Rent Shifting, Exclusion, and Market-Share Discounts

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Abstract

We study the economics of rent-shifting using a three-party sequential contracting environment in which two sellers negotiate with a common buyer. We find that overall joint payoff is maximized in this environment under a variety of plausible restrictions on contracts, even though, in equilibrium, surplus extraction may not be complete. The division of surplus depends on the set of feasible contracts, the relationship between the sellers' products, and the bargaining power of the firms. We use the model to evaluate the distributional consequences and welfare effects of legal restrictions on below-cost pricing and market-share discounts. We find that such restrictions may actually induce exclusion in the long run if the buyer is able to commit to purchase from a single seller.

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1 Introduction

In traditional price theory, buyers are price takers. They choose whether and how much to purchase, taking the price as given. In many settings, this is a reasonable approximation. For example, we are all familiar with buying groceries at the local supermarket. In other settings, however, the approximation is not a good one. For example, Wal-Mart and Toys "R" Us often negotiate their terms of trade when they purchase from manufacturers.

Settings in which the terms of trade are negotiated often occur in intermediate-goods markets, where buyers and sellers jointly participate in creating value for the end user. This creates a tension that does not arise in traditional price theory: individual buyers and sellers are partners in creating value, but adversaries in determining how the surplus is apportioned. When a single buyer negotiates a contract with a single seller, the terms of trade play dual roles. They determine how much overall value is created and how that value is divided. Furthermore, if, as in most intermediate-goods settings, a buyer negotiates contracts with multiple sellers whose payoffs are, or can be made, interrelated, then the terms of trade also play another role: they affect the buyer's future negotiations with all other sellers.

In this paper, we study the economics of rent shifting, where a buyer and one seller choose the terms of their contract with an eye towards extracting surplus from a second seller. One way for them to do this is with a schedule of quantity discounts. Whereas traditional price theory might emphasize how such discounts encourage a buyer to purchase the efficient quantity of a seller's product and divide the surplus, these discounts, if structured appropriately, may also help the buyer in its subsequent negotiations with other sellers. For example, a retailer who can purchase additional quantities of Pepsi from PepsiCo, a maker of soft drinks, at discounted prices, will be in a better position to negotiate more favorable terms of trade from the Coca Cola, Co., a rival maker of soft drinks, than one who cannot.[1]

Examples of rent shifting abound and are not limited to quantity discounts. Any con-

[1] Sellers often claim to be harmed by their rivals' quantity discounts. For example, Anti-Monopoly, Inc. argued in *Anti-Monopoly, Inc. v. Hasbro* that, when selling to Toys "R" Us, it was disadvantaged by Hasbro's practice of offering volume discounts because its sales would "cut into TRU's sales of Hasbro products, which will reduce the percentage of TRU's volume discount," 958 F. Supp. 895 at 901 (S.D.N.Y.), 1997.

tractual provision between a buyer and seller that increases the buyer's opportunity cost of purchasing from other sellers can jointly increase the buyer and first seller's payoff. One can interpret Philip Morris' Retail Leaders program, in which retailers were offered increasingly generous discounts based on the percentage of shelf space they gave to Philip Morris' products, in this context. Under the program, surplus was effectively transferred from the rival manufacturers to the retailer and Philip Morris.[2] Other companies offer discounts that are explicitly tied to the sales of its rivals' products. With market-share discounts, for instance, a buyer's purchases in a given product category over a period of time are totaled and its discount is determined by what percentage of these purchases came from the manufacturer's product. The less the retailer buys from other manufacturers, the larger will be its discount.[3]

It is our view that rent-shifting contracts have often been misunderstood, particularly when they are used by dominant sellers against smaller rivals. Antitrust challenges to contracts that exhibit generous quantity discounts and/or contracts that are based on market shares typically claim that their intent is exclusionary; the dominant seller (with the buyer's acquiescence) is seeking to drive its rival out of the market.[4] But this view runs counter to the essence of rent-shifting. Under rent-shifting, you want the rival seller to be in the market in order to capture the surplus created by the sales of its product. In contrast, if the dominant seller were to induce its rival to exit, there would be no rents to shift.

Under the rent-shifting view, contracts that exhibit below-cost pricing off the equilibrium path may be required to extract the rival seller's surplus. However, under antitrust law, offers to sell at below-cost prices are considered to be predatory and thus illegal.[5] Under the rent-

[2] See *R.J. Reynolds Tobacco Co. v. Philip Morris, Inc.*, Civ. No. 1:99CV00185 (M.D.N.C. May 1, 2002), in which the federal district court granted summary judgement against R.J. Reynolds. The court noted that the Retail Leaders program was successful in that it forced R.J. Reynolds and other competitors to respond by increasing their own promotional discounts and merchandising payments to retailers. However, the court found no evidence that the Retail Leaders program caused rival manufacturers to lose market share.

[3] See Mills (2004) for examples of companies that use market-share discounts. The fear in antitrust is that these discounts may harm competition. However, our approach is to look at market-share discounts as a means of shifting rents from rival sellers. Similarly, Mills develops a non-foreclosure based model of market-share discounts. In his model, a dominant firm uses them to induce brand-specific retail services.

[4] For example, it was alleged by R.J. Reynolds, Lorillard, and Brown & Williamson that Philip Morris' Retail Leaders program was an attempt at monopolizing cigarette sales through retail outlets.

[5] The court ruled against *Anti-Monopoly* (see previous footnote) stating that an antitrust plaintiff cannot argue that its competitor's prices are too low unless it can prove that the prices are below-cost. *Id.* at 905.

shifting view, contracts that are contingent on the buyer's purchases of a rival seller's product promote the extraction of surplus when below-cost pricing alone does not suffice. However, under antitrust law, such contracts are subject to a rule-of-reason analysis and may be illegal if the seller is found to have market power (see Tom, Balto, and Averitt, 2000).

These differences matter. When rent-shifting is unconstrained, we know from Aghion and Bolton (1987) that a buyer and seller can extract all the surplus from a second seller when there is complete information. In this case, inefficient exclusion does not occur. However, when rent-shifting is constrained, e.g., when below-cost pricing and market-share discounts are not allowed, full extraction will often not be possible. Ironically, in this case, inefficient exclusion may occur. We show this in the context of a buyer who can capture the value to each seller from moving first. When the full complement of rent-shifting contracts is feasible, the buyer is able to capture all the surplus available from sales of the two sellers' products. When the means of rent-shifting are restricted, however, the buyer will sometimes find it optimal to adopt a "second-best strategy" of committing ex-ante to buy from only one seller. This is inefficient when the seller's products are imperfect substitutes. The irony is that instead of reducing the incidence of exclusion, antitrust law may be exacerbating it.

The use of contracts by sellers who are first-movers in negotiating with a buyer to extract surplus from sellers who are second-movers was first studied by Aghion and Bolton (1987). In a model in which the buyer purchases at most one unit from one seller, they show that, with complete information, the buyer and first seller can extract all the surplus from the second seller by agreeing to an exclusive-dealing contract in which the buyer must pay a lump-sum penalty to the first seller if it buys from the second seller.[6] However, their model is limited in two key ways. First, they allow the sellers to have all the bargaining power, which greatly improves the ability of the buyer and first seller to extract all the surplus from the second seller. Second, their assumption that the buyer can purchase at most one unit means that contracts that penalize the buyer for purchasing one unit of the second seller's product are equivalent to contracts that reward the buyer for purchasing one unit of the

[6]Exclusion and rent-shifting are linked in their model, but only in the case of incomplete information, when the buyer and first seller overestimate the surplus that can be extracted from the second seller.

first seller's product. This makes it difficult to disentangle the role played by contracts that are contingent on the rival seller's market-share (of which exclusive dealing represents an extreme) and contracts that rely solely on individual-seller quantity discounts.

We study the simplest multiple player, sequential contracting environment that captures the key ingredients of rent shifting: there are three players (a buyer and two sellers), two bilateral negotiations, and interdependencies between the sellers' payoffs. We assume complete information and focus on the effect of legal restrictions on contracts. To study market-share discounts, we allow for contracts that can depend in a general way on both sellers' quantities. To study the effectiveness of quantity discounts only, we also consider contracts that are restricted to depend only on a seller's own quantity. Unlike in Aghion and Bolton's model, however, we allow for continuous quantities, general cost functions, trade with one or both sellers, any interactions (any manner of substitution or complementarity) among the units sold by the sellers, and any distribution of bargaining power among the contracting parties.[7]

Our first main result is that the ability of the buyer and first seller to shift rents from the second seller depends on the contracting environment. It depends on the distribution of bargaining power, with surplus extraction increasing in the first and second seller's bargaining power. It depends on the relationship between the sellers' products, with greater surplus extraction when the products are substitutes than when they are complements. And it depends on the feasible set of contracts, with surplus extraction often being strictly greater when the buyer and first seller can negotiate discounts that can depend on both sellers' quantities than when they cannot, and when below-cost pricing is feasible than when it is not.

Our second main result is that overall joint payoff is maximized in all non-Pareto dominated equilibria for all contracting environments. Thus, the rent-shifting methods that arise in our model do not distort the buyer's equilibrium quantity choices; the buyer will choose

[7] Segal (1999) considers a general contracting environment in which the common player makes simultaneous take-it-or-leave-it offers to multiple agents. The main difference between our paper's setting and his lies in the sequential nature of our contracting, which introduces a new "pecuniary" externality in that the second seller's equilibrium payoff depends on the prior contract between the buyer and first seller.

the same quantities that a fully-integrated monopolist would choose.[8] This result holds for any distribution of bargaining power, any relationship between the sellers' products, and whether or not below-cost pricing and contracts with market-share discounts are feasible.

