Bargaining, bundling, and clout: the portfolio effects of horizontal mergers

Daniel P. O'Brien*  
and  
Greg Shaffer**

July 2004

Abstract

This paper examines the output and profit effects of horizontal mergers between up-steam firms in intermediate-goods markets. We consider market settings in which the upstream firms sell differentiated products to a downstream retail monopolist. We find that if the merging firms can bundle their products, transfer pricing is efficient before and after the merger. The merging firms gain and the retailer loses, but absent any cost efficiencies, consumer and total welfare do not change. If the merging firms cannot bundle their products, the effects of the merger depend on the merged firm’s bargaining power. If the merged firm’s bargaining power is low, the welfare effects are the same as in the case with bundling; if its bargaining power is high, and there are no offsetting cost efficiencies, the merger typically reduces welfare. We evaluate the profit effects of mergers on the rival firms and retailer for the case of two-part tariff contracts. Contrary to conventional wisdom, we find that a merger that harms the retailer may increase welfare.

*Economist, U.S. Federal Trade Commission, 601 New Jersey Avenue, N.W., Washington, D.C., 20001, USA. The views expressed herein are my own and do not purport to represent the views of the Federal Trade Commission or any Commissioner. O’Brien can be reached at dobrien@ftc.gov or danobrien@cox.net.

**Professor of Economics and Management, William E. Simon Graduate School of Business, University of Rochester, Rochester, N.Y. 14627, USA. Shaffer can be reached at shaffer@simon.rochester.edu.

We thank Gabriella Chiesa, Morten Hviid, Mark McCabe, Chris Snyder, seminar participants at the 2004 International Industrial Organization Conference in Chicago, IL, and two anonymous referees for helpful comments.
1 Introduction

Merger policy in the industrialized countries is largely motivated by classical theories of oligopoly whose roots trace to the theories developed by Cournot (1838) and Bertrand (1883). The Merger Guidelines in the U.S., for example, rely heavily on modern variants of these theories, which predict that a merger between competitors in concentrated markets can raise prices significantly unless the merger generates offsetting cost efficiencies or attracts sufficient post-merger entry (Deneckere and Davidson, 1985; Farrell and Shapiro, 1990; and Willig, 1991). Unfortunately, however, the classical theories and their progeny generally make no distinction between final-goods and intermediate-goods markets. The theories assume that firms set take-it or leave-it prices that apply to all buyers, which is a reasonable assumption for most final-goods markets and some intermediate-goods markets, but it is not descriptive of pricing in many intermediate-goods markets.

A common feature of pricing in many intermediate-goods markets is that the terms of supply are negotiated. For example, manufacturers of products sold through retail outlets like supermarkets, convenience stores, and mass merchants often negotiate different supply contracts with each distributor. Moreover, these contracts are a far cry from the simple, linear price set unilaterally by firms in the classical theories. The pricing schedules one observes in reality are often highly non-linear, with features like slotting allowances, minimum quantity thresholds, and quantity discounts. They may also involve variants of bundling, such as aggregate rebates and full-line forcing.

In this paper we incorporate nonlinear supply contracts, bargaining, and bundling (defined as inter-dependent price schedules) into a model of upstream competition to examine the effects of horizontal mergers. We focus on the simplest market setting in which these factors are present: \( N \geq 2 \) upstream manufacturers and a single downstream retailer, with no uncertainty, no asymmetric

\footnote{The 1992 Horizontal Merger Guidelines can be found at http://www.ftc.gov/be/docs/horizmer.htm. The unilateral effects section of the Guidelines discusses markets in which firms are distinguished by their capacities, and markets in which they are distinguished by product differentiation. The former is motivated by Cournot’s model of oligopoly among producers of homogeneous products, while the latter is motivated by Bertrand competition among producers of differentiated products. The coordinated effects section of the Guidelines follows closely the ideas in Stigler’s (1964) theory of oligopoly, which is understood today as a repeated game among Cournot or Bertrand competitors.}

\footnote{Aggregate rebates are discounts given to a buyer based on the buyer’s cumulative purchases of all products in the seller’s product line. With full-line forcing, the seller requires the buyer to purchase all of the seller’s products.}
information, and no moral hazard. We find that the effects of horizontal mergers in this simple environment differ substantially from the effects predicted by the classical theories of oligopoly.

One key difference between an environment in which firms distribute their products through a common retailer and an environment in which firms sell their products directly to consumers is that, in the former environment, all pricing externalities can be internalized. As Bernheim and Whinston (1985; 1998) and O’Brien and Shaffer (1997) have shown, when two single-product firms make simultaneous take-it-or-leave-it offers to a common retailer, the vertically-integrated outcome is obtained (overall joint profit is maximized). In this paper, we show that this result also holds under simultaneous Nash bargaining for the case of \( N \geq 2 \) single-product firms, and for the case of \( N \) firms selling single or multiple products if the multi-product firms can bundle their products.

This has strong implications for merger policy. If bundling is feasible, it means that a merger between two single-product firms that sell differentiated products to a common retailer need not have any effect on input choices, output choices, wholesale prices, or final-goods prices. Thus, in contrast to conventional wisdom, a merger between upstream competitors in concentrated markets need not lead to higher prices even if there are no offsetting cost efficiencies or post-merger entry. In this instance, the U.S. Merger Guidelines would fail to offer much in the way of guidance. Nor would it be possible to deduce appropriate public policy based on whether the retailer feels it would benefit or be harmed. The merging firms would have an incentive to merge even absent cost savings because the combined entity would be able to extract more surplus from the retailer. Although the retailer would oppose the merger in this case, consumer and total welfare would be unchanged.

The analysis is more complicated if bundling is not feasible (or is prohibited). In that case, the effect of a merger on final-goods prices, and hence on consumer and total welfare, depends on the relative bargaining powers of the merged firm and retailer, as measured by their bargaining weights in an asymmetric Nash bargaining solution. If the merged firm’s bargaining power is sufficiently low, post-merger contracts are efficient and the welfare effects are the same as in the case with bundling: welfare is typically higher if there are cost-efficiencies related to the merger, lower if the merger raises production costs, and otherwise there is no change. But if the merged firm has
sufficiently high bargaining power, it no longer has an incentive to negotiate an efficient contract. This result contrasts with what one might expect from models of common agency. However, it has a simple, intuitive interpretation. The distortion arises because the retailer can threaten to drop one of the merged firm’s products if the merged firm attempts to extract more than the incremental surplus generated by each product. In order to reduce the retailer’s profit if it were to carry through with this threat, the merged firm’s incentive is to negotiate higher than efficient marginal transfer prices. Although this allows the merged firm to extract more surplus from the retailer than it would otherwise obtain, it also tends to reduce total and consumer welfare. Unless there are offsetting cost efficiencies, the merger would typically lead to higher final-goods prices. Thus, when post-merger bundling is not possible and the merged firm’s bargaining power is sufficiently high, the qualitative tradeoffs of mergers predicted by the classical theories of oligopoly re-emerge.

In addition to its implications for mergers, our model has implications for the on-going policy debate over the use of “bundled discounts” by multi-product firms. Several high-profile cases in the U.S. and Europe involving vitamins, tires, soft drinks, and transparent tape have hinged on the alleged use of these discounts by larger firms to exclude smaller rivals. To satisfy the EC Commission, for example, Cola Cola Export was forced to remove from its contracts with large distributors any rebates that were conditioned on the buyer’s purchases of other beverages in addition to its purchases of Coca Cola. Bundled discounts are viewed skeptically by some because they may raise barriers to entry, allowing the larger firm to charge higher prices. However, in our setting bundled discounts can lead to lower transfer prices if the bundling firm’s bargaining power with respect to the retailer is high enough. Thus, a policy of prohibiting bundled discounts may lead to higher final-goods prices and lower welfare if it fails to induce additional entry. Prohibiting

---

3In contrast, when bundling is possible, the contract can be structured so that the retailer effectively does not have the option of dropping just one of the merged firm’s products. It will either sell both products or neither product.


5In the case of LePage’s Inc. v. 3M, the court said that “the principal anticompetitive effect of bundled rebates as offered by 3M is that when offered by a monopolist they may foreclose portions of the market to a potential competitor who does not manufacture an equally diverse group of products and who therefore cannot make a comparable offer” (Id. at 155).
such discounts can also make mergers less profitable, preventing efficient mergers from taking place. A complete assessment of the effects of bundled discounts requires balancing the potential static benefits we identify against the potential dynamic effects on firm entry and exit.

The literature on mergers in intermediate-goods markets is surprisingly sparse. The seminal work in this area is Horn and Wolinsky (1988), who use the Nash bargaining solution to analyze incentives for mergers in markets where competing downstream firms acquire inputs from independent suppliers, and in which they acquire inputs from a monopoly supplier. Horn and Wolinsky differ from us in that their upstream suppliers do not compete, bargaining takes place over linear prices only, and the inputs are assumed to be homogeneous.\textsuperscript{6} von Ungern-Sternberg (1996) and Dobson and Waterson (1997) also use the Nash bargaining solution to analyze the effects of mergers on input prices. They too, however, restrict attention to linear prices and do not consider bundling. The market structure they consider consists of a single upstream firm. Other papers in this area look at different market structures, do not allow for bargaining, and do not consider bundling.\textsuperscript{7}

Much of the literature on multi-product pricing focuses on the use of bundling to extract surplus from heterogeneous buyers (Adams and Yellen, 1976; McAfee, et al. 1989; Mathewson and Winter, 1997) or to leverage monopoly power across markets (Whinston, 1990; Choi and Stefanadis, 2001; and Carlton and Waldman, 2002). In contrast, bundling is profitable in our model even when there is a single buyer (the downstream retail monopolist) and no opportunity to leverage across markets. Bundling also takes place with substitute goods in our model, in contrast to the well-studied cases of bundling with independent or complementary goods. The closest paper to ours in the literature on multi-product pricing is Shaffer (1991), who considers bundling in a bilateral monopoly setting with take-it-or-leave-it offers. However, his model does not allow for upstream rivalry, mergers, or bargaining, nor does he consider the welfare implications of a policy prohibiting bundling.

The remainder of the paper is organized as follows. Section 2 presents the model and solves for the pre-merger equilibrium. Section 3 solves for the post-merger equilibrium and discusses output

\textsuperscript{6}See also Lommerud et al. (2003), where each downstream firm is locked in a bilateral monopoly relationship with its own independent supplier. In their model, the downstream firms set quantities and bundling is not considered.\textsuperscript{7}Ziss (1995) considers mergers-to-monopoly in a market setting with two manufacturer-retailer pairs. Colangelo (1995) looks at pre-emptive horizontal and vertical mergers. Neither considers bundling or allows for bargaining.
and welfare effects with and without bundling. Section 4 examines the profit effects of mergers and explores the effects of cost efficiencies. Section 5 discusses extensions and concludes the paper.

