

ALL-UNITS DISCOUNTS IN RETAIL CONTRACTS

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All-units discounts in retail contracts refer to discounts that lower a retailer's wholesale price on every unit purchased when the retailer's purchases equal or exceed some quantity threshold. These discounts pose a challenge to economic theory because it is difficult to understand why a manufacturer ever would charge less for a larger order if its intentions were benign. In this paper, we show that all-units discounts may profitably arise absent any exclusionary motive. All-units discounts eliminate double marginalization in a complete information setting, and they extract more profit than would a menu of two-part tariffs in the standard incomplete information setting with two types of buyers. All-units discounts may improve or may reduce welfare (relative to menus of two-part tariffs) depending on demand parameters.

1. INTRODUCTION

All-units discounts in retail contracts refer to discounts that lower a retailer's wholesale price on every unit purchased when the retailer's purchases equal or exceed some quantity threshold. The discount usually is specified in terms of a percentage off list price, and it also sometimes is referred to as a "target rebate" because, when the target threshold is

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reached, the retailer receives a rebate on all units previously purchased. Thus, all-units discounts have the odd property that purchases at higher quantities actually may be cheaper for the buyer than purchases at lower quantities.

Although the use of such discounts in intermediate-goods markets is common,¹ all-units discounts for the most part have been ignored in the economics and business literatures. And when they are mentioned in these literatures, it often is asserted simply that their use is irrational² or that they are anticompetitive because they provide a strong incentive for a retailer to promote the sale of products on which it is eligible to earn a rebate at the expense of other, substitute products.³ Indeed, all-units discounts pose a challenge to economic theory because it is difficult to understand why a manufacturer ever would charge less for a larger order if its intentions were benign.

In this paper we consider the use of all-units discounts in retail contracts and assess their welfare implications. To rule out exclusion of rivals as a possible motive for the manufacturer a priori, we restrict attention to the case in which a single upstream firm (manufacturer) sells its output to a single downstream firm (retailer). We consider a setting of complete information in which demand is known by both firms at the time of contracting and a standard setting of incomplete information with two types of retailers. In each setting, we compare and contrast the manufacturer's profit-maximizing menu of all-units discounts to its profit-maximizing menu of two-part tariffs.

We establish three main results. First, we show that all-units discounts, like two-part tariffs, can eliminate double marginalization when demand is known by both firms at the time of contracting.⁴ Instead of selling its product to the retailer at a per-unit price of marginal cost plus a fixed fee, which is one way of eliminating double marginalization, the manufacturer can offer the retailer an all-units discount contract and can eliminate double markups by choosing the target quantity to induce the joint-profit maximizing retail price and the percentage discount off list price to divide the surplus. Compared to contracts with linear pricing, all-units discount contracts in this case are Pareto improving. They benefit firms with higher profits and consumers with lower prices.

1. The list of companies using them in their contracts include Coca-Cola, Irish Sugar British Airways, and Michelin.

2. Nahmias (2001, p. 216) states "The all-units schedule appears irrational in some respects. Why would . . . actually charge less for a larger order? One reason would be to provide an incentive for the purchaser to buy more."

3. See, for example, the law journal article by Tom, Balto, and Averitt (2000).

4. The problem of double marginalization was pointed out first by Spengler (1950). The use of two-part tariffs as a possible solution has been noted by Moorthy (1987), Tirole (1988, pp. 174–176), Katz (1989, pp. 664–665), and many others.

Second, we show that when the retailer has private information about consumers' demand, the manufacturer earns higher profit under its profit-maximizing menu of all-units discount contracts than under its profit-maximizing menu of two-part tariff contracts. All-units discounts in this case are more efficient than two-part tariffs at inducing the retailer to reveal the state of demand (they are a better screening device) and hence lead to greater surplus extraction for the manufacturer.

Third, we show that when the retailer has private information about consumers' demand, welfare may be higher or lower with all-units discounts than with two-part tariffs. The direction of change depends on the shape of consumer demand. For example, if the demand curves in the different states of nature are linear and have a common vertical intercept, then all-units discounts lead to lower consumer prices and are welfare improving. On the other hand, if they have a common horizontal intercept, then all-units discounts lead to higher consumer prices and are welfare reducing.

In summary, our results imply that all-units discounts need not be irrational, as some have asserted, nor are they used necessarily by manufacturers for exclusionary purposes. Instead, we find that all-units discounts may arise profitably in retail contracts even in the absence of an exclusionary motive. The discounts can eliminate double marginalization in a complete information setting when demand is known by both firms at the time of contracting, and they can act as a screening device to induce the retailer to reveal the state of demand and to extract surplus when the retailer has private information.⁵ Consumer welfare can be higher or lower as a result.

The paper is organized as follows. In section 2, we introduce the model, discuss the problem of double marginalization, and present our first main result. In section 3, we extend the model to allow for incomplete information and present our second main result. Our third main result and illustrative examples are presented in section 4. We offer concluding remarks in section 5.

2. THE CASE OF COMPLETE INFORMATION

We begin with the case of complete information (Demand is known at the time of contracting). There is a single upstream firm that produces a good at constant marginal cost, c . The upstream firm distributes the good through a downstream firm, which then resells the good to final

5. The problem we consider is equivalent formally to one of implementing second-degree price discrimination among independent retailers with different levels of demand. In this case, the manufacturer offers a menu of options to each retailer and prices the menu so as to induce each retailer to reveal its level of demand by the option it selects.

consumers. We will refer to the upstream firm as the *manufacturer* and the downstream firm as the *retailer*. For simplicity, we assume the only cost the retailer incurs is the payment it makes to the manufacturer when it purchases $q \geq 0$ units of the good. Let $T(q)$ denote the retailer's payment. Let p denote the retail price and $p = p(q)$ denote the consumers' inverse demand, where for all q such that $p(q) > 0$, we assume $p(q)$ is downward sloping. Then the manufacturer's profit is $\pi^m = T(q) - cq$, the retailer's profit is $\pi^r = p(q)q - T(q)$, and the overall joint profit is $\pi^{int} = p(q)q - cq$, which is the profit an integrated firm would earn if it produced and distributed an amount q of the product.

Let $q^*(w) \equiv \operatorname{argmax}_q p(q)q - wq$. Then it is well known that the manufacturer can maximize its profit by offering a two-part tariff contract such that (1) the per-unit price induces the retailer to purchase the integrated quantity, $q^*(c)$; and (2) the fixed fee fully extracts the retailer's surplus:

$$T(q) = \begin{cases} 0 & \text{if } q = 0, \\ cq + F & \text{if } q > 0, \end{cases} \quad (1)$$

where $F = (p(q^*(c)) - c)q^*(c)$. The total outlay of the retailer for any quantity it might purchase under this contract is shown in Figure 1A. The jump-up in cost at $q = 0$ represents the fixed amount the retailer must pay if it purchases from the manufacturer. The upward-sloping line with constant slope c represents the additional payment the retailer makes as q increases. Given this contract, the retailer chooses the integrated quantity, $q^*(c)$, and pays $F + cq^*(c)$ to the manufacturer.

Alternatively, the manufacturer can maximize its profit by offering a contract with a high inframarginal per-unit price, as long as the retailer's per-unit price at the margin is set at cost:

$$T(q) = \begin{cases} p(0)q & \text{if } q < \hat{q}, \\ c(q - \hat{q}) + p(0)\hat{q} & \text{if } q \geq \hat{q}, \end{cases} \quad (2)$$

where \hat{q} is implicitly defined by $(p(0) - c)\hat{q} = p(q^*(c)) - c)q^*(c)$. In this contract, which we illustrate in Figure 1B, the manufacturer offers the initial units up to \hat{q} at a per-unit price of $p(0) > c$. For additional units beyond \hat{q} , the retailer's per-unit price falls to c . At $q = \hat{q}$, the retailer's total outlay is $p(q^*(c)) - c)q^*(c) + c\hat{q}$, which is the same outlay the retailer would make if it purchased \hat{q} units and faced the contract in Figure 1. For $q > \hat{q}$, the two curves are the same. For $q < \hat{q}$, the total outlay of the retailer is $p(0)q$, which is less than what the same quantity would cost in Figure 1A.

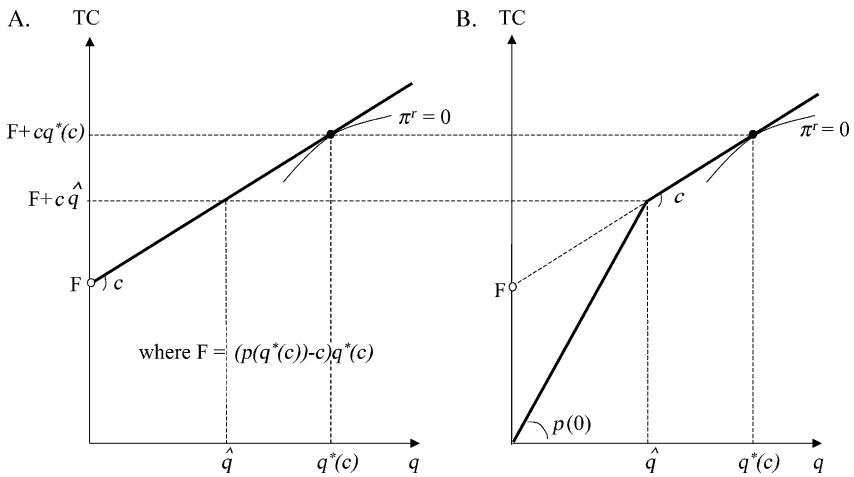


FIGURE 1. TWO-PART TARIFF AND INCREMENTAL-UNITS DISCOUNT

Since the retailer never would purchase $q < \hat{q}$ when faced with the contract in (2) (because at a per-unit price of $p(0)$ its cost would exceed its revenue), it follows that the retailer will purchase the same quantity and that the manufacturer will earn the same profit in Figure 1B as in Figure 1A.⁶

The contracts in Figures 1A and 1B have the property that the retailer’s average per-unit payment exceeds its marginal payment at the profit-maximizing quantity. This allows the manufacturer to solve the double marginalization problem by extracting surplus from the retailer with inframarginal payments without distorting its incentives at the margin.⁷ In contrast, all-units discount contracts do not allow separation between the retailer’s average and marginal payment. They have the form

$$T(q) = \begin{cases} wq & \text{if } q < \bar{q}, \\ \lambda wq & \text{if } q \geq \bar{q}, \end{cases} \quad (3)$$

6. In Figure 1B, the manufacturer offers an initially high wholesale price and then discounts the wholesale price on the retailer’s incremental purchases beyond the quantity, \hat{q} . This is sometimes referred to as an incremental-units discount, and it is equivalent to the lower envelope of an appropriately chosen menu of two-part tariffs. For example, the contract in Figure 1B is the lower envelope of the two-part tariff contract $T(q) = p(0)q$, where the fixed fee is zero, and the two-part tariff contract $T(0) = 0, T(q) = cq + F$ for all $q > 0$, where the fixed fee is the same as in Figure 1A.