We can draw policy implications from these results because they allow us to consider the effects of different legal environments on the distribution of payoffs and consumer surplus. Pepsi-Co's recent acquisition of Gatorade, a non-carbonated sports drink, provides a case in point. Our results lend support to Coca Cola's claim that it is harmed by the acquisition. If one believes that PepsiCo's acquisition gives it more bargaining power with retailers than it had before the acquisition, then our theory of rent shifting suggests that Coca Cola has reason to worry: PepsiCo will be better off as a result of the merger and Coca Cola will be worse off. However, our result that overall joint payoff is unaffected by rent shifting implies that there will be no effect on the prices consumers pay, so while Coca Cola may lose, the theory predicts that the merger will not harm competition. Our results also suggest that contracts based on market shares, and contracts with below-cost prices, need not raise rivals' costs or drive them out of business. To the contrary, we show that antitrust laws against such practices may, for some environments, be inducing exclusion instead of preventing it.[9]

In related literature, Spier and Whinston (1995) add an investment stage to Aghion and Bolton's (1987) model of incomplete information and find that inefficient exclusion occurs even when the initial contract can be renegotiated. In their model, the first seller overinvests in cost-reduction, after the initial contract is written but before the second seller makes its offer, in an attempt to extract more surplus from the second seller. In our model, the buyer sometimes wants to 'underinvest in shelf space' as a means of extracting more surplus.

Gans and King (2002) consider a model with two upstream firms, which have decreasing average production costs, and multiple large and small buyers. In equilibrium, one upstream firm offers below-cost contracts to the large buyers, which allows it to extract greater surplus

[8] In contrast, it has been shown in an environment with one seller and two buyers that the seller is often unable to commit not to engage in rent shifting, reducing the overall joint payoff and thus preventing overall joint payoff from being maximized (O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994; Segal, 1999).

[9] This suggests that antitrust intervention against such practices may be unwarranted. Mills (2004) reaches a similar policy conclusion about market-share discounts, even when they are adopted by dominant sellers.

from small buyers and denies its upstream competitor the ability to achieve minimum efficient scale. Our results do not rely on decreasing average costs for the upstream firm.

The rest of the paper is organized as follows. We describe the model in Section 2. In Section 3, we offer some preliminary results on the role of market-share contracts and below-cost pricing in facilitating rent shifting. In Section 4, we solve the model under different contracting environments and show that overall joint payoff is maximized in all non-Pareto dominated equilibria. In Section 5, we show that the buyer might want to commit to carrying only one product if market-share contracts are infeasible. In Section 6 we conclude.

2 Model

We consider a sequential contracting environment with complete information in which there are two sellers, X and Y, and a single buyer. Seller X sells product X and seller Y sells product Y. If the buyer purchases quantity x of seller X's product and quantity y of seller Y's product, then sellers X and Y incur costs $c_X(x)$ and $c_Y(y)$, respectively. We assume that $c_X(\cdot)$ and $c_Y(\cdot)$ are strictly increasing, continuous, and unbounded, with $c_X(0) = c_Y(0) = 0$.[10]

The model has three stages. In stage one, the buyer and seller X negotiate a contract T_X for the purchase of product X. In stage two, the buyer and seller Y negotiate a contract T_Y for the purchase of product Y. In stage three, the buyer makes its quantity choices and pays the sellers according to contracts T_X and T_Y. We consider environments in which the sellers are prohibited from selling their products at below-cost prices, and we consider environments in which there is no legal prohibition of below-cost pricing. We also consider cases in which contracts can depend on both sellers' quantities (i.e., market-share contracts are feasible), and we consider cases in which contracts can depend only on a seller's own quantity.

We let Ω denote the set of contracts, where $\Omega \in \{\Omega^0, \Omega^M, \Omega^I\}$. Set Ω^0 is our base case with no restrictions. In this case, T_X specifies a payment from the buyer to seller X as a

[10] Our results hold equally if sellers X and Y each sell multiple products and x and y are vectors, or if sellers X and Y each sell a single product and x and y are scalars.

function of the quantities (x, y) purchased by the buyer:

$$\Omega^0 \equiv \{T_X : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R} \cup \{\infty\}\}.$$

We allow the contract to specify payments in $\mathbb{R} \cup \{\infty\}$, but given later boundedness assumptions, a range that includes large finite values suffices. Contracts in Ω^M and Ω^I do not allow below-cost pricing, but differ as to whether a seller's contract can depend on both sellers' quantities (these are the multi-seller contracts of Ω^M) or whether a seller's contract can depend only on its own quantity (these are the individual-seller contracts of Ω^I). Thus,

$$\Omega^M \equiv \{T_X \in \Omega^0 \mid T_X(x, y) \geq c_X(x) \quad \forall x, y\}$$

and

$$\Omega^I \equiv \left\{T_X \in \Omega^M \mid T_X(x, y) = T_X(x, y') \ \forall y, y' \geq 0\right\}.$$

We make similar assumptions for contract T_Y; however, because T_Y is negotiated after T_X, rent-shifting outcomes are unaffected by whether T_Y depends on both quantities x and y or only on the quantity of y chosen by the buyer (see Lemma 1 below).

If a seller does not have a contract with the buyer, the seller's net payoff is zero. If a seller does have a contract with the buyer, and the buyer purchases the quantities (x, y), then seller X's net payoff is $T_X(x, y) - c_X(x)$ and seller Y's net payoff is $T_Y(x, y) - c_Y(y)$.

Let $R(x, y)$ denote the buyer's maximized gross payoff if it purchases the quantities (x, y).[11] Then the buyer's net payoff is $R(x, y) - T_X(x, y) - T_Y(x, y)$. If negotiations with seller Y fail, the buyer's net payoff is $R(x, 0) - T_X(x, 0)$. If negotiations with seller X fail, the buyer's net payoff is $R(0, y) - T_Y(0, y)$. If negotiations with both sellers fail, the buyer's net payoff is zero. We assume that $R(\cdot, \cdot)$ is continuous and bounded, with $R(0, 0) = 0$.

Let $\Pi(x, y) \equiv R(x, y) - c_X(x) - c_Y(y)$ denote overall joint payoff, $\Pi_{XY} \equiv \max_{x,y \geq 0} \Pi(x, y)$ denote its maximized value, and $Q_{XY} \equiv \arg\max_{x,y \geq 0} \Pi(x, y)$ denote the set of maximizing quantity pairs. Similarly, denote the monopoly value and quantities for the buyer and seller X by $\Pi_X \equiv \max_{x \geq 0} \Pi(x, 0)$ and $Q_X \equiv \arg\max_{x \geq 0} \Pi(x, 0)$, and the monopoly value and

[11] We do not force the buyer to use all that it purchases. Thus, we have $R(x, y) = \max_{x', y'} \tilde{R}(x', y')$, where $0 \leq x' \leq x$ and $0 \leq y' \leq y$ and $\tilde{R}(x', y')$ denotes the buyer's utility (revenue) if it consumes (resells) (x', y').

quantities for the buyer and seller Y by $\Pi_Y \equiv \max_{y\geq 0} \Pi(0,y)$ and $Q_Y \equiv \arg\max_{y\geq 0} \Pi(0,y)$. Given our assumptions on $R(\cdot,\cdot)$ and $c_i(\cdot)$, these values and quantities are well defined.

In the negotiation between the buyer and seller i, we assume that the two players choose T_i to maximize their joint payoff, and that the division of surplus is such that each player receives its disagreement payoff plus a share of the incremental gains from trade (the joint payoff of the buyer and seller i if they trade minus their joint payoff if negotiations fail), with proportion $\lambda_i \in [0,1]$ going to seller i.[12] Our assumption of a fixed division of the gains from trade admits several interpretations. For example, if seller i makes a take-it-or-leave-it offer to the buyer, then $\lambda_i = 1$. If the buyer makes a take-it-or-leave-it offer to seller i, then $\lambda_i = 0$. And if the buyer and seller i split the gains from trade equally, then $\lambda_i = \frac{1}{2}$.

We solve for the equilibrium strategies (we restrict attention to pure strategies) of the buyer and sellers X and Y by working backwards, taking our assumptions about the outcome of negotiations as given. The equilibrium we identify corresponds to the subgame-perfect equilibrium of the related three-stage game in which the assumed bargaining solution is embedded in the players' payoff functions. For subgame perfection, we must restrict attention to contracts such that optimal quantity choices for the buyer in stage three exist.

3 Preliminary Results

To gain some intuition, we start by considering the simplest case, a multiple-units extension of Aghion and Bolton's (1987) model with complete information in which both sellers can make take-it-or-leave-it offers and in which overall joint payoff is maximized when only product Y is sold ($\Pi_{XY} = \Pi_Y > \Pi_X$). In this case, the inefficient seller (seller X) and buyer can extract all the surplus by agreeing to a contract that penalizes the buyer if it purchases

[12] These assumptions are consistent with most commonly used bargaining solutions, which typically require players to maximize their bilateral joint payoffs and divide the incremental gains from trade. For example, the bargaining solutions proposed in Nash (1953) and Kalai-Smorodinsky (1975) satisfy these conditions. However, the bargaining solution proposed in Binmore, Shaked, and Sutton (1989) does not since the additional surplus above the two players' disagreement payoffs is not always divided in fixed proportion.

from seller Y. To see this, suppose seller X offers, and the buyer accepts, the contract:

$$T_X(x,y) = \begin{cases} c_X(x) + \Pi_Y, & \text{if } y > 0 \\ c_X(x) + \Pi_X, & \text{if } y = 0. \end{cases} \tag{1}$$

Then all of the gains from trade between the buyer and seller Y are extracted by seller X : given T_X, the joint payoff of the buyer and seller Y if the buyer purchases from seller Y is $\max_{x\geq 0, y>0} \Pi(x,y) - c_X(x) - c_Y(y) - \Pi_Y = 0$, which implies that it is optimal for seller Y to offer the buyer the contract $T_Y(x,y) = c_Y(y)$ in stage two (the buyer rejects Y's offer rather than pay more). Given T_X and T_Y, the buyer purchases from seller Y, the buyer and seller Y's payoff is zero, and seller X's payoff is Π_Y. Seller X has no incentive to offer any other contract, since the contract in (1) extracts all the surplus, and the buyer has no incentive to reject seller X's offer, since then seller Y would offer $T_Y(x,y) = c_Y(y) + \Pi_Y$ and seller Y would extract all the surplus. Thus, the contract in (1) is an equilibrium contract.