2 The model and pre-merger equilibria

\(N \geq 2\) upstream firms (manufacturers) each distribute a single differentiated product through a downstream monopolist.\(^8\) Manufacturer \(i\)'s production cost is \(C_i(q_i) \geq 0\), where \(C_i(0) = 0\) and \(q_i \geq 0\) is the quantity it produces. The downstream firm (retailer) resells the manufacturers’ products to final consumers. Its net revenue from selling \(q \equiv (q_1, q_2, \ldots, q_N)\) units, after subtracting all costs other than what it pays the manufacturers, is \(R(q)\). We assume that \(R(q) - \sum_i C_i(q_i)\) is concave and differentiable for all \(q_i > 0\), and that it has a unique maximum at \(q^I \equiv (q^I_1, q^I_2, \ldots, q^I_N)\). We refer to the quantity vector \(q^I\) and associated joint profit as “the fully-integrated outcome.”

Competitive interactions are modeled as a two-stage game. In stage one, the manufacturers simultaneously negotiate contracts with the retailer over the price of their products. In stage two, with all contracts in place, the retailer decides how much of each product to buy, and hence how much to resell to final consumers. As in Horn and Wolinsky (1988), we assume the outcomes of bargaining are determined by the set of simultaneous, asymmetric Nash bargaining solutions between the retailer and each manufacturer. In the event a negotiation breaks down, each firm in the negotiation earns its disagreement payoff. For the manufacturer, who has only the retailer as a trading partner, we normalize this payoff to zero. In the case of the retailer, we assume its disagreement payoff with each manufacturer is the profit it could earn without that manufacturer.\(^9\)

Manufacturer \(i\)'s contract with the retailer is a function \(T_i(\cdot)\), which specifies the retailer’s payment for any quantity \(q_i\) purchased. We assume \(T_i(0) = 0\) and \(T_i(q_i) \geq C_i(q_i)\) for all \(q_i\). Additional restrictions on contracts are discussed as needed. This formulation permits most contractual forms

\(^8\)Many of our results go through with \(M > 2\) retailers if the retailers do not observe each others’ contracts and have passive beliefs (O’Brien and Shaffer, 1992; McAfee and Schwartz, 1994). This is discussed further in section 5.

\(^9\)The simultaneous Nash bargaining solution also arises in the literature on labor market negotiations, which are analogous to negotiations between upstream and downstream firms. For example, Davidson (1988) considers bargaining between a union and two employers, and Jun (1989) examines bargaining between two unions and one or two employers. These papers show that the simultaneous Nash bargaining solution is equivalent to perfect equilibria of natural alternating offer bargaining games in the limit as the time between offers goes to zero.
observed in practice, including linear wholesale prices, two-part tariffs, and quantity forcing. It does not permit payments to depend on final-goods prices (resale price maintenance), or on the retailer’s purchases of other manufacturers’ products (e.g., we do not allow exclusive dealing provisions).

We now characterize the bargaining equilibrium. Given the vector of contracts \( T \equiv (T_1(\cdot), T_2(\cdot), ..., T_N(\cdot)) \), the retailer chooses quantities to maximize profits. Let \( \Omega(T) \) be the set of quantity vectors that maximize the retailer’s profit given the contract vector \( T \). That is,

\[
\Omega(T) \equiv \arg \max_q R(q) - \sum_j T_j(q_j).
\]

In the first stage, the retailer and each manufacturer negotiate their contract recognizing that the retailer will subsequently choose quantities from the set \( \Omega(T) \). Let \( T_{-i} \) denote the vector of contracts of firm \( i \)’s rivals, e.g., \( T_{-1} \equiv (T_2(\cdot), T_3(\cdot), ..., T_N(\cdot)) \). Then, given \( T_{-i} \), we can define the feasible set of quantity-contract combinations available to manufacturer \( i \) and the retailer as

\[
A_i(T_{-i}) \equiv \{(q_i, T_i(\cdot)) \mid q \in \Omega(T), T_i(0) = 0, T_i(q_i) \geq C_i(q_i)\}.
\]

The Nash bargaining solution between manufacturer \( i \) and the retailer solves

\[
\max_{(q_i, T_i(\cdot)) \in A_i(T_{-i})} \left( \pi_i - d_i \right)^{\alpha_i} \left( \pi_r - d_r \right)^{1-\alpha_i},
\]

where \( \pi_i = T_i(q_i) - C_i(q_i) \) is manufacturer \( i \)’s profit; \( \pi_r = R(q) - \sum_j T_j(q_j) \) is the retailer’s profit; \( d_i \) and \( d_r \) are the disagreement profits of manufacturer \( i \) and the retailer, respectively; and \( \alpha_i \in [0, 1] \) is manufacturer \( i \)’s bargaining weight. As discussed previously, the disagreement profit for manufacturer \( i \) is \( d_i = 0 \). The disagreement profit of the retailer with manufacturer \( i \) is

\[
d_r = \max_{q_{-i}} R(0, q_{-i}) - \sum_{j \neq i} T_j(q_j),
\]

where, for ease of exposition, we use \( R(0, q_{-i}) \) in place of \( R(q_1, ..., q_{i-1}, 0, q_{i+1}, ..., q_N) \).

A bargaining equilibrium is a set of quantities and contracts that solve (1) for each product \( i \). We refer to an equilibrium in which \( K \) products are sold as “K-product” equilibria, \( K \in \{1, 2, ..., N\} \).
Characterization of equilibrium quantities and payoffs

Unfortunately, the maximization problem in (1) is not easy to work with because it involves the choice of a function $T_i(\cdot)$ and hence it is not amenable to calculus. However, as we show below, we can nevertheless characterize equilibrium quantities and payoffs by solving an equivalent problem in which manufacturer $i$ and the retailer choose a quantity-forcing contract with two parameters,

$$T_i^F(q_i) = \begin{cases} 
0 & \text{if } q_i = 0 \\
F_i & \text{if } q_i = q'_i \\
\infty & \text{otherwise}
\end{cases}$$

and quantity $q_i$, from the feasible set of quantity-contract combinations

$$A_i^F(T_{-i}) \equiv \{(q_i, F_i, q'_i) \mid q \in \arg \max \limits_q R(q) - T_i^F(q_i) - \sum_{j \neq i} T_j(q_j), \ F_i \geq C_i(q'_i)\}.$$

With this restriction to quantity-forcing contracts, the maximization problem in (1) becomes

$$\max_{(q_i, F_i, q'_i) \in A_i(T_{-i})} \left( T_i^F(q_i) - C_i(q_i) \right)^{\alpha_i} \left( R(q_i) - T_i^F(q_i) - \sum_{j \neq i} T_j(q_j) - d_{ri} \right)^{1-\alpha_i}. \quad (2)$$

In what follows, we assume that this maximization problem has a unique solution.

The idea is to create a simpler, constrained, bargaining problem that yields the same solution (same quantities and payoffs) as any solution to the unconstrained problem in (1). As we show in Lemma 1, since the choice set in the constrained problem (2) is a subset of the choice set in the unconstrained problem (1), and since any solution to the unconstrained problem is a feasible choice in the constrained problem, the solution to the constrained problem is equivalent to any solution to the unconstrained problem.

**Lemma 1** Suppose $(\hat{q}_i, \hat{F}_i, \hat{q}'_i) \in A_i^F(T_{-i}^*)$ is the unique solution to the maximization problem in (2) given the vector of rival contracts $T_{-i}^*$. Suppose $(q_i^*, T_i^*(\cdot)) \in A_i(T_{-i}^*)$ is a solution to the maximization problem in (1) given the vector of rival contracts $T_{-i}^*$. Then $q_i = q_i^*$ and $F_i = T_i^*(q_i^*)$.

**Proof.** Suppose $(q_i^*, T_i^*(\cdot)) \in A_i(T_{-i}^*)$ is a solution to (1) given the vector of contracts $T_{-i}^*$, and let $q_* \in \Omega(T^*)$. Note that $A_i^F(T_{-i}) \subset A_i(T_{-i})$, and that the quantities $q^*$ and payments $T_i^*(q_i^*)$ can be obtained when quantity-contract combinations are restricted to the set $A_i^F(T_{-i})^*$ by setting $q_i = q_i^*$. 

7
where $q_i$ and payoffs for manufacturer $i$ for manufacturer $i$ are such that $q_i = q_i^*$. Thus, $(q_i^*, T_i^*(q_i^*), q_i^*) \in A_i^F(T_{-i})$ solves the maximization problem in (2) for manufacturer $i$ and the retailer, and the two maximization problems yield the same quantities and payoffs for manufacturer $i$ and the retailer, conditional on contracts $T_{-i}^*$. Q.E.D.

Suppose $(q^*, T^*)$ is a vector of quantities and contracts that form a bargaining equilibrium, where $q^* \equiv (q_1^*, ..., q_N^*)$ and $T^* \equiv (T_1^*, ..., T_N^*)$. Then Lemma 1 implies that we can characterize the equilibrium quantity and payoffs for manufacturer $i$ and the retailer by solving the problem:

$$
\max_{(q_i, F_i, q_{-i})} \left( F_i - C_i(q_i) \right)^{\alpha_i} \left( R(q) - F_i - \sum_{j \neq i} T_j^*(q_j) - d_{ri} \right)^{(1-\alpha_i)}
$$

$$
= \max_{q_i, F_i, q_{-i}} \left( F_i - C_i(q_i) \right)^{\alpha_i} \left( R(q) - F_i - \sum_{j \neq i} T_j^*(q_j) - d_{ri} \right)^{(1-\alpha_i)}
$$

(3)

such that

$$
F_i \geq C_i(q_i),
$$

(4)

$$
R(q) - F_i - \sum_{j \neq i} T_j^*(q_j) \geq d_{ri},
$$

(5)

where constraints (4) and (5) ensure that manufacturer $i$ and the retailer earn at least their disagreement profits. The equality in (3) follows because the constraint $(q_i, F_i, q_{-i}) \in A_i^F(T_{-i})$ requires that $q_{-i}$ maximize $R(q) - \sum_{j \neq i} T_j^*(q_j)$. Since $F_i - C_i(q_i)$ is independent of $q_{-i}$, and $F_i$ and $d_{ri}$ are fixed when the retailer chooses $q_{-i}$, this amounts to choosing $q_{-i}$ to maximize the Nash product.

The first-order conditions for $F_i$ and $q_i$ at an interior solution of (3) are

$$
\alpha_i \pi_i^{(\alpha_i-1)} (\pi_r - d_{ri})^{(1-\alpha_i)} - (1 - \alpha_i) \pi_i^{\alpha_i} (\pi_r - d_{ri})^{-\alpha_i} = 0.
$$

(6)

$$
-C_i'(q_i) \alpha_i \pi_i^{(\alpha_i-1)} (\pi_r - d_{ri})^{(1-\alpha_i)} + \frac{\partial R(q)}{\partial q_i} (1 - \alpha_i) \pi_i^{\alpha_i} (\pi_r - d_{ri})^{-\alpha_i} = 0.
$$

(7)

Substituting (6) into (7) and simplifying yields

$$
\frac{\partial R(q)}{\partial q_i} - C_i'(q_i) = 0,
$$

(8)

which implies that $q_i^*$ maximizes the joint profit of manufacturer $i$ and the retailer given $T_{-i}^*$. Since this must be true for all $i$, the bargaining equilibrium quantities must maximize overall joint profits, i.e., $q_i^* = q_i^T$, provided (3) has an interior solution for each $i$. This proves the following proposition.