7. Contracts such as $T(q) = wq$ suffer from a double marginalization problem because in order to extract surplus, the manufacturer must charge $w > c$, but in doing so, it necessarily distorts downward the retailer’s quantity choice.

where $w \geq 0$ and $\lambda \in (0, 1)$. In this contract, if the retailer purchases $q < \bar{q}$, its average per-unit price and marginal price equals w , whereas if the retailer purchases $q \geq \bar{q}$, its average per-unit price and marginal price equals λw . Since $\lambda < 1$, the retailer is rewarded for purchasing at least \bar{q} units. One can think of w as the manufacturer's list price and $1 - \lambda$ as the percentage discount off list price, which is applied to every unit the retailer purchases if the retailer reaches the target quantity.

The most salient characteristic of contracts with all-units discounts is that the retailer's total outlay jumps down at \bar{q} , which implies that purchases at higher quantities can be cheaper than purchases at lower quantities. This property has raised concern that these contracts may be exclusionary when used by dominant firms because the downward discontinuity in the total outlay schedule at \bar{q} allegedly unfairly induces the retailer to promote the sale of the manufacturer's product at the expense of smaller competitors who may not be able to offer such potentially large rebates.⁸ Indeed, the effective marginal wholesale price in these contracts at the target quantity is negative, which under some definitions of predation might be construed as predatory pricing.

However, we now show that contracts with all-units discounts have an efficiency rationale. They, like the contracts in Figures 1A and 1B, can eliminate double marginalization. To see this, consider the set of contracts that offer an initial wholesale price of $p(0)$ for all q less than some quantity threshold $q^*(c)$, and a discounted wholesale price of $\lambda p(0)$, where $\lambda \in [\frac{c}{p(0)}, \frac{p(q^*(c))}{p(0)}]$, thereafter

$$T(q) = \begin{cases} p(0)q & \text{if } q < q^*(c), \\ \lambda p(0)q & \text{if } q \geq q^*(c). \end{cases} \quad (4)$$

With these contracts, the retailer never would purchase less than $q^*(c)$ because at a per-unit price of $p(0)$ its cost would exceed its revenue, and we know by the definition of $q^*(c)$ that the retailer never would purchase more than $q^*(c)$ because $\lambda p(0) \geq c$ implies that $q^*(\lambda p(0)) \leq q^*(c)$. Thus, the contracts in (4) induce the retailer to choose $q = q^*(c)$, realizing the integrated outcome. When λ is at its upper bound of $\frac{p(q^*(c))}{p(0)}$, as we illustrate in Figure 2A, the manufacturer earns all the surplus. When λ is at its lower bound of $\frac{c}{p(0)}$, as we illustrate in Figure 2B, the retailer earns

8. The European Commission has brought several cases against upstream firms for, among other things, offering all-units discounts in their contracts. To our knowledge, the U.S. antitrust agencies have not condemned contracts explicitly with all-units discounts, although their concerns are similar (see Tom, Balto, and Averitt, 2000).

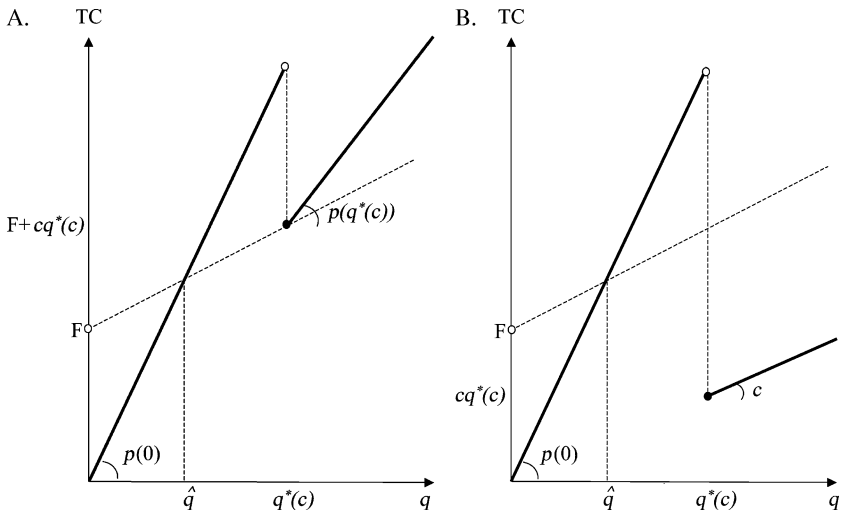


FIGURE 2. ALL-UNITS DISCOUNT ($\lambda w = p(q^*(c))$) AND $\lambda w = c$)

all the surplus. Intermediate levels of λ between these bounds reflect a more balanced sharing of the overall surplus.

PROPOSITION 1: *The all-units discount contracts in (4) eliminate double marginalization.*

The contracts in (4) induce the integrated outcome because the quantity threshold $q^*(c)$ effectively caps the retailer’s price to consumers at $p(q^*(c))$. The price is capped because if the retailer were to charge more than $p(q^*(c))$, it would not be able to sell $q^*(c)$ units and thus would not qualify for the target rebate. Of course, the retailer also does not want to charge less than $p(q^*(c))$ because, under the contracts in (4), it never wants to sell more than $q^*(c)$ units (This follows because the retailer’s wholesale price is everywhere weakly larger than c). If the manufacturer makes a take-it-or-leave-it offer, the double-marginalization problem is solved by eliminating the retailer’s markup. If the retailer makes a take-it-or-leave-it offer, the double marginalization problem is solved by eliminating the manufacturer’s markup. Otherwise, the contracts in (4) ensure that the sum of the firms’ markups is equal to the markup that an integrated firm would have. To the extent that both firms have some bargaining power, there will be agreement on the quantity threshold, $q^*(c)$, but disagreement on the percentage discount off list price. The manufacturer will prefer a lower percentage discount, and the retailer will prefer a higher percentage discount.

3. THE CASE OF INCOMPLETE INFORMATION

We now extend the model to allow for the possibility that the manufacturer may not know what demand will be at the time of contracting. This is an important case to consider because demand sometimes may be unpredictable (e.g., Sales of ice cream will depend on the weather), and because the model with complete information does not have predictive power to explain why one contract form may be chosen over another. As we shall see, the equivalence among two-part tariffs, incremental-units discounts, and all-units discounts no longer holds when demand is unknown.

We assume, for simplicity, that consumer demand can take one of two forms. It may be high or low depending on the state of nature. The low-demand state of nature occurs with probability $\alpha \in [0, 1]$ and the high-demand state of nature occurs with probability $(1 - \alpha)$. We denote the low-demand state by L and the high-demand state by H . The retailer faces the inverse demand $p = p_L(q)$ under state L and the inverse demand $p = p_H(q)$ under state H , with $p_H(\cdot) \geq p_L(\cdot)$.

Let $R_i(q) \equiv p_i(q)q$ denote the retailer's revenue from the sale of $q \geq 0$ units of the good in the i th state of nature, where $i \in \{L, H\}$. Our assumptions on inverse demands imply that the retailer's revenue in the high-demand state is weakly larger than the retailer's revenue in the low-demand state for a given quantity sold: $R_H(q) \geq R_L(q)$. We also make the following assumptions on $R_i(q)$:

$$\frac{\partial^2 R_i(q)}{\partial q^2} < 0, \quad \frac{\partial R_H(q)}{\partial q} > \frac{\partial R_L(q)}{\partial q}. \quad (5)$$

The first assumption in (5) implies that the retailer's revenue in each state is concave (This ensures that firm i 's marginal revenue is downward sloping). The second assumption in (5) implies that the retailer's marginal revenue in the high-demand state always is greater than its marginal revenue in the low-demand state (This property sometimes is referred to as the single-crossing condition).

The timing of the game is as follows. In the first stage, the manufacturer specifies the terms at which it will sell its good to the retailer. It does so without knowing whether consumer demand will be high or low. In the second stage, the uncertainty is resolved, and the retailer learns the state of nature. It then chooses whether to purchase, and how much to purchase, from the manufacturer and pays the manufacturer according to the terms of its contract. The retailer resells this quantity to final consumers, earning revenue $R_i(q)$, where $i \in \{H, L\}$ is the state of nature.

We assume the manufacturer cannot contract on the state of nature (If it could, we are back to the model in the previous section) and so cannot prevent a retailer facing one demand state from pretending that another demand state has occurred. Thus, the manufacturer must choose its contract in stage one to induce the retailer to reveal the true state of demand in stage two.⁹

This situation has been studied in the literature on self-selection and price discrimination (In our model, the manufacturer offers a menu of options to the retailer and prices the menu so as to induce the retailer to reveal the state of demand by the option it selects).¹⁰ Conventional wisdom is that (1) the retailer will earn zero surplus in the low-demand state and positive surplus in the high-demand state; and (2) the manufacturer will distort downward the quantity chosen by a low-demand retailer but not the quantity chosen by a high-demand retailer.¹¹ We will show these results using the manufacturer's profit-maximizing menu of two-part tariffs, or, equivalently, incremental-units discount, as a benchmark. We then will compare consumer prices and social welfare under this benchmark with those of the manufacturer's optimal menu of all-units discounts.