In this example, seller X extracts all the surplus in equilibrium, leaving none for seller Y or the buyer. On the other hand, if seller Y were to make the first offer, then seller Y would extract all the surplus in equilibrium. In both cases the seller that moves first gets Π_Y and the seller that moves second gets zero. Since the buyer plays such a critical role in facilitating the transfer of surplus, one might think that the buyer should be able to capture some of the surplus for itself. And indeed our example suggests that if the buyer can capture the value to each seller of moving first (in this case Π_Y) by assigning first-mover rights, then the buyer will be able to capture all the surplus for itself, leaving both sellers with zero.

More generally, however, surplus may be split between the buyer and first seller according to each player's bargaining power, or among all three players if surplus extraction is limited. To see why it might be limited, we now consider the role of market-share contracts and below-cost pricing in facilitating rent shifting under some perturbations of this simple case.

Role of market-share contracts in facilitating rent shifting

Contracts that depend on both sellers' quantities are sometimes referred to as market-share contracts; the buyer's payment to seller X depends not only on how much the buyer purchases from seller X but also on how much the buyer purchases from seller Y. An extreme example of this is the contract in (1), where the buyer must pay a penalty of $\Pi_Y - \Pi_X$ to seller X if it purchases any amount from seller Y. More generally, the penalty will depend on how much the buyer purchases from seller Y and how much it is purchasing from seller X. Contracts in which sellers offer discounts to buyers that are based on the share they receive of the buyer's total purchases in the product category are common and are called market-share discounts.

These contracts can be instrumental in shifting rents from one seller to another, and thus their feasibility has important rent-shifting implications. When they are infeasible, or when they are prohibited by antitrust authorities, the ability of the buyer and first seller to extract surplus from the second seller may be impaired. For example, in the simple case above, there is no way for the buyer and seller X to extract all of seller Y's surplus without using market-share contracts (see Proposition 3 below). An astute reader might think that, to the contrary, seller Y's surplus can be fully extracted with the non-market-share contract

$$T_X(x,y) = \begin{cases} c_X(x) + \Pi_Y, & \text{if } x = 0 \\ c_X(x) + \Pi_X, & \text{if } x > 0, \end{cases} \tag{2}$$

because, given T_X, the most the buyer can earn from purchasing solely from seller X or Y is zero. However, in this case, if the buyer purchases from both sellers, its payoff is $\max_{x,y>0} R(x,y) - c_X(x) - T_Y(x,y) - \Pi_X$, which is strictly positive for some $T_Y(x,y) > c_Y(y)$. This implies that seller Y earns positive surplus. In other words, given T_x defined in (2), there exists $T_y(x,y) > c_Y(y)$ such that the buyer is induced to purchase positive amounts of both x and y, giving surplus to seller Y. Other non-market-share contracts fail similarly.

Role of below-cost pricing in facilitating rent shifting

The ability of firms to engage in rent shifting is also affected by laws against predatory pricing.[13] To see this, suppose that $\Pi_{XY} = \Pi_Y > \Pi_X$, market-share contracts are feasible,

[13] Predatory pricing is a violation of Section 2 of the Sherman Act and, if it occurs in an intermediate goods market, of Section 2(a) of the Robinson-Patman Act. To establish a violation, seller Y must show that its rival's prices are below "an appropriate measure of its rival's costs." Although there is some disagreement in the lower courts about whether sunk costs should be included in the definition of below-cost pricing, all courts would find below-cost pricing if $T_X(x,y) < c_X(x)$, given that $c_X(x)$ is an incremental cost.

and the buyer can make a take-it-or-leave-it offer to seller X in stage one. Then, if the buyer is to capture all the surplus from seller Y, seller X must accept a contract offer such as

$$T_X(x,y) = \begin{cases} c_X(x) & \text{if } y > 0 \\ c_X(x) + \Pi_X - \Pi_Y, & \text{if } y = 0. \end{cases} \tag{3}$$

The contract in (3), if accepted, would eliminate the buyer's gains from trade with seller Y, allowing the buyer to capture all the surplus.[14] But notice that because $\Pi_Y > \Pi_X$, the rent-shifting mechanism requires the buyer to purchase seller X's product at below-cost prices in the event the buyer does not purchase from seller Y. Such contracts are not credible because if the buyer were to purchase seller X's product at below-cost prices, the purchase would be illegal. It would be illegal for seller X to sell its product at below-cost prices, and it would be illegal for the buyer knowingly to induce seller X to sell its product at below-cost prices.

Not only can laws against predatory pricing reduce the ability of firms to shift rents, they can also reduce the incidence of market-share contracts even when the latter are feasible. It turns out that the best the buyer can do in this example is to offer seller X the non-market-share contract $T_X(x,y) = c_X(x)$ (see Section 4.2), earning for itself a payoff of Π_X. Given this T_x, seller Y's best is to offer $T_Y(x,y) = c_Y(y) + \Pi_Y - \Pi_X$, implying that seller X earns zero and seller Y earns $\Pi_Y - \Pi_X$ in equilibrium. If the buyer were to negotiate with seller Y first, the equilibrium payoffs would be unchanged. Thus, in this example, the buyer's payoff does not depend on the order of negotiations and neither does the payoff of either seller.

The common theme of the examples in this section is that rent shifting can take many forms and that the feasibility of certain kinds of contracts can have important effects on the distribution of surplus. In the next section, we extend the model by allowing for any relationship among the units sold by the sellers (substitutes, complements, or independent in demand), and any distribution of bargaining power between the buyer and each seller.

[14] The buyer's opportunity cost of purchasing from seller Y is $R(x,0) - c_X(x) - \Pi_X + \Pi_Y = \Pi_Y$.

4 Solving the Model

Stage three—Buyer's quantity choices

We use two stars to denote the buyer's quantity choices when contracts are in place with both sellers. Thus, if the buyer has contracts with both sellers, we denote the buyer's quantity choices by $(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y))$. We use one star to denote the buyer's quantity choice when a contract is in place with only one seller. Thus, if the buyer only has a contract with seller X, we denote the buyer's quantity choice by $x^*(T_X)$, and if the buyer only has a contract with seller Y, we denote the buyer's quantity choice by $y^*(T_Y)$. For now, we assume that x^{**}, y^{**}, x^*, and y^* are well defined. Later we verify this for the equilibrium contracts.

Consider first the case in which at stage three the buyer has contracts with both sellers. Then the buyer chooses quantities $(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y))$, where

$$(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y)) \in \arg\max_{x,y \geq 0} R(x, y) - T_X(x, y) - T_Y(x, y). \tag{4}$$

If instead the buyer only has a contract with seller X, it chooses $x^*(T_X)$, where

$$x^*(T_X) \in \arg\max_{x \geq 0} R(x, 0) - T_X(x, 0). \tag{5}$$

We refer to $R(x^*, 0) - T_X(x^*, 0)$ as the buyer's disagreement payoff with seller Y.

If the buyer only has a contract with seller Y, it chooses $y^*(T_Y)$, defined analogously to $x^*(T_X)$. We refer to $R(0, y^*) - T_Y(0, y^*)$ as the buyer's disagreement payoff with seller X.

Stage two—Negotiations with seller Y

Given the buyer's equilibrium behavior in stage three, and assuming the buyer and seller X negotiate contract T_X in stage one, the buyer and seller Y choose T_Y in stage two to solve

$$\max_{T_Y} R(x^{**}, y^{**}) - T_X(x^{**}, y^{**}) - c_Y(y^{**}), \tag{6}$$

such that seller Y earns λ_Y times the difference between the joint payoff of the buyer and seller Y when contract T_Y is in place and the buyer's disagreement payoff with seller Y:[15]

$$\pi_Y = \lambda_Y \left(R(x^{**}, y^{**}) - T_X(x^{**}, y^{**}) - c_Y(y^{**}) - (R(x^*, 0) - T_X(x^*, 0)) \right). \tag{7}$$

[15] Note that seller Y's payoff, π_Y, depends on $(x^{**}, y^{**}, x^*, T_X, T_Y)$; we suppress the arguments in the text.

Given T_X, it follows from (6) and (7) that the buyer and seller Y have no incentive to choose a contract that would distort the buyer's quantity decisions for products X and Y.

Lemma 1 *Given any contract T_X such that $x^*(T_X)$ and $(x^{**}(T_X, c_Y), y^{**}(T_X, c_Y))$ are well defined, if T_Y solves* (6) *subject to* (7)*, then*

$$(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y)) \in \arg\max_{x,y\geq 0} R(x, y) - T_X(x, y) - c_Y(y).$$

Proof. See the Appendix.

Given T_X, Lemma 1 implies that the buyer and seller Y will choose a contract in stage two that induces the buyer to choose their joint payoff-maximizing quantities in stage three. For example, the buyer and seller Y might agree on a contract in which seller Y offers to sell its units to the buyer at cost plus a fixed fee, where the fixed fee is chosen to satisfy (7).

If there is no first-stage contract between the buyer and seller X, i.e., if negotiations with seller X fail, the buyer and seller Y negotiate T_Y to solve $\max_{T_Y} \Pi(0, y^*)$, subject to seller Y's earning payoff $\pi_Y = \lambda_Y \Pi(0, y^*)$. It is straightforward to show that for any optimal T_Y in this case, the buyer chooses $y^* \in Q_Y$ in stage three. The buyer's payoff is then $(1 - \lambda_Y)\Pi_Y$.

Stage one—Negotiations with seller X

In stage one, the buyer and seller X negotiate T_X to maximize their joint payoff, subject to dividing the surplus so that each player receives its disagreement payoff plus a share of the gains from trade, with proportion λ_X going to seller X. Thus, given the buyer and seller Y's equilibrium strategies in stages two and three, the buyer and seller X choose T_X to solve

$$\max_{T_X \in \Omega} \; \Pi(x^{**}, y^{**}) - \pi_Y, \tag{8}$$

subject to the buyer's preferring to negotiate with seller Y in stage two,[16]

$$R(x^{**}, y^{**}) - T_X(x^{**}, y^{**}) - c_Y(y^{**}) \geq R(x^*, 0) - T_X(x^*, 0), \tag{9}$$

[16] Given our assumptions, it is never optimal for the buyer and seller X to negotiate a contract in stage one that precludes negotiations with seller Y in stage two.

seller X's earning a payoff of

$$T_X(x^{**}, y^{**}) - c_X(x^{**}) = \lambda_X \left(\Pi(x^{**}, y^{**}) - \pi_Y - (1 - \lambda_Y)\Pi_Y\right), \tag{10}$$

and, from Lemma 1,

$$(x^{**}, y^{**}) \in \arg\max_{x,y \geq 0} R(x, y) - T_X(x, y) - c_Y(y). \tag{11}$$

Note that, using Lemma 1, x^{**}, y^{**}, and π_Y can be viewed as functions only of T_X.