---

\[\text{Note that this does not imply that the forcing contracts that solve the restricted problem and the contracts } T_{-i}^* \text{ form a bargaining equilibrium because the best responses of rivals to the forcing contract } T_{-i}^* \text{ may differ from } T_{-i}^* \text{. The appeal to forcing contracts is only a means to characterize the bargaining equilibrium quantities and profits.}\]
Proposition 1 When \( N \) upstream firms each sell a single differentiated product to a common downstream retailer, the fully-integrated outcome is realized in all \( N \)-product bargaining equilibria.

Proposition 1 extends to bargaining with \( N \) upstream firms the well-known result in the agency literature that a common retailer internalizes all pricing externalities when manufacturers make take-it-or-leave-it offers (Bernheim and Whinston, 1985, 1998; and O’Brien and Shaffer, 1997). The result that all bargaining equilibria replicate the fully-integrated outcome implies that transfer pricing is efficient. This has an intuitive interpretation. Fix the contracts of all manufacturers other than \( i \), and consider negotiations between the retailer and manufacturer \( i \). Since nonlinear contracts are feasible, they can choose their quantity to maximize their bilateral profits and divide the surplus with a non-distortional transfer. Note that choosing \( q_i \) to maximize their bilateral profits, \( R(q) - \sum_{j \neq i} T_j^*(q_j) - C_i(q_i) \), is the same as choosing \( q_i \) to maximize overall joint profit.

3 Post-merger equilibria and welfare effects

Suppose manufacturers 1 and 2 merge. This alters negotiations in potentially three ways. First, it affects the retailer’s disagreement profit with the merged firm. After the merger, the retailer’s disagreement profit is the profit it would earn if it did not sell products 1 and 2.\(^{11}\) Second, it may affect the retailer’s bargaining power. After the merger, the retailer’s bargaining weight in the Nash bargaining solution with respect to the newly merged firm, \( \alpha_m \in [0, 1] \), may differ from what it was with respect to each firm separately. Third, it affects the contracts the merged firm may be able to negotiate. After the merger, contracts in which the payments for \( q_1 \) and \( q_2 \) are interdependent may be feasible. We say that the merged firm engages in bundling if the payment for one of its products depends on the amount purchased of the other. Formally, let \( T_m(q_1, q_2) \) be the merged firm’s contract with the retailer, where \( q_1 \) is the quantity the retailer purchases of product 1 and \( q_2 \) is the quantity the retailer purchases of product 2. Then \( T_m(q_1, q_2) \) exhibits bundling if and only if there does not exist \( T_1(q_1) \) and \( T_2(q_2) \) such that \( T_m(q_1, q_2) = T_1(q_1) + T_2(q_2) \) for all \( q_1, q_2 \geq 0 \).

\(^{11}\)A merged firm in our model is larger than its rivals because it now controls two products. This is unlike Salant et al. (1983) type models in which the only effect of a merger is to reduce the number of firms in the market.
Mergers with bundling

Let \( C_m(q_1, q_2) \) be the post-merger cost of producing \( q_1 \) units of product 1 and \( q_2 \) units of product 2. For now, we assume the merger does not affect costs so that \( C_m(q_1, q_2) = C_1(q_1) + C_2(q_2) \). Later, we will discuss how our results would differ if the merger created cost efficiencies or inefficiencies.

Let \( \pi_m \) denote the merged firm’s post-merger profit. Then \( \pi_m = T_m(q_1, q_2) - C_m(q_1, q_2) \). As before, the retailer will choose quantities from the set \( \Omega(T) \) in stage two, where \( T \) now equals \( (T_m(\cdot, \cdot), T_3(\cdot), ..., T_N(\cdot)) \). Thus, given the rivals’ contracts, \( T_{-1,2} \equiv (T_3(\cdot), ..., T_N(\cdot)) \), we can define the feasible set of quantity-contract combinations available to the merged firm and retailer as

\[
A_m(T_{-1,2}) \equiv \{(q_1, q_2, T_m(\cdot, \cdot)) \mid q \in \Omega(T), T_m(0, 0) = 0, T_m(q_1, q_2) \geq C_m(q_1, q_2) \}.
\]

The feasible set of quantity-contract combinations available to rival firm \( j \) and the retailer is \( A_j(T_{-j}) \), which is the same as their premerger feasible set with \( T_1(\cdot) \) and \( T_2(\cdot) \) replaced by \( T_m(\cdot, \cdot) \).

Suppose \( q^B \equiv (q_1^B, ..., q_N^B) \) and \( T^B \equiv (T_m^B, T_3^B, ..., T_N^B) \) form a bargaining equilibrium with bundling. Then the Nash bargaining solution between the merged firm and retailer solves

\[
\max_{(q_1, q_2, T_m(\cdot, \cdot)) \in A_m(T_{-1,2}^B)} (\pi_m - d_m)^{\alpha_m} (\pi_r - d_{rm})^{(1 - \alpha_m)},
\]

where \( d_m \) and \( d_{rm} \) are the disagreement profits of the merged firm and retailer. The disagreement profit of the merged firm is \( d_m = 0 \). The disagreement profit of the retailer with the merged firm is

\[
d_{rm} = \max_{q \in A_{-1,2}} R(0, 0, q_{-1,2}) - \sum_{j \neq 1, 2} T_j(q_j).
\]

To characterize equilibrium quantities and payoffs, we can use the same method that we used in the previous section. In particular, let \( T_m^F(\cdot, \cdot) \) be a quantity-forcing contract with \( T_m^F(0, 0) = 0, T_m^F(q_1^2, q_2^2) = F_m \), and \( T_m^F(q_1, q_2) = \infty \) otherwise. Then, as we show in the appendix, we can characterize the equilibrium quantities and payoffs for the merged firm and retailer by solving

\[
\max_{q_1, q_2, F_m, q_{-1,2}} (F_m - C_m(q_1, q_2))^{\alpha_m} \left( R(q) - F_m - \sum_{j \neq 1, 2} T_j^B(q_j) - d_{rm} \right)^{1 - \alpha_m}
\]

such that

\[
F_m \geq C_m(q_1, q_2),
\]
\[ R(q) - F_m - \sum_{j \neq 1,2} T^B_j(q_j) \geq d_{rm}. \]  

(12)

where constraints (11) and (12) ensure that the merged firm and retailer earn at least their disagreement profits when \( q_1, q_2 > 0 \). The constraints that the retailer would rather choose \( q_1, q_2 > 0 \) than \( q_1 = 0, q_2 > 0 \), or \( q_1 > 0, q_2 = 0 \), are not binding because \( T^F_m(q_1, q_2) \) in these cases equals \( \infty \).

The first-order conditions for \( F_m \) and \( q_i \) at an interior solution of (10) are

\[ \alpha_m \pi_m (\alpha_m - 1) (\pi_r - d_{rm})^{1-\alpha_m} - (1 - \alpha_m) \alpha_m \pi_m^{\alpha_m} (\pi_r - d_{rm})^{-\alpha_m} = 0. \]  

(13)

\[ -\frac{\partial C_m(q_1, q_2)}{\partial q_i} \alpha_m \pi_m (\alpha_m - 1) (\pi_r - d_{rm})^{1-\alpha_m} + \frac{\partial R(q)}{\partial q_i} (1 - \alpha_m) \pi_m^{\alpha_m} (\pi_r - d_{rm})^{-\alpha_m} = 0. \]  

(14)

Substituting (13) into (14) and simplifying yields

\[ \frac{\partial R(q)}{\partial q_i} - \frac{\partial C_m(q_1, q_2)}{\partial q_i} = 0, \]  

(15)

which implies that \( q^B_i \) maximizes the joint profit of the merged firm and retailer given \( T^B_{1,2} \). Since this must be true for \( i = 1, 2 \), and since rival manufacturers solve the same problem as before (pre-merger), it must be that the bargaining equilibrium quantities maximize overall joint profit, i.e., \( q^B_j = q^I_j \), provided (3-5) and (10-12) have interior solutions. This proves the following proposition.

**Proposition 2** Suppose two manufacturers merge. Then, if the merged firm can bundle its products, the fully-integrated outcome is realized in all N-product bargaining equilibria.

Proposition 2 extends the well-known result in the agency literature to the case of bargaining with N upstream firms selling one or more differentiated products to a common retailer if the multi-product upstream firms can bundle their products. It implies, among other things, that a merger between upstream firms in concentrated markets need not lead to higher prices for consumers even if there are no offsetting cost efficiencies. In the absence of any effect on costs, we see from Propositions 1 and 2 that a merger between two upstream rivals will have no effect on input choices, output choices, wholesale prices, or final-goods prices. These results and the implications that

\[ \text{12 Since nothing in its derivation relies on the absence of cost savings, it follows that Proposition 2 holds also for the case in which } C_m(q_1, q_2) \neq C_1(q_1) + C_2(q_2). \text{ But in that case the post-merger fully-integrated quantities could differ from the pre-merger fully-integrated quantities, in which case the merger would affect welfare.} \]
follow from them differ substantially from the effects of mergers that arise in the classical theories of oligopoly. Perhaps the biggest difference is the diminished role of concentration indices and cross-price elasticities in our model as predictors of the competitive effects of a merger. When bundling is feasible, for example, a merger that does not affect costs is benign regardless of the concentration level in the upstream market or the degree of substitution between the merging firms’ products. In this setting, the U.S. Merger Guidelines fail to offer much in the way of guidance.

**Mergers without bundling**

Suppose bundling is not feasible. Then the merged firm and retailer must negotiate a contract that is additively separable in \( q_1 \) and \( q_2 \), i.e., \( T_m(q_1, q_2) = T_1(q_1) + T_2(q_2) \). Recall that \( T_i^F(0) = 0 \), \( T_i^F(q'_i) = F_i \), and \( T_i^F(q_i) = \infty \) otherwise. Suppose \( q^{NB} \equiv (q_1^{NB},...,q_N^{NB}) \) and \( T^{NB} \equiv (T_1^{NB},...,T_N^{NB}) \) form a bargaining equilibrium when bundling is infeasible. Then, as we show in the appendix, we can characterize the equilibrium quantities and payoffs for the merged firm and retailer by solving

\[
\max_{q_1,q_2,F_1,F_2,q_{-1,2}} (F_1 + F_2 - C_m(q_1,q_2))^{\alpha_m} \left( R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{NB}(q_j) - d_{rm} \right)^{(1-\alpha_m)}
\]

such that

\[
F_1 + F_2 \geq C_m(q_1,q_2),
\]

\[
R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{NB}(q_j) \geq d_{rm},
\]

\[
R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{NB}(q_j) \geq \max_{q_1,q_{-1,2}} R(0,q_2,q_{-1,2}) - T_2^F(q_2) - \sum_{j \neq 1,2} T_j^{NB}(q_j),
\]

\[
R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{NB}(q_j) \geq \max_{q_1,q_{-1,2}} R(q_1,0,q_{-1,2}) - T_1^F(q_1) - \sum_{j \neq 1,2} T_j^{NB}(q_j),
\]

where constraints (17) and (18) ensure that the merged firm and retailer earn at least their disagreement profits when \( q_1, q_2 > 0 \). Constraints (19) and (20) are incremental-profit constraints that ensure that the retailer earns weakly higher profit by choosing \( q_1, q_2 > 0 \) than by dropping product 1 (constraint 19) or product 2 (constraint 20). The right-hand sides of (19) and (20) are weakly larger than the right-hand side of (18). With bundling, these constraints are always satisfied because in these cases \( T_m^F(\cdot,\cdot) = \infty \). Without bundling, however, these constraints may bind.
**Lemma 2** There exists \( \alpha_m \in (0, 1) \) such that for all \( \alpha_m > \bar{\alpha}_m \) constraints (19) and (20) bind.