3.1 MENU OF TWO-PART TARIFFS, INCREMENTAL-UNITS DISCOUNT

Suppose the manufacturer offers a menu of two-part tariffs $((w_L, F_L), (w_H, F_H))$, where w_i denotes the per-unit price and F_i is the fixed fee, with (w_L, F_L) meant for the retailer facing low demand and (w_H, F_H) meant for the retailer facing high demand. Let $q_L^*(w)$ denote the low-demand retailer's quantity choice if it faces a per-unit price of w and purchases a positive quantity, and let $q_H^*(w)$ be defined similarly, i.e., $q_i^*(w)$ solves $\frac{\partial R_i(q)}{\partial q} = w$. Then the manufacturer's problem is

$$\begin{aligned} \max_{w_L, w_H, F_L, F_H} \quad & \alpha(w_L - c)q_L^*(w_L) + (1 - \alpha)(w_H - c)q_H^*(w_H) + \alpha F_L \\ & + (1 - \alpha)F_H, \end{aligned} \tag{6}$$

9. In what follows, we assume that the manufacturer wants to serve both retailer types. That is, we assume that selling to the high-demand retailer only is less profitable than selling to both types. See Salant (1989) for a characterization of the necessary and sufficient conditions for discrimination to be optimal in the two-type case.

10. See, for example, Willig (1978), Spence (1977, 1980), Roberts (1979), Goldman, Leland, and Sibley (1984), Maskin and Riley (1984), and the excellent survey on second-degree price discrimination in Tirole (1988, pp. 142–158).

11. This result depends on the assumption that the retailer's demand in the high-demand state is independent of its demand in the low-demand state. Ordovery and Panzar (1982) show that if the demands across states of nature, or among different types of consumers, are interrelated, then a distortion may arise even in the high-demand state.

subject to the low-demand retailer choosing a positive quantity under (w_L, F_L) ,

$$R_L(q_L^*(w_L)) - w_L q_L^*(w_L) - F_L \geq 0, \quad (7)$$

and the high-demand retailer choosing to purchase under (w_H, F_H) rather than (w_L, F_L) ,¹²

$$R_H(q_H^*(w_H)) - w_H q_H^*(w_H) - F_H \geq R_H(q_H^*(w_L)) - w_L q_H^*(w_L) - F_L. \quad (8)$$

Our assumptions on the revenue functions, and the fact that the maximand in (6) is increasing in F_L and F_H , imply that these two constraints will be binding. Specifically, it follows that F_L will be chosen to satisfy (7) with equality and that F_H will be chosen to satisfy (8) with equality; i.e., the low-demand retailer will be pushed to indifference between purchasing a positive quantity and not purchasing, and the high-demand retailer will be pushed to indifference between choosing a quantity under contract (w_H, F_H) and choosing a quantity under contract (w_L, F_L) . Thus, a low-demand retailer will earn zero profit, while a high-demand retailer will earn positive profit.¹³

Substituting the fixed fees that satisfy (7) and (8) with equality into the maximand, and assuming the new maximand is concave in w_L and w_H , we have that the optimal w_H and w_L solve

$$\left(\frac{\partial R_H(q_H^*(w_H))}{\partial q} - c \right) \frac{\partial q_H^*(w_H)}{\partial w_H} = 0, \quad (9)$$

$$\alpha \left(\frac{\partial R_L(q_L^*(w_L))}{\partial q} - c \right) \frac{\partial q_L^*(w_L)}{\partial w_L} + (1 - \alpha)(q_H^*(w_L) - q_L^*(w_L)) = 0. \quad (10)$$

We see from the expression in (9) that the manufacturer should charge $w_H = c$ to the high-demand retailer, thereby inducing it to purchase and to resell $q_H^*(c)$, the integrated quantity when demand is high. However, because of the term $(1 - \alpha)(q_H^*(w_L) - q_L^*(w_L))$, the expression in (10) implies that the manufacturer will want to distort the low-demand retailer's quantity. Since the high-demand retailer's marginal revenue is everywhere above the low-demand retailer's marginal revenue, it follows that $q_H^*(w_L) > q_L^*(w_L)$, for all $w_L \geq 0$, which implies that the left

12. Two constraints are suppressed: A high-demand retailer must purchase a positive quantity, and a low-demand retailer must purchase under (w_L, F_L) rather than (w_H, F_H) (These constraints are trivially satisfied).

13. To see this, note that the high-demand retailer's profit is $R_H(q_H^*(w_L)) - w_L q_H^*(w_L) - F_L$, which is strictly positive because $R_H(q_H^*(w_L)) - w_L q_H^*(w_L) - F_L \geq R_H(q_L^*(w_L)) - w_L q_L^*(w_L) - F_L > R_L(q_L^*(w_L)) - w_L q_L^*(w_L) - F_L = 0$.

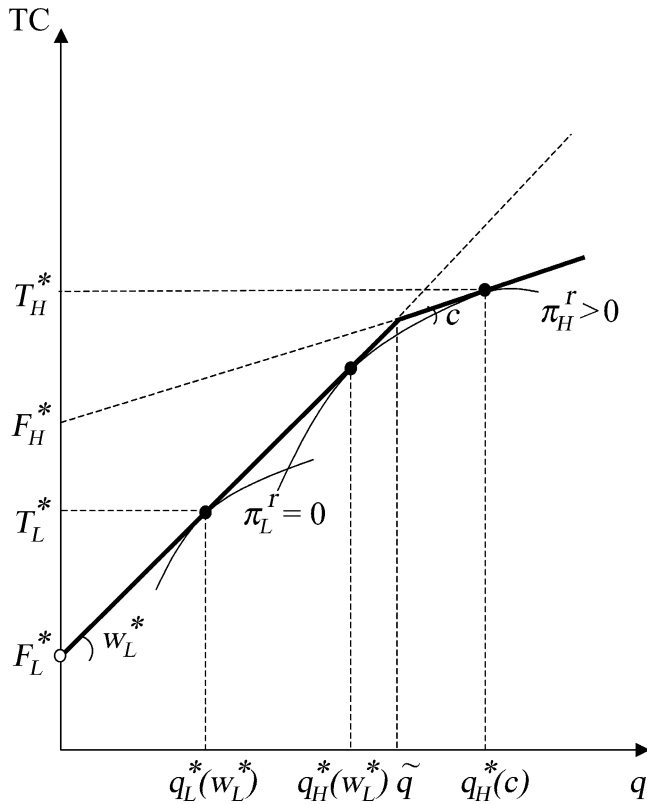


FIGURE 3. MENU OF TWO-PART TARIFFS

side of (10) is positive when evaluated at $w_L = c$. Thus, the manufacturer should charge $w_L > c$ and should induce the retailer to purchase and to resell less than $q_L^*(c)$, the integrated quantity when demand is low.

Let $((w_L^*, F_L^*), (w_H^*, F_H^*))$ solve the manufacturer's problem in (6) through (8), and let $T_i^* = w_i^* q_i^*(w_i^*) + F_i^*$ denote the amount a retailer facing demand state i pays to the manufacturer, $i = H, L$. Then, the manufacturer's maximized profit given the profit-maximizing menu of two-part tariffs is

$$\pi^{2PT} = \alpha(T_L^* - cq_L^*(w_L^*)) + (1 - \alpha)(T_H^* - cq_H^*(w_H^*)).$$

Figure 3 depicts the total outlay of the retailer for any quantity it might purchase under the manufacturer's profit-maximizing menu of two-part tariffs. The two-part tariff meant for the low-demand retailer is given by the upward-sloping line beginning at F_L^* on the vertical axis,

with slope w_L^* . The two-part tariff meant for the high-demand retailer is given by the upward-sloping line beginning at F_H^* on the vertical axis, with slope $w_H^* = c$. The cost-minimizing outlay for any quantity the retailer might purchase is given by the lower envelope of these two lines, with a kink-point at \tilde{q} , which is defined implicitly by $w_L^*\tilde{q} + F_L^* = c\tilde{q} + F_H^*$. If demand turns out to be low, the retailer will choose $q_L^*(w_L^*)$ from this envelope and will pay the manufacturer T_L^* . Since (7) is satisfied with equality, it follows that the retailer in this case is indifferent between purchasing $q_L^*(w_L^*)$ and zero, earning profit $\pi_L^r = 0$. If demand turns out to be high, the retailer will choose $q_H^*(c)$ from this envelope and will pay the manufacturer T_H^* . Since (8) is satisfied with equality, it follows that the retailer in this case is indifferent between purchasing $q_H^*(c)$ and $q_H^*(w_L^*)$, earning profit $\pi_H^r > 0$.

Since the retailer only will choose points from along the lower envelope of the menu of two-part tariffs, as depicted in Figure 3 (the cost-minimizing outlay), the manufacturer instead could have induced the same profit-maximizing outcome with a single contract that traces out this curve:

$$T^{IU}(q) \equiv \begin{cases} w_L^*q + F_L^* & \text{if } q < \tilde{q}, \\ c(q - \tilde{q}) + w_L^*\tilde{q} + F_L^* & \text{if } q \geq \tilde{q}. \end{cases} \quad (11)$$

With this contract, the retailer faces a per-unit price of w_L^* for all units purchased up to \tilde{q} . For incremental units purchased beyond \tilde{q} , the retailer pays a per-unit price of c . If demand is low, the retailer purchases $q_L^*(w_L^*)$ and pays the manufacturer T_L^* . If demand is high, the retailer purchases $q_H^*(c)$ and pays the manufacturer T_H^* . Since the quantities and payments are the same as in the case of the profit-maximizing menu of two-part tariffs, it follows that each firm's profit also will be the same. This establishes that contracts with incremental-units discounts can obtain the same outcome as contracts with menus of two-part tariffs. Since any contract with incremental-units discounts can be replicated with a menu of two-part tariffs, the converse also can be shown. Thus, the equivalence between these two types of contracts extends to the case of unknown demand.

3.2 ALL-UNITS DISCOUNTS

We now show that there exists a contract with all-units discounts that induces the retailer to choose the same quantities as in Figure 3 but yields strictly higher profit than π^{2PT} . Thus, the equivalence among different types of contracts does not extend to contracts with all-units discounts.