In this stage, the buyer and seller X choose their contract $T_X \in \Omega$ taking into account its effect on the buyer's subsequent negotiations with seller Y. We now consider whether the contract will induce $(x^{**}, y^{**}) \in Q_{XY}$ and whether surplus extraction will be complete.

4.1 Market-share contracts with below-cost pricing

When market-share contracts and below-cost pricing are feasible, $T_x \in \Omega^0$, the buyer and seller X can induce $(x^{**}, y^{**}) \in Q_{XY}$ in stage three (this maximizes $\Pi(x^{**}, y^{**})$) and extract all of seller Y's surplus (this minimizes π_Y) by negotiating the contract

$$T_X(x, y) = \begin{cases} c_X(x) + F_1, & \text{if } y > 0 \\ c_X(x) + F_0, & \text{if } y = 0, \end{cases} \tag{12}$$

where $F_0 = F_1 + \Pi_X - \Pi_{XY}$. Given T_x in (12), and an equilibrium contract T_Y, if the buyer purchases from both sellers, then the joint payoff of the buyer and seller Y is $\Pi_{XY} - F_1$. And if the buyer purchases only from seller X, then the joint payoff of the buyer and seller Y is also $\Pi_X - F_0 = \Pi_{XY} - F_1$. This implies that the buyer gains nothing from trading with seller Y, so any equilibrium involving the contract in (12) must result in zero payoff for seller Y.

If the buyer can make a take-it-or-leave-it offer to seller X, it would offer $F_1 = 0$ and $F_0 = \Pi_X - \Pi_{XY}$ to extract all the surplus. If seller X can make a take-it-or-leave-it offer to the buyer, it would offer $F_1 = \Pi_{XY} - (1 - \lambda_Y)\Pi_Y$ and $F_0 = \Pi_X - (1 - \lambda_Y)\Pi_Y$ to extract all the surplus.[17] For intermediate bargaining power, seller Y's surplus is fully extracted in

[17] If seller X attempted to extract more surplus by asking for a payment of more than $F_1 = \Pi_{XY} - (1 - \lambda_Y)\Pi_Y$, the buyer would reject seller X's offer and earn its disagreement payoff of $(1 - \lambda_Y)\Pi_Y$.

equilibrium using $F_0 = F_1 + \Pi_X - \Pi_{XY}$ and $F_1 = \lambda_X(\Pi_{XY} - (1-\lambda_Y)\Pi_Y)$. Thus, we have the following proposition.

Proposition 1 *Assume $\Omega = \Omega^0$. Then, for any distribution of bargaining power and relation among Π_{XY}, Π_X, and Π_Y, equilibria exist and overall joint payoff is maximized in all equilibria. Letting π_b^0, π_X^0, and π_Y^0 denote respectively the buyer's payoff, seller X's payoff, and seller Y's payoff, then $\pi_b^0 = \Pi_{XY} - \pi_X^0$, $\pi_X^0 = \lambda_X\left(\Pi_{XY} - (1-\lambda_Y)\Pi_Y\right)$, and $\pi_Y^0 = 0$.*

Proposition 1 establishes that when the buyer and seller X negotiate a contract that depends on both sellers' quantities, and there are no constraints on below-cost pricing, overall joint payoff is maximized and seller Y's surplus is fully extracted. This follows because the contract in (12) eliminates the buyer's gains from trade with seller Y and together with the optimal T_Y from Lemma 1 induces the buyer to choose $(x^{**}, y^{**}) \in Q_{xy}$ in stage three.

4.2 Market-share contracts without below-cost pricing

The analysis differs when there are constraints on below-cost pricing. Although a buyer might like to make a take-it-or-leave-it offer to seller X with $F_1 = 0$ and $F_0 = \Pi_X - \Pi_{XY}$, this would involve seller X's selling at below-cost prices if negotiations with seller Y were to fail and $\Pi_{XY} > \Pi_X$. Similarly, although seller X might like to make a take-it-or-leave-it offer to the buyer with $F_1 = \Pi_{XY} - (1-\lambda_Y)\Pi_Y$ and $F_0 = \Pi_X - (1-\lambda_Y)\Pi_Y$, this would involve below-cost pricing if the buyer's negotiations with seller Y were to fail and $\Pi_X < (1-\lambda_Y)\Pi_Y$.

More generally, unless $\lambda_Y = 0$, it must be that $F_0 = F_1 + \Pi_X - \Pi_{XY}$ and $F_1 = \lambda_X(\Pi_{XY} - (1-\lambda_Y)\Pi_Y)$ if seller Y's surplus is to be fully extracted in equilibrium.[18] Substituting this value of F_1 into F_0, we have that full extraction occurs if and only if $\Delta_Y \leq 0$, where

$$\Delta_Y \equiv \Pi_{XY} - \Pi_X - \lambda_X\left(\Pi_{XY} - (1-\lambda_Y)\Pi_Y\right).$$

In other words, seller Y's contribution to the available surplus must be less than the profit that seller X earns in a full-extraction equilibrium, so that seller X can credibly offer to cut its profit by an amount equal to seller Y's contribution if the buyer were to exclude seller Y.

[18] If $\lambda_Y = 0$, then seller Y earns zero payoff in any equilibrium and full extraction is trivially achieved.

Full extraction is not assured for $\Omega = \Omega^M$ because the emphasis of the optimal rent-shifting contract changes when the buyer's bargaining power with each seller changes. When the sellers have most of the bargaining power, the emphasis of surplus extraction is on the component of T_x in which the buyer is penalized when it purchases from seller Y. This was the intuition, for example, in Aghion and Bolton (1987). In contrast, when the buyer has most of the bargaining power, the emphasis of surplus extraction is on the component of T_x in which seller X offers a good deal to the buyer if the buyer does *not* purchase from seller Y (prices are lower on out-of-equilibrium quantities). Both methods are effective at extracting surplus from seller Y because both increase the buyer's opportunity cost of purchasing from seller Y, but at some point the latter method runs up against the legal constraint on below-cost pricing and additional surplus extraction is not possible. For $\Pi_{XY} > \Pi_X$, this happens when the buyer's bargaining power with each seller is sufficiently large (when λ_X and λ_Y are sufficiently small). For perspective, in Aghion and Bolton (1987), where $\lambda_X = \lambda_Y = 1$, it follows that $\Delta_Y < 0$, and so seller Y always earns zero payoff under complete information.

As long as the below-cost pricing constraint is satisfied, the contract in (12) suffices to eliminate the buyer's gains from trade with seller Y and together with the optimal T_Y from Lemma 1 induce the buyer to choose $(x^{**}, y^{**}) \in Q_{xy}$ in stage-three. However, when the below-cost pricing constraint is violated, so that full extraction cannot be achieved, it is not obvious whether the buyer and seller X should still try to induce $(x^{**}, y^{**}) \in Q_{xy}$ in stage three or whether they should try to distort the buyer's equilibrium quantity choices.

To reduce the dimensionality of the problem, we begin by proving the following lemma.

Lemma 2 *Assume $\Omega = \Omega^M$. Then contract T_X is an equilibrium contract if and only if $(x_2, y_2, x_1, t_2, t_1) = (x^{**}(T_X),\, y^{**}(T_X),\, x^*(T_X),\, T_X(x^{**}, y^{**}),\, T_X(x^*, 0))$ solves*

$$\max_{x_2 \geq 0, y_2 \geq 0, x_1 \geq 0, t_2, t_1} \Pi(x_2, y_2) - \tilde{\pi}_Y \tag{13}$$

subject to

$$R(x_2, y_2) - t_2 - c_Y(y_2) \geq R(x_1, 0) - t_1, \tag{14}$$

$$t_1 \geq c_X(x_1) \text{ and } t_2 \geq c_X(x_2), \tag{15}$$

$$\tilde{\pi}_Y = \lambda_Y \left(R(x_2, y_2) - t_2 - c_Y(y_2) - (R(x_1, 0) - t_1)\right), \tag{16}$$

$$t_2 - c_X(x_2) = \lambda_X \left(\Pi(x_2, y_2) - \tilde{\pi}_Y - (1 - \lambda_Y)\Pi_Y\right). \tag{17}$$

Proof. See the Appendix.

Lemma 2 simplifies the task of choosing contract T_X to the easier task of choosing x^{**}, y^{**}, x^*, $T_X(x^{**}, y^{**})$, and $T_X(x^*, 0)$ to maximize the buyer and seller X's joint payoff in (13) subject to the buyer and seller Y's having non-negative gains from trade, (14), seller X's earning non-negative payoff on and off the equilibrium path, (15), and each seller's earning its bargaining share of the buyer's gains from trade with it, (16) and (17).

The buyer and seller X cannot always achieve full extraction because they cannot always choose (x^*, $T_X(x^*, 0)$) to eliminate the buyer's gains from trade with seller Y. From (15) and (16), we see that the buyer and seller X extract surplus from seller Y by decreasing $T_X(x^*, 0)$ as long as $T_X(x^*, 0) \geq c_X(x^*)$ is satisfied. If this constraint binds before surplus extraction is complete, then the best the buyer and seller X can do is to choose (x^*, $T_X(x^*, 0)$) so that the buyer earns Π_X if negotiations with seller Y fail. Thus, in any equilibrium in which full extraction is not achieved, the buyer's disagreement payoff with seller Y is fixed. Since the buyer's disagreement payoff with seller X, $(1 - \lambda_Y)\Pi_Y$, is also fixed, it follows that the buyer and seller X have no incentive to choose their contract T_X to distort overall joint payoff.