**Proof.** See the appendix.

Lemma 2 says that if the manufacturer’s bargaining weight is sufficiently high, (19) and (20) must bind in any \( N \)-product bargaining equilibrium. To see this intuitively, suppose the merged firm had all the bargaining power \( (\alpha_m = 1) \). If constraints (19) or (20) did not bind, the merged firm would raise one of the fixed fees to the point where the retailer earns its disagreement profit \( d_{rm} = \max_{q_{-1,2}} R(0,0,q_{-1,2}) - \sum_{j \neq 1,2} T_j^{NB}(q_j) \). Since \( d_{rm} \) is weakly smaller than the right-hand sides of (19) and (20), this contradicts the assumption that one of the constraints does not bind.

When the constraints do not bind \( (\alpha_m < \bar{\alpha}_m) \), the problem in (16-20) is equivalent to the problem in (10-12) with \( F_m = F_1 + F_2 \). In this case, the fully-integrated outcome is realized even without bundling. However, when the constraints bind \( (\alpha_m \geq \bar{\alpha}_m) \), the fully-integrated outcome will typically not be realized. In this case, the equilibrium quantities for the merged firm and retailer can be found by substituting the binding constraints into (16). For convenience, define

\[
\begin{align*}
  v_1(q_1) &= \max_{q_{-1,2}} R(q_1,0,q_{-1,2}) - F_1 - \sum_{j \neq 1,2} T_j^{NB}(q_j), \\
  v_2(q_2) &= \max_{q_{-1,2}} R(0,q_2,q_{-1,2}) - F_2 - \sum_{j \neq 1,2} T_j^{NB}(q_j).
\end{align*}
\]

The function \( v_i(q_i) \) is the profit of the retailer if it purchases product \( i \) but drops product \( j \). Substituting these definitions into constraints (19) and (20), and then substituting the constraints into the objective in (16), the merged firm and retailer’s maximization problem becomes

\[
\max_{q_1,q_2} \left( R(q) - \frac{v_1(q_1) + v_2(q_2)}{2} - \sum_{j \neq 1,2} T_j^{NB}(q_j) - C_m(q_1,q_2) \right) \left( \frac{v_1(q_1) + v_2(q_2)}{2} - d_{rm} \right)^{(1-\alpha_m)}. \tag{23}
\]

To gain insight into the solution, assume that \( T_j^{NB}(q_j) \) is continuously differentiable (i.e., \( v_1(q_1) \) and \( v_2(q_2) \) are differentiable). After some algebra, the first-order condition for \( q_1 \) can be written as

\[
\frac{\partial R(q)}{\partial q_1} - \frac{\partial C_m(q_1,q_2)}{\partial q_1} = \frac{1}{2} \left[ 1 - \frac{(1-\alpha_m)\bar{\pi}_m}{\alpha_m(\pi_r - d_{rm})} \right] \frac{\partial v_1(q_1)}{\partial q_1}. \tag{24}
\]

A symmetric condition holds for \( q_2 \). The term on the left-hand side of (24) is the derivative of overall joint profit with respect to \( q_1 \). Since profits are concave and single-peaked, this derivative equals
zero at the fully-integrated outcome, exceeds zero if $q_1^{NB} < q_1^I$, and is less than zero if $q_1^{NB} > q_1^I$. The term in square brackets on the right-hand side of (24) is positive. This is true because when the constraints (19) and (20) bind, the merged firm receives less than its “fair share” of the profits from Nash bargaining, i.e., $(1 - \alpha_m)\pi_m < \alpha_m(\pi_r - d_r)$. Therefore, the sign of $\frac{\partial R(q)}{\partial q_1} - \frac{\partial C_m(q_1, q_2)}{\partial q_1}$ is the same as the sign of $\frac{\partial v_1(q_1)}{\partial q_1}$. Using the envelope theorem, the derivative of $v_1(q_1)$ is

$$\frac{\partial v_1(q_1)}{\partial q_1} = \frac{\partial R(q_1, 0, \tilde{q}_{-1,2}(q_1))}{\partial q_1} > 0.$$  (25)

where $\tilde{q}_{-1,2}(q_1) \equiv \arg\max_{q_{-1,2}} R(q_1, 0, q_{-1,2}) - \sum_{j \neq 1,2} T^{NB}(q_j)$. Therefore, at the bargaining equilibrium, $\frac{\partial R(q)}{\partial q_1} - \frac{\partial C_m(q_1, q_2)}{\partial q_1} > 0$, which implies that $q_1^{NB} < q_1^I$. An analogous argument implies that $q_2^{NB} < q_2^I$. This proves Proposition 3 for the case in which $T^{NB}(q_j)$ is continuously differentiable.

**Proposition 3** Suppose two manufacturers merge. If the merged firm cannot bundle its products, then whether the fully-integrated outcome is realized depends on $\alpha_m$: (i) if $\alpha_m < \bar{\alpha}_m$, the fully-integrated outcome is obtained in all N-product bargaining equilibria; (ii) if $\alpha_m > \bar{\alpha}_m$, the merged firm’s quantities are distorted downward and the fully-integrated outcome is not obtained.

**Proof.** See the RJE Supplementary Material section at http://www.rje.org/main/sup-mat.html for the case in which $T^{NB}(q_j)$ is not continuously differentiable.

Proposition 3 contains the main result of the paper. It says that if the merged firm’s bargaining weight *vis-a-vis* the retailer is sufficiently low, then the constraints (19) and (20) do not bind, and the incentives of the two firms are to maximize bilateral joint profit. However, if the merged firm has a lot of bargaining power ($\alpha_m > \bar{\alpha}_m$), then maximizing bilateral joint profit is not optimal because the negotiated $F_1$ and $F_2$ will be constrained by the ability of the retailer to drop one or both of the products. For example, if the merged firm attempts to extract ‘too much’ surplus by raising $F_1$, then the retailer can drop product 1 (constraint (19) is violated), and similarly, it can drop product 2 if the merged firm attempts to extract ‘too much’ surplus by raising $F_2$ (constraint (20) is violated). To relax these constraints, it is optimal for the merged firm to induce an upward distortion in its input prices (decrease its quantities). This allows $F_1$ and $F_2$ to rise, increasing
the Nash product. The intuition for why a reduction in quantities relaxes the incremental-profit constraints is clearest when the demands for products 1 and 2 are symmetric. In this case, a small reduction $\epsilon$ in $q_1$ and $q_2$ below the efficient quantity has no first-order effect on the bilateral joint profit of the merged firm and the retailer, but it relaxes both incremental-profit constraints because products 1 and 2 are (imperfect) substitutes.\(^{13}\) The decrease in the retailer’s profit from the reduction in $q_1$ ($q_2$) when it does not carry product 2 (product 1) more than offsets the decrease in the retailer’s profit from an equal reduction in $q_1$ and $q_2$ when both products are carried.

This result contrasts with the common intuition that overall joint profits tend to be maximized in situations of common agency and complete information. We have shown that this intuition does not extend to a negotiations setting in which the upstream firm has sufficiently high bargaining power and bundling is not possible. In that case, the merged firm (or any multi-product upstream firm) will find it optimal to knowingly reduce the overall joint profit pie because in doing so it can capture a larger share for itself. With a larger share of a smaller pie, the upstream firm can gain.

An analogy is useful for understanding the difference between the bundling and no-bundling regimes in our model. Consider the pre-merger situation, where each upstream firm produces multiple units of its own, homogeneous, product. Note that the ability to write nonlinear contracts effectively allows each firm to “bundle” these units. For example, each firm can use a forcing contract to effectively bundle the efficient number of units of its product for sale to the retailer, leading to the fully-integrated outcome. If a firm cannot bundle its units with a nonlinear contract, but instead must charge a separate price for each, then its best strategy is to charge the same price for each unit.\(^{14}\) This results in a linear price and causes a double-marginalization distortion.

In our model, we allow firms to bundle homogeneous units using nonlinear contracts for each product. This prevents the usual double-marginalization distortion. However, in our no-bundling regime, we assume the merged firm cannot bundle by combining units of differentiated products. This leads to a distortion somewhat analogous to double marginalization. Although products 1 and

\(^{13}\)More generally, one can show that a small reduction in $q_1$ and $q_2$ in some direction in $(q_1, q_2)$-space will relax both incremental-profit constraints.

\(^{14}\)Formally, the retailer’s incremental-profit constraints in each firm’s bargaining problem require that all units have the same price—the manufacturer must use a linear price. We thank an anonymous referee for pointing this out.
2 are not perfect substitutes, the retailer will still have a credible threat to drop one of them if the
merged firm’s bargaining power is high enough to cause the retailer’s incremental-profit constraints
to bind. The merged firm then distorts its input prices in an effort to extract more surplus.

Welfare effects

Our results have strong implications for the output and welfare effects\(^\text{15}\) of mergers (subject to
the caveats discussed in Section 5). If bundling is feasible, a merger will not affect output unless
it alters marginal costs. It is easy to see that cost changes affect the merged firm’s outputs in
the normal way: if marginal costs decrease, the merged firms’ outputs will rise, and if marginal
costs increase, the merged firms’ outputs will fall. Post-merger welfare will typically be higher with
cost savings, lower with cost increases, and unchanged otherwise. If bundling is infeasible (or is
prohibited) and the merged firm’s bargaining power is sufficiently low \((\alpha_m < \overline{\alpha}_m)\), then the results
are the same as when bundling is feasible. But if the merged firm’s bargaining power is sufficiently
high, the merged firm will no longer have an incentive to negotiate an efficient contract. Absent cost
savings, the merger will reduce the merged firms’ outputs, and welfare will typically fall. Because
the products are substitutes, rival firms would respond by increasing their quantities, but typically
not by enough to offset the negative welfare effect of the reduction in the merged firms’ quantities.