PROPOSITION 2: *When demand is unknown at the time of contracting, the manufacturer can earn higher profit with a menu of all-units discounts than with a menu of two-part tariffs.*

Proof: Let $\epsilon > 0$ be arbitrarily small, and consider the menu of contracts $T_1(q)$ and $T_2(q)$, where

$$T_1(q) \equiv \begin{cases} p_H(0)q & \text{if } q < q_L^*(w_L^*) \\ \frac{T_L^*}{q_L^*(w_L^*)}q & \text{if } q \geq q_L^*(w_L^*) \end{cases},$$

$$T_2(q) \equiv \begin{cases} p_H(0)q & \text{if } q < q_H^*(c) \\ \frac{T_H^* + \epsilon}{q_H^*(c)}q & \text{if } q \geq q_H^*(c), \end{cases} \tag{12}$$

or, equivalently, the contract that corresponds to the cost-minimizing outlay given this menu:

$$T^{AU}(q) \equiv \begin{cases} p_H(0)q & \text{if } q < q_L^*(w_L^*), \\ \frac{T_L^*}{q_L^*(w_L^*)}q & \text{if } q_L^*(w_L^*) \leq q < q_H^*(c), \\ \frac{T_H^* + \epsilon}{q_H^*(c)}q & \text{if } q \geq q_H^*(c). \end{cases} \tag{13}$$

Figure 4 depicts the retailer’s total cost for any quantity it might purchase under $T^{AU}(q)$. For all units purchased up to $q_L^*(w_L^*)$, the retailer’s per-unit price is $p_H(0)$. For purchase quantities of $q_L^*(w_L^*)$ or between $q_L^*(w_L^*)$ and $q_H^*(c)$, the retailer’s per-unit price on all units is $\frac{T_L^*}{q_L^*(w_L^*)}$, and for purchase quantities that equal or exceed $q_H^*(c)$, the retailer’s per-unit price on all units is $\frac{T_H^* + \epsilon}{q_H^*(c)}$.

To expedite the proof, it is useful to begin with a couple of observations. First, we note that, when faced with contract $T^{AU}(q)$, the retailer never will purchase $q < q_L^*(w_L^*)$ because at a per-unit price of $p_H(0)$ its marginal revenue is everywhere below its marginal cost. Second, we note that $T^{AU}(q) = T^{IU}(q)$ at $q = q_L^*(w_L^*)$. Third, we note that for all $q > q_L^*(w_L^*)$, $T^{AU}(q) > T^{IU}(q)$.¹⁴

These observations imply that the retailer will choose $q_L^*(w_L^*)$ and will pay T_L^* when the low-demand state occurs. Recall that $q_L^*(w_L^*) = \arg \max_q R_L(q) - T^{IU}(q)$. Hence, it also must be the case that

14. To see this, note that for all units between $q_L^*(w_L^*)$ and $q_H^*(c)$, the per-unit price in $T^{IU}(q)$ is either w_L^* or c , which is less than the per-unit price of $\frac{w_L^* q_L^*(w_L^*) + F_L^*}{q_L^*(w_L^*)}$ in $T^{AU}(q)$. And, for all units greater than or equal to $q_H^*(c)$, the per-unit price in $T^{IU}(q)$, which is c , is less than the per-unit price in $T^{AU}(q)$, which is $\frac{c q_H^*(c) + F_H^* + \epsilon}{q_H^*(c)}$.

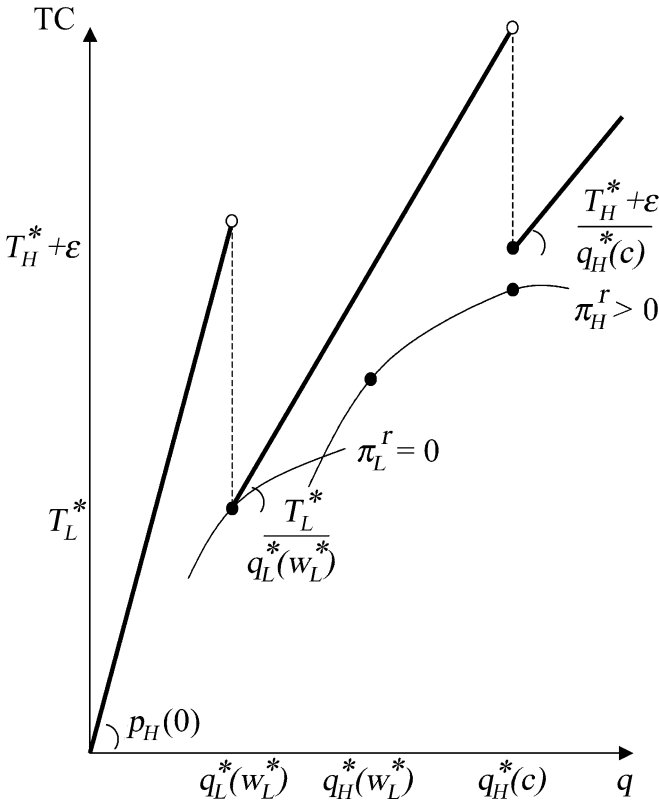


FIGURE 4. MENU OF ALL-UNITS DISCOUNTS

$q_L^*(w_L^*) = \arg \max_q R_L(q) - T^{AU}(q)$. At this quantity, the low-demand retailer earns zero profit.

These observations also imply that the retailer will choose $q_H^*(c)$ and will pay $T_H^* + \epsilon$ when the high-demand state occurs. To see this, recall that $q_H^*(c) = \arg \max_q R_H(q) - T^{IU}(q)$ and $T^{IU}(q_H^*(c)) = T_H^*$. Then, it follows that $R_H(q_H^*(c)) - T^{AU}(q_H^*(c)) > R_H(q) - T^{AU}(q)$, for all $q \neq q_H^*(c)$, because

$$\begin{aligned} R_H(q_H^*(c)) - T^{AU}(q_H^*(c)) &= R_H(q_H^*(c)) - T^{IU}(q_H^*(c)) - \epsilon \\ &> R_H(q) - T^{IU}(q) \quad \text{if } q \neq q_H^*(c) \\ &\geq R_H(q) - T^{AU}(q) \quad \text{for all } q \geq q_L^*(w_L^*), \end{aligned}$$

where the first inequality follows because ϵ is arbitrarily small, and the second inequality follows because $T^{AU}(q) \geq T^{IU}(q)$ in the relevant range. At $q_H^*(c)$, the high-demand retailer earns positive profit, but not as much

as it would have earned if the manufacturer had offered the contract $T^{IU}(q)$.

Since the manufacturer earns $T_L^* - cq_L^*(w_L^*)$ when the low-demand state occurs, and $T_H^* + \epsilon - cq_H^*(w_H^*)$ when the high-demand state occurs, its expected payoff under contract $T^{AU}(q)$ is

$$\pi^{AU} = \alpha(T_L^* - cq_L^*(w_L^*)) + (1 - \alpha)(T_H^* + \epsilon - cq_H^*(w_H^*)).$$

Comparing π^{AU} and π^{2PT} , we see that $\pi^{AU} = \pi^{2PT} + (1 - \alpha)\epsilon$, which implies that the manufacturer earns strictly higher profit under the all-units discount contract in (13). \square

The manufacturer can extract a higher profit with an all-units discount contract because these contracts are more efficient at inducing the retailer to reveal the state of demand. Put simply, the flexibility afforded by the discontinuous outlay schedule makes it possible for the manufacturer to charge higher prices for all quantities greater than $q_L^*(w_L^*)$ while still inducing a retailer in the high-demand state to purchase $q_H^*(c)$. As can be seen from Figure 4, the retailer in the high-demand state cannot reach the same isoprofit curve that it did in Figure 3 because the profitability of purchasing any quantity between $q_L^*(w_L^*)$ and $q_H^*(c)$, and in particular of purchasing $q_H^*(w_L^*)$, is lower when the high-demand retailer faces the contract $T^{AU}(q)$ than when it faces the manufacturer's profit-maximizing menu of two-part tariffs. Thus, choosing to purchase under the contract option meant for the low-demand retailer— $T_1(q)$ —is less profitable than it was before, implying that the manufacturer does not have to leave the retailer as large an informational rent in this case.

4. PROFIT-MAXIMIZING QUANTITIES WITH ALL-UNITS DISCOUNTS

As we have seen, all-units discounts are the preferred choice of the manufacturer relative to contracts with incremental-units discounts (or contracts with a menu of two-part tariffs) when demand is unknown at the time of contracting. This is because the manufacturer always can choose an all-units discount contract that induces the retailer to purchase the same quantities and to set the same prices as under the optimal incremental-units discount while extracting a higher payment. Thus, there exists an all-units discount contract that increases its profit without affecting welfare.

In this section, we extend the analysis to consider the welfare effects of all-units discounts vis à vis incremental-units discounts when the manufacturer chooses its optimal all-units discount contract. Our main

result is that welfare can be higher or lower depending on demand parameters.