Proposition 2 *Assume $\Omega = \Omega^M$. Then, equilibria exist for any distribution of bargaining power and relation among Π_{XY}, Π_X, and Π_Y. If $\lambda_Y = 1$, overall joint payoff is maximized in all non-Pareto dominated equilibria. For all other λ_Y, overall joint payoff is maximized in all equilibria. Letting π_b^M, π_X^M, and π_Y^M denote respectively the buyer's payoff, seller X's payoff, and seller Y's payoff in any non-Pareto dominated equilibrium with $\Omega = \Omega^M$, then*

$$\begin{aligned}
\pi_b^M &= \Pi_{XY} - \pi_X^M - \pi_Y^M \\
\pi_X^M &= \lambda_X \left(\Pi_{XY} - \pi_Y^M - (1 - \lambda_Y)\Pi_Y\right) \\
\pi_Y^M &= \max\left\{0,\ \frac{\lambda_Y}{1 - \lambda_X \lambda_Y} \Delta_Y\right\}.
\end{aligned}$$

Proof. See the Appendix.

Proposition 2 implies that in the design of the optimal rent-shifting contract, joint-profit maximization considerations can be separated from surplus extraction considerations in all non-Pareto dominated equilibria, even if full extraction is not achieved. If $\lambda_Y < 1$, the buyer and seller X's payoff is increasing in $\Pi(x^{**}, y^{**})$, whether or not full extraction is achieved, and thus the buyer and seller X have an incentive to choose T_X to induce $(x^{**}, y^{**}) \in Q_{XY}$ (choosing $(x^{**},\ T_X(x^{**}, y^{**}))$ to distort the buyer's quantity choices in equilibrium would lower overall joint payoff with no offsetting gain to either player). If $\lambda_Y = 1$, seller Y captures any gains from inducing the buyer to choose $(x^{**}, y^{**}) \in Q_{XY}$, and so the buyer and seller X are indifferent to choosing T_X in stage one to induce $(x^{**}, y^{**}) \in Q_{XY}$ or not.

In all non-Pareto dominated equilibria, the buyer earns the difference between the joint payoff maximum and the sum of the sellers' payoffs. Seller X earns its share of the buyer's gains from trade with it, which in equilibrium are given by the joint payoff maximum minus the sum of seller Y's payoff and the buyer's disagreement payoff. Seller Y earns the maximum of zero and $\frac{\lambda_Y}{(1-\lambda_X \lambda_Y)}\Delta_Y$. Thus, we see from Propositions 1 and 2 the distributional consequences of a law prohibiting sellers from charging below-cost prices. If $\Delta_Y \leq 0$, then a law against predatory pricing has no distributional effect and the contract in (12) can be used by the buyer and seller X to extract all of seller Y's surplus. Otherwise, if $\Delta_Y > 0$, then seller Y gains from a law against predatory pricing and, if $\lambda_X \in (0, 1)$, seller X and the buyer lose. Surprisingly, there is no short-run effect on the quantities purchased or on overall joint payoff, and hence no effect on the prices end-users pay. Although the constraint affects the players engaged in rent-shifting, it neither helps nor harms consumers in the short run.

4.3 Individual-seller contracts without below-cost pricing

Contracts that depend on both sellers' quantities may be infeasible if a seller cannot observe how much the buyer purchases from its rival. There may also be legal constraints that prohibit making contracts contingent on such information. Thus, in this section, we extend the analysis to consider rent shifting in an environment in which contracts can only depend on a seller's own quantity, $\Omega = \Omega^I$. We refer to contracts in Ω^I as individual-seller contracts.

Because Ω^I is a subset of Ω^M, we know that Lemma 1 continues to hold and, because T_Y is negotiated second, similar restrictions on the domain of T_Y do not affect equilibrium outcomes. However, unlike the case of multi-seller contracts, with individual seller contracts, there is no direct way to penalize the buyer for choosing $y > 0$. As we have shown in the section on preliminary results, this sometimes further limits the ability of the buyer and seller X to extract surplus.

As in the previous section, we begin by simplifying the buyer and seller X's task of choosing contract T_X to the easier task of choosing x^{**}, y^{**}, x^*, $T_X(x^{**}, y^{**})$, and $T_X(x^*, 0)$.

Lemma 3 *Assume $\Omega = \Omega^I$. Then contract T_X is an equilibrium contract if and only if $(x_2, y_2, x_1, t_2, t_1) = (x^{**}(T_X), y^{**}(T_X), x^*(T_X), T_X(x^{**}, y^{**}), T_X(x^*, 0))$ solves* (13) *subject to* (14) – (17) *and*

$$y_2 \in \arg\max_{y \geq 0} R(x_2, y) - c_Y(y). \tag{18}$$

$$R(x_1, 0) - t_1 \geq R(x_2, 0) - t_2. \tag{19}$$

$$R(x_2, y_2) - t_2 - c_Y(y_2) \geq \max_{y \geq 0} R(x_1, y) - t_1 - c_Y(y) \tag{20}$$

Proof. See the Appendix.

With individual-seller contracts, there are three additional constraints that must be satisfied: y^{**} must maximize $R(x^{**}, y) - c_Y(y)$, the buyer must choose $(x^*, 0)$ over $(x^{**}, 0)$ when it only has a contract with seller X, and the buyer must choose (x^{**}, y^{**}) over (x^*, y) for any y when it has contracts with both sellers. The first requirement has no effect on surplus extraction, and the second requirement does not bind in equilibrium since the incentives of the buyer and seller X are to decrease payments for x^*. But the requirement that (x^{**}, y^{**}) be chosen over (x^*, y) for $y \geq 0$, while never binding for $\Omega = \Omega^M$,[19] may be binding with

[19] Contracts in Ω^M can penalize the buyer for choosing x^* together with any positive y.

individual-seller contracts. Thus, for $\Omega = \Omega^I$, the following constraint can bind:

$$R(x^{**}, y^{**}) - T_X(x^{**}, y^{**}) - c_Y(y^{**}) \geq \max_{y \geq 0} R(x^*, y) - T_X(x^*, 0) - c_Y(y). \tag{21}$$

We refer to the constraint in (21) as the individual-seller-IC constraint. If this constraint binds, then the definition of π_Y in (7) implies that seller Y's payoff satisfies[20]

$$\pi_Y = \lambda_Y \left(\max_{y \geq 0} \Pi(x^*, y) - \Pi(x^*, 0) \right).$$

If the individual-seller-IC constraint binds, then the joint payoff of the buyer and seller X is $\Pi(x^{**}, y^{**}) - \lambda_Y \left(\max_{y\geq 0} \Pi(x^*, y) - \Pi(x^*, 0)\right)$, which is maximized by choosing T_X such that $(x^{**}, y^{**}) \in Q_{XY}$ and $x^* \in \arg\min_{x \geq 0} \lambda_Y \left(\max_{y \geq 0} \Pi(x, y) - \Pi(x, 0)\right)$. Inducing the buyer to choose $(x^{**}, y^{**}) \in Q_{XY}$ in stage three also relaxes the individual-seller-IC constraint (see 21), allowing the buyer and seller X to extract more surplus from seller Y by decreasing $T_X(x^*, 0)$. Thus, even with individual-seller contracts, the buyer and seller X can separate the maximization of overall joint payoff from the minimization of seller Y's payoff. This leads to a result for individual-seller contracts that is analogous to Proposition 2.

Proposition 3 *Assume $\Omega = \Omega^I$. Then, equilibria exist for any distribution of bargaining power and relation among Π_{XY}, Π_X, and Π_Y. If $\lambda_Y = 1$, overall joint payoff is maximized in all non-Pareto dominated equilibria. For all other λ_Y, overall joint payoff is maximized in all equilibria. Letting π_b^I, π_X^I, and π_Y^I denote respectively the buyer's payoff, seller X's payoff, and seller Y's payoff in any non-Pareto dominated equilibrium with $\Omega = \Omega^I$, then*

$$\begin{aligned} \pi_b^I &= \Pi_{XY} - \pi_X^I - \pi_Y^I \\ \pi_X^I &= \lambda_X \left(\Pi_{XY} - \pi_Y^I - (1 - \lambda_Y)\Pi_Y \right) \\ \pi_Y^I &= \max \left\{ \pi_Y^M,\ \lambda_Y \min_{x \geq 0} \max_{y \geq 0} \left(\Pi(x, y) - \Pi(x, 0) \right) \right\}. \end{aligned}$$

Proof. See the Appendix.

[20] From condition (7), we have that $\pi_Y = \lambda_Y \left(R(x^{**}, y^{**}) - T_X(x^{**}, y^{**}) - c_Y(y^{**}) - (R(x^*, 0) - T_X(x^*, 0))\right)$. It follows that if the constraint in (21) binds, then $\pi_Y = \lambda_Y \left(\max_{y \geq 0} R(x^*, y) - T_X(x^*, 0) - c_Y(y) - (R(x^*, 0) - T_X(x^*, 0)\right)$, which, after adding and subtracting $c_X(x)$, simplifies to the displayed math in the text.

Previous literature on rent-shifting and individual-seller contracts suggests that the buyer's quantity choices will be distorted (McAfee and Schwartz, 1994; Marx and Shaffer, 1999). In both cases, however, these authors restrict attention to two-part tariff contracts. Marx and Shaffer, for example, find that seller X will offer the buyer a wholesale price that is below its marginal cost in order to increase the buyer's disagreement payoff with seller Y. This distorts the buyer's quantity choices. The distortion occurs both on and off the equilibrium path in their model because seller X only has a wholesale price and fixed fee to control three objectives (maximizing overall joint payoff, dividing surplus between seller X and the buyer, and extracting surplus from seller Y). Proposition 3 implies that the class of individual-seller contracts we consider, although more restrictive than the class of market-share contracts, is sufficiently less restrictive than the class of two-part tariff contracts that it is still possible to separate the maximization of overall joint payoff from how much surplus is extracted and how it is divided. What is surprising is that this is so even when full extraction is not achieved (either because the constraint on below-cost pricing binds, or because the constraint that the buyer must choose x^* over x^{**} when it has contracts in place with both sellers binds).

