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INDEXATION, THE RISK-FREE ASSET, AND CAPITAL MARKET EQUILIBRIUM

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I. INTRODUCTION AND SUMMARY

INDEXATION REFERS TO THE linking of the return on an asset or the payment on a contract to a price level index. Historically, Edgeworth, Jevons, Keynes, Marshall, and Irving Fisher have been among the economists advocating the use of indexed assets.¹ Keynes (1927) argued that indexed bonds actually enhance investors' opportunities. This argument has recently been supported by Sarnat (1973) who attempts to demonstrate within the context of the two-parameter asset-pricing model of Sharpe (1964) and Lintner (1965) that in a perfect capital market, government issuance of an index-linked bond expands the opportunity set which investors face.

This paper is divided into two parts. In the first part we explore the nature of the real riskless asset. In the simple economy we construct, the existence of assets which are riskless in real terms depends upon the underlying productive technology and the return on these assets may be either endogenously or exogenously determined. When we apply this analysis to the Keynes/Sarnat contention that indexation expands investment opportunities, we find that their conclusions do not necessarily apply. In the second part of the paper, the nature of the nominal riskless asset, which always exists, is examined, and the Sharpe/Lintner model is employed to determine the equilibrium returns on both linked and unlinked assets. We determine that the Fisherian relationship, i.e., that the *ex ante* expected real return on an asset be equal to the nominal return less the expected rate of inflation, is valid only under very restrictive assumptions.²

II. THE NATURE OF THE REAL RISK-FREE ASSET

Let us assume that there is a production process F which transforms one unit of commodity input at time t into an uncertain quantity of commodity output at time $t + 1$, represented by the random variable \tilde{C}_{t+1} . The random return of this process is then defined as $\tilde{C}_{t+1} - 1$. A *risk-free asset* is defined as a contract which can

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1. A volume by Collier (1969) discusses in detail the historical development of indexation in various countries.

2. Both Roll (1973) and Fischer (1975) construct more elaborate frameworks in order to derive similar conditions. Also see Long (1974).

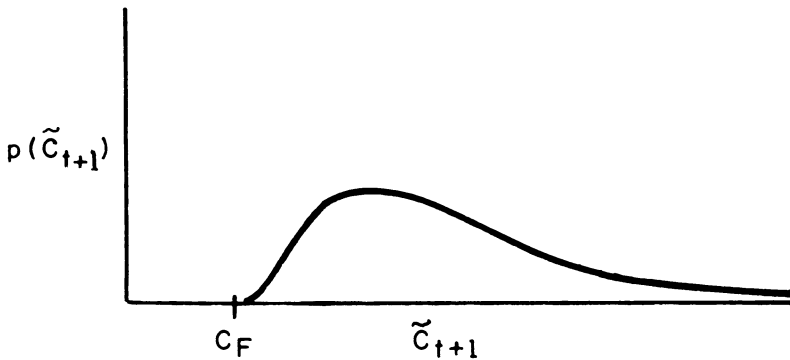


FIGURE 1. The Production Process, F

guarantee the delivery of a positive quantity of commodities in period $t + 1$.³ The production process F will admit a risk-free asset only if the probability that $\tilde{C}_{t+1} < \epsilon$ for some $\epsilon > 0$ is 0, or that the production process will not yield a return of -100 percent. Figure 1 represents such a production process, with $C_F > 0$ being the minimum production outcome.

Although the underlying production process is uncertain, two securities can be formed from the distribution shown in Figure 1. One is a claim to an amount C_F and the other is the claim to the residual, if any. The sum of the prices of both claims must equal one, the amount of commodity input into the process. However, each part of the claim will be priced according to the risk preferences of the public.