Let $(T_L(q), T_H(q))$ denote the menu of all-units discount contracts offered by the manufacturer, where T_L is meant for the low-demand retailer and T_H is meant for the high-demand retailer, and

$$T_L(q) = \begin{cases} w_L q & \text{if } q < \bar{q}_L \\ \lambda_L w_L q & \text{if } q \geq \bar{q}_L, \end{cases} \quad T_H(q) = \begin{cases} w_H q & \text{if } q < \bar{q}_H \\ \lambda_H w_H q & \text{if } q \geq \bar{q}_H. \end{cases}$$

Let $q_L^{**}(w_i, \lambda_i, \bar{q}_i) \in \arg \max_q R_L(q) - T_i(q)$ denote the low-demand retailer's quantity choice if it purchases under contract $T_i(q)$ and define $q_H^{**}(w_i, \lambda_i, \bar{q}_i)$ similarly. Then the manufacturer solves

$$\max_{w_L, \lambda_L, \bar{q}_L, w_H, \lambda_H, \bar{q}_H} \alpha(T_L(q_L^{**}(w_L, \lambda_L, \bar{q}_L)) - cq_L^{**}(w_L, \lambda_L, \bar{q}_L)) + (1 - \alpha)(T_H(q_H^{**}(w_H, \lambda_H, \bar{q}_H)) - cq_H^{**}(w_H, \lambda_H, \bar{q}_H)), \quad (14)$$

subject to the low-demand retailer choosing a positive quantity under $T_L(q)$,

$$R_L(q_L^{**}(w_L, \lambda_L, \bar{q}_L)) - T_L(q_L^{**}(w_L, \lambda_L, \bar{q}_L)) \geq 0, \quad (15)$$

and the high-demand retailer choosing to purchase under $T_H(q)$ rather than $T_L(q)$,

$$R_H(q_H^{**}(w_H, \lambda_H, \bar{q}_H)) - T_H(q_H^{**}(w_H, \lambda_H, \bar{q}_H)) \geq R_H(q_H^{**}(w_L, \lambda_L, \bar{q}_L)) - T_L(q_H^{**}(w_L, \lambda_L, \bar{q}_L)). \quad (16)$$

The solution to the manufacturer's problem is not as straightforward here as it is in the case of two-part tariffs because, with all-units discounts, there are no fixed fees to equate the two sides of (15) and (16). Instead surplus must be extracted indirectly through the choice of λ_i and \bar{q}_i . Nevertheless, as we now show, even without fixed fees, (15) and (16) will bind in any solution.

LEMMA 1: *Let $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}), (w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ be a solution to the manufacturer's problem. Then the low-demand retailer earns zero profit and the high-demand retailer is indifferent between purchasing $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ under contract $T_H(q)$ and purchasing $q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ under contract $T_L(q)$:*

$$R_L(q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) - T_L(q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) = 0,$$

$$R_H(q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})) - T_H(q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})) =$$

$$R_H(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) - T_L(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})).$$

For the proof, see the Appendix.

We can understand Lemma 1 as follows. The manufacturer can induce a retailer in the low-demand state to purchase zero or at least \bar{q}_L by choosing w_L sufficiently high. And for a finite w_L , the manufacturer can induce the retailer to purchase at most \bar{q}_L by choosing λ_L sufficiently high. It follows that the manufacturer can induce the low-demand retailer to purchase exactly \bar{q}_L and to extract all surplus by choosing $\lambda_L w_L$ appropriately. For example, the manufacturer can extract all surplus and can induce the low-demand retailer to purchase $q_L^{**}(w_L, \lambda_L, \bar{q}_L) = \bar{q}_L$ by choosing $w_L \geq p_H(0)$ and $\lambda_L w_L = \frac{R_L(\bar{q}_L)}{\bar{q}_L} = p_L(\bar{q}_L)$. The extraction of surplus from a retailer in the high-demand state can be achieved similarly, except that instead of choosing $\lambda_H w_H$ to extract all of the high-demand retailer's surplus, the manufacturer chooses $\lambda_H w_H$ to make the retailer indifferent between purchasing $q_H^{**}(w_H, \lambda_H, \bar{q}_H) = \bar{q}_H$ under $T_H(q)$ and purchasing $q_H^{**}(w_L, \lambda_L, \bar{q}_L)$ under $T_L(q)$.

Lemma 2 shows that the manufacturer must choose its contract terms in this way.

LEMMA 2: *Let $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}), (w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ be a solution to the manufacturer's problem. Then it must be that $w_L^{**} \geq p_L(0), \lambda_L^{**} w_L^{**} = p_L(\bar{q}_L^{**}), q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}) = \bar{q}_L^{**}$ and $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = \bar{q}_H^{**}$.*

For the proof, see the Appendix.

Lemma 2 implies that at the optimum the manufacturer must choose its contract terms to induce the retailer to purchase at the quantity threshold that corresponds to each demand state. Since we know that for any \bar{q}_L the manufacturer can choose $w_L \geq p_L(0)$ and $\lambda_L w_L = p_L(\bar{q}_L)$ to induce $q_L^{**}(w_L, \lambda_L, \bar{q}_L) = \bar{q}_L$ and satisfy (15) with equality, and since we know that for any \bar{q}_H the manufacturer can choose w_H and $\lambda_H w_H$ to induce $q_H^{**}(w_H, \lambda_H, \bar{q}_H) = \bar{q}_H$ and satisfy (16) with equality,¹⁵ we can use Lemmas 1 and 2 to rewrite the manufacturer's problem in (14)–(16) as

$$\begin{aligned} \max_{w_L, \lambda_L, \bar{q}_L, \bar{q}_H} & \alpha(R_L(\bar{q}_L) - c\bar{q}_L) - (1 - \alpha)(R_H(q_H^{**}(w_L, \lambda_L, \bar{q}_L)) \\ & - T_L(q_H^{**}(w_L, \lambda_L, \bar{q}_L))) + (1 - \alpha)(R_H(\bar{q}_H) - c\bar{q}_H), \end{aligned} \tag{17}$$

such that

$$w_L \geq p_L(0), \quad \text{and} \quad \lambda_L w_L = p_L(\bar{q}_L). \tag{18}$$

15. For example, the manufacturer can choose w_H such that $R_H(q_H^*(w_H)) - w_H q_H^*(w_H) < R_H(q_H^{**}(w_L, \lambda_L, \bar{q}_L)) - T_L(q_H^{**}(w_L, \lambda_L, \bar{q}_L))$, and $\lambda_H w_H$ such that $\lambda_H w_H \bar{q}_H = R_H(\bar{q}_H) - (R_H(q_H^{**}(w_L, \lambda_L, \bar{q}_L)) - T_L(q_H^{**}(w_L, \lambda_L, \bar{q}_L)))$.

It follows that the profit-maximizing \bar{q}_H solves $\max_q(1 - \alpha)(R_H(q) - cq)$, implying that, for $\alpha \neq 1$, there is no distortion in the quantity purchased by a retailer in the high-demand state.

PROPOSITION 3: *Let $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ be a solution to the manufacturer's problem. Then it must be that a retailer in the high-demand state will choose $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = q_H^*(c)$.*

For the proof, see the Appendix.

Proposition 3 establishes that a retailer in the high-demand state will choose the integrated quantity, which is the same quantity that a high-demand retailer would choose under the manufacturer's profit-maximizing menu of two-part tariffs. Thus, the welfare comparison between the profit-maximizing menu of two-part tariffs and the profit-maximizing menu of all-units discounts will depend solely on the relation between the quantities purchased by the low-demand retailer in the two cases. From the manufacturer's problem in (17)–(18), we see that, in the case of the profit-maximizing menu of all-units discounts, the manufacturer will want to distort downward the quantity purchased by a retailer in the low-demand state. There are two subcases to consider.

PROPOSITION 4: *Let $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ be a solution to the manufacturer's problem. Then it must be that if $q_H^*(\lambda_L^{**} w_L^{**}) > \bar{q}_L^{**}$, then $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ solves*

$$\alpha \left(\frac{\partial R_L(q)}{\partial q} - c \right) + (1 - \alpha) \frac{\partial p_L(q)}{\partial q} q_H^*(p_L(q)) = 0, \quad (19)$$

and if $q_H^*(\lambda_L^{**} w_L^{**}) \leq \bar{q}_L^{**}$, then $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ solves

$$\alpha \left(\frac{\partial R_L(q)}{\partial q} - c \right) + (1 - \alpha) \left(\frac{\partial R_L(q)}{\partial q} - \frac{\partial R_H(q)}{\partial q} \right) = 0. \quad (20)$$

For the proof, see the Appendix.

Proposition 4 characterizes the manufacturer's choice of \bar{q}_L^{**} , and hence the retailer's induced choice $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, for the two subcases that arise depending on whether or not a retailer in the high-demand state would be constrained by \bar{q}_L^{**} if it purchased under contract $T_L(q)$. As the linear-demand examples below make clear, this will depend on exogenous parameters that determine the gap between consumers' inverse demands in the low and high demand states. If the high-demand retailer would be unconstrained by \bar{q}_L^{**} , then \bar{q}_L^{**} must solve (19). If the high-demand retailer would be constrained by \bar{q}_L^{**} , then \bar{q}_L^{**} must solve (20). In both cases, we see that \bar{q}_L^{**} , and hence $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, will be less than $q^*(c)$, the integrated quantity in the low-demand state.

4.1 THE WELFARE EFFECTS OF ALL-UNITS DISCOUNTS

To determine whether welfare is higher with all-units discounts or with two-part tariffs, we compare the low-demand retailer’s quantity choice as characterized in Proposition 4 with its quantity choice under the profit-maximizing menu of two-part tariffs, $q_L^*(w_L^*)$, where w_L^* is characterized in (10).

4.1.1 SUBCASE 1: $q_H^*(\lambda_L^{**} w_L^{**}) > \bar{q}_L^{**}$.

If we evaluate the left-hand sides of (19) and (10) at $q_L^*(w_L^*)$ and w_L^* , respectively, subtract (10) from (19), and use the fact that retailer optimality implies $w_L^* = \frac{\partial R_L(q_L^*(w_L^*))}{\partial q}$, so that

$$\frac{\partial q_L^*(w_L^*)}{\partial w_L^*} = \frac{1}{\frac{\partial^2 R_L(q_L^*(w_L^*))}{\partial q^2}}, \tag{21}$$

then the left-hand side of (19) can be written as

$$\begin{aligned} & - (1 - \alpha) (q_H^*(w_L^*) - q_L^*(w_L^*)) \frac{\partial^2 R_L(q_L^*(w_L^*))}{\partial q^2} \\ & + (1 - \alpha) \frac{\partial p_L(q_L^*(w_L^*))}{\partial q} q_H^*(p_L(q_L^*(w_L^*))). \end{aligned} \tag{22}$$

It follows that if $q_H^*(\lambda_L^{**} w_L^{**}) > \bar{q}_L^{**}$, so that the condition in (19) applies, and if the left-hand side of (19) is decreasing in q , then $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}) > q_L^*(w_L^*)$ if and only if (22) is positive.