If $\lambda_Y = 1$ or the individual-seller-IC constraint does not bind, then the problem of choosing T_X to maximize the joint payoff of the buyer and seller X is the same for individual-seller contracts as for contracts that can depend on both firms' quantities, so the equilibrium payoffs are the same under individual-seller contracts as under the more general contracts. (This is apparent from the proof of Proposition 3.) However, if $\lambda_Y < 1$ and the individual-seller-IC constraint binds, then seller Y's payoff is $\min_{x \geq 0} \lambda_Y \left(\max_{y \geq 0} \Pi(x, y) - \Pi(x, 0) \right)$.

Comparing the equilibrium payoffs in Propositions 2 and 3, we have that, since

$$\pi_Y^M = \max\{0, \frac{\lambda_Y}{1 - \lambda_X \lambda_Y} \left(\Pi_{XY} - \Pi_X - \lambda_X \left(\Pi_{XY} - (1 - \lambda_Y) \Pi_Y \right) \right) \},$$

the infeasibility of market-share contracts has distributional consequences if and only if

$$\begin{aligned} & \lambda_Y \min_{x \geq 0} \max_{y \geq 0} \left(\Pi(x, y) - \Pi(x, 0) \right) \\ > \; & \max \left\{ 0, \; \tfrac{\lambda_Y}{1 - \lambda_X \lambda_Y} \left(\Pi_{XY} - \Pi_X - \lambda_X \left(\Pi_{XY} - (1 - \lambda_Y) \Pi_Y \right) \right) \right\}. \end{aligned} \tag{22}$$

There are many environments in which this inequality is satisfied. For example, if the sellers' products are independent ($\Pi_{XY} = \Pi_X + \Pi_Y$), or if costs $c_X(x)$ and $c_Y(y)$ are zero, then (22) is satisfied as long as λ_X, λ_Y, Π_X, and Π_Y are positive.[21] In these cases, a restriction to individual-seller contracts reduces the amount of surplus the buyer and seller X can extract from seller Y. However, there are also environments in which the inequality is not satisfied. If the sellers' products are perfect complements, then the left side of (22) is no greater than zero, which implies that seller Y's payoff is the same as it is with market-share contracts. And if the sellers' products are perfect substitutes, then both the left and right sides of (22) are zero, which implies that full extraction from seller Y is achieved in all equilibria.

5 Strategic Buyers and Exclusion

Up to now the buyer has played a passive role in choosing which seller to negotiate with first. But clearly this decision has value, which the buyer may be able to capture.[22] If the buyer can get the sellers to compete for the right to negotiate first, then the buyer may be able to capture an additional amount equal to the difference between what the first seller earns by negotiating first and what it would have earned by negotiating second. In this case, the buyer's payoff is equal to Π_{XY} minus what each seller would earn if it negotiated second.

Corollary 1 *Assume the buyer can capture the value to seller X from moving first. Then the buyer's payoff is*

$$\pi_b = \begin{cases} \Pi_{XY}, & \text{if } \Omega = \Omega^0 \\ \Pi_{XY} - \max\left\{0,\ \frac{\lambda_X}{1-\lambda_X\lambda_Y}\Delta_X\right\} - \max\left\{0,\ \frac{\lambda_Y}{1-\lambda_X\lambda_Y}\Delta_Y\right\}, & \text{if } \Omega = \Omega^M \\ \Pi_{XY} - \max\left\{0,\ \frac{\lambda_X}{1-\lambda_X\lambda_Y}\Delta_X, \lambda_X\Gamma_X\right\} - \max\left\{0,\ \frac{\lambda_Y}{1-\lambda_Y\lambda_X}\Delta_Y, \lambda_Y\Gamma_Y\right\}, & \text{if } \Omega = \Omega^I, \end{cases}$$

[21] If the sellers' products are independent, the left side of (22) is $\lambda_Y\Pi_Y$ and the right side of (22) is $\lambda_Y\Pi_Y - \frac{\lambda_Y\lambda_X}{1-\lambda_Y\lambda_X}\Pi_X$. If the sellers' costs $c_X(x)$ and $c_Y(y)$ are zero, the left side of (22) simplifies to $\lambda_Y(\Pi_{XY} - \Pi_X)$, which, for positive λ_X, λ_Y, exceeds the right side of (22). If the sellers' products are perfect complements, the left side of (22) is no larger than zero ($\max_{y\geq 0}(\Pi(0,y) - \Pi(0,0)) = 0$).

[22] One can show that the sellers' have strict preferences over the order of negotiations if both sellers have some bargaining power and products X and Y are not perfect complements or independent in demand.

where $\Delta_X \equiv \Pi_{XY} - \Pi_Y - \lambda_Y (\Pi_{XY} - (1-\lambda_X)\Pi_X)$, $\Gamma_X \equiv \min_{y \geq 0} \max_{x \geq 0} (\Pi(x,y) - \Pi(0,y))$, *and* $\Gamma_Y \equiv \min_{x \geq 0} \max_{y \geq 0} (\Pi(x,y) - \Pi(x,0))$.

Corollary 1 is a straightforward application of Propositions 1, 2, and 3, where seller Y's payoff is unchanged and seller X's payoff is what it would earn if it negotiated second.

5.1 Committing to carry at most one product

In the long run, the buyer may be able to shift rents further in its favor by manipulating investment decisions prior to the start of the game. For example, the buyer may be a retailer who purchases products from manufacturers and resells them to final consumers. In this case, we have implicitly assumed that the buyer has adequate shelf space to carry both products. However, one can imagine that the buyer might want to limit its shelf space ex-ante in order to commit credibly to carry only one of the two products. The question we ask in this subsection is will the buyer ever want to commit at the start of the game to carry at most one product, and how does the feasible set of contracts affect its decision?

In order to keep the focus on rent-shifting, we assume there is no cost of investing in shelf space, so that any decision by the buyer to limit its shelf space is not cost related. This allows us to modify the game with a minimal change in notation. As before, if the buyer is able to carry both products, then $\Pi_{XY} = \max_{x,y \geq 0} \Pi(x,y)$. If, however, the buyer commits to carrying at most one product, then $\Pi(x,y) = \{\Pi(x,0), \Pi(0,y)\}$ and $\Pi_{XY} = \max\{\Pi_X, \Pi_Y\}$.

By committing to carry at most one product, the buyer hopes to be able to extract more surplus from the sellers than it otherwise could (assuming it does not already extract all the surplus). However, this potential for greater surplus extraction comes at the expense of a decrease in overall joint payoff of which the buyer receives a share. Thus, whether such commitments are profitable for the buyer depends first on whether a tradeoff exists, and if a tradeoff does exist, on whether it can gain from having a larger share of a smaller total pie.

Our first result is that for $\Omega = \Omega^0$, the buyer has nothing to gain from committing to carry only one product. The buyer earns Π_{XY} if it can capture the value to seller X from moving first, and it earns $(1-\lambda_X)\Pi_{XY}+\lambda_X(1-\lambda_Y)\Pi_Y$ otherwise. In both cases, there is no tradeoff for the buyer since seller Y's surplus is fully extracted without any commitment.

Similarly, there is no tradeoff for the buyer for contracts in $\Omega=\Omega^M$ if the constraint on below-cost pricing does not bind. If the constraint does bind, then a tradeoff exists but the decrease in overall joint payoff that occurs if the buyer commits to carrying only one product is only partially offset by the gain to the buyer from greater surplus extraction. To see this, note that differentiating seller X's and Y's payoff in Corollary 1 with respect to Π_{XY} when the below-cost pricing constraint binds yields $\frac{\lambda_X}{1-\lambda_X\lambda_Y}(1-\lambda_Y)+\frac{\lambda_Y}{1-\lambda_X\lambda_Y}(1-\lambda_X)$, which is less than one. The gain to the buyer from reducing the sellers' payoffs is even less if it cannot capture the value to seller X from moving first. Thus, in both cases when the constraint binds, the tradeoff is resolved in favor of the buyer carrying both products.

The tradeoff facing the buyer may be resolved differently, however, for contracts in $\Omega=\Omega^I$ if the individual-seller-IC constraint binds. Suppose the buyer can capture the value to seller X from moving first and that, relative to market-share contracts, individual-seller contracts reduce the ability of the buyer and first seller to extract surplus from the second seller. Let $\Pi_Y\geq\Pi_X$. Then, from Corollary 1, the buyer's payoff if it can carry both products is

$$\max_{x,y\geq0}\Pi(x,y)-\lambda_X\left(\min_{y\geq0}\max_{x\geq0}\left(\Pi(x,y)-\Pi(0,y)\right)\right)-\lambda_Y\left(\min_{x\geq0}\max_{y\geq0}\left(\Pi(x,y)-\Pi(x,0)\right)\right), \quad (23)$$

while its payoff if it can commit ex-ante to carrying at most one product is[23]

$$\Pi_Y-\lambda_Y\left(\Pi_Y-\Pi_X\right). \quad (24)$$

If the payoff in (24) is larger than the payoff in (23), the buyer will find it profitable to exclude one of the sellers. Clearly, the buyer suffers a loss in payoff when overall joint payoff decreases from $\max_{x,y\geq0}\Pi(x,y)$ to Π_Y, but this may be more than offset by the increased surplus that can be extracted from sellers X and Y. The simplest way to show that either effect can dominate is to consider the case of independent products, which translates into $\Pi(x,y)=\Pi(x,0)+\Pi(0,y)$ when both products can be sold. In this case, (23) simplifies to

[23]Since $\Pi_Y\geq\Pi_X$, seller X earns zero payoff in any equilibrium in which the buyer is committed to carrying at most one product. If $\Pi_X\geq\Pi_Y$, then seller Y would earn zero payoff.

$(1-\lambda_X)\Pi_X + (1-\lambda_Y)\Pi_Y$, which is less than (24) if and only if $\lambda_X + \lambda_Y > 1$. Intuitively, the larger is $\lambda_X + \lambda_Y$, the more weight the buyer will place on extracting surplus from the sellers, and therefore the more likely the buyer will want to commit to exclude one of them.