Our results also have implications for policy toward bundled discounts. If the bargaining power
of the merged firm is high enough, prohibiting bundling leads to higher marginal transfer prices for
the merged firm’s products.\(^\text{16}\) Any attempt by authorities to prevent a multi-product firm from
increasing its “clout” through bundling may therefore result in higher prices for final consumers.

This finding suggests that antitrust concerns with bundling by dominant, multi-product firms may
be misguided unless there is reason to believe that bundling has foreclosed, or is likely to foreclose
rivals. In our model bundling arises not to foreclose rivals but to extract profit from the retailer.

\(^{15}\) Unqualified welfare conclusions are not possible at this level of generality because of the usual tradeoffs involving price and product variety. Thus, our welfare discussion proceeds under the assumption that in the “normal case” a reduction in the marginal transfer price of one or both of the merged firm’s products leads to an increase in welfare.

\(^{16}\) If the prohibition of bundling extends to the bundling of homogeneous units (i.e., nonlinear pricing is not allowed), in addition to the bundling of imperfect substitutes, then the distortion would likely be even worse.
4 Profit effects

Expressions for equilibrium profits can be derived for each case by solving the restricted (quantity-forcing) negotiations of each firm for its optimal fixed fee and then substituting back into the expressions for profits. The resulting equilibrium profit expressions for the pre-merger case are

\[ \pi_i^* = \alpha_i \left( R(q_i^1) - C_i(q_i^1) - \sum_{j \neq i} T_j^*(q_j^1) - d_{ri}^* \right), \quad i = 1, ..., N \]  

\[ \pi_r^* = R(q^1) - \sum_i C_i(q_i^1) - \sum_i \pi_i^* \]  

\[ d_{ri}^* = \max_{q_{-i}} R(0, q_{-i}) - \sum_{j \neq i} T_j^*(q_j) \]

where \( \pi_i^* \) is manufacturer \( i \)'s equilibrium profit, \( \pi_r^* \) is the retailer's equilibrium profit, and \( d_{ri}^* \) is the retailer's disagreement profit with manufacturer \( i \) under the equilibrium contracts \( T_j^*(\cdot), \forall j \neq i \).

Notice that manufacturer \( i \)'s profits are expressed in terms of rival firms' equilibrium contracts and not just in terms of the revenue and cost primitives. This is because the contracts are not uniquely determined in equilibrium. The reason for this is that there are many different contracts for product \( j \) that induce the retailer to select a given quantity \( q_j^* \), and the disagreement profit of the retailer in negotiations with firm \( i \) depend on the contract with firm \( j \) at quantities other than \( q_j^* \). Thus, firm \( i \)'s equilibrium profits depend on the type of equilibrium contracts employed by rival firms \( j \neq i \). The non-uniqueness of profits means that it is not possible to compare pre-merger and post-merger profits at this level of generality. Further restrictions are needed to make this comparison. In the remainder of this section we restrict attention to two-part tariff contracts, and we assume that the manufacturers have constant marginal costs, i.e., \( C_i''(q_i) = 0, \quad i = 1, ..., N \).

Before proceeding we need some more notation. Let \( w_i^I = C_i'(q_i), \quad i = 1, ..., N \), be the constant per-unit prices (wholesale prices) that yield the vertically-integrated outcome. Define

\[ \Pi \equiv R(q^1) - \sum_i w_i^I q_i^I \]  

\[ \Pi_{-i} \equiv \max_{q_{-i}} R(0, q_{-i}) - \sum_{j \neq i} w_j^I q_j \]  

\[ \Pi_{-1,2} \equiv \max_{q_{-1,2}} R(0, q_{-1,2}) - \sum_{k \neq 1,2} w_k q_k \]

17
These are the retailer’s maximized profits (net of fixed fees) if it sells all $N$ products, all but product $i$, and all but products 1 and 2, respectively, when its marginal costs are given by $w_i, i = 1, ..., N$.

**Profit effects when the merging firms’ cost structure does not change**

Using (26) and the definitions (29)-(31), manufacturer $i$’s pre-merger profit under two-part tariffs is

$$ \pi_i^* = \alpha_i \left( R(q_i^1) - C_i(q_i^1) - \sum_{j \neq i} w_j q_j^1 - \Pi_{-i} \right). $$

(32)

The profit of the merged firm when it bundles its products can be found by solving the first-order conditions for (10) and then substituting them into the expression for profit. This gives

$$ \pi_m^B = \alpha_m \left( R(q_1^1, q_2^1) - C_m(q_1^1, q_2^1) - \sum_{j \neq 1, 2} w_j q_j^1 - \Pi_{-1,2} \right). $$

(33)

Suppose the merger does not lead to greater bargaining power ($\alpha_i = \alpha_m$) or affect production costs ($C_m(q_1, q_2) = C_1(q_1) + C_2(q_2)$). Then the benefit to manufacturers 1 and 2 from merging is

$$ \Delta \pi_m^B = \pi_m^B - \pi_1^* - \pi_2^* = \alpha_m \left( [\Pi - \Pi_{-1,2}] - [\Pi - \Pi_{-1}] - [\Pi - \Pi_{-2}] \right). $$

(34)

The term $\Pi - \Pi_{-1,2}$ is the cost to the retailer (net of fixed fees) of failing to reach an agreement with the merged firm. The term $\Pi - \Pi_{-i}, i \in \{1, 2\}$, is the cost to the retailer of failing to reach an agreement with manufacturer $i$ prior to the merger. Equation (34) indicates that the merger will be profitable if the expression in parenthesis is positive, i.e., if the retailer’s cost of failing to reach an agreement with the merged firm is greater than the sum of the costs of failing to reach agreement with of each of the merging firms prior to the merger. This is intuitive. A manufacturer’s bargaining strength comes in part from its ability to inflict a loss on the retailer by refusing an agreement. If the loss imposed by the merged firm exceeds the sum of the losses imposed by the merging firms prior to the merger, then the merged firm will extract greater profit from the retailer. In general, the concavity of joint profits ensures that this will be the case. Since the products are substitutes, the loss imposed by the merged firm will indeed exceed the sum of the losses imposed
by the merging firms prior to the merger (see the proof of Proposition 4 below). Thus, we have
that $\Delta \pi_m^B > 0$, implying that mergers are profitable for the merging firms when bundling is feasible.

If bundling is infeasible and the merged firm’s bargaining weight is less than $\alpha_m$, then the
constraints (19) and (20) do not bind and the maximization problem in (16) is the same as the
maximization problem in (10) with $F_m = F_1 + F_2$. In this case, the merger is profitable for the
merging firms and the profit of the merged firm is the same with or without bundling. However, if
$\alpha_m > \alpha_m$, then at the integrated quantities the merged firm is constrained from capturing its share
of the incremental profits from its products. That is, an unconstrained Nash bargaining solution
would require $(1 - \alpha_m)\pi_m = \alpha_m(\pi_r - d_r)$, but constraints (19) and (20) force $(1 - \alpha_m)\pi_m < \alpha_m(\pi_r - d_r)$. This establishes an upper bound on $\pi_m$. Since the wholesale price of each non-
merging firm is unchanged in all equilibria whether or not bundling is feasible, it follows that the
merged firm is worse off when $\alpha_m > \alpha_m$ and bundling is infeasible than when bundling is feasible.

To determine whether the merger itself is profitable when bundling is infeasible and $\alpha_m > \alpha_m$, let $\pi_m^{NB}$ denote the profit of the merged firm in this case. Then, using the fact that the constraints
(19) and (20) will bind in any bargaining equilibrium, and that when $\alpha_m > \alpha_m$ the merged firm
prefers to introduce a distortion by inducing the retailer to choose the vector of quantities $q_{\text{I}}$ in
equilibrium rather than $q_{\text{NB}}$, it can be shown that (see the proof of Proposition 4 below)

$$\pi_m^{NB} > \sum_{i=1,2} \left( R(q_{\text{I}}) - \sum_{j \neq i} w_{ij} q_j - \Pi_{-i} \right) - C_m(q_{\text{I}}, q_{\text{I}}).$$

(35)

Assuming, as before, that the merger does not affect relative bargaining weights or the merged
firm’s costs ($C_m(q_1, q_2) = C_1(q_1) + C_2(q_2)$), the benefit to manufacturers 1 and 2 from merging is

$$\Delta \pi_m^{NB} = \pi_m^{NB} - \pi_1^{*} - \pi_2^{*}$$

$$> \sum_{i=1,2} (1 - \alpha_i) \left( R(q_{\text{I}}) - C_1(q_{\text{I}}) - \sum_{j \neq i} w_{ij} q_j - \Pi_{-i} \right),$$

(36)

which is positive if pre-merger profits are positive and $\alpha_i < 1$. Intuitively, there are two reasons
why the merger is profitable even when bundling is infeasible and $\alpha_m > \alpha_m$. First, the merged

\[\text{Inderst and Wey (2003;14) also obtain this result, in a different context, using the Shapley value as their solution concept to multilateral bargaining. As they note, “Broadly speaking, a merger shifts bargaining away from the margin. If the created net surplus is smaller at the margin ... the respective market side prefers to become integrated.”} \]
firm’s fixed fees rise until (19) and (20) bind, whereas they do not bind prior to the merger unless $\alpha_i = 1$. Second, the merged firm earns additional profit by reducing its output of each product (raising its wholesale price) in order to capture more profit from selling the other product.

We summarize these results for the bundling and no-bundling cases in the following proposition.

Proposition 4 A merger between two manufacturers is profitable whether or not bundling is feasible, even if there are no cost efficiencies from the merger and no increase in their collective bargaining weight. If $\alpha_m < \pi_m$, then the merged firm’s profit is the same with and without bundling. If $\alpha_m > \pi_m$, then the merged firm’s profit is higher with bundling than without bundling.

Proof. See the appendix.

The result that mergers are profitable in our model even if there are no cost efficiencies contrasts with results in the standard models of horizontal mergers in final-goods markets where the profitability of a merger turns on whether the firms’ strategies are strategic substitutes or strategic complements. In the latter case, we know from Deneckere and Davidson (1985) and others that mergers of any size are profitable because, in addition to the usual gains from coordination, they induce less aggressive pricing by the non-merging firms.\(^{18}\) In the former case, however, we know from Salant et al. (1983) and others that, in the absence of cost efficiencies, mergers may not be profitable because they induce rival firms to respond by increasing their outputs. In our model, mergers are profitable even without cost efficiencies because (a) they allow the merging firms to impose losses on the retailer by jointly withholding their products, (b) when bundling is infeasible, they allow the merging firms to earn additional profit by reducing their output of each product in order to capture more profit from selling the other product, and (c) the non-merging firms’ wholesale prices weakly increase post merger (that is, the rival firms do not respond aggressively).