4.1.2 SUBCASE 2: $q_H^*(\lambda_L^{**} w_L^{**}) \leq \bar{q}_L^{**}$.

If we evaluate the left-hand sides of (20) and (10) at $q_L^*(w_L^*)$ and w_L^* , respectively, subtract (10) from (20), and use (21), and the fact that retailer optimality implies

$$\frac{\partial R_L(q_L^*(w_L^*))}{\partial q} = \frac{\partial R_H(q_H^*(w_L^*))}{\partial q},$$

then the left-hand side of (20) can be written as

$$(1 - \alpha) \int_{q_L^*(w_L^*)}^{q_H^*(w_L^*)} \left(\frac{\partial^2 R_H(q)}{\partial q^2} - \frac{\partial^2 R_L(q_L^*(w_L^*))}{\partial q^2} \right) dq. \tag{23}$$

It follows that if $q_H^*(\lambda_L^{**} w_L^{**}) \leq \bar{q}_L^{**}$, so that the condition in (20) applies, and if the left-hand side of (20) is decreasing in q , then $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}) > q_L^*(w_L^*)$ if and only if (23) is positive.

4.2 LINEAR DEMANDS

With linear demands, the left-hand sides of (19) and (20) are decreasing in q , implying that the signs of (22) and (23) suffice to determine whether the low-demand retailer's quantity choice under the profit-maximizing menu of all-units discounts is greater than, less than, or equal to the low-demand retailer's quantity choice under the profit-maximizing menu of two-part tariffs. Since the high-demand retailer's quantity choice is the same for both, we have the following proposition.

PROPOSITION 5: *Suppose consumer demands are linear and $\alpha \neq 1$. Then, if $q_H^*(\lambda_L^{**} w_L^{**}) > \bar{q}_L^{**}$, consumer welfare is higher (lower) with all-units discounts than with two-part tariffs if and only if*

$$2(q_H^*(w_L^*) - q_L^*(w_L^*)) > (<) q_H^*(p_L(q_L^*(w_L^*))). \quad (24)$$

If $q_H^(\lambda_L^{**} w_L^{**}) \leq \bar{q}_L^{**}$, consumer welfare is higher (lower) if and only if*

$$\frac{\partial^2 R_H(q)}{\partial q^2} > (<) \frac{\partial^2 R_L(q)}{\partial q^2}. \quad (25)$$

Proof. With linear demands, (24) follows by substituting $\frac{\partial^2 R_L(q)}{\partial q^2} = 2 \frac{\partial p_L(q)}{\partial q}$ into (22). And, with linear demands, (25) follows by noting that $\frac{\partial^2 R_i(q)}{\partial q^2}$, $i \in \{H, L\}$, are constants in (23). \square

As we now show, consumer welfare can be higher or lower with all-units discounts. To see this, suppose demand in the high and low demand states are sufficiently close that $q_H^*(\lambda_L^{**} w_L^{**}) \leq \bar{q}_L^{**}$. Then (25) implies that consumer welfare is higher (lower) if and only if the slope of the marginal-revenue curve in the high-demand state is larger (smaller) than the slope of the marginal-revenue curve in the low-demand state. The different outcomes are illustrated in Figures 5 and 6.

4.3 EXAMPLE IN WHICH WELFARE IS HIGHER WITH ALL-UNITS DISCOUNTS

Suppose the inverse demands are given by $p_i(q) = a - \frac{q}{\theta_i}$, where $i \in \{H, L\}$ and $\theta_H > \theta_L$, so that when plotted (see Figure 5), the inverse demand is everywhere flatter in the high-demand state. Since this implies that the corresponding marginal-revenue curve is everywhere flatter in the high-demand state, it follows immediately from (25) that, when $q_H^*(\lambda_L^{**} w_L^{**}) \leq \bar{q}_L^{**}$, consumer prices are lower and social welfare is higher with all-units discounts than with two-part tariffs. As we show

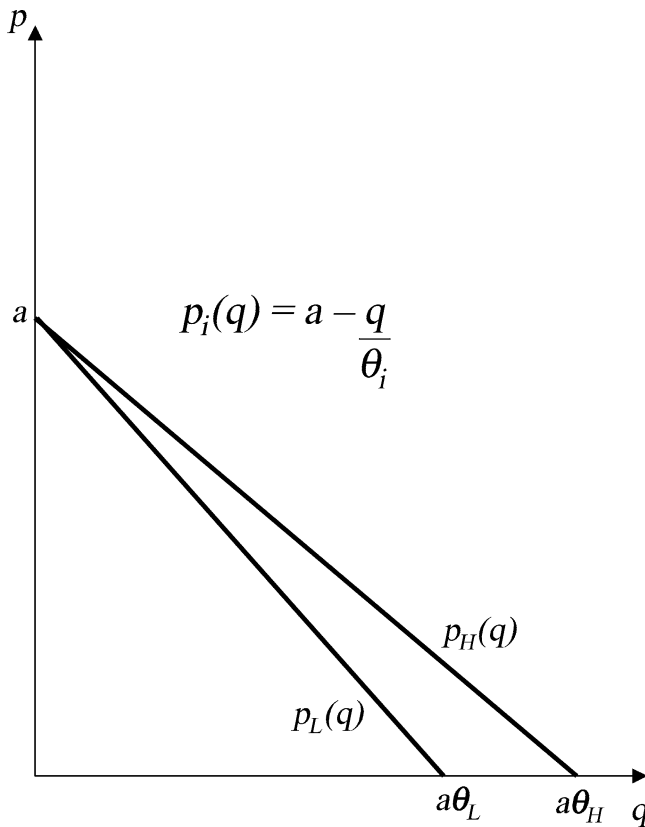


FIGURE 5. WELFARE IS HIGHER WITH ALL-UNITS DISCOUNTS

in the Appendix, a similar conclusion also holds when (24) applies, i.e., when $q_H^*(\lambda_L^{**} w_L^{**}) > \bar{q}_L^{**}$.

This case might arise, for example, when there is uncertainty about the size of the market, but firms have knowledge of the proportion of customers with a given willingness to pay. For example, suppose the manufacturer sells a product in a locale that depends on tourism, which in turn depends on the weather (Tourism is higher when the weather is good and is lower when the weather is bad). Then one can think of the high-demand state as corresponding to an increase in the population size of a market relative to the low-demand state with no change in the underlying distribution of consumer preferences. At the choke price, the quantity demanded in either state is zero, but for any price with positive demand, the high-demand quantity is a multiple of the low-demand

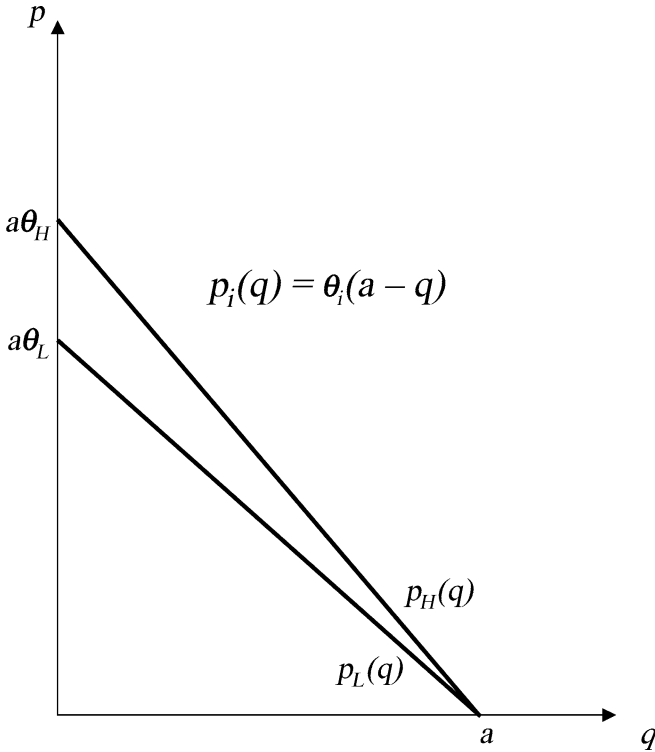


FIGURE 6. WELFARE IS LOWER WITH ALL-UNITS DISCOUNTS

quantity reflecting its larger population size. Our findings imply that in this case the manufacturer's profit with all-units discounts will be higher and so will consumer surplus and social welfare.

4.4 EXAMPLE IN WHICH WELFARE IS LOWER WITH ALL-UNITS DISCOUNTS

Suppose the inverse demands are given by $p_i(q) = \theta_i(a - q_i)$, where $i \in \{H, L\}$ and $\theta_H > \theta_L$, so that when plotted (see Figure 6), the inverse demand is everywhere steeper in the high-demand state. Since this implies that the corresponding marginal-revenue curve is everywhere steeper in the high-demand state, it follows immediately from (25) that, when $q_H^*(\lambda_L^{**}, w_L^{**}) \leq \bar{q}_L^{**}$, consumer prices are higher and social welfare is lower with all-units discounts than with two-part tariffs. As we show in the Appendix, a similar conclusion holds when (24) applies, i.e., when $q_H^*(\lambda_L^{**}, w_L^{**}) > \bar{q}_L^{**}$.

This case might arise, for example, when firms have knowledge of the number of potential customers, but there is uncertainty about the distribution of their willingness to pay. For example, suppose the manufacturer sells a product for which demand depends primarily on consumers' discretionary income. In good times, consumers will have more to spend, while in bad times, consumers will have less to spend. Thus, when the product's price is zero, demand is the same in the two states of nature, but as the price increases, demand is everywhere higher in the state of the world where consumers have higher aggregate incomes. Our findings imply that in this case although the manufacturer's profit will be higher, consumer surplus and social welfare will be lower.

5. CONCLUSION

A discussion of the purpose and competitive effects of all-units discounts until now has been left to policymakers and legal scholars, many of whom view the discounts as exclusionary. On the one hand, it is not surprising that all-units discounts are viewed with suspicion because it seems odd that a manufacturer ever would want to charge less for a larger order if its intention is benign. On the other hand, policymakers and legal scholars have overlooked plausible efficiency rationales.

