x$, so that $p^m_x = p^m_y$, and in which the market is covered ($a_x - v_x \leq 2h_x$).

In contrast, if a monopolist were to sell both products $X$ and $Y$, then its profit is maximized by choosing a quantity $x$ of product $X$ and a quantity $y$ of product $Y$ to solve

$$\max_{x,y} p_x(x) + p_y(y) \text{ such that } y = 1,$$  \hspace{1cm} (4)

where we have assumed that products $X$ and $Y$ are substitutes in demand at the monopoly prices, i.e., the monopolist serves all consumer types $\theta \in [0, 1]$. Solving yields

$$p^*_x = v_x + y^* a_x + x^* b_x, \quad p^*_y = v_y + y^* a_y + x^* b_y,$$

$$x^* = (v_x - v_y + a_x + b_x - 2a_y)/(2(a_x + b_y - b_x - a_y)), \quad y^* = 1 - x^*.$$  \hspace{1cm} (5)

At the quantities that maximize the monopolist's profit, prices are $(p^*_x, p^*_y)$, consumers of type $\theta \in [0, x^*)$ purchase product $X$, and consumers of type $\theta \in [x^*, 1]$ purchase product $Y$. Let $\Pi_{xy} = p^*_x x^* + p^*_y y^*$ denote the monopoly profit when both products $X$ and $Y$ are sold. This profit is illustrated in Figure 6 for the symmetric case $(p^*_x = p^*_y)$.

We will refer to $\Pi_{xy}$ as the total channel profit. If firm $X$ produced product $X$, firm $Y$ produced product $Y$, and the retailer distributed both products to consumers, then $\Pi_{xy}$ would be the maximum profit to be allocated among the retailer, firm $X$, and firm $Y$. The difference between $\Pi_{xy}$ and $\Pi_x$ is the difference between the maximum profit that could be earned by a monopolist selling both products and the profit it could earn by selling only product $X$, i.e., the incremental contribution of product $Y$ to total channel profit. Similarly, the difference between $\Pi_{xy}$ and $\Pi_y$ is the incremental contribution of product $X$ to total channel profit.

### 3.2. Firms

We now “break up” the firm and consider a game in which manufacturers $X$ and $Y$ make “take-it or leave-it” offers to a retailer that may carry one or both products. The retailer is a monopolist.\(^5\) In stage 1 each manufacturer specifies how much the retailer must pay as a function of how much the retailer buys from it. Let $T_x(\cdot)$ denote firm $X$’s contract and $T_y(\cdot)$ denote firm $Y$’s contract. In stage 2 the retailer chooses quantities $x$ and $y$, such that $x + y = 1$. The

\(^5\)Allowing for downstream competition does not change the qualitative nature of our results. See §6.
where the retailer then resells these quantities to final consumers. The retailer’s profit is \( p_i(x) + p_y(y) - T_i(x) - T_y(y) \), manufacturer X’s profit is \( T_i(x) \), and manufacturer Y’s profit is \( T_y(y) \).

We impose two restrictions on the domain of contracts. First, we assume that \( T_i(0) = 0, i = x, y \), which says that the retailer cannot be coerced into making a payment to a manufacturer. If the retailer buys zero, it pays zero. Second, we assume that the total payment from the retailer to a manufacturer is nondecreasing in the quantity purchased from the manufacturer, i.e., \( T_i(a) >= T_i(b) \) for all \( a > b \), which means that the retailer cannot buy additional units from a manufacturer at below cost (doing so would violate antitrust laws against predatory pricing).

Given these assumptions and extensive form we can establish Proposition 1.

**Proposition 1.** Subgame perfect equilibria to the game in which two manufacturers sell their products through a common retailer exist, and in all equilibria, the retailer buys \( x^* \) of X and \( y^* \) of Y. Let \( \pi_i^*, \pi_y^*, \) and \( \pi_y^* \) denote, respectively, manufacturer X, manufacturer Y, and retailer’s equilibrium profit. Then the unique equilibrium division of channel profits is given by

\[
\pi_i^* = \Pi_{xy} - \Pi_y, \quad (6)
\]

\[
\pi_y^* = \Pi_{yx} - \Pi_x, \quad (7)
\]

\[
\pi_y^* = \Pi_{xy} - \pi_x^* - \pi_y^*. \quad (8)
\]

In equilibrium, the retailer’s quantity choices maximize overall channel profit, i.e., they are the same as those which a multiproduct integrated monopolist would choose, and each manufacturer earns the difference between this profit and the monopoly profit of the retailer and its rival. Note that manufacturer \( i \)’s profit is strictly positive in equilibrium unless products X and Y are perfect substitutes. Note also that unless the products are independent in demand, the two manufacturers cannot extract everything from the retailer, no matter how differentiated their products become. Intuitively, each manufacturer captures what is unique about its product. If products X and Y are perfect substitutes, the retailer earns \( \Pi_{xy} \) and each manufacturer earns zero. As the products become less substitutable, each manufacturer’s profit increases, and the retailer’s profit decreases. In the limit, if products X and Y are independent in demand, the retailer would choose not to sell product X since it would be better off just selling product Y. If firm X were to charge less than \( \Pi_{yx} - \Pi_y \), it would be leaving money on the table, since it could increase its profit by increasing \( T_x \) by a small amount and still serve as a supplier to the retailer. Similarly, firm Y extracts in equilibrium payment \( \Pi_{yx} - \Pi_x \) from the retailer.
mand, each manufacturer earns the monopoly profit on its product, and the retailer earns zero.

4. Product Information and the Division of Channel Profits

We now consider how each firm’s profit is affected by third-party sources of information of the kind typically found on the Internet and in other media. Our main result is that seemingly very similar pieces of third-party information can have dramatically different implications for which firms gain and which firms lose. This will highlight a crucial distinction between types of third-party information, namely, the group of consumers for whom they are primarily relevant.

To derive our results, we proceed by determining how the information affects total channel profit and how the information affects the monopoly profits of the retailer and each firm. Knowing these changes in profits, we can then use Proposition 1 to determine which firms gain and which firms lose. For example, to analyze the impact on firm i’s profit, it suffices to compute $\Delta \Pi_i = \Delta \Pi_x y - \Delta \Pi_p$, i.e., the change in total channel profit minus the change in the monopoly profit of the retailer and rival firm. For simplicity, we illustrate our results using a symmetric benchmark.

4.1. Product Information Primarily Relevant to Non-Core Consumers

We begin with information about product X that is primarily relevant to firm X’s non-core consumers (refer back to Figure 3). To analyze the effect of this type of information, first note that the retailer will increase its price on products X and Y (decrease its prices, if the information is negative) to reflect the increase in consumers’ valuations. This causes total channel profit to increase and implies that $\Delta \Pi_{xy} > 0$. As for the effect of the information on $\Pi_i$ and $\Pi_p$, we can see from Figure 3 that the retailer’s monopoly profit with firm Y unchanged ($\Delta \Pi_y = 0$) but that its monopoly profit with firm X increases ($\Delta \Pi_x > 0$). In particular, $\Delta \Pi_x = \Delta \Pi_x y > 0$. Since the retailer cannot raise equilibrium prices by $\Delta \Pi_x$ without causing some consumers to drop out of the market, $\Delta \Pi_x > \Delta \Pi_{xy}$. Thus, for positive information about product X that is primarily relevant to firm X’s non-core consumers, we have $\Delta \Pi_x > \Delta \Pi_{xy} > \Delta \Pi_y$

PROPOSITION 2. Suppose consumers learn positive (negative) information about product X that is primarily relevant to firm X’s non-core consumers, $\Delta \Pi_x > 0$ ($\Delta \Pi_x < 0$). Then, firm X and the retailer’s profit increases (decreases), and firm Y’s profit decreases (increases).