Effects on the non-merging firms’ profits. The nature of the bargaining problem between the non-merging firms and the retailer does not change after the merger. A non-merging firm’s profit in

\(^{18}\)However, it should be noted that in the Deneckere and Davidson (1985) setup, each firm would prefer to remain an outsider and allow its rivals to merge. Thus, as pointed out by a referee, most current models of mergers make it difficult to explain why mergers ever occur (unless they significantly alter the merging firms’ cost structure).
any regime has the same structure as (32) but is evaluated at the wholesale prices and quantities corresponding to the particular regime. In all the efficient regimes—pre-merger, post-merger with bundling, and post-merger with no bundling and $\alpha_m < \bar{\alpha}_m$—the wholesale prices and quantities are the same. Thus, we have that a non-merging firm’s profits are the same across these regimes.

In the post-merger regime with no bundling and $\alpha_m > \bar{\alpha}_m$, the merged firm reduces its outputs by raising its wholesale prices above the fully-integrated prices. To see the effect of this on rival firms, let $q^e(w_1, w_2) \equiv (q^e_1(w_1, w_2), ..., q^e_N(w_1, w_2))$ denote the vector of bargaining equilibrium quantities, where $q^e_j(w_1, w_2)$ is the bargaining equilibrium quantity of product $j$ when the merged firm has wholesale prices $w_1$ and $w_2$. Then we can write a non-merging firm $i$’s profit as

$$\pi_i = \alpha_i(R(q^e_i(w_1, w_2)) - C_i(q^e_i(w_1, w_2))$$

$$- w_1 q^e_1(w_1, w_2) - w_2 q^e_2(w_1, w_2) - \sum_{j \neq 1,2,i} w_j q^e_j(w_1, w_2)$$

$$- \max_{q_{-i}}[R(0, q_{-i}) - w_1 q_1 - w_2 q_2 - \sum_{j \neq 1,2,i} w_j q_j],$$

(37)

where we have substituted $w_j^f$ for $w_j$, $j \neq 1,2,i$, because the wholesale prices of each non-merging firm equals marginal cost in any bargaining equilibrium. Let $\tilde{q}_j(w_1, w_2)$, for all $j \neq i$, solve the maximization problem in the third line of (37). Then, using the envelope theorem, we have

$$\frac{\partial \pi_i}{\partial w_1} = \alpha_i(\tilde{q}_1(w_1, w_2) - q^e_1(w_1, w_2)).$$

(38)

Since $\tilde{q}_j(w_1, w_2) - q^e_j(w_1, w_2) > 0$ for all $j$ when the products are substitutes, it follows that a non-merging firm $i$’s profit will be increasing in $w_1$. A symmetric argument implies that a non-merging firm $i$’s profit will be increasing in $w_2$. Since $w_1$ and $w_2$ both rise after the merger if $\alpha_m > \bar{\alpha}_m$ and bundling is infeasible, the non-merging firms will benefit from the merger in this case.

We summarize these results for the bundling and no-bundling cases in the following proposition.

**Proposition 5** A merger between two manufacturers that does not affect the merging firms’ costs has no effect on the non-merging firms’ profits if bundling is feasible or if the merged firm’s bargaining power is sufficiently low ($\alpha_m < \bar{\alpha}_m$). However, the same merger raises the non-merging firms’ profits if bundling is infeasible and the merged firm’s bargaining power is high ($\alpha_m > \bar{\alpha}_m$).
Proposition 5 implies that the non-merging firms will oppose bundling by a merged firm when the merged firm has a lot of ‘clout’ *vis-a-vis* the retailer. However, we know from Proposition 3 that prohibiting bundling in this case leads to higher wholesale prices (lower outputs) and thus such opposition does not necessarily coincide with the interests of social welfare. When bundling is allowed, Proposition 5 implies that the non-merging firms will be indifferent to the merger. When bundling is prohibited, our results imply that the non-merging firms will be in favor of the merger.

**Effects on the retailer’s profit.** When the merger does not affect costs, we know from Proposition 4 that the merging firms always benefit from the merger, and we know from Proposition 5 that the non-merging firms either benefit or are not affected. Since overall joint profits are maximized pre-merger, it follows that the retailer is harmed by the merger if there are no cost efficiencies.

We can also rank the retailer’s preferences over bundling versus no bundling in the post-merger regime. The two cases can be nested by writing (19) and (20) for the no-bundling regime as

\[
\pi_r \geq \max_{q_2, q_{-2}} R(0, q_2, q_{-2}) - T_2^F(q_1) - \sum_{j \neq 1, 2} T_j^{NB}(q_j) - b, \tag{39}
\]

\[
\pi_r \geq \max_{q_1, q_{-1, 2}} R(q_1, 0, q_{-1, 2}) - T_1^F(q_1) - \sum_{j \neq 1, 2} T_j^{NB}(q_j) - b. \tag{40}
\]

In constraints (39) and (40), \(b\) is a parameter that represents the tightness of the constraints. The no-bundling case occurs when \(b = 0\). The bundling case occurs when \(b\) is large enough that the constraints (39) and (40) do not bind. The effects of prohibiting bundling on the retailer can be found by evaluating the derivative of \(\pi_r\) with respect to \(b\) starting from the value of \(b\) at which the constraints just begin to bind.\(^{19}\) As we show in the appendix, we find that the derivative is negative at this point, implying that the retailer is typically worse off under bundling than it is with no bundling. Mathematically, tightening the no-bundling constraint (decreasing \(b\)) has a first-order positive effect on the retailer’s profits, as shown in (39) and (40). It also has a second-order effect that comes through equilibrium adjustments in wholesale prices and quantities as the bundling constraint is tightened. However, the second-order effects are outweighed by the first-order effects.

We summarize these results for the bundling and no-bundling cases in the following proposition.

\(^{19}\)Note that either both constraints bind or neither constraint binds. If only one constraint was binding, the merged firm could adjust \(F_1\) and \(F_2\) to raise \(F_1 + F_2\) and at the same time relax the binding constraint.
**Proposition 6** A merger between two manufacturers that does not affect the merging firms’ costs reduces the retailer’s profit. The retailer’s profit is the same whether or not bundling is feasible if \( \alpha_m < \overline{\alpha}_m \). The retailer’s profit is lower with bundling than without bundling if \( \alpha_m > \overline{\alpha}_m \).

**Proof.** See the appendix.

As in the previous case with the non-merging firms, the retailer will oppose bundling by the merged firm when the latter has a lot of bargaining ‘clout’. In particular, the retailer will oppose bundling when \( \alpha_m > \overline{\alpha}_m \). However, as we have seen, such opposition by interested third-parties (in this case the buyer) does not necessarily coincide with the interests of social welfare. Opposition may also arise with respect to the merger itself. Whether or not bundling is feasible, Proposition 6 implies that we should expect the retailer to oppose any merger that does not generate significant cost efficiencies. But this opposition is driven solely by the retailer’s desire to preserve its share of the profit; a merger between upstream rivals need not have any social welfare consequences.

**Profit effects when the merging firms’ cost structure changes**

To investigate the effects of cost changes that arise when manufacturers 1 and 2 merge, we rewrite the merged firm’s costs as \( C_m(q_1, q_2; \theta) \), where \( \theta \) is a cost-shift parameter on the merged firm’s products. We assume \( \frac{\partial C_m(q_1, q_2; \theta)}{\partial \theta} > 0, \frac{\partial^2 C_m(q_1, q_2; \theta)}{\partial q_1 \partial \theta} \geq 0, \) and \( \frac{\partial^2 C_m(q_1, q_2; \theta)}{\partial q_i \partial \theta} \geq 0 \) for all \( i \). Thus, a decrease in \( \theta \) decreases the merged firm’s total cost of producing \( q_1 \) units of product 1 and \( q_2 \) units of product 2 and, for some types of cost shifts, may also decrease its marginal cost of producing product \( i \).

In the regimes where joint profits are maximized, the merged firm’s profit can be written as

\[
\pi_m = \alpha_m \left[ R(q_1^I) - C_m(q_1^I, q_2^I; \theta) - \sum_{j \neq 1, 2} w_j^I q_j^I - \max_{q_{-1,2}} [R(0, 0, q_{-1,2}) - \sum_{j \neq 1, 2} w_j^I q_j] \right]
\]  

(41)

Differentiating the expression in (41) with respect to \( \theta \) and using the envelope theorem gives

\[
\frac{\partial \pi_m}{\partial \theta} = -\alpha_m \frac{\partial C_m(q_1^I, q_2^I; \theta)}{\partial \theta}
\]

(42)

Since \( \frac{\partial C_m(q_1, q_2; \theta)}{\partial \theta} > 0 \), condition (42) implies that \( \frac{\partial \pi_m}{\partial \theta} < 0 \), which is what one would expect when the merged firm’s costs change. The merged firm gets an additional boost to profit if the merger
generates cost efficiencies, but suffers a decrease in its profit if the merger increases costs. The effect on the retailer is the same as the effect on the manufacturer. Given Nash bargaining between the merged firm and retailer, the retailer shares in any cost savings or suffers from any cost increases.

The effects of changes in the merged firm’s costs of producing products 1 and 2 on the non-merging firms’ profits depend on whether the cost changes alter fixed or marginal costs. Marginal-cost savings (increases) have the same effect on the non-merging firms’ profits as a reduction (increase) in $w_1$, as expressed in (38), and therefore reduce (increase) the non-merging firms’ profits. Fixed cost savings (increases), on the other hand, do not affect the non-merging firms’ profits.

It can be shown that cost changes in the no-bundling regime with $\alpha_m > \bar{\alpha}_m$ have the same qualitative effects. The retailer benefits from fixed and marginal-cost savings; non-merging firms are harmed when the savings reduce the merged firm’s marginal costs and are not affected otherwise. The retailer is harmed by fixed and marginal-cost increases; non-merging firms benefit when the merged firm’s marginal costs increase and are unaffected by increases in the firm’s fixed costs.

We summarize these results for fixed and marginal-cost changes in the following proposition.

**Proposition 7** The merged firm and retailer gain (lose) from any cost savings (increases) that are generated by the merger. The non-merging firms lose (gain) if the merger decreases (increases) the merged firm’s marginal costs. They are unaffected by changes in the merged firm’s fixed costs.

The results in Proposition 7 can be combined with those in Propositions 5 and 6 to give a complete picture of the incentives of the non-merging firms and retailer. If bundling is feasible, or infeasible with $\alpha_m < \bar{\alpha}_m$, we would expect the non-merging firms to oppose the merger if and only the merger would generate marginal-cost efficiencies. But if bundling is infeasible and $\alpha_m > \bar{\alpha}_m$, we would expect the non-merging firms to oppose the merger only if the marginal-cost efficiencies are sufficiently large. In contrast, we would expect the retailer to oppose the merger unless the expected cost savings (fixed or marginal) are sufficiently large. This means that the preferences of the non-merging firms and retailer on when to oppose the merger will sometimes coincide and sometimes conflict.

24
These results have implications regarding the inferences that can be drawn from third-party testimony in mergers between upstream firms. In our setting, contrary to conventional wisdom, retailer (customer) complaints do not necessarily indicate that a merger is likely to harm welfare. Because the merger transfers surplus from the retailer to the merged firm, the retailer can be worse off even if the merger is efficient. On the other hand, the non-merging firms’ preferences are a good indicator in our model of the effects of the merger on final-goods prices. They will oppose mergers that reduce final prices and support mergers that raise prices. Of course, these observations ignore any strategic incentives third parties may have in attempting to influence merger authorities.