In this paper, we have shown that all-units discounts can arise in the absence of any exclusionary motive. They can eliminate double marginalization when demand is known at the time of contracting, and they can profitably induce second-degree price discrimination when retailers have private information about demand. The welfare effects of all-units discounts are ambiguous. Compared to linear pricing, all-units discounts lead to lower prices for consumers and are welfare improving. Compared to menus of two-part tariffs and contracts with incremental-units discounts, all-units discounts can lead to higher or lower consumer prices in equilibrium. We showed that with linear demands the latter case occurs when the uncertainty about demand is over market size, while the former case occurs when the uncertainty about demand applies to consumers' willingness to pay.

Although we have assumed there are only two states of nature, this assumption can be relaxed with no change in our qualitative results. Our results extend as long as the number of menu options offered by the manufacturer equal or exceed the number of states of nature. To see this, suppose there are three states of nature, corresponding to high, medium, and low demand. Then, our results suggest that a menu of all-units discounts can do strictly better than a menu of two-part tariffs (implementation and complexity costs aside) because regardless

of the induced quantity choices of the retailer in the low-, medium-, and high-demand states given the optimal menu of two-part tariffs, the manufacturer can do strictly better by offering these same quantities as targets in an all-units contract at prices such that the amount collected from the retailer in the low- and medium- demand states is unchanged and such that the amount collected from the retailer in the high-demand state is increased by a small amount. This method of proof extends to any finite number of states of nature.

As the number of states of nature increases, however, it may be that the relative effectiveness of all-units discounts in reducing the retailer's informational rent decreases. Intuitively, with a finite number of states of nature, there is money on the table the manufacturer can claim by restricting the retailer's choice using all-units discounts. However, with a continuum of states, and assuming that the "hazard rate" of the distribution of types is monotonic, it has been shown that the optimal nonlinear payment schedule can be implemented by letting consumers choose among a continuum of two-part tariffs (see Maskin and Riley, 1984). In this case, it can be shown that the optimal menu of all-units discounts yields the same outcome as the optimal menu of two-part tariffs.¹⁶

The problem we consider is equivalent formally to one of implementing second-degree price discrimination among independent retailers with different demands. For example, the manufacturer may know that it faces retailers of different sizes but, given existing antitrust laws, e.g., the Robinson-Patman Act, it may not be possible directly to offer discriminatory terms to the large and small retailer. However, as many commentators have pointed out, it often is possible to satisfy antitrust laws by offering a menu of options and letting each retailer self select.¹⁷ As long as the various options functionally are available to all, our results suggest that a manufacturer can do better by offering a menu of all-units discounts than by offering a menu of two-part tariffs.

APPENDIX

Proof of Lemma 1. To prove Lemma 1, we must show that (15) and (16) are satisfied with equality at $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$. We proceed by contradiction. Suppose first that $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ are such that (16) is not satisfied with equality. Then it must be that the profit the

16. As pointed out by a referee, when there is a continuum of states, but with a finite number of peaks and little mass in between, the hazard rate is not monotonic, and the superiority of all-units discounts is established once again.

17. For a comprehensive legal treatment of this issue in the United States see American Bar Association Antitrust Section, 1997.

manufacturer earns in the high-demand state, $T_H(q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})) - cq_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$, is less than Ω_H , where $\Omega_H \equiv R_H(q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})) - cq_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) - R_H(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) + T_L(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}))$.

To show the manufacturer has a profitable deviation, consider an alternative menu of contracts $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(p_H(0), \hat{\lambda}_H, q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}))$, where

$$\hat{\lambda}_H = \frac{R_H(q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})) - R_H(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) + T_L(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}))}{p_H(0)q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})}. \quad (\text{A.1})$$

With these contracts, there is no change in the manufacturer's profit in the low-demand state and the constraint in (15) is satisfied (The option meant for the low-demand retailer is unchanged). To see that the constraint in (16) is satisfied, note that because $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ is the same the right-hand side is unchanged, and because the retailer always can choose $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$, the left-hand side must be at least as large as $R_H(q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})) - p_H(0)\hat{\lambda}_H q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$. It follows that the constraint in (16) is satisfied and thus that the manufacturer's profit in the high-demand state is $(p_H(0)\hat{\lambda}_H - c)q = \Omega_H \frac{q}{q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})} \geq \Omega_H$, for all $q \geq q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$. Since the high-demand retailer never would purchase less than $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ under the new menu of contracts (because its per-unit price of $p_H(0)$ would be prohibitively high), it follows that the manufacturer will earn profit of at least Ω_H in the high-demand state. Thus, the deviation is profitable, contradicting the supposition that $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ are such that (16) is not satisfied with equality.

Now suppose $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ are such that (15) is not satisfied with equality. Then it must be that the profit the manufacturer earns in the low-demand state, $T_L(q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) - cq_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, is less than Ω_L , where $\Omega_L \equiv R_L(q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) - cq_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$.

To show the manufacturer has a profitable deviation, consider an alternative menu of contracts $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$, $(p_H(0), \hat{\lambda}_L, q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}))$, where

$$\hat{\lambda}_L = \frac{R_L(q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}))}{p_H(0)q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})}. \quad (\text{A.2})$$

With these contracts, there is no change in the manufacturer's profit in the high-demand state and the constraint in (16) is satisfied (The menu meant for the high-demand retailer is unchanged, and it is now more costly to mimic the low-demand retailer). To see that the

constraint in (15) is satisfied, note that because the retailer always can choose $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, the left-hand side must be at least as large as $R_L(q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) - p_H(0)\hat{\lambda}_L q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$. It follows that the constraint in (15) is satisfied and thus that the manufacturer's profit in the low-demand state is $(p_H(0)\hat{\lambda}_L - c)q = \Omega_L \frac{q}{q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})} \geq \Omega_L$, for all $q \geq q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$. Since the low-demand retailer never would purchase less than $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ under the new menu of contracts (because its per-unit price of $p_H(0)$ would be prohibitively high), it follows that the manufacturer will earn profit of at least Ω_L in the low-demand state. Thus, the deviation is profitable, contradicting the supposition that $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ are such that (15) is not satisfied with equality. \square

Proof of Lemma 2. We know from Lemma 1 that (15) must be satisfied with equality at $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$. This implies that $w_L^{**} \geq p_L(0)$ because otherwise a retailer in the low-demand state could earn positive profit by purchasing $q < \bar{q}_L^{**}$ for some q . Given $w_L^{**} \geq p_L(0)$, Lemma 1 also implies that $\lambda_L^{**} w_L^{**} = p_L(\bar{q}_L^{**})$ because if $\lambda_L^{**} w_L^{**} < p_L(\bar{q}_L^{**})$, a retailer in the low-demand state could earn positive profit by purchasing $q = \bar{q}_L^{**}$, and if $\lambda_L^{**} w_L^{**} > p_L(\bar{q}_L^{**})$, a retailer in the low-demand state would earn negative profit for all $q > 0$. Given $w_L^{**} \geq p_L(0)$ and $\lambda_L^{**} w_L^{**} = p_L(\bar{q}_L^{**})$, it follows that the retailer chooses $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}) = \bar{q}_L^{**}$ because for all other $q > 0$ it would earn negative profit.

To finish the proof, it remains only to establish that $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = \bar{q}_H^{**}$. We know from Lemma 1 that (16) must be satisfied with equality at $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, $(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$. This implies that w_H^{**} must be such that $\max_{q < \bar{q}_H^{**}} R_H(q) - w_H^{**}q \leq K$, where K is the right-hand side of (16) evaluated at $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$, because otherwise a retailer in the high-demand state would strictly prefer to purchase $q < \bar{q}_H^{**}$ for some q under contract $T_H(q)$ than purchase under contract $T_L(q)$. Given w_H^{**} , Lemma 1 also implies that $\lambda_H^{**} w_H^{**}$ must be such that $\max_{q \geq \bar{q}_H^{**}} R_H(q) - \lambda_H^{**} w_H^{**}q \leq K$ because otherwise a retailer in the high-demand state would strictly prefer to purchase $q = \max\{\bar{q}_H^{**}, q_H^*(\lambda_H^{**} w_H^{**})\}$ under contract $T_H(q)$ than purchase under contract $T_L(q)$. Finally, Lemma 1 implies that at least one of these two inequalities must be satisfied with equality because otherwise a high-demand retailer would strictly prefer to purchase under contract $T_L(q)$. It follows that $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) \in \{q_H^*(w_H^{**}), \max\{\bar{q}_H^{**}, q_H^*(\lambda_H^{**} w_H^{**})\}\}$ if both inequalities are satisfied with equality, $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = q_H^*(w_H^{**})$ if only the first inequality is satisfied with equality, and $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = \max\{\bar{q}_H^{**}, q_H^*(\lambda_H^{**} w_H^{**})\}$ if only the second inequality is satisfied with equality.