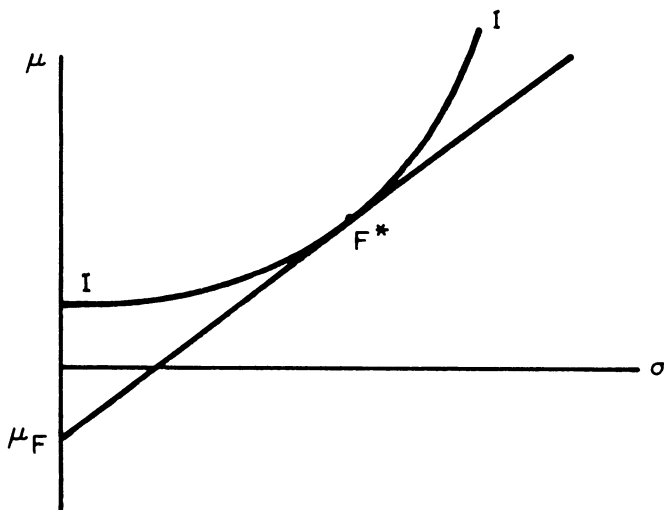


FIGURE 2. Determination of the Rate of Return, μ_F , on the Riskless Asset

3. As demonstrated by Black (1972), the existence of a risk-free asset is not needed for most results in capital asset pricing theory. However, portfolio choices are most easily described by combining a risk-free asset with a mutual fund of risky securities, so the former represents a “benchmark” asset.

Assume this is the only production technique in the economy, and investors' risk preferences can be represented by indifference maps in mean-variance space. In Figure 2, F^* represents the expected return $\mu = E(\bar{C}_{t+1}) - 1$ and standard deviation $\sigma = \sigma(\bar{C}_{t+1})$ of the single, economy-wide production process and II represents the investors' indifference curve passing through F^* . The tangent to that indifference curve intersects the return axis at μ_F , which may be positive or negative. A sure claim would then carry this endogenously determined return if traded in the market. However, if all investors are identical and have preferences represented by indifference curve II in Figure 2, everyone will hold portfolio F^* and no trading in riskless claims will take place.

Risk-free assets will be traded, however, as preferences differ. Trades of the risk-free portion of the distribution will arise between those investors who are relatively risk-averse and those who are relatively risk-preferring but still absolutely risk-averse. Figure 3 illustrates the indifference curves of the "risk-aversers" (I_A) and "risk-prefersers" (I_P). μ_A is the incipient return on a sure asset to the risk-aversers and μ_P is the incipient return to the risk-prefersers. Trades arise between