Firm X gains because the positive product information applies to its product and because the monopoly profit of the retailer and firm Y is unchanged. Thus, $\Delta \Pi_x = \Delta \Pi_{xy} > 0$, i.e., firm X can appropriate the entire added channel profits. Firm Y is not as fortunate. Recall that if the retailer sells both products, it cannot raise prices by the full amount of the increase in consumers’ valuations for product X without causing some consumers to drop out of the market, but if the retailer were to sell only product X, it could in fact raise prices by the full amount. This means that total channel profits increase by less than the monopoly profits of the retailer and firm Y, implying that $\Delta \Pi_y = \Delta \Pi_{xy} - \Delta \Pi_x < 0$, i.e., that product Y’s incremental contribution to total channel profit has decreased. As a result, the new equilibrium contract between firm Y and the retailer will result in better terms for the retailer. Notice that the retailer’s added profit in this case stems purely from a redistribution of profit with firm Y.

Compare this to the case of positive information about product X that is equally relevant to all consumers (refer back to Figure 4). In this case, a similar result and intuition apply.

PROPOSITION 3. Suppose consumers learn positive (negative) information about product X that is equally relevant to all consumers, $\Delta \Pi_{xy} > 0$ ($\Delta \Pi_{xy} < 0$). Then, firm X’s profit and the retailer’s profit increase (decrease), and firm Y’s profit decreases (increases).

This information leads to an increase in total channel profit ($\Delta \Pi_{xy} > 0$), since the utility of the marginal consumer has increased. However, the retailer
cannot raise prices by $\Delta v_x$ without causing some consumers to drop out of the market; consequently, total channel profit increases by less than $\Delta v_x$. If the retailer were to sell only product $X$, profit would rise by $\Delta v_x$.\(^\text{11}\) If the retailer were to sell only product $Y$, profit would be unchanged. Thus, as in the previous example, we have $\Delta \Pi_x > \Delta \Pi_{xy} > \Delta \Pi_y$ implying that the qualitative results are the same.

It is tempting to conclude from Propositions 2 and 3 that firm $X$ gains because the information is positive about its own product, the retailer gains because it can adjust its pricing to reflect consumers’ higher valuations, and firm $Y$ loses because the information causes some consumers to switch to product $X$. However, as we now show, this intuition is incomplete.

4.2. Product Information Primarily Relevant to Core Consumers

A critical component of the change in each firm’s profit is the change in the retailer’s monopoly payoff with the rival firm. If this increases by less than the increase in total channel profit, then a firm can be better off even if the information about its rival’s product causes its own market share and sales to decrease, and the retailer can be worse off even though its price-cost margins have increased. For example, this is the case for positive information about product $X$ that is primarily relevant to firm $X$’s core consumers (refer back to Figure 2). Here, the retailer’s monopoly profit with each manufacturer is unchanged, $\Delta \Pi_x = \Delta \Pi_y = 0$, but $\Delta \Pi_{xy} > 0$.\(^\text{12}\)

**Proposition 4.** Suppose consumers learn positive (negative) information about product $X$ that is primarily relevant to firm $X$’s core consumers, $\Delta a_x > 0$ ($<0$). Then, firm $X$’s profit and firm $Y$’s profit increase (decrease), and the retailer’s profit decreases (increases).

To understand why both manufacturers gain at the expense of the retailer, notice that although the retailer can profitably raise prices when it sells both products, thus increasing total channel profit, the retailer cannot profitably raise its price if it only sells one product (since the marginal consumer’s valuation at each end point is unchanged). This means that channel profit increases if and only if the retailer sells both products. In equilibrium, each manufacturer captures the full increase in channel profit. Not only can the retailer not appropriate any of the increase in profit, the retailer actually loses by the same amount as the profit increases.

4.3. Discussion

We have shown our first key finding, namely, that a retailer can lose from third-party information that increases consumers’ valuations for one of its products, even if it increases total channel profit. CompUSA, for example, would be harmed if consumers learn that “Windows 98 now works reliably with all common e-mail servers,” since this positive information about Windows is primarily relevant to Microsoft’s core consumers. In contrast, CompUSA would benefit if consumers learn that “Apple’s networking support is now good enough to allow a seamless integration of Macs into existing PC networks,” since this positive information about the MacOS is primarily relevant to Apple’s non-core consumers. Intuitively, overall channel profit increases as the information in each example allows the retailer to segment consumers better. Yet how this profit is allocated among the channel members depends in part on the role each party plays in segmenting the market. In the example of positive information about product $X$ that is of relevance to firm $X$’s core consumers, the retailer is unable to use the third-party information to increase channel profit unless it sells both manufacturers’ products. This obviously works to the advantage of the manufacturers and allows them to gain vis-à-vis the retailer.

We have also shown our second key finding, namely, that positive information about a rival’s product can benefit a manufacturer if the information is primarily relevant to the rival’s core consumers.\(^\text{13}\) For example, Apple may benefit from a report

\(^\text{11}\)We present the case in which $a_x - v_x \leq 2h_x$. For $a_x - v_x > 2h_x$, the argument proceeds with $\Delta \Pi_{xy} < \Delta \Pi_x < \Delta \Pi_y$.

\(^\text{12}\)The analogue to this result is that negative information about a rival’s product need not be beneficial to a manufacturer if the information is primarily relevant to the rival’s core consumers.

\(^\text{13}\)We present the case in which $a_x - v_x \leq 2h_x$. For $a_x - v_x > 2h_x$, the argument proceeds with $\Delta \Pi_{xy} > \Delta \Pi_x > \Delta \Pi_y$. 

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that concludes that "Windows 98 now works reliably with all common e-mail servers," because CompUSA can only take advantage of the increase in consumers’ valuations for Microsoft if it also sells Apple computers. In contrast, Apple would be harmed by a review that praises improvements in the ease-of-use of Windows 98. This kind of third-party information would decrease the retailer’s dependence on Apple to serve consumers who care primarily about the ease of use of an operating system.

5. Category Information and the Division of Channel Profits

The distinction between core and non-core consumers is also important in understanding how category information affects channel profits. Consider the following two examples. First, suppose consumers read that "because of its platform-independence, the widespread adoption of the Java programming language is making compatibility concerns a thing of the past." Second, suppose consumers learn of a laptop comparison test that finds that "the new generation of LCD displays has improved the legibility and ease of use of both MacOS and Windows 98-based notebooks." Although in both examples the information is positive about all of the products in the category, the information in the first example is more important to Microsoft’s core consumers (it refers to an increase in compatibility), while the information in the second example is more important to Apple’s core consumers (it refers to an increase in ease of use).

Who gains and who loses in these examples? Since we have shown that each manufacturer always gains from third-party information that is positive about its own product, one might think that both manufacturers would gain from the information in these examples and that, if anyone loses, it would be the retailer. In addition, one might think that although both manufacturers would gain, Microsoft would gain the most from having compatibility improved and that Apple would gain the most from having LCD displays improved (since these improvements, respectively, benefit each firm’s core consumers). Instead, we will show that the opposite is true. In many cases, Apple’s profit decreases and Microsoft’s profit increases when the information is more important to Apple’s core consumers and vice versa when the information is more important to Microsoft’s core consumers. Ironically, in both examples, it is the retailer that is the only sure winner. Its profit always increases from the information in these examples.

5.1. Category Information: The Core vs. Noncore Distinction

Category information that increases consumers’ valuations of all products but does so more for firm X’s core consumers than for its non-core consumers corresponds to a rotation of both products’ consumers’ valuation curves around their intersections with the vertical line at 1 (which corresponds to shifting consumers’ valuations of product X and product Y such that the new curves become $v_x + (1 - \theta)(a_x + \Delta a_x) + \theta b_x$ and $v_y + (1 - \theta)(a_y + \Delta a_y) + \theta b_y$, where $\Delta a_x = \Delta a_y > 0$). See Figure 7 for the case in which the market is covered, i.e., $a_x - v_x < 2b_x$.