5 Conclusion

A phrase often used by business people in explaining the reasons for a merger is that it will increase the merging firms’ ‘clout’ in negotiations with buyers or suppliers. As such, antitrust investigations of upstream mergers in intermediate-goods markets often focus on the effects of the merger on the combined entity’s bargaining strength vis-a-vis the retailer, and whether the retailer will be harmed as a result. In this paper, additional clout may come from three sources: increased bargaining power, as measured by the merged firm’s bargaining weight in its asymmetric Nash bargaining solution; the merged firm’s ability to negotiate contracts on products jointly rather than separately; and the merged firm’s ability to bundle products in its product line via interdependent price schedules, for example, by offering discounts and rebates that are applied ‘across-the-board.’

We find that merging firms may benefit from an increase in clout even if it does not affect their marginal transfer prices. Absent cost efficiencies, for the reasons noted above, mergers with bundling increase the merging firms’ profits, decrease the retailer’s profit, and leave rivals’ profits unchanged. The profit effects are purely rent transfers; there is no effect on consumer or total welfare. In contrast, with cost efficiencies, mergers with bundling increase the merging firms’ profits and decrease rival firms’ profits. If the cost savings are small, the retailer’s share of the cost

savings will be less than the rent transfer to the merged firm and the retailer’s profit will decrease. Otherwise, if the cost savings are large enough, the retailer will gain. Thus, it is possible for the merger to increase welfare while harming both rival firms and the retailer, or just rival firms. Basing policy on the retailer’s perceived benefit or harm is uninspired and uninformative in this case.

When bundling is not feasible, the reason for the increase in the merging firms’ profits depends on whether the merged firm’s bargaining power is high or low. If it is low, the increase in the merging firms’ profits can be attributed to the benefits of negotiating terms on products jointly rather than separately. Because the products are substitutes, the loss imposed on the retailer by the merged firm if it withholds its products exceeds the sum of the losses that can be imposed by each merging firm prior to the merger. If the merged firm’s bargaining power is high, the increase in the merging firms’ profits follows from its ability to coordinate its pricing decisions across its product line. By cutting back on its output of each product (relative to the pre-merger case), the merged firm is able to limit the loss the retailer can impose on it by dropping one of its products.

When bundling is feasible, marginal transfer prices are unaffected by the merger in the absence of any cost efficiencies. When bundling is infeasible, and the merged firm’s bargaining power is sufficiently high, transfer prices will rise, all else equal. In this case, the benefits of merging are smaller than they are when bundling is allowed. Rival firms benefit if the merging firms’ transfer prices increase, but the retailer is always worse off than with no merger, and worse off with bundling than without it after the merger if the merged firm’s bargaining power is sufficiently high.

We view this paper as a first step toward understanding the effects of mergers in intermediate-goods markets when contracts are negotiated. The model is too simple at this point to yield robust policy conclusions. However, it is rich enough to show that the effects of mergers in this environment can be substantially different than the effects predicted by the classical oligopoly models.

An important simplification in this paper is the restriction to a single downstream firm. Under this assumption, equilibrium contracts are efficient (in the sense of replicating the fully-integrated outcome) before and after the merger except when bundling is prohibited and the merging firm’s bargaining power is sufficiently high. This result has strong implications for the effects of mergers. If
bundling is allowed, so that contracts are efficient before and after the merger, the merger increases the merged firm’s output if and only if it reduces marginal costs. This result is independent of the level of concentration in the upstream market and the degree of substitution among products. The merger also increases the merged firms’ clout in negotiations with the retailer by increasing the loss the merging firms can impose by refusing to sell. An implication is that a merger with small cost savings enhances welfare even though it reduces the profits of rival firms and the retailer.

An obvious next step would be to extend the model to an environment with downstream oligopoly. Once there is downstream competition, the rents to be split by a manufacturer and retailer will depend inter-alia on the amount of competition the retailer faces from rival retailers who sell the same product. In this case, contracts generally will not lead to the vertically-integrated outcome. An additional complication is that the nature of the equilibrium will depend on whether downstream firms can observe each others’ contracts. If contracts are not observable, it can be shown that per-unit transfer prices will still equal marginal cost in a bargaining equilibrium. In this case, many of the results in this paper carry through. However, the implication that per-unit transfer prices equal marginal cost does not appear to be consistent with pricing in many intermediate-goods markets in which non-linear contracts are negotiated. If contracts are observable, then firms have incentives to negotiate contracts that dampen competition so as to increase the size of the total surplus to be split.\(^{21}\) This generally leads to per-unit transfer prices that exceed production marginal costs. The analysis of mergers in these environments awaits further work.

Finally, we have abstracted from two additional factors that could affect the policy implications of our model: non-contractible investments by upstream or downstream firms, and upstream contracting externalities. The desire to give manufacturers incentives to make ongoing, non-contractible investments in marketing or quality is another reason for contracts to specify positive wholesale margins, as one often observes in practice. A change in market structure due to merger may change the margins that arise in equilibrium to promote such investments. Upstream contracting externalities can arise if input supplies are scarce, in which case an upstream firm’s

\(^{21}\)One factor that tends to make supply contracts more observable is the Robinson-Patman Act, which constrains the ability of manufacturers to discriminate in prices. See O’Brien and Shaffer (1994).
decisions could affect its rivals’ costs by affecting input prices. Or, if there are scale economies, upstream decisions may affect rivals’ costs and entry decisions and thereby affect competition in the future or in other markets. A merger will internalize such externalities and may affect marginal prices.
Appendix

This appendix characterizes equilibrium quantities and payoffs with and without bundling, proves Lemma 2, and proves Propositions 4 and 6.

Characterization of equilibrium quantities and payoffs with bundling

To characterize equilibrium quantities and payoffs, we solve an equivalent problem to the one in (9). In the equivalent problem, the merged firm and retailer choose a quantity-forcing contract

\[ T^F_m(q_1, q_2) = \begin{cases} 
0 & \text{if } q_1 = q_2 = 0 \\
F_m & \text{if } q_1 = q'_1 \text{ and } q_2 = q'_2 \\
\infty & \text{otherwise} 
\end{cases}, \]

and quantities \( q_1 \) and \( q_2 \), from the feasible set of quantity-contract combinations

\[ A^F_m(T_{-1,2}) \equiv \{(q_1, q_2, F_m, q'_1, q'_2) \mid q \in \arg \max_q R(q) - T^F_m(q_1, q_2) - \sum_{j \neq 1,2} T_j(q_j), F_m \geq C_m(q'_1, q'_2)\}. \]

With this restriction to quantity-forcing contracts, the maximization problem in (9) becomes

\[
\max_{(q_1, q_2, F_m, q'_1, q'_2) \in A^F_m(T_{-1,2})} \left( T^F_m(q_1, q_2) - C_m(q_1, q_2) \right)^{\alpha_m} \left( R(q) - T^F_m(q_1, q_2) - \sum_{j \neq 1,2} T_j(q_j) - d_{rm} \right)^{1-\alpha_m}.
\]

(A1)

We will henceforth assume that the Nash product in (A1) has a unique solution. Then, since \( A^F_m(T_{-1,2}) \subset A_m(T_{-1,2}) \), and the choices \( q'_1 = q'_2 = q_B^* \), and \( F_m = T^B_m(q'_1, q'_2) \) are feasible when the merged firm and retailer choose a quantity-forcing contract, it follows that the solution to (A1) yields the same quantities and payoffs for the merged firm and retailer as the solution to the problem in (9), conditional on contracts \( T^B_{-1,2} \). This means that we can characterize the equilibrium quantities and payoffs for the merged firm and retailer by solving the restricted problem:

\[
\max_{(q_1, q_2, F_m, q'_1, q'_2) \in A^F_m(T_{-1,2})} (F_m - C_m(q_1, q_2))^{\alpha_m} \left( R(q) - F_m - \sum_{j \neq 1,2} T^B_j(q_j) - d_{rm} \right)^{1-\alpha_m} = \max_{q_1, q_2, F_m, q_{-1,2}} (F_m - C_m(q_1, q_2))^{\alpha_m} \left( R(q) - F_m - \sum_{j \neq 1,2} T^B_j(q_j) - d_{rm} \right)^{1-\alpha_m} \]

(A2)

such that

\[ F_m \geq C_m(q_1, q_2), \]
\[ R(q) - F_m - \sum_{j \neq 1,2} T_j^B(q_j) \geq d_r, \]

which correspond to (10)–(12), respectively. The rest follows from the discussion in the text.

**Characterization of equilibrium quantities and payoffs without bundling**

When bundling is not feasible, the merged firm and retailer must negotiate a contract that is additively separable in \( q_1 \) and \( q_2 \): \( T_m(q_1, q_2) = T_1(q_1) + T_2(q_2) \). In this case, we define the feasible set of quantity-contract combinations available to the merged firm and retailer as

\[ \hat{A}_m(T_{-1,2}) \equiv \{ (q_1, q_2, T_1(\cdot), T_2(\cdot)) \mid q \in \Omega(T), T_1(0) + T_2(0) = 0, T_1(q_1) + T_2(q_2) \geq C_m(q_1, q_2) \}. \]

The feasible set of quantity-contract combinations available to rival firm \( j \) is still \( A_j(T_{-j}) \).