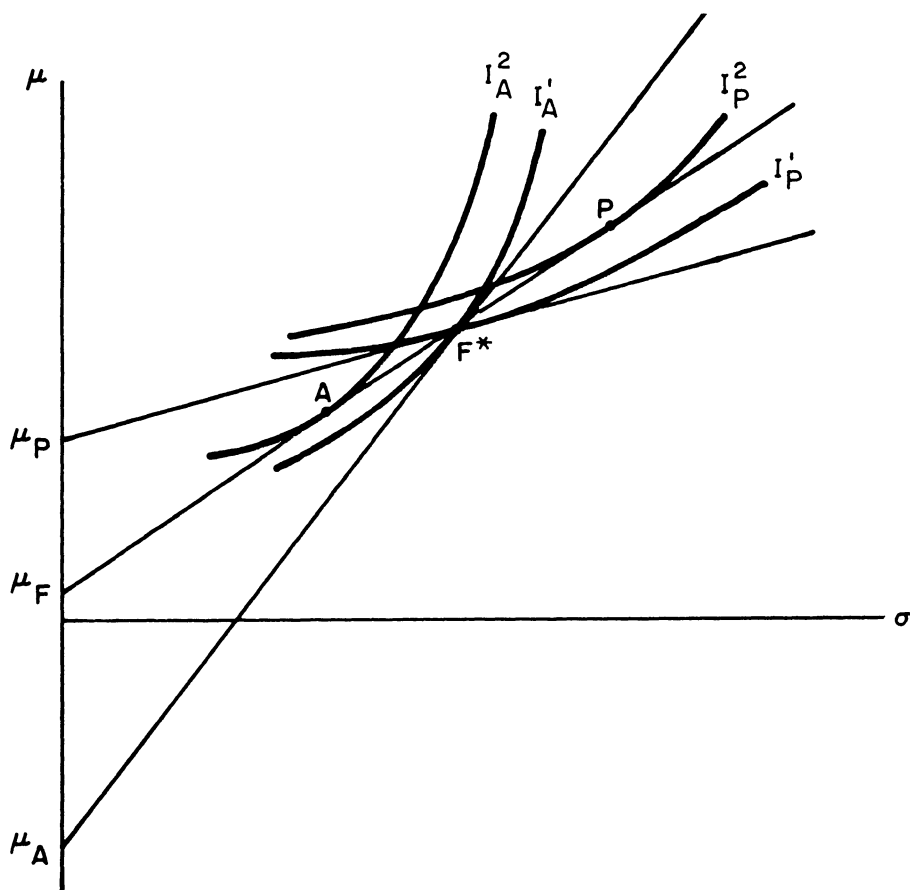


FIGURE 3. Trading in Riskless Assets When Preferences Differ

these groups and determine the equilibrium sure rate μ_F such that $\mu_A < \mu_F < \mu_P$ and a higher level of utility (I_P^2 and I_A^2) is achieved for each group at P and A . μ_F is thus determined so that the moments of the aggregated portfolio A and P , weighted by the wealth of each group, equal the moments of portfolio F^* . Any individual investor can obtain any combination of risk and return along $\mu_F F^*$, but, in the aggregate, the *net* demand for the sure financial asset must be zero since the supply is zero. This is true since investors holding the claim to the residual output (return above C_F in Figure 1) are in effect holding the entire distribution and short the sure portion. At any rate above μ_F , there will be excess demand for the sure asset and, at a rate below μ_F , there will be excess supply. Therefore, only at μ_F will the markets clear.⁴

In contrast to the above endogenously determined sure rate there may be a second production process in the economy, let us call it S , which is a storage process. S transforms a unit of commodity at time t into less than a unit, say C_S , with certainty in the next period.⁵ The return on the storage function is $\mu_S = C_S - 1$, and as long as $\mu_S > \mu_F$, investment in the storage process will be made. Figure 4 depicts a case where the existence of a storage process expands the efficient frontier from $\mu_F F^*$ to $\mu_S F^*$ which allows the indifference curve I_2 to be attained. Also, the quantity of the riskless asset in this case is limited only by the total supply of

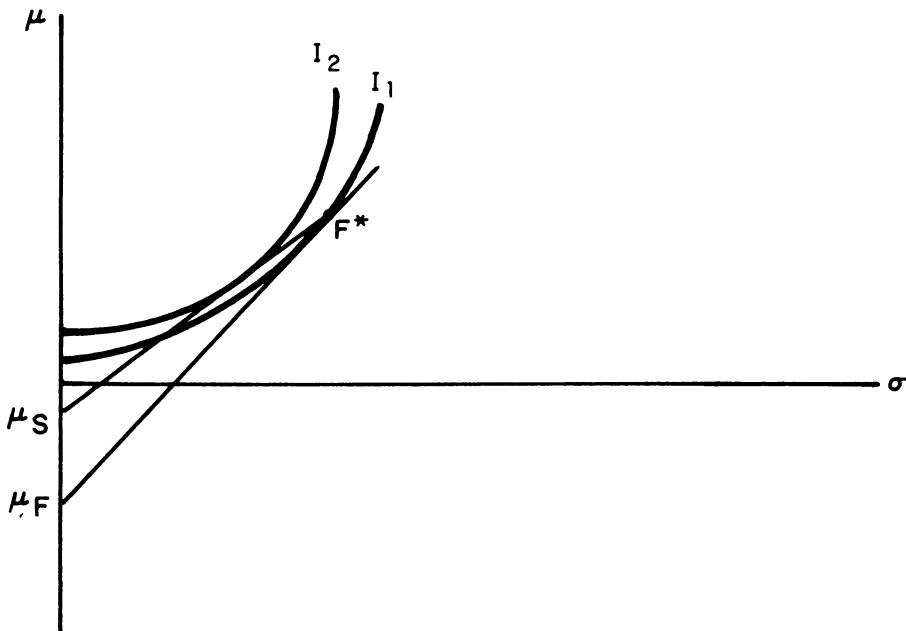


FIGURE 4. Determination of Equilibrium When a Storage Process, S , Exists

4. Note that the guarantee of delivery of goods may not, in a multi-commodity world, guarantee a *value* of goods if tastes change. Therefore, if one wishes to guarantee purchasing power, one must also know or be able to insure against utility function changes, a situation which may be impossible.