**Proposition 5.** Suppose consumers learn positive (negative) category information that is primarily relevant to firm X’s core consumers, $\Delta a_x = \Delta a_y > 0 (<0)$. Then, firm Y’s profit and the retailer’s profit increase (decrease) and firm X’s profit decreases (increases).

To see why this holds, note that the price the retailer will charge when selling only product Y is determined by the consumer of type $\theta = 0$, i.e., the consumer who cares most about ease of use. This consumer is the one whose valuation will rise most as a result of the new information, namely, by $\Delta v_x = \Delta v_y > 0$, which raises the value to the retailer of selling only product Y by $\Delta v$. Thus, $\Delta \Pi_y = \Delta v > 0$. The prices the retailer charges when it sells both products, however, must rise by less than $\Delta v$, for

\[2(a_x - (a_x - v_x)/\sqrt{2}) < 2b_x < a_x - v_x,\]

then the qualitative results are unchanged. If $b_x < a_x - (a_x - v_x)/\sqrt{2}$, then whether firm X gains or loses depends on the size of the valuation changes (see the Appendix). For large valuation changes, the change in firm X’s profit is the same as in Proposition 5, but for small valuation changes, firm X’s profit increases with positive information. The proposition holds generally for firm Y and the retailer.
otherwise some consumers would drop out of the market. This implies that \( \Delta \Pi_y > \Delta \Pi_{xy} \) and, thus, \( \Delta \pi_x = \Delta \Pi_{xy} - \Delta \Pi_y < 0 \). Since the information does not affect the consumer of type \( \theta = 1 \), \( \Delta \Pi_x = 0 \), and since total channel profits increase, firm \( Y \) gains. The retailer gains as well, since its profit is \( \Delta \pi_y = \Delta \Pi_{xy} - \Delta \Pi_y > 0 \). In this case, the retailer gains solely from the redistribution of profit with firm \( X \).

Since the non-core case is identical to the core case with only the names of firms \( X \) and \( Y \) interchanged, the analogue to Proposition 5 also holds. If consumers learn positive (negative) category information that is primarily relevant to firm \( X \)'s non-core consumers, then firm \( X \) and the retailer’s profit increases (decreases) and firm \( Y \)'s profit decreases (increases). These results imply that a manufacturer can lose from positive category information. In particular, they imply that manufacturers gain if the category information is more relevant to their non-core consumers and lose if the information is more relevant to their core consumers.

### 5.2. Category Information of Equal Relevance to All Consumers

In this subsection, we consider which firms gain and which firms lose if the category information is equally relevant to all consumers. Such information corresponds to an equal upward shift in consumers’ valuations for products \( X \) and \( Y \) (which corresponds to shifting consumers’ valuations of product \( X \) and product \( Y \) such that the new curves become \( v_x + \Delta v_x + (1 - \theta) a_x + \theta b_x \) and \( v_y + \Delta v_y + (1 - \theta) a_y + \theta b_y \), where \( \Delta v_x = \Delta v_y > 0 \); see Figure 8). For example, suppose a study finds that “the SAT scores of children who have a personal computer at home are on average higher than the SAT scores of children without a personal computer.” Since the study pertains both to the MacOS and to Windows 98, and the associated utility increase is uncorrelated with ease of use or compatibility, this information affects all parties equally.

**Proposition 6.** Suppose consumers learn positive (negative) category information that is equally relevant to all consumers, \( \Delta v_x = \Delta v_y > 0 \) (\(<0\)). If \( a_i - v_i < 2b_i \), then firm \( X \)'s profit and firm \( Y \)'s profit are unchanged, and the retailer captures the entire increase (decrease) in channel profits. If \( a_i - v_i > 2b_i \), then all channel members’ profits increase (decrease).

The information shifts all consumers’ gross utilities from consuming products \( X \) and \( Y \) upward by \( \Delta v = \Delta v_x = \Delta v_y \). Since the retailer will set prices in equilibrium to make the marginal consumer just indifferent between purchasing and not doing so, we know that both prices will rise by \( \Delta v \), resulting in an increase in total channel profit of \( \Delta \Pi_{xy} = \Delta \Pi > 0 \).

If the retailer sells only one firm’s product, we distinguish between two cases, depending on whether a retailer would serve all consumers \( (a_i - v_i < 2b_i) \) or leave some consumers unserved \( (a_i - v_i > 2b_i) \). If \( a_i - v_i < 2b_i \), the retailer would raise prices by \( \Delta v \), leading to an identical increase in its monopoly profit with manufacturer \( i \), \( \Delta \Pi_i = \Delta \Pi_{xy} = \Delta v \). Hence, neither manufacturer can appropriate any of the added channel profit, since \( \Delta \pi_i = \Delta \Pi_{xy} - \Delta \Pi_i = 0 \), and thus the retailer is the sole beneficiary of the shift in consumers’ valuations. On the other hand, if \( a_i - v_i > 2b_i \), a price increase would only lead to increased revenues from a subset of consumers. Thus, each firm would be able to appropriate some of the added channel profit.\textsuperscript{15}

### 5.3. Discussion

Propositions 5 and 6 imply that the retailer gains irrespective of the consumer group for whom the information is primary.
formation is most important. Thus, positive category information is always good for the retailer in that it increases the retailer’s profit. In contrast, with category information, positive information does not imply that the manufacturers are better off. In fact, if the information is equally relevant to all consumers, it may be that the retailer is the sole beneficiary. This shows our third key finding, namely, that positive information about a product category need not benefit manufacturers, even though consumers’ valuations of their products have increased.

We have found that a manufacturer always benefits (loses) from positive category information if such information primarily affects its rival’s core (non-core) consumers. For example, Apple benefits if consumers read that “because of its platform-independence the widespread adoption of the Java programming language is making compatibility concerns a thing of the past,” precisely because this is not what its customers are primarily concerned with. Although such news also increases consumers’ valuations for Windows 98, the information increases CompUSA’s dependence on Apple to serve consumers whose primary concern is the ease of use of an operating system, and thus Microsoft stands to lose from this kind of information.

6. Extensions and Limitations

In this section, we consider extensions of the model to downstream competition, bargaining, and non-information-based changes in consumers’ valuations.

6.1. Retail Competition

The most common way of modeling downstream competition is to have each manufacturer sell its product through an exclusive retailer, which then competes noncooperatively either in prices or quantities. A representative example is Gabrielsen and Sorgard (1999).16

Gabrielsen and Sorgard show that when the retailers can choose from which manufacturer to buy, and the manufacturers can make take-it-or-leave-it two-part tariff offers, each manufacturer earns its product’s incremental contribution to overall channel profit. This division of profit between the manufacturers and retailers is analogous to what we find in Proposition 1 with one difference. In our model, the overall channel profit is maximized by the common retailer. In Gabrielsen and Sorgard’s model, downstream competition among the retailers prevents overall channel profit from being maximized. Thus, the baseline of overall channel profit is different in the two models. Nonetheless, this difference is not crucial for our comparative static results, since our results rely only on the changes in each manufacturer’s incremental contribution to channel profit, not on the initial absolute level of the overall channel profit. For example, in both our model and Gabrielsen and Sorgard’s model, to know whether information benefits or harms manufacturer \(j\), one only needs to know whether \(\Delta \Pi_{ij} > 0\) is greater than or less than \(\Delta \Pi_i\). This suggests that our model can be extended in a straightforward way; there exist cases in which firm \(Y\) benefits from positive information about firm \(X\) while the retailer loses from such information, and vice versa, and thus there is no change in our qualitative results.

The case of multiple multiproduct retailers is more difficult to model explicitly when the contract space allows for nonlinear contracts, and we are unaware of any literature that does so. However, we expect our qualitative results to hold in this case as well, as long as each retailer has some downstream market power. To see this, consider a reduced form model of competition in which there are \(n \geq 2\) symmetrically differentiated retailers. Suppose that in stage one each manufacturer can make a take-it-or-leave-it two-part tariff offer to each retailer. Assume the offers are made simultaneously and let \(w_i\) denote man-

---

manufacturer $i$’s wholesale price and $F_i$ its fixed fee. Under symmetry, manufacturer $i$ will charge the same wholesale price and fixed fee to each retailer. Suppose that in Stage 2 the retailers choose which products to sell, and in Stage 3, the retailers compete noncooperatively either in prices or quantities.