Suppose \( (q^{NB}, T^{NB}) \) form a bargaining equilibrium when bundling is infeasible. Then the Nash bargaining solution between the merged firm and retailer solves

\[
\max_{(q_1, q_2, T_1(\cdot), T_2(\cdot)) \in \hat{A}_m(T_{-1,2}^{NB})} \left( \pi_m - d_m \right)^{\alpha_m} \left( \pi_r - d_r \right)^{1-\alpha_m}, \tag{A3}
\]

Following the technique used to characterize the equilibrium in the bundling case, suppose that the merged firm and retailer restrict attention to a pair of quantity-forcing contracts in the set

\[ \hat{A}_m^F(T_{-1,2}) \equiv \{ (q_1, q_2, F_1, F_2, q_1', q_2') \mid q \in \arg \max_q R(q) - \sum_{i=1,2} T_i^F(q_i) - \sum_{j \neq 1,2} T_j(q_j), \]

\[ F_1 + F_2 \geq C_m(q_1', q_2') \} . \]

With this restriction to quantity-forcing contracts, the maximization problem in (A3) becomes

\[
\max_{(q_1, q_2, F_1, F_2, q_1', q_2') \in \hat{A}_m^F(T_{-1,2}^{NB})} \left( \sum_{i=1,2} T_i^F(q_i) - C_m(q_1, q_2) \right)^{\alpha_i} \left( R(q) - \sum_{i=1,2} T_i^F(q_i) - \sum_{j \neq 1,2} T_j(q_j) - d_r \right)^{1-\alpha_i} \tag{A4}
\]

We will henceforth assume that the Nash product in (A4) has a unique solution. Then, since \( \hat{A}_m^F(T_{-1,2}) \subset \hat{A}_m(T_{-1,2}) \), and the choices \( q_1' = q_1^{NB} \), \( q_2' = q_2^{NB} \), \( F_1 = T_1(q_1^{NB}) \) and \( F_2 = T_2(q_2^{NB}) \) are feasible when the merged firm and retailer choose quantity-forcing contracts, it follows that the solution to (A4) yields the same quantities and payoffs for the merged firm and retailer as
the solution to (A3), conditional on contracts $\mathbf{T}_{1,2}^{\text{NB}}$. This means that we can characterize the equilibrium quantities and payoffs for the merged firm and retailer by solving the restricted problem:

$$\max_{(q_1, q_2, F_1, F_2, q_1', q_2') \in A_m^{F}(\mathbf{T}_{1,2}^{\text{NB}})} (F_1 + F_2 - C_m(q_1, q_2))^{\alpha_m} \left( R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j) - d_r \right) \quad (1-\alpha_m)$$

$$= \max_{q_1, q_2, F_1, F_2, q_{-1,2}} (F_1 + F_2 - C_m(q_1, q_2))^{\alpha_m} \left( R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j) - d_r \right) \quad (1-\alpha_m) \quad (A5)$$

such that

$$F_1 + F_2 \geq C_m(q_1, q_2),$$

$$R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j) \geq d_r,$$

$$R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j) \geq \max_{(q_2, q_{-1,2})} R(0, q_2, q_{-1,2}) - T_2^{F}(q_2) - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j),$$

$$R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j) \geq \max_{(q_1, q_{-1,2})} R(q_1, 0, q_{-1,2}) - T_1^{F}(q_1) - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j),$$

which correspond to (16)–(20), respectively. The rest follows from the discussion in the text.

**Proof of Lemma 2.** Suppose (19) or (20) does not bind. Without loss of generality, let (19) be the non-binding constraint. Then the merged firm and retailer will negotiate $F_1$ to maximize the objective in (16). After some algebra, the first-order condition for $F_1$ can be written as

$$F_1 + F_2 = \alpha_m (R(q) - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j) - d_r) + (1 - \alpha_m) C_m(q_1, q_2). \quad (A6)$$

Substituting (A6) into the expression for the retailer’s profit gives

$$\pi_r = R(q) - F_1 - F_2 - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j)$$

$$= (1 - \alpha_m) (R(q) - C_m(q_1, q_2) - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j)) + \alpha_m d_r, \quad (A7)$$

Note that

$$\lim_{\alpha_m \to 1} \pi_r = d_r \quad (A8)$$

Since (19) does not bind by assumption, condition (A8) implies that for sufficiently large $\alpha_m$,

$$d_r = \max_{q_{-1,2}} R(0, 0, q_{-1,2}) - \sum_{j \neq 1,2} T_j^{\text{NB}}(q_j)$$
which implies that the merger is profitable when bundling is feasible or \( \alpha \) is achieved pre and post merger if bundling is feasible or \( \alpha \). We know from Propositions 1, 2, and 3 that the fully-integrated outcome is a contradiction. Q.E.D.

**Proof of Proposition 4.** We know from Propositions 1, 2, and 3 that the fully-integrated outcome is achieved pre and post merger if bundling is feasible or \( \alpha_m < \bar{\alpha}_m \). In these cases, when there are no cost savings from the merger, the benefit to manufacturers 1 and 2 from merging is given by

\[
\Delta \pi_m^B = \pi_m^B - \pi_1^* - \pi_2^*
\]

\[
= \alpha_m \left( (\Pi - \Pi_{-1,2}) - (\Pi - \Pi_{-1}) - (\Pi - \Pi_{-2}) \right).
\]

(A9)

To see that the right-hand side of (A9) is positive, define

\[
M(q_1, q_2) \equiv \max_{q_{-1,2}} R(q_1, q_2, q_{-1,2}) - w_1^I q_1 - w_2^I q_2 - \sum_{j \neq 1, 2} w_j^I q_j. \tag{A10}
\]

Since the objective in (A10) is concave in \( (q_1, q_2, q_{-1,2}) \), it follows that \( M \) is concave in \( (q_1, q_2) \).

Let \( \tilde{q}_1 \equiv \arg \max_{q_1} M(q_1, 0) \) and \( \tilde{q}_2 \equiv \arg \max_{q_2} M(0, q_2) \). Using these definitions along with the definitions of \( \Pi, \Pi_{-1,2}, \Pi_{-1}, \) and \( \Pi_{-2} \) in the text, we have

\[
\Pi - \Pi_{-1,2} = M(q_1^I, q_2^I) - M(0, 0) \quad \text{(by definition)}
\]

\[
> [M(q_1^I, q_2^I) - M(0, q_2^I)] + [M(q_1^I, q_2^I) - M(q_1^I, 0)] \quad \text{(by concavity and uniqueness)}
\]

\[
\geq [M(q_1^I, q_2^I) - M(0, \tilde{q}_2)] + [M(q_1^I, q_2^I) - M(\tilde{q}_1, 0)] \quad \text{(by the definition of } \tilde{q}_1 \text{ and } \tilde{q}_2) \]

\[
= (\Pi - \Pi_{-1}) + (\Pi - \Pi_{-2}) \quad \text{(by definition)},
\]

which implies that the merger is profitable when bundling is feasible or \( \alpha_m < \bar{\alpha}_m \).

If bundling is infeasible and \( \alpha_m > \bar{\alpha}_m \), then Lemma 1 implies that constraints (19) and (20) will bind in any bargaining equilibrium. Suppose \( q^{NB} = (q_1^{NB}, ..., q_N^{NB}) \) and \( T^{NB} = (T_1^{NB}, ..., T_N^{NB}) \) form a bargaining equilibrium. Then, after some algebra, we can rearrange constraint (19) as

\[
F_1 = R(q^{NB}) - \sum_{j \neq 1, 2} w_j^I q_{j,}^{NB} - \left( \max_{q_{-1,2}} \left( R(0, q_2^{NB}, q_{-1,2}) - \sum_{j \neq 1, 2} w_j^I q_j \right) \right)
\]

32
\[
\begin{align*}
R(q^{NB}) - \sum_{j \neq 1} w^I_j q^B_j & - \left( \max_{q_{-1,2}} \left( R(0, q^{NB}_{2}, q_{-1,2}) - w^I_2 q^B_{2} - \sum_{j \neq 1,2} w^I_j q_j \right) \right) \\
> R(q^{NB}) - \sum_{j \neq 1} w^I_j q^B_j & - \left( \max_{q_{-1,2}} \left( R(0, q_{-1,2}) - w^I_2 - \sum_{j \neq 1,2} w^I_j q_j \right) \right) \\
= R(q^{NB}) - \sum_{j \neq 1} w^I_j q^B_j - \Pi_{-1},
\end{align*}
\]

where we have used the fact that the non-merging firms offer their products at marginal cost to the retailer whether or not bundling is feasible. Similarly, we can rearrange constraint (20) as

\[
F_2 > R(q^{NB}) - \sum_{j \neq 2} w^I_j q^B_j - \Pi_{-2}.
\]

It follows that the profit of merged firm when bundling is infeasible and \( \alpha_m > \alpha_m \) is

\[
\pi_{NB}^{m} = F_1 + F_2 - C_m(q_1^{NB}, q_2^{NB}) > \sum_{i=1,2} \left( R(q^I) - \sum_{j \neq i} w^I_j q^I_j - \Pi_{-i} \right) - C_m(q_1^{IB}, q_2^{IB}).
\]

The first inequality follows from (A11) and (A12). The second inequality follows from the observation that the merged firm’s profit increases when it induces the retailer to choose quantities \( q^{NB} \) rather than \( q^I \). Note that (A13) corresponds to (35) in the text, as was to be proved. \( Q.E.D. \)

**Proof of Proposition 6.** Let \( w^I_1(b) \) and \( w^I_2(b) \) be the bargaining equilibrium wholesale prices for firms 1 and 2, respectively, and let \( q^I_i(b) \), for all \( i \), be the bargaining equilibrium quantities. Rearranging (39) and (40), the upstream profits for products 1 and 2 can be written as

\[
\begin{align*}
\pi_1 &= F_1 + w_1 q_1 - C_1(q_1) \\
&= R(q^I_1(b)) - C_1(q^I_1(b)) - w^I_2(b) q^I_2(b) - \sum_{j \neq 1,2} w^I_j q^I_j(b) \\
&- \max_{q_{-1}} \left( R(0, q_{-1}) - w^I_2(b) q_2 - \sum_{j \neq 1,2} w^I_j q_j \right) + b, \\
\pi_2 &= F_2 + w_2 q_2 - C_2(q_2)
\end{align*}
\]
\[ R(q^e(b)) - C_2(q^e_2(b)) - w_1^e(b)q_1^e(b) - \sum_{j \neq 1,2} w_j^f q_j(b) \]
\[ - \max_{q_{-2}} \left( R(0, q_{-2}) - w_1^e(b)q_1 - \sum_{j \neq 1,2} w_j^f q_j + b \right). \]

(A15)

Using (37), the profit of a rival firm \(i\) can be written as

\[ \pi_i = \alpha_i [R(q^e(b)) - C_i(q^e_i(b)) - w_1^e(b)q_1^e(b) - w_2^e(b)q_2^e(b) - \sum_{j \neq 1,2} w_j^f q_j(b)] \]
\[ - \max_{q_{-1}} \left( R(0, q_{-1}) - w_1^e(b)q_1 - w_2^e(b)q_2 - \sum_{j \neq 1,2,i} w_j^f q_j \right), \quad \forall i \neq 1, 2. \]  

(A16)

Total profits can be written as

\[ \pi = R(q^e(b)) - \sum_i C_i(q^e_i(b)). \]  

(A17)

The retailer’s profits are given by

\[ \pi_r = \pi - \pi_1 - \pi_2 - \sum_{i \neq 1,2} \pi_i. \]  

(A18)

Let \(q_i^j\) maximize the retailer’s profits when the retailer drops product \(j\). For example, \(q_1^2\) is the quantity of product 2 that solves the maximization term in equation (A14). Substituting (A14)-(A17) into (A18), differentiating \(\pi_r\) with respect to \(b\), and using the envelope theorem gives

\[ \frac{\partial \pi_r}{\partial b} = -2 - \left\{ \sum_{i \neq 1} \alpha_i (q_i^1 - q^e_i) \frac{\partial w_1^e(b)}{\partial b} + \sum_{i \neq 2} \alpha_i (q_i^2 - q^e_i) \frac{\partial w_2^e(b)}{\partial b} \right\}. \]

The terms involving \(q_i^j - q^e_i\) for all \(i, j \neq i\), are positive because the products are substitutes and an increase in \(b\) induces increases in \(w_1\) and \(w_2\). It follows that \(\frac{\partial \pi_r}{\partial b} < 0. \)  

Q.E.D.
REFERENCES