Proposition 4 *There exist environments in which the buyer can increase its payoff by committing to carry at most one product. A necessary condition for this is that market-share contracts must be infeasible and the sellers' must have sufficient bargaining power.*

Proposition 4 is surprising and has important policy implications. A common view in antitrust is that market-share contracts may be exclusionary, particularly when they are adopted by dominant sellers, and that they may lead to reduced product variety for consumers. Under this view, the appropriate antitrust policy is to ban these contracts when certain prerequisites are met. However, if one takes the view that market-share contracts are about rent-shifting, then writing consent decrees that ban them may have undesirable long-run consequences. In the short run, a ban on market-share contracts helps the sellers and hurts the buyer but does not harm competition. In the long run, the ban can actually increase the incidence of exclusion by inducing the buyer to adopt less efficient means of surplus extraction (e.g., having the sellers compete for limited shelf space). The more dominant are the sellers in the sense of having lots of bargaining power, the more likely this 'buyer-induced' exclusion will occur if market-share contracts are banned. Thus, while the intended purpose of the ban may be to mitigate the effects of alleged 'supplier-induced' exclusion, the actual effects on consumers may nonetheless be perverse. Interestingly, a ban on below-cost pricing by itself does not lead to buyer-induced exclusion, and in that sense its effect may be more in line with traditional antitrust thinking, or at least not opposed to it.

6 Conclusion

In this paper, we provide a framework for analyzing the use of contracts between vertically related firms to shift rents. We focus on a three-party sequential contracting environment in which two sellers negotiate with a common buyer. For example, the sellers might be manufacturers and the buyer a retailer. We find that overall joint payoff is maximized in this environment, even though, in equilibrium, contracts are used for rent shifting. Although the presence of a common buyer results in a coordinated outcome under general conditions, the division of surplus depends on the contracting environment, including the set of feasible contracts, the relationship between the sellers' products, and each firm's bargaining power.

We use the model to gain a new perspective on business practices such as exclusive dealing, market-share contracts, and below-cost pricing, which are often viewed by antitrust authorities as ways that a dominant seller can raise rivals' costs and induce market foreclosure. Our model offers a different perspective. As in O'Brien and Shaffer (1997) and Bernheim and Whinston (1998), we find that it is not optimal for one seller to exclude another when doing so would lower overall joint payoff—because this prevents the seller from extracting rents from the excluded firm. Similarly, we find that tactics by a seller that are designed to raise its rival's costs are also not optimal because they destroy surplus. Instead, firms prefer to use contractual provisions to maximize overall joint payoff and extract as much surplus as possible rather than to obtain a larger share of a reduced overall payoff.

This new perspective allows us to draw some nonobvious policy implications. In the short run, when things like product design, shelf space, and production technology are fixed, the use of exclusive dealing, market-share discounts, and below-cost pricing in rent-shifting contracts between manufacturers and retailers have no effect on social welfare. For example, in the case of market-share discounts, offering discounts that are contingent on how much the retailer buys from a rival manufacturer benefit the offering firm and retailer and harms the rival manufacturer, but there is no effect on the prices consumers pay, product variety, or welfare, and similarly for the practices of exclusive dealing and below-cost pricing.

In the long run, however, when investments can vary, firms have more leeway to affect the extent of rent shifting. We consider the incentive of the buyer to underinvest in shelf space as a means of committing to carry at most one product, and found that when market-share contracts are feasible, the buyer has no incentive to restrict shelf space. Thus, using shelf space as a means of rent-shifting is a second-best alternative. However, if market-share contracts are not feasible, then committing to carry at most one product may be a profitable option for the retailer. Surprisingly, in this case, our results suggest that antitrust laws aimed at reducing exclusionary behavior, may do more harm than good. For example, a rule-of-reason policy in which the use of market-share contracts by dominant sellers is banned may have the perverse effect of actually increasing the incidence of exclusion, as industry participants seek to maximize their payoffs subject to prevailing legal constraints. These long-run implications of our theory of rent shifting are in need of further analysis.

A Appendix

Proof of Lemma 1. Let T_X be such that $x^*(T_X)$ and $(x'', y'') \equiv (x^{**}(T_X, c_Y), y^{**}(T_X, c_Y))$ are well defined. Assume T_Y solves (6) subject to (7). Then $(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y))$ solves (4). Suppose that

$$(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y)) \notin \arg\max_{x,y \geq 0} R(x, y) - T_X(x, y) - c_Y(y). \tag{A1}$$

Then

$$\begin{aligned} & R\left(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y)\right) - T_X\left(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y)\right) - c_Y(y^{**}(T_X, T_Y)) \\ < \;& R(x'', y'') - T_X(x'', y'') - c_Y(y''). \end{aligned} \tag{A2}$$

Define $T_Y^e(x, y) \equiv c_Y(y) + \tilde{F}$, where

$$\tilde{F} \equiv \lambda_Y \max\{0,\ R\left(x'', y''\right) - T_X\left(x'', y''\right) - c_Y(y'') - (R(x^*(T_X), 0) - T_X(x^*(T_X), 0)))\}.$$

Note that $(x^{**}(T_X, T_Y^e), y^{**}(T_X, T_Y^e))$ is well defined, $T_Y^e(x'', y'') - c_Y(y'') \geq 0$, and

$$\begin{aligned} & R\left(x'', y''\right) - T_X\left(x'', y''\right) - c_Y(y'') \\ = \;& R\left(x^{**}(T_X, T_Y^e), y^{**}(T_X, T_Y^e)\right) - T_X\left(x^{**}(T_X, T_Y^e), y^{**}(T_X, T_Y^e)\right) - c_Y\left(y^{**}(T_X, T_Y^e)\right). \end{aligned} \tag{A3}$$

Then, using the definition of $\tilde{F}$, T_Y^e satisfies (7). Expressions (A2) and (A3) imply

$$\begin{aligned} & R(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y)) - T_X(x^{**}(T_X, T_Y), y^{**}(T_X, T_Y)) - c_Y\left(y^{**}(T_X, T_Y)\right) \\ < \;& R(x^{**}(T_X, T_Y^e), y^{**}(T_X, T_Y^e)) - T_X(x^{**}(T_X, T_Y^e), y^{**}(T_X, T_Y^e)) - c_Y\left(y^{**}(T_X, T_Y^e)\right), \end{aligned}$$

which contradicts our assumption that T_Y solves (6) subject to (7). Q.E.D.

Proof of Lemma 2. Suppose $T_X \in \Omega^M$ is an equilibrium contract. Then T_X solves (8) subject to (9), (10), and (11), where π_Y is given by (7). Consider $(x_2, y_2, x_1, t_2, t_1) \equiv (x^{**}(T_X),$ $y^{**}(T_X)$, $x^*(T_X)$, $T_X(x^{**}, y^{**})$, $T_X(x^*, 0))$. Constraint (9) implies that (14) is satisfied. Constraint (10) implies that (17) is satisfied, where $\tilde{\pi}_Y$ and π_Y are defined analogously. Since $T_X \in \Omega^M$, the definitions of x^{**} and x^* imply that (15) is satisfied. Thus, $(x_2, y_2, x_1, t_2, t_1)$ is a feasible solution. Suppose $(x_2, y_2, x_1, t_2, t_1)$ does not solve the program in (13)–(17). Then there exists $(x_2', y_2', x_1', t_2', t_1')$ satisfying the constraints in (14)–(17) such that (13) is greater at $(x_2', y_2', x_1', t_2', t_1')$ than at $(x_2, y_2, x_1, t_2, t_1)$. Consider contract T_X' defined by:

$$T_X'(x,y) \equiv \begin{cases} t_2', & \text{if } (x,y) = (x_2', y_2') \\ t_1', & \text{if } (x,y) = (x_1', 0) \\ \infty, & \text{otherwise.} \end{cases}$$

Because $(x_2', y_2', x_1', t_2', t_1')$ satisfies the constraints in (14)–(17) and $(x^{**}(T_X'), y^{**}(T_X')) = (x_2', y_2')$ and $x^*(T_X') = x_1'$, it follows that $T_X' \in \Omega^M$ and that T_X' satisfies (9), (10), and (11). Thus, T_X' is a feasible contract and gives the buyer and seller X higher joint payoff than T_X, a contradiction.

Suppose T_X is not an equilibrium contract. If $x^{**}(T_X)$, $y^{**}(T_X)$, or $x^*(T_X)$ is not well defined, then there does not exist x_2, y_2, or x_1 satisfying (15) when $(t_2, t_1) = (T_X(x^{**}, y^{**}), T_X(x^*, 0))$. Because T_X is not an equilibrium contract, the contract T_X'', where

$$T_X''(x,y) \equiv \begin{cases} T_X(x^{**}(T_X), y^{**}(T_X)), & \text{if } (x,y) = (x^{**}(T_X), y^{**}(T_X)) \\ T_X(x^*(T_X), 0), & \text{if } (x,y) = (x^*(T_X), 0) \\ \infty, & \text{otherwise,} \end{cases}$$

is also not an equilibrium contract. Since $T_X \in \Omega^M$, it follows that $T_X'' \in \Omega^M$, so if T_X'' is not feasible, then either (9), (10), or (11) is violated. Consider $(x_2, y_2, x_1, t_2, t_1) \equiv (x^{**}(T_X), y^{**}(T_X), x^*(T_X), T_X(x^{**}, y^{**}), T_X(x^*, 0))$. If T_X'' violates (9), then $(x_2, y_2, x_1, t_2, t_1)$ violates (14). If T_X'' violates (10), then $(x_2, y_2, x_1, t_2, t_1)$ violates (17). If T_X'' violates (11), then $(x_2, y_2, x_1, t_2, t_1)$ violates (14). If T_X'' is feasible, then there exists $T_X''' \in \Omega^M$ also feasible but giving higher payoff to the buyer and seller X. Then $(x_2, y_2, x_1, t_2, t_1)$ and $(x_2''', y_2''', x_1''', t_2''', t_1''') \equiv (x^{**}(T_X'''), y^{**}(T_X'''), x^*(T_X'''), T_X'''(x^{**}, y^{**}), T_X'''(x^*, 0))$ satisfy the constraints in (14)–(17), but $(x_2''', y_2''', x_1''', t_2''', t_1''')$ results in a higher value of the maximand in (13) than $(x_2, y_2, x_1, t_2, t_1)$. Thus, $(x^{**}(T_X), y^{**}(T_X), x^*(T_X), T_X(x^{**}, y^{**}), T_X(x^*, 0))$ does not solve the program in (13)–(17). Q.E.D.