Let $\Pi_i(w_i, w_j)$ denote the reduced-form equilibrium profit of retailer $i$ gross of fixed fees if it sells products $X$ and $Y$ and all other retailers sell products $X$ and $Y$, let $\Pi_i^*(w_i, w_j)$ denote the reduced-form equilibrium profit of retailer $i$ gross of fixed fees if it sells only product $X$ and all other retailers sell products $X$ and $Y$, and let $\Pi_i(w_i, w_j)$ denote the reduced-form equilibrium profit of retailer $i$ gross of fixed fees if it sells only product $Y$ and all other retailers sell products $X$ and $Y$. Then, in any subgame perfect equilibrium in which each retailer sells both manufacturers’ products, it must be that manufacturer $X$ earns $\sum_{i=1}^m (\Pi_i^* - \Pi_i)$, manufacturer $Y$ earns $\sum_{i=1}^m (\Pi_i^* - \Pi_i^*)$, and retailer $i$ earns $\Pi_i^* + \Pi_i^* - \Pi_i$. To know whether firm $Y$ is better off or worse off with positive third-party information about product $X$, one only needs to know whether $\Delta\Pi_i > 0$ if the information leads to $\Delta\Pi_i > \Delta\Pi_i^*$, then firm $Y$ gains. If the information leads to $\Delta\Pi_i > \Delta\Pi_i^*$, then firm $Y$ loses. The effect of positive third-party information on firm $X$’s and the retailer’s profit can be analyzed similarly.

6.2. Retailer Bargaining Power

Our assumption that manufacturers can make take-it-or-leave-it offers may at first sight seem inconsistent with our assumption that the retailer is a monopolist. However, this is not so. The ability of the manufacturers to make take-it-or-leave-it offers depends on their ability to commit to their contract offers; whether there are one or more retailers determines whether the manufacturers have outside options. The two concepts (commitment and outside options) are quite distinct. In particular, the degree to which a manufacturer can commit to its contract offer reflects its degree of bargaining power, which is related to its degree of patience and other exogenous factors, whereas whether a manufacturer has an outside option reflects whether it has a positive disagreement payoff. To illustrate the difference between these concepts, suppose both Procter & Gamble and Kraft are selling a macaroni and cheese dinner to a “mom-and-pop” store with local monopoly power. In the absence of this retailer, suppose Procter & Gamble and Kraft have no other way to reach the retailer’s customers. Then, in negotiations with this particular retailer, each manufacturer’s disagreement payoff is essentially zero. However, one might reasonably argue that in reality both Procter & Gamble and Kraft are likely to be much more patient than the “mom-and-pop” store to reach an agreement and therefore that they are likely to have a much greater degree of bargaining power vis-à-vis the mom-and-pop store.

Our assumption that the manufacturers can make take-it-or-leave-it offers has the property that it yields a division of profit in equilibrium in which all three firms share in the overall channel profit. The manufacturers make positive profit because they have bargaining power and can extract the incremental surplus of their product, while the retailer makes positive profit because even though it does not have any bargaining power, its outside options are positive (i.e., it can still earn positive payoff by saying no to either manufacturer). Notice that these results differ from those of Bernheim and Whinston (1985), who found that two manufacturers selling through a common retailer can extract all the surplus. The difference between our results and theirs lies in their assumption that rejected manufacturers can contract with any number of other, homogeneous retailers. Thus, there is nothing unique about

\[ k = \frac{p_i - d_j}{p_j - d_j} \]

\[ \lambda = p_i \]

\[ \Pi_i = F_i \]

\[ \Pi_i^* = F_i \]

\[ \Pi_i^* = F_i \]

\[ \Pi_i^* = F_i \]

\[ \Pi_i^* = F_i \]
the common retailer in their model, and so the common retailer in their model does not have any ability to extract rents.

Given that each player earns positive profit in our model, the assumption of take-it-or-leave-it offers provides a useful simplification in which to conduct the comparative statics of our paper. However, it should be emphasized that our qualitative results do not depend on this assumption. For example, using a simultaneous Nash bargaining model, one can show that if each player has some bargaining power then manufacturer $X$ earns $\pi_x^* = \lambda_x (\Pi_x - \Pi_r)$, manufacturer $Y$ earns $\pi_y^* = \lambda_y (\Pi_y - \Pi_z)$, and the retailer earns $\pi_r^* = \Pi_x - \pi_x^* - \pi_y^* = \pi_y^* - \pi_x^*$. Where $\lambda_x \in [0, 1]$ and $\lambda_y \in [0, 1]$ are measures of manufacturer $X$ and $Y$’s bargaining power. 19 If the manufacturers have all the bargaining power ($\lambda_x = \lambda_y = 1$) then the division of profit is the same as in Proposition 1. In this case, the manufacturers do not capture all the profit because, even though they make the offers, the retailer’s disagreement payoff in each negotiation is positive. In contrast, if the retailer has all the bargaining power ($\lambda_x = \lambda_y = 0$) then the retailer earns all the profit. In this case, the retailer captures all the profit because the manufacturers’ disagreement payoffs are zero. For $\lambda_x \in (0, 1)$ and $\lambda_y \in (0, 1)$, the change in each firm’s profit depends on the change in $\Pi_{xy}$, $\Pi_{yx}$, and $\Pi_z$ in the same manner as when $\lambda_x = \lambda_y = 1$. Thus, our comparative static results regarding the manufacturers’ profits go through unchanged. For $\lambda_x$’s sufficiently large, our comparative static results regarding the retailer’s profit are also unchanged. For smaller $\lambda_x$’s the change in the retailer’s profit will always be positive for positive product information.

6.3. Non-Information-Based Changes in Consumers’ Valuations
Evaluating the effect of information shocks on the division of profit in the distribution channel is a two-step process. First, we model the effect of new information on consumers’ valuations for all products in the market. Second, we perform comparative statics on each firm’s profit. The separation of these two steps enhances the applicability of the model. Since the division of channel profits depends on consumers’ valuations, we can analyze the effect on channel profits of other variables that influence consumers’ valuations for products in the market. For example, our framework can be used to analyze the profit implications of a new product introduction by a firm. Or we can use the framework to assess any value added that a retailer might offer.

To illustrate, suppose retailers perform services that add to consumers’ valuations of each manufacturer’s product an amount equal to $\Delta$. To analyze who gains and who loses, one can think of the retailer’s value added as corresponding to the case of positive category information that is equally relevant to all consumers. In this case, the retailer is the sole beneficiary of the change in consumers’ valuations. This has some interesting implications. It suggests that any value added by the retailer that is not biased between manufacturers’ products can be fully captured by the retailer. On the other hand, if the value added of the retailer is product specific, say it benefits Apple’s products but not Microsoft’s products, then we know from the analogous case of positive product information of equal relevance to all consumers that the retailer and Apple will share in the gains but that Microsoft will lose. In order to induce the appropriate level of retail service (from Apple’s perspective), therefore, it may be necessary for Apple to engage in cooperative subsidies with the retailer. We leave these topics for future research, but stress that a variety of such extensions can be handled within our framework.

7. Concluding Remarks
In this paper we have analyzed how third-party information about a product category or a firm’s product affects the division of profit in a distribution channel in which retailers carry multiple products. Our main observation is that information that may seem similar can nevertheless have very different implications for which firms gain and which firms lose. Table 1 summarizes the profit implications of each of the information types we consider. We see that
Manufacturers always benefit from positive product information about their own product. Retailers always benefit from positive category information. A manufacturer benefits from positive product information about its rival’s product, and from positive category information, if such information pertains to its rival’s core consumers. Retailers benefit from positive product information if such information pertains to all consumers equally or to the product’s non-core consumers.