To narrow the possibilities, suppose $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = q_H^*(w_H^{**})$. Then the manufacturer earns profit $w_H^{**} q_H^*(w_H^{**}) - c q_H^*(w_H^{**}) = R_H(q_H^*$

$(w_H^{**}) - cq_H^*(w_H^{**}) - K$ in the high-demand state. Since $q_H^*(w_H^{**}) < q_H^*(c)$ because $w_H^{**} > c$, this profit is less than $R_H(q_H^*(c)) - cq_H^*(c) - K$, the profit it could earn in the high-demand state by choosing w_H such that $\max_{q < \bar{q}_H} R_H(q) - w_H q < K$, $\lambda_H w_H$ such that $R_H(q_H^*(c)) - \lambda_H w_H q_H^*(c) = K$, and $\bar{q}_H = q_H^*(c)$. Thus, there is a profitable deviation, contradicting the supposition that $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = q_H^*(w_H^{**})$. Suppose $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = q_H^*(\lambda_H^{**} w_H^{**})$. Then, since $\lambda_H^{**} w_H^{**} > c$, implying that $q_H^*(\lambda_H^{**} w_H^{**}) < q_H^*(c)$, the same argument also establishes the existence of a profitable deviation. Thus, it must be that $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = \bar{q}_H^{**}$. \square

Proof of Proposition 3. To prove Proposition 3, we must show that $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = q_H^*(c)$. The proof is by contradiction. Using Lemma 1, we can write the manufacturer’s maximized profit as

$$\begin{aligned} & \alpha(R_L(q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) - cq_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) \\ & + (1 - \alpha)(R_H(q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})) - cq_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})) \\ & - (1 - \alpha)(R_H(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) - T_L(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}))). \end{aligned} \tag{A.3}$$

Suppose $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) \neq q_H^*(c)$. Then, since $q_H^*(c) = \arg \max_q R_H(q) - cq$, it must be that the manufacturer’s profit in (A.3) is less than $\tilde{\Omega}_H$, where

$$\begin{aligned} \tilde{\Omega}_H \equiv & \alpha(R_L(q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) - cq_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) \\ & + (1 - \alpha)(R_H(q_H^*(c)) - cq_H^*(c)) \\ & - (1 - \alpha)(R_H(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) - T_L(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}))). \end{aligned} \tag{A.4}$$

To show the manufacturer has a profitable deviation, consider an alternative menu of contracts $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}), (p_H(0), \tilde{\lambda}_H, q_H^*(c))$, where

$$\tilde{\lambda}_H = \frac{R_H(q_H^*(c)) - R_H(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})) + T_L(q_H^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}))}{p_H(0)q_H^*(c)}. \tag{A.5}$$

With these contracts, there is no change in the manufacturer’s profit in the low-demand state and the constraint in (15) is satisfied (the option meant for the low-demand retailer is unchanged). Also, because $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ is the same, the right-hand side of (16) is unchanged, and because the retailer always can choose $q_H^*(c)$, the left-hand side must be at least as large as $R_H(q_H^*(c)) - p_H(0)\tilde{\lambda}_H q_H^*(c)$. It follows that the constraint in (16) is satisfied and thus that the manufacturer’s profit in the high-demand state is $(p_H(0)\tilde{\lambda}_H - c)q$, for all $q \geq q_H^*(c)$. Since the high-demand retailer never will purchase less than $q_H^*(c)$ under the new menu of contracts (because its per-unit price of $p_H(0)$ would be prohibitively high), it follows that the sum of the manufacturer’s profit in the low-demand state and high demand state will be at least $\tilde{\Omega}_H$. Thus, the deviation is profitable, contradicting the supposition that $(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}), (w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**})$ are such that $q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) \neq q_H^*(c)$. \square

Proof of Proposition 4. To prove Proposition 4, we begin by considering the high-demand retailer’s quantity choice if it were to purchase under the contract meant for the low-demand retailer. Since $w_L \geq p_H(0)$ is optimal, the high-demand retailer never will purchase $q < \bar{q}_L$.¹⁸ This means that the high-demand retailer will either choose $q = \bar{q}_L$ if the threshold is binding, or $q > \bar{q}_L$ if the threshold is not binding. If the threshold is not binding, then the high-demand retailer will choose $q_H^{**}(w_L, \lambda_L, \bar{q}_L) = q_H^*(p_L(\bar{q}_L))$. Substituting the retailer’s quantity choice into (17), we can write the manufacturer’s problem as

$$\max_{\bar{q}_L} \alpha(R_L(\bar{q}_L) - c\bar{q}_L) - (1 - \alpha)(R_H(q_H^*(p_L(\bar{q}_L))) - p_L(\bar{q}_L)q_H^*(p_L(\bar{q}_L))). \tag{A.6}$$

Differentiating (A.6) with respect to \bar{q}_L , and using the envelope theorem, we have that \bar{q}_L^{**} solves the first-order condition in (19). If the threshold binds, then the high-demand retailer will choose $q_H^{**}(w_L, \lambda_L, \bar{q}_L) = \bar{q}_L$. Substituting this into (17), we can write the manufacturer’s problem as

$$\max_{\bar{q}_L} \alpha(R_L(\bar{q}_L) - c\bar{q}_L) - (1 - \alpha)(R_H(\bar{q}_L) - p_L(\bar{q}_L)\bar{q}_L). \tag{A.7}$$

Differentiating (A.7) with respect to \bar{q}_L , we have that \bar{q}_L^{**} solves the first-order condition in (20). We conclude the proof by noting that Lemma 2 implies that $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}) = \bar{q}_L^{**}$. \square

A.1 EXAMPLE IN WHICH WELFARE IS HIGHER WITH ALL-UNITS DISCOUNTS

Suppose the inverse demands are given by $p_i(q) = a - \frac{q}{\theta_i}$, where $i \in \{H, L\}$ and $\theta_H > \theta_L$, so that when plotted (see Figure 5), the inverse demand is everywhere flatter in the high-demand state. Solving for $q_H^*(\cdot), q_L^*(\cdot), \frac{\partial R_i(\cdot)}{\partial q}$, and $\frac{\partial q_i^*(\cdot)}{\partial q}$, and substituting these expressions into (9) and (10), we obtain the retailer’s quantity choices under the profit-maximizing menu of two-part tariffs:

$$q_H^*(c) = \frac{\theta_H(a - c)}{2}; \quad q_L^*(w_L^*) = \frac{\theta_L^2 \alpha(a - c)}{2[(1 - \alpha)\theta_H - (1 - 2\alpha)\theta_L]}.$$

In the case of all-units discounts, we will have different solutions for the two subcases that may arise. If $q_H^*(\lambda_L^{**}w_L^{**}) > \bar{q}_L^{**}$, then we can use Proposition 4 and condition (19) to solve for the retailer’s quantity choices under the profit-maximizing menu of all-units

18. Note that w_L affects the maximand in (17) only through the term $R_H(q_H^{**}(w_L, \lambda_L, \bar{q}_L)) - T_L(q_H^{**}(w_L, \lambda_L, \bar{q}_L))$. Since the maximand is decreasing in this term, and since this term is decreasing weakly in w_L , it follows that the manufacturer will choose w_L such that a high-demand retailer never would purchase $q < \bar{q}_L$, e.g., $w_L \geq p_H(0)$.

discounts:

$$q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = \frac{\theta_H(a - c)}{2}; \quad q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}) = \frac{\theta_L^2 \alpha (a - c)}{2[\alpha \theta_L + \frac{(1-\alpha)}{4} \theta_H]}.$$

In this subcase, the high-demand retailer’s quantity choice is the same in both cases (as expected), while the low-demand retailer’s quantity choice is higher if and only if $\theta_H > \frac{4}{3} \theta_L$. Since this subcase arises if and only if $\theta_H > 2\theta_L > \frac{4}{3} \theta_L$, it follows that welfare is higher with all-units discounts.

If $q_H^*(\lambda_L^{**} w_L^{**}) \leq \bar{q}_L^{**}$, then we can use Proposition 4 and condition (20) to solve for the retailer’s quantity choices under the profit-maximizing menu of all-units discounts:

$$q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) = \frac{\theta_H(a - c)}{2}; \quad q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}) = \frac{\theta_H \theta_L \alpha (a - c)}{2[\theta_H - (1 - \alpha) \theta_L]}.$$

Comparing $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ and $q_L^*(w_L^*)$, it can be shown that $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ is higher than $q_L^*(w_L^*)$ for all $\theta_H \leq 2\theta_L$, implying that, once again, welfare is higher with all-units discounts.

A.2 EXAMPLE IN WHICH WELFARE IS LOWER WITH ALL-UNITS DISCOUNTS

Suppose the inverse demands are given by $p_i(q) = \theta_i(a - q_i)$, where $i \in \{H, L\}$, and $\theta_H > \theta_L$, so that when plotted (see Figure 6), the inverse demand is everywhere steeper in the high-demand state. Solving for $q_H^*(\cdot)$, $q_L^*(\cdot)$, $\frac{\partial R_i(\cdot)}{\partial q}$, and $\frac{\partial q_i^*(\cdot)}{\partial q}$, and substituting these expressions into (9) and (10), we obtain the retailer’s quantity choices under the profit-maximizing menu of two-part tariffs:

$$q_H^*(c) = \frac{1}{2} \left(a - \frac{c}{\theta_H} \right); \quad q_L^*(w_L^*) = \frac{1}{2} \left(a - \frac{\alpha \theta_H c}{\theta_L [(1 - \alpha) \theta_L - (1 - 2\alpha) \theta_H]} \right).$$

In the case of all-units discounts, we will have different solutions for the two subcases that may arise. If $q_H^*(\lambda_L^{**} w_L^{**}) > \bar{q}_L^{**}$, then we can use Proposition 4 and condition (19) to solve for the retailer’s quantity choices under the profit-maximizing menu of all-units discounts:

$$\begin{aligned} q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) &= \frac{1}{2} \left(a - \frac{c}{\theta_H} \right); \quad q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}) \\ &= \frac{a \theta_L [(3\alpha - 1) \theta_H + (1 - \alpha) \theta_L] - 2\alpha \theta_H c}{\theta_L [4\alpha \theta_H + (1 - \alpha) \theta_L]}. \end{aligned}$$

In this subcase, the high-demand retailer's quantity choice is the same in both cases (as expected), while the low-demand retailer's quantity choice is higher if and only if

$$\frac{a}{c} < \frac{\alpha(2\theta_H - \theta_L)}{\theta_L(\theta_L - (1 - \alpha)\theta_H)}.$$

If this condition holds, then welfare is lower with lower with all-units discounts.

If $q_H^*(\lambda_L^{**}, w_L^{**}) \leq \bar{q}^{**}$, then we can use Proposition 4 and condition (20) to solve for the retailer's quantity choices under the profit-maximizing menu of all-units discounts:

$$\begin{aligned} q_H^{**}(w_H^{**}, \lambda_H^{**}, \bar{q}_H^{**}) &= \frac{1}{2} \left(a - \frac{c}{\theta_H} \right); \quad q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**}) \\ &= \frac{1}{2} \left(a - \frac{\alpha c}{\theta_L - (1 - \alpha)\theta_H} \right). \end{aligned}$$

Comparing $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ and $q_L^*(w_L^*)$, it can be shown that $q_L^{**}(w_L^{**}, \lambda_L^{**}, \bar{q}_L^{**})$ is always lower than $q_L^*(w_L^*)$, implying that, once again, welfare is lower with all-units discounts.

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