Proof of Proposition 2. Assume $\Omega = \Omega^M$. Suppose $\lambda_Y < 1$. Using Lemma 2, we consider

$(x_2, y_2, x_1, t_2, t_1)$ solving the program in (13)–(17). Note that (16) and (17) imply

$$t_2 = c_X(x_2) + \frac{\lambda_X}{1 - \lambda_X \lambda_Y} \left((1 - \lambda_Y)\Pi(x_2, y_2) - (1 - \lambda_Y)\Pi_Y + \lambda_Y \left(R(x_1, 0) - t_1 \right) \right) \qquad \text{(A4)}$$

and

$$\tilde{\pi}_Y = \lambda_Y \left(\frac{1 - \lambda_X}{1 - \lambda_X \lambda_Y} \Pi(x_2, y_2) + \frac{\lambda_X (1 - \lambda_Y)}{1 - \lambda_X \lambda_Y} \Pi_Y - \frac{1}{1 - \lambda_X \lambda_Y} \left(R(x_1, 0) - t_1 \right) \right). \qquad \text{(A5)}$$

Substituting in for t_2 and $\tilde{\pi}_Y$, the program in (13)–(17) is

$$\max_{x_2, y_2, x_1, t_1} \frac{1 - \lambda_Y}{1 - \lambda_X \lambda_Y} \Pi\left(x_2, y_2\right) + \frac{\lambda_Y}{1 - \lambda_X \lambda_Y} \left(R(x_1, 0) - t_1 \right) - \frac{\lambda_X \lambda_Y (1 - \lambda_Y)}{1 - \lambda_X \lambda_Y} \Pi_Y \qquad \text{(A6)}$$

subject to

$$R(x_1, 0) - t_1 \leq (1 - \lambda_X)\Pi(x_2, y_2) + \lambda_X (1 - \lambda_Y)\Pi_Y, \qquad \text{(A7)}$$

$$\lambda_X \lambda_Y \left(R(x_1, 0) - t_1 \right) \geq \lambda_X (1 - \lambda_Y)\Pi_Y - \lambda_X (1 - \lambda_Y)\Pi(x_2, y_2), \qquad \text{(A8)}$$

$$t_1 \geq c_X(x_1). \qquad \text{(A9)}$$

Because choosing $(x_2, y_2) \in Q_{XY}$ maximizes the first term in the maximand and maximally relaxes the constraints, and because a feasible solution exists with $(x_2, y_2) \in Q_{XY}$, choosing $(x_2, y_2) \in Q_{XY}$ is optimal. This completes the proof that all equilibria are efficient when $\lambda_Y < 1$. If $\lambda_Y = 1$, then the joint payoff of the buyer and seller X does not depend on (x_2, y_2), so there exists an equilibrium in which $(x_2, y_2) \in Q_{XY}$.

One can calculate the equilibrium payoffs to the players by using (7), (10), the above efficiency result, the result from the text that full extraction is achieved if and only if $\lambda_Y = 0$ or $\Delta_Y \leq 0$, and the fact that the buyer's disagreement payoff with seller Y is Π_X when full extraction is not achieved. Q.E.D.

Proof of Lemma 3. Define Program I to be $\max_{T_X \in \Omega^I} \Pi(x^{**}, y^{**}) - \pi_Y$ subject to (9)–(11), and define Program II to be (13) subject to (14)–(20). Suppose $\hat{T}_X \in \Omega^I$ is an equilibrium individual-seller contract. Then $\hat{T}_X$ solves Program I. Letting

$$(x_2, y_2, x_1, t_2, t_1) \equiv (x^{**}(\hat{T}_X), y^{**}(\hat{T}_X), x^*(\hat{T}_X), \hat{T}_X(x^{**}), \hat{T}_X(x^*)),$$

constraint (9) implies that (14) is satisfied, constraint (10) implies (17) is satisfied, constraint (11) implies that (18) is satisfied. Since $\hat{T}_X \in \Omega^I$, the definitions of x^{**} and x^* imply that (15), (19), and (20) are satisfied. Thus, $(x_2, y_2, x_1, t_2, t_1)$ is a feasible solution to Program II. Suppose $(x_2, y_2, x_1, t_2, t_1)$ does not solve Program II. Then there exists $(x_2', y_2', x_1', t_2', t_1')$ satisfying (14)–(20) such that the maximand in (13) is greater at $(x_2', y_2', x_1', t_2', t_1')$ than at $(x_2, y_2, x_1, t_2, t_1)$. Consider contract T_X' defined by:

$$T_X'(x) \equiv \begin{cases} t_2', & \text{if } x = x_2' \\ t_1', & \text{if } x = x_1' \\ \infty, & \text{otherwise.} \end{cases}$$

Because $(x_2', y_2', x_1', t_2', t_1')$ satisfies (14)–(20) it follows that $T_X' \in \Omega^I$ and that $(x^{**}(T_X'), y^{**}(T_X')) = (x_2', y_2')$ and $x^*(T_X') = x_1'$, and so T_X' satisfies (9)–(11). Thus, T_X' is a feasible contract in Program I and gives the buyer and seller X higher joint payoff than $\hat{T}_X$, a contradiction. Thus, $\hat{T}_X$ solves Program II.

Now suppose $\hat{T}_X$ is not an equilibrium contract. Then $\hat{T}_X$ does not solve Program I. If $x^{**}(\hat{T}_X)$, $y^{**}(\hat{T}_X)$, or $x^*(\hat{T}_X)$ is not well defined, then there does not exist x_2, y_2, or x_1 satisfying (15) when $(t_2, t_1) = (T_X(x^{**}), T_X(x^*))$. So suppose they are well defined. Because $\hat{T}_X$ is not an equilibrium contract, the contract T_X'', where

$$T_X''(x) \equiv \begin{cases} \hat{T}_X(x^{**}(\hat{T}_X)), & \text{if } x = x^{**}(\hat{T}_X) \\ \hat{T}_X(x^*(\hat{T}_X)), & \text{if } x = x^*(\hat{T}_X) \\ \infty, & \text{otherwise,} \end{cases}$$

is also not an equilibrium contract. Since $\hat{T}_X \in \Omega^I$, it follows that $T_X'' \in \Omega^I$, and so if T_X'' is not a feasible solution to Program I, then at least one of (9)–(11) is violated. Consider $(x_2, y_2, x_1, t_2, t_1) \equiv (x^{**}(\hat{T}_X), y^{**}(\hat{T}_X), x^*(\hat{T}_X), \hat{T}_X(x^{**}), \hat{T}_X(x^*))$. If T_X'' violates (9), then $(x_2, y_2, x_1, t_2, t_1)$ violates (14). If T_X'' violates (10), then $(x_2, y_2, x_1, t_2, t_1)$ violates (17). If T_X'' violates (11), then $(x_2, y_2, x_1, t_2, t_1)$ violates at least one of (14)–(20). If T_X'' is a feasible solution to Program I, then there exists $T_X''' \in \Omega^I$ also feasible but giving a higher value of the maximand. Then $(x_2, y_2, x_1, t_2, t_1)$ and

$$(x_2''', y_2''', x_1''', t_2''', t_1''') \equiv (x^{**}(T_X'''), y^{**}(T_X'''), x^*(T_X'''), T_X'''(x^{**}), T_X'''(x^*))$$

both satisfy the constraints of Program II, but $(x_2''', y_2''', x_1''', t_2''', t_1''')$ results in a higher value of the maximand in (13) than $(x_2, y_2, x_1, t_2, t_1)$. Thus, $(x^{**}(\hat{T}_X), y^{**}(\hat{T}_X), x^*(\hat{T}_X), \hat{T}_X(x^{**}), \hat{T}_X(x^*))$ does not solve Program II. Q.E.D.

Proof of Proposition 3. Assume $\lambda_Y < 1$. Using Lemma 3, we consider $(x_2, y_2, x_1, t_2, t_1)$ solving (13) subject to (14)–(20), which we refer to as Program II. As in the proof of Proposition 2, (16) and (17) imply (A4) and (A5). Using (A4) and (A5) to substitute in for t_2 and $\tilde{\pi}_Y$, Program II can be rewritten as (A6) subject to (A7)–(A9), (18),

$$\lambda_X \lambda_Y \left(R(x_1, 0) - t_1\right) \leq (1 - \lambda_X)\, \Pi(x_2, y_2) + \lambda_X (1 - \lambda_Y) \Pi_Y - (1 - \lambda_X \lambda_Y) \left(\max_{y \geq 0} R(x_1, y) - t_1 - c_Y(y) \right),$$

and

$$R(x_1, 0) - t_1 \geq (1 - \lambda_X \lambda_Y)\, \Pi(x_2, 0) - \lambda_X \left((1 - \lambda_Y)\Pi(x_2, y_2) - (1 - \lambda_Y)\Pi_Y\right). \tag{A10}$$

A feasible solution exists with $(x_2, y_2) \in Q_{XY}$, and choosing $(x_2, y_2) \in Q_{XY}$ maximizes the first term in (A6) and maximally relaxes all the constraints except (A10). One can easily confirm that (A10) does not bind because it specifies a lower bound for $R(x_1, 0) - t_1$ and the objective function requires that (x_1, t_1) be chosen to maximize $R(x_1, 0) - t_1$. This completes the proof that all equilibria are efficient when $\lambda_Y < 1$. If $\lambda_Y = 1$, then the joint payoff of the buyer and seller X does not depend on (x_2, y_2), so there exists an equilibrium in which $(x_2, y_2) \in Q_{XY}$. Q.E.D.

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