To illustrate, information such as “Apple’s networking support now allows a seamless integration of Macs into existing PC networks” is more important to Apple’s non-core consumers, information such as “the newest upgrade of the MacOS has eliminated the last remaining quirks in the user interface” is more important to Apple’s core consumers, and information such as “the SAT scores of children whose families own a Macintosh are on average higher than the SAT scores of children without a Macintosh” is equally relevant to all consumers. In each example, the information is positive about the MacOS and thus leads to higher profit for Apple. However, the effects on Apple’s rival and retailer differ across information types. The information in the first and third examples benefits the retailer and harms Microsoft while the information in the second example benefits Microsoft and may be harmful for the retailer.

There are three novel features of the analysis. First, we have derived the equilibrium division of profit among firms when a retailer sells the products of competing manufacturers, and we have done so while placing few restrictions on the feasible set of contracts. Second, we have shown how this equilibrium division of profit lends itself to a simple graphical interpretation which depicts which firms gain and which firms lose from third-party information. Third, we have provided a taxonomy of information types and identified the key features of each type that cause profit incentives to vary. In particular, we have conceptualized information as having three components, namely (1) the products to which the information pertains, (2) whether the information is positive or negative, and (3) the consumers to whom the information is relevant. All three information components play a role in determining the change in each firm’s profit.

We believe that our framework can also be used to analyze a variety of other settings of interest, for example, it can be used to analyze profit incentives when there is downstream competition, and when there are non-information-based changes in consumers’ valuations. In addition, our framework may be used both to analyze the effects on profits of persuasive advertising and to predict advertising content. For example, for a given advertising budget, one might ask whether a firm should structure its advertising communications to favor its core or non-core consumers, how it should structure its advertising communications if it wants to obtain retailer support.

<table>
<thead>
<tr>
<th>Table 1 Profit Implications of Information</th>
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<tr>
<td>Positive Product Information About Product X Affects</td>
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<tr>
<td>Primarily X’s core consumers</td>
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<tr>
<td>All consumers equally</td>
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<tr>
<td>Primarily Y’s core consumers</td>
</tr>
<tr>
<td>Positive Category Information Affects (cases of less than full-market coverage in parentheses)</td>
</tr>
<tr>
<td>Primarily X’s core consumers</td>
</tr>
<tr>
<td>All consumers equally</td>
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for its advertising campaign, and how it should structure its advertising communications if it wants to deter entry or otherwise punish its rival. We leave these questions for future research.

Acknowledgments

We are indebted to the area editor and Meghan Busse for detailed comments and helpful suggestions. We also thank Wujin Chu, Ganesh Iyer, Barry Nalebuff, Miguel Villas-Boas, Birger Wernerfelt, two anonymous referees, and seminar participants at Northwestern University, UC Berkeley, the University of Chicago, Yale University, and the Marketing Science Conference in Syracuse (1999).

Appendix

Proof of Proposition 1

The proof proceeds in three parts. In part one, we derive necessary and sufficient conditions for subgame perfect equilibrium. In part two, we show that, in any equilibrium, these conditions imply that consumers will purchase \( x' \) of product X and \( y' \) of product Y at prices \( p_x' \) and \( p_y' \), and that manufacturer X, manufacturer Y, and the retailer will earn \( \pi_x' \), \( \pi_y' \), and \( \pi_r' \), respectively. In part three, we prove that a subgame perfect equilibrium exists.

1. Necessary and Sufficient Conditions for Subgame Perfect Equilibrium

Since our solution concept is subgame perfection, we begin by solving for retailer optimality in the second stage. The retailer chooses \((x, y)\) to maximize its profit given the contracts \(T_x(x)\) and \(T_y(y)\) offered in the first stage. All such \((x, y)\) must belong to the set \(\Omega\), where

\[
\Omega = \{ (x, y) \in \mathbb{R}^2 : p_x(x) + p_y(y) - T_x(x) - T_y(y) \}
\]

such that \(x + y \leq 1\).

Now go to the first stage and let \((T_x', T_y')\) be a pair of first-stage contracts that induce the retailer to purchase \((x', y')\) \(\in \Omega\), yielding a maximized profit for the retailer of \(p_x(x') + p_y(y') - T_x'(x') - T_y'(y')\). Furthermore, let the retailer’s maximized profit if it is constrained to sell only product \(x\) be \(\max_{y \geq 0} p_x(x) - T_x'(x)\), and let \(\max_{x \geq 0} p_y(y) - T_y'(y)\) be its maximized profit if it is constrained to sell only product \(y\). Let \(\beta = \{x, y\} + x + y \leq 1\).

We now characterize necessary and sufficient conditions for subgame perfect equilibrium.

**Lemma 1.** \((T_x', T_y')\) arise in a subgame perfect equilibrium if and only if the following three conditions hold:

\[
\begin{align*}
\max_{x \geq 0} p_x(x) + p_y(y) & - T_x'(x) - T_y'(y) \\
= & p_x(x') + p_y(y') - T_x'(x') - T_y'(y'),
\end{align*}
\]

(A1)

\[
\begin{align*}
\max_{y \geq 0} p_x(x) + p_y(y) & - T_x'(x) - T_y'(y) \\
= & p_x(x') + p_y(y') - T_x'(x') - T_y'(y'),
\end{align*}
\]

(A2)

\[
\begin{align*}
\max_{x \geq 0} p_x(x) + p_y(y) & - T_x'(x) - T_y'(y) \\
= & \max_{0 \leq y \leq 1} p_y(y) - T_y'(y).
\end{align*}
\]

(A3)

**Proof of Lemma 1.**

Necessity. Consider first the necessity of condition (A3). Because the retailer can always choose not to sell one of the products when it has the option of selling both products, we know that the retailer will always be at least as well off when it has the option of selling both products than when it is constrained to sell only one product. Thus, we know that

\[
\begin{align*}
p_x(x') + p_y(y') - T_x'(x') - T_y'(y') & \\
\geq & \max_{0 \leq y \leq 1} p_y(y) - T_y'(y).
\end{align*}
\]

and

\[
\begin{align*}
p_x(x') + p_y(y') - T_x'(x') - T_y'(y') & \\
\geq & \max_{0 \leq y \leq 1} p_y(y) - T_y'(y).
\end{align*}
\]

Now suppose one of the inequalities were strict. Then the manufacturer failing to extract all the incremental surplus from its product could increase its profit by charging a fixed fee, or by increasing its fixed fee if it already has one. Next, consider the necessity of condition (A1). Suppose \((T_x', T_y')\) arise in a subgame perfect equilibrium, but

\[
\max_{x \geq 0} p_x(x) + p_y(y) - T_x'(x) \neq p_x(x') + p_y(y') - T_x'(x') - T_y'(y').
\]

Using \(p_x(x') + p_y(y') - T_x'(x') - T_y'(y') = \max_{0 \leq y \leq 1} p_y(y) - T_y'(y)\), this inequality becomes

\[
\max_{x \geq 0} p_x(x) + p_y(y) - T_x'(x) \neq \max_{0 \leq y \leq 1} p_y(y) - T_y'(y).
\]

(A4)

If the left-hand side of condition (A4) were less than the right-hand side, then

\[
\max_{x \geq 0} p_x(x) + p_y(y) - T_x'(x) \leq \max_{0 \leq y \leq 1} p_y(y) - T_y'(y),
\]

(A5)

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implying that since \((x', y') \in \beta\),

\[
p_x(x')x' + p_y(y')y' - T_x(y') - T_y(y') < \max_{x', y'} p_x(y)y - T_y(y'), \quad (A6)
\]

the retailer would earn less by buying \(x'\) and \(y'\) than by buying only product \(Y\), contradicting condition \((A3)\). Now suppose the left-hand side of \((A4)\) were greater than the right-hand side.

Then for some positive \(\omega\),

\[
\max_{x, y} p_x(x)x + p_y(y)y - T_x(y) - T_y(y) > \max_{x', y'} p_x(y)y - T_y(y'),
\]

\(\text{Note that condition } (A7) \implies \text{that } x > 0, \text{ for if it involved } x = 0, \text{ then } (A7) \text{ would be violated. This implies that under the alternative contract } T_\omega(x), \text{ where}

\[
T_\omega(x) = \begin{cases} 
0 & \text{if } x = 0, \\
T(x') + \omega & \text{if } x > 0,
\end{cases}
\]

the retailer would also choose \(x > 0\), because if it chose \(x = 0\), it would earn \(\max_{x < x'} p_x(y) y - T_x(y)\), whereas if it chooses \(x > 0\), it gets the left-hand side of \((A7)\). Thus, it is strictly profitable for the retailer to purchase a positive amount of product \(X\) under \(T_\omega(x)\). Since manufacturer \(X\) earns more profit under \(T_\omega(x)\) than under \(T_\omega(y)\), the latter cannot be a best response to \(T_\omega(x)\), a contradiction. The necessity of condition \((A2)\) is similarly established.

\textbf{Sufficiency.} Suppose conditions \((A1)\) through \((A3)\) hold, but that \(T(x')\) and \(T(y)\) do not arise in a subgame perfect equilibrium. This means that at least one manufacturer can increase its profit. Without loss of generality, suppose manufacturer \(X\) can do so. Then there exists \(T_\omega(x)\) that induces the retailer to choose \(x > 0\) and makes manufacturer \(X\) better off. That is,

\[
\max_{x, y} p_x(x)x + p_y(y)y - T_\omega(x) - T_y(y) > \max_{x', y'} p_x(y)y - T_y(y'),
\]

\(\text{and } T_\omega(x) > T(x')\), for all \((x, y) \in \Omega(T_\omega(x), T(y)).\) Let \((\bar{x}, \bar{y})\) be the retailer’s choice of \((x, y)\). Then there exists some \(\hat{\omega} > 0\) such that

\[
T_\hat{\omega}(x') = T_\hat{\omega}(x) - \hat{\omega}
\]

Subtracting this expression from both sides of condition \((A1)\), and using \(p_x(x')x' + p_y(y')y' - T_\omega(x') - T_y(y') = \max_{x < x'} p_x(y) y - T_y(y)\) from condition \((A3)\), yields

\[
\max_{x, y} p_x(x)x + p_y(y)y - T_\hat{\omega}(y) - T_\hat{\omega}(x) + \hat{\omega} = \max_{x', y'} p_x(y)y - T_\hat{\omega}(y).
\]

\(\text{The left-hand side of condition } (A9) \text{ can be evaluated at } (\bar{x}, \bar{y}) \text{ to yield}

\[
p_x(\bar{x}) \bar{x} + p_y(\bar{y}) \bar{y} - T(x') - T(y) + \hat{\omega} \leq \max_{x < x'} p_x(y) y - T_y(y),\)

\(\text{which means that}

\[
p_x(\bar{x}) \bar{x} + p_y(\bar{y}) \bar{y} - T(x') - T(y) + \hat{\omega} < \max_{x', y'} p_x(y)y - T_y(y).
\]

\(\text{However, by the definition of } (\bar{x}, \bar{y}), \text{ condition } (A10) \text{ contradicts condition } (A8). \) \text{Q.E.D.}

\textbf{2. Interpretation of the Conditions}

We begin by showing that the retailer will serve all consumers in the market in any equilibrium. We then use this fact to show that conditions \((A2)\) and \((A3)\) imply that the retailer will choose quantities \((x, y) = (x', y')\) in every subgame perfect equilibrium. Finally, we show that manufacturer \(X\), manufacturer \(Y\), and the retailer will earn \(\pi_x, \pi_y, \text{ and } \pi_\omega\) respectively.

\textbf{Lemma 2.} The retailer’s quantity choices in any subgame perfect equilibrium are such that \(x' + y' = 1.\)

\textbf{Proof of Lemma 2.} Suppose not. Then because \(x' + y' > 1\) is infeasible, it must be that \(x' + y' < 1\). Using the fact that products \(X\) and \(Y\) are substitutes in demand at the monopoly prices (see subsection 3.1), it implies that there exists \(\epsilon > 0\) such that \(x' + y' + \epsilon < 1\) and either \(p_x(x') + p_y(y) - p_x(x') + p_y(y) + \epsilon < p_x(x') + p_y(y) + \epsilon\) or \(p_x(x') + p_y(y) > p_x(x') + p_y(y) + \epsilon\). It follows that either condition \((A1)\) or \((A2)\) is violated, a contradiction. \text{Q.E.D.}

\textbf{Lemma 3.} The retailer’s quantity choices in any subgame perfect equilibrium solve \(\max_{x, y} p_x(x)x + p_y(y)y\) such that \(x + y = 1.\) That is, in any equilibrium, \((x', y') = (x', y').\)

\textbf{Proof of Lemma 3.} Suppose there exists a subgame perfect equilibrium with \((x', y') \neq (x', y').\) Then from conditions \((A1)\) and \((A2)\) we know that

\[
p_x(x')x' + p_y(y')y' - T(x')') \geq p_x(x')x' + p_y(y')y' - T(y')\]

\(\text{and}

\[
p_x(x')x' + p_y(y')y' - T(x')') \geq p_x(x')x' + p_y(y')y' - T(x').\]

\(\text{Since } p_x(x')x' + p_y(y')y' > p_x(x')x' + p_y(y')y'\) whenever \((x', y') \neq (x', y')\) by the definition of \((x', y')\), it follows that \(T(x') > T(y')\) and \(T(x') > T(x')\), and thus \(y' + x' > y' + x'\) (where we have used the fact that \(T(b) > T(a)\) for all \(b > a\)). Since \(y' + x' = 1\), it follows that \(y' + x' < 1\), contradicting Lemma 2. \text{Q.E.D.}

Condition \((A3)\) determines the division of profit among the three firms. It requires that the retailer be indifferent in equilibrium between buying both products, only product \(X\), and only
product $Y$. Using the fact that the retailer will purchase $(x', y')$, we have

$$p_i(x')x' + p_i(y')y' - T_i'(x') - T_i'(y') = \Pi_{i,y} - T_i'(x') - T_i'(y').$$  
(A13)

Suppose $x''$ solves $\max_{0 < x < \bar{y}} p_i(x)y - T_i(x)$. Then it follows that

$$p_i(x'')x'' - T_i'(x'') = \Pi_{i,y} - T_i'(x'') = \Pi_{i} - T_i(x').$$  
(A14)

where the first inequality follows from the definition of $\Pi_{i,y}$ (recall $\Pi_{i,y} = \max_{0 < x < \bar{y}} p_i(x)y$) and the second inequality follows because $x'' \geq x'$ from the definition of substitute goods and our assumption that $T_i(x) > T_i(0)$, for all $a > b$. Using conditions (A3), (A13), and (A14) yields

$$\Pi_{i,y} - T_i'(x') - T_i'(y') = \Pi_{i} - T_i'(x').$$  
(A15)

Rearranging (A15) gives a lower bound on the equilibrium profits of manufacturer $Y$:

$$\Pi_{i,y} - \Pi_i = T_i'(y').$$  
(A16)

Using (A1) we can derive an upper bound on the equilibrium profits of manufacturer $Y$. The left-hand side of (A1) is greater than or equal to $\Pi_i$. The right-hand side of (A1) equals $\Pi_{i,y} - T_i'(y')$ and so an upper bound on manufacturer $Y$'s profit is

$$T_i'(y') = \Pi_{i,y} - \Pi_i.$$  
(A17)

On inspection of (A16) and (A17), it follows that

$$T_i'(y') = \Pi_{i,y} - \Pi_i,$$  
(A18)

which is manufacturer $Y$'s profit in any subgame perfect equilibrium.

Similarly, we can derive manufacturer $X$'s profit in any subgame perfect equilibrium. The steps are as follows. Suppose $y''$ solves $\max_{0 < y < \bar{x}} p_i(y)x - T_i(y)$. Then it follows that

$$p_i(y'')y'' - T_i'(y'') = \Pi_{i,y} - T_i'(y'') = \Pi_{i} - T_i'(y'),$$  
(A19)

where the first inequality follows by the definition of $\Pi_{i,y}$ (recall $\Pi_{i,y} = \max_{0 < x < \bar{y}} p_i(x)y$) and the second inequality follows because $y'' \geq y'$ from the definition of substitute goods and our assumption that $T_i(b) > T_i(0)$, for all $b > a$. Using conditions (A3), (A13), and (A19) yields

$$\Pi_{i,y} - T_i'(y') - T_i'(x') = \Pi_{i} - T_i'(y').$$  
(A20)

Rearranging (A20) gives a lower bound on the equilibrium profits of manufacturer $X$:

$$\Pi_{i,y} - \Pi_i = T_i'(x').$$  
(A21)

Using (A2) we can derive an upper bound on the equilibrium profits of manufacturer $X$. The left-hand side of (A2) is greater than or equal to $\Pi_i$. The right-hand side of (A2) equals $\Pi_{i,y} - T_i'(x')$. Thus, (A2) implies $\Pi_{i,y} \leq \Pi_{i,y} - T_i'(x')$ and so an upper bound on manufacturer $X$'s profit is

$$T_i'(x') = \Pi_{i,y} - \Pi_i.$$  
(A22)

On inspection of (A21) and (A22), it follows that

$$T_i'(x') = \Pi_{i,y} - \Pi_i,$$  
(A23)

which is firm $X$'s profit in any subgame perfect equilibrium.

The retailer’s equilibrium profit is obtained by subtracting from total channel profits the equilibrium profits of firm $X$ and firm $Y$. This yields

$$\pi_r' = \Pi_{i,y} - \pi_i' - \pi_y',$$  
(A24)

which is the retailer’s profit in any subgame perfect equilibrium.

3. Existence of Equilibria

To prove the existence of a pair of contracts that satisfy (A1), (A2), and (A3), consider

$$T_r(x) = \begin{cases} 0 & \text{if } x = 0, \\ \Pi_{i,y} - \Pi_i & \text{if } x > 0. \end{cases}$$  
(A25)

and

$$T_r(y) = \begin{cases} 0 & \text{if } y = 0, \\ \Pi_{i,y} - \Pi_i & \text{if } y > 0. \end{cases}$$

Each firm charges a fixed payment equal to its product’s incremental contribution to overall channel profit and sells marginal units to the retailer at zero cost (this is because marginal production costs have been normalized to zero). With these contracts, it is easy to show that $(x', y') \in \partial T_i'(1), T_r'(1)$ and the conditions in Lemma 1 are satisfied. This proves the existence of a subgame perfect equilibrium. Parts 1, 2, and 3 jointly prove Proposition 1. Q.E.D.

**Proof of Propositions 2 Through 6**

Recall from Equation (5) that in equilibrium the retailer prices at

$$p_i' = v_i + y'a_i + x'b_i, \quad p_i'' = v_i + y''a_i + x''b_i$$

where

$$x' = \frac{(v_i - v_i + a_i + b_i - 2a_i)}{2(a_i + b_i - b_i - a_i)}, \quad y' = 1 - x'.$$
The maximum revenue of the retailer or total channel profit is given by \( \Pi_y = \rho x^* + \rho'(1 - x^*) \). Substituting equilibrium prices and market shares and after some simplifications we obtain:

\[
\Pi_y(v, u, a, c, b) = 4(a - b + h)(v + a - b - v - h) + 4(a - b + h)(v + a - b - v - h)
\]

For symmetry of the benchmark case assume that \( ax, bx, vy, vy, p = 0 \) by inspection. By \( \mu < \delta, \Delta \pi_y = \Delta \pi_y < 0 \), and \( \Delta \pi_y = -\Delta \pi_y > 0 \). Hence the function is maximized at one of the endpoints \( y = \delta, y = 28 \). Evaluating \( \Delta \pi_y \) at \( y = \delta, y = 28 \) shows \( \Delta \pi_y < 0 \). Since \( \Delta \Pi_y = 0, \Delta \pi_y = -\Delta \pi_y > 0 \). Assume \( y = 28 \). Then

\[
\{ \Delta \pi_y, \Delta \pi_y, \Delta \pi_y \}
\]

By \( \mu < \delta, \Delta \pi_y = \Delta \pi_y < 0 \), and \( \Delta \pi_y = -\Delta \pi_y > 0 \). Hence the proposition holds.

**Proposition 3.** Let \( \Delta \Pi_y = \mu_1 + \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = 0 \), where \( \mu < 0 \). Assume \( y < 28, \mu < 28 - y \). Then

\[
\{ \Delta \pi_y, \Delta \pi_y, \Delta \pi_y \}
\]

By \( \mu < \delta, \Delta \pi_y = \Delta \pi_y < 0 \), and \( \Delta \pi_y = -\Delta \pi_y > 0 \). Hence the proposition holds.

**Proposition 4.** Let \( \Delta \Pi_y = \mu_1 + \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = 0 \), where \( \mu < 0 \). Assume \( y < 28 \). Then

\[
\{ \Delta \pi_y, \Delta \pi_y, \Delta \pi_y \}
\]

To avoid degenerate solutions assume \( l \Delta \pi_y < 1/2 \). Since \( x^* = \mu / (4 \delta + \mu), \mu < 28 \). Thus, \( \Delta \pi_y = \Delta \pi_y < 0 \), and \( \Delta \pi_y = -\Delta \pi_y > 0 \). Hence the proposition holds.

**Proposition 5.** Let \( \Delta \Pi_y = \mu_1 + \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = \Delta \pi_y = 0 \), where \( \mu < 0 \). Assume \( y < 28 \). Then

\[
\{ \Delta \pi_y, \Delta \pi_y, \Delta \pi_y \}
\]

By \( \mu < \delta, \Delta \pi_y < 0 \). Also, \( \delta \Delta \pi_y < 0 \) by \( \mu < \delta \) and \( \mu < \delta \). Hence, \( \Delta \pi_y < 0 \). Since \( \Delta \Pi_y = 0, \Delta \pi_y = -\Delta \pi_y > 0 \). Assume \( y < 28, \mu < \delta, \delta = y / 2 \). Then

\[
\{ \Delta \pi_y, \Delta \pi_y, \Delta \pi_y \}
\]

By \( \mu < \delta, \Delta \pi_y > 0 \). Also, \( \delta \Delta \pi_y < 0 \) by \( \mu < \delta \) and \( \mu < \delta \). Evaluate \( \Delta \pi_y \) at the smallest admissible \( \mu = \delta - \gamma / 2 \). The resulting function has a unique minimum at \( \gamma = x^* \) where \( \delta < \gamma < 28 \). Hence, the function is maximized at one of the endpoints \( y = \delta, y = 28 \). Evaluating \( \Delta \pi_y \) at \( y = \delta, y = 28 \) shows \( \Delta \pi_y < 0 \). Since \( \Delta \Pi_y = 0, \Delta \pi_y = -\Delta \pi_y > 0 \). Assume \( y = 28 \). Then

\[
\{ \Delta \pi_y, \Delta \pi_y, \Delta \pi_y \}
\]
Assume $\gamma \geq 2\delta$, $\mu \geq \gamma - 2\delta$. Then

$$\{\Delta_\pi, \Delta_\pi, \Delta_\pi\} = \left\{ \frac{\mu}{4}, \frac{-\gamma + 2\delta + \mu}{4(\delta + \mu)}, \frac{-\gamma + 2\delta + \mu}{4(\delta + \mu)} \right\}$$

$\Delta_\pi > 0$ by inspection. Also, $\partial \Delta_\pi/\partial \mu > 0$. Evaluating at the smallest admissible $\mu = \gamma - 2\delta$, $\Delta_\pi < 0$. Since $\Delta_\pi = 0 \Delta_\pi = -\Delta_\pi < 0$.

Assume $\gamma \geq 2\delta, \mu < \gamma - 2\delta$. Then

$$\{\Delta_\pi, \Delta_\pi, \Delta_\pi\} = \left\{ \frac{\mu}{4}, \frac{-\gamma + 2\delta + \mu}{4(\delta + \mu)} \right\}$$

Clearly, $\Delta_\pi = \Delta_\pi = \Delta_\pi < 0$. Hence the proposition holds.

**Proposition 5.** Let $\Delta_\pi = \Delta_\pi = \mu$ and $\Delta_\pi = \Delta_\pi = \Delta_\pi = \Delta_\pi = 0$, where $\mu > 0$. We assume nonnegative demand for product $i$ if the retailer sells product $i$ only, which is equivalent to a restriction on information shocks applied to the benchmark case, $\mu < \delta$.

Assume $\gamma < 2\delta, \mu < \gamma - 2\delta$. Then

$$\{\Delta_\pi, \Delta_\pi, \Delta_\pi\} = \left\{ \frac{\mu}{2} + \frac{2\gamma^2}{\delta + \mu}, \frac{\mu(2\gamma^2 - 4\gamma + 4\delta^2 + 3\delta \mu + \mu^2)}{8\delta(\delta + \mu)}, \frac{\mu(-4\delta^2 + 4\gamma^2 \mu + \mu^2 + 2\delta(4\gamma + \delta^2 \mu)(\delta + \mu))}{8\delta(\delta + \mu)(\delta + \mu)} \right\}$$

$\Delta_\pi < 0$ at the largest admissible $\mu$. Let $\delta < \gamma < \sqrt{2\delta}$. One can easily show that there exists exactly one $\mu > 0$ such that $\Delta_\pi = 0$ at $\mu$. Also, $\partial \Delta_\pi/\partial \mu > 0$ at $\mu = 0$. Let $\sqrt{2\delta} < \gamma < 2\delta$. Then there is no $\mu > 0$ such that $\Delta_\pi = 0$ at $\mu$. Also, $\partial \Delta_\pi/\partial \mu < 0$ at $\mu = 0$. The claims regarding $\Delta_\pi$ hold by continuity of $\Delta_\pi$. $\Delta_\pi$ is minimized at $\gamma = 0$. Evaluating $\Delta_\pi$ at $\gamma = 0$ shows that $\Delta_\pi > 0$. The numerator of $\Delta_\pi$ is minimized in the admissible parameter range at $\gamma = 0$ and is positive. By $\mu < \Delta_\pi < 0$.

Assume $\gamma < 2\delta, \mu = \delta - \gamma/2$. Then

$$\{\Delta_\pi, \Delta_\pi, \Delta_\pi\} = \left\{ \frac{2\gamma^2 + 8\delta^2 + \mu^2 - 4\gamma(2\gamma + \mu)}{8\delta(\delta + \mu)}, \frac{\mu(2\gamma^2 - 4\gamma + 4\delta^2 + 3\delta \mu + \mu^2)}{8\delta(\delta + \mu)}, \frac{-\delta - 4\gamma^2 + \mu^2}{4(\delta + \mu)} \right\}$$

$\Delta_\pi < 0$ at the largest admissible $\mu$. Let $\delta < \gamma < 10\delta/9$. One can easily show that there exists exactly one $\mu > \delta - \gamma/2$ such that $\Delta_\pi = 0$ at $\mu$. Also, $\Delta_\pi > 0$ at $\mu = \delta - \gamma/2$. Let $10\delta/9 < \gamma < 2\delta$. Then there is no $\mu > \delta - \gamma/2$ such that $\Delta_\pi = 0$ at $\mu$. Also, $\partial \Delta_\pi/\partial \mu < 0$ at $\mu = \delta - \gamma/2$. The claims regarding $\Delta_\pi$ hold by continuity of $\Delta_\pi$. $\Delta_\pi$ is minimized at $\gamma = \delta$. Evaluating $\Delta_\pi$ at $\gamma = \delta$ shows $\Delta_\pi > 0$. Approximating $\partial \Delta_\pi/\partial \mu$ from below using $\delta - \gamma/2 < \mu < \delta$ and $\gamma < 2\delta$ shows $\partial \Delta_\pi/\partial \mu > 0$. Evaluate $\Delta_\pi$ at the smallest admissible $\mu = \delta - \gamma/2$. The denominator of the resulting function is negative by $\gamma < 2\delta$. The numerator is negative for $\delta < \gamma < 2\delta$. Hence, $\Delta_\pi > 0$.

Assume $\gamma < 2\delta, \mu = \gamma - 2\delta$. Then

$$\{\Delta_\pi, \Delta_\pi, \Delta_\pi\} = \left\{ \frac{\mu(-4\delta + \mu)}{8\delta}, \frac{\mu(2\gamma^2 - 4\gamma + 4\delta^2 + 3\delta \mu + \mu^2)}{8\delta(\delta + \mu)}, \frac{-\gamma + 2\delta + \mu}{4(\delta + \mu)} \right\}$$

$\Delta_\pi < 0$ and $\partial \Delta_\pi/\partial \mu < 0$. $\Delta_\pi, \Delta_\pi < 0$ at $\gamma = 0$. Evaluating $\Delta_\pi$ at the smallest admissible $\gamma = 2\delta$ shows $\Delta_\pi > 0$. Notice that by $\gamma > 2\delta$ and $\gamma < 0$. Notice that by $\gamma > 2\delta$ shows $\Delta_\pi > 0$. This shows $\Delta_\pi > 0$.

Assume $\gamma > 2\delta, \mu < \gamma - 2\delta$. Then

$$\{\Delta_\pi, \Delta_\pi, \Delta_\pi\} = \left\{ \frac{\mu(-4\delta + \mu)}{8\delta}, \frac{\mu(2\gamma^2 - 4\gamma + 4\delta^2 + 3\delta \mu + \mu^2)}{8\delta(\delta + \mu)}, \frac{-\gamma + 2\delta + \mu}{4(\delta + \mu)} \right\}$$

Since $\mu < \delta$, $\Delta_\pi = \Delta_\pi = \Delta_\pi < 0$, and $\Delta_\pi = -\Delta_\pi > 0$. Hence the proposition holds.

**Proposition 6.** Let $\Delta_\pi = \Delta_\pi = \mu$ and $\Delta_\pi = \Delta_\pi = \Delta_\pi = \Delta_\pi = 0$, where $\mu > 0$.

Assume $\gamma < 2\delta, \mu < 2\gamma - \gamma$. Then

$$\{\Delta_\pi, \Delta_\pi, \Delta_\pi\} = \left\{ \frac{\mu(-4\delta + \mu)}{8\delta}, \frac{\mu(2\gamma^2 - 4\gamma + 4\delta^2 + 3\delta \mu + \mu^2)}{8\delta(\delta + \mu)}, \frac{-\gamma + 2\delta + \mu}{4(\delta + \mu)} \right\}$$

$\partial \Delta_\pi/\partial \mu > 0$, $\Delta_\pi < 0$. Assume $\gamma < 2\delta, \mu = 2\gamma - \gamma$. Then

$$\{\Delta_\pi, \Delta_\pi, \Delta_\pi\} = \left\{ \frac{(\gamma - 2\delta)^2}{8\delta} - \frac{(\gamma - 2\delta)^2}{2\delta - 4\delta + 2\gamma + \mu} \right\}$$

Clearly, $\Delta_\pi = \Delta_\pi = 0$ and $\Delta_\pi = \Delta_\pi > 0$. Hence the proposition holds. Q.E.D.
References


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