5. In reality, storage activity does entail risk of fire, theft, infestation, etc.

commodities in the economy rather than by C_F as is the case without a storage process.

The previous analysis can be extended to the case where there are two factors of production, labor and capital, and one homogeneous output. In this case, workers contract to deliver a specified quantity of input in the current period. In exchange, they are given a claim to a specified quantity of output in the next period. Assume that the production process is stochastic and minimum output is just sufficient to cover the contracted wage, but the residual, the return to capital, is uncertain. In this case, only workers can issue risk-free claims against their labor income, while owners of capital cannot since income accruing solely from the residual is not certain.

Suppose we now introduce government. Assume that the government engages in no production of either commodities or money balances, but merely effects wealth and income transfers. The government can issue indexed financial claims, backed by its power to tax next-period income, and transfer the proceeds from the sale of those claims to any group it chooses. A government "index" bond is a bond whose principal and interest are linked to (i.e., multiplied by) the price level of a representative bundle of commodities. Hence a government index bond is analogous to the real assets we have analyzed, since the government is actually guaranteeing the delivery of a certain amount of "purchasing power".

It is clear from the above discussion that the government's ability to produce a riskless real asset will be limited by the same production processes that are available to the private sector. If wage income is certain, then only index bonds based on a wage tax will be riskless, while those based on taxing income from capital are inherently risky. There is no way that a government can guarantee the delivery of commodities (which is what an index bond in essence assures) if the fundamental production process cannot guarantee such a delivery.⁶

It is important to note that the possibility that the government can tax not only current production, but also the stock of accumulated real wealth, does not alter this result. Only if some positive amount of real wealth (being produced via the storage process) is riskless will either individuals or the government be able to issue riskless claims. Unless there are differential costs of issuing the claims, the fact that the government rather than the private sector happens to issue them will not change the set of investment opportunities which would be available to investors if they themselves sold riskless claims.⁷

6. If the government has a monopoly on the money creation process, the government may obtain real resources by printing nominal money and hence levying an "inflation tax". However, the government's ability to guarantee future delivery still depends on the economy's fundamental production processes, which may include the provision of transaction services by the government.

7. Whether or not indexation will in actual fact have any benefits which are not already available to the private sector depends critically upon the assumed costs of issuing riskless claims. In the above case, where it is assumed to be costless for both individuals and the government to issue riskless claims, the argument that government indexation enhances investment opportunities is not justified. However, in practice it may be less costly for the government to issue claims against future (riskless) income or wealth than for individuals to do so. To the extent that such differential costs are present, there may be benefits associated with government indexation which are not available to the private sector.

III. THE NATURE OF THE NOMINAL RISK-FREE ASSET

In contrast to a real asset, a nominal asset is one in which contingent payment in period $t + 1$ is made in the form of government or private money instead of either commodities or the money equivalent of a bundle of commodities. If the nominal bonds permit payment in government fiat money, the government can pay any nominal rate the market requires, regardless of the production techniques or the taxing powers available to it, by simply printing money.⁸

Without exploring the reasons why the private sector contracts in nominal terms,⁹ the pricing of money-denominated assets can be readily determined from the equilibrium rates of return derived for real assets in the preceding section. To establish the equilibrium relationship between returns on real and nominal assets we use the capital asset pricing model (CAPM) first developed by Sharpe and Lintner. A fundamental result of Sharpe and Lintner is that, in equilibrium, assets are priced such that the *ex ante* expected return on any asset or portfolio is equal to the risk-free rate plus a premium related to the asset's covariance with the market portfolio. Let R_f be the real risk-free rate (the "indexed" rate of short-term securities), \tilde{R}_m be the stochastic real market rate of return. Let R_n be the stated, sure, *nominal* return on an asset. The real return on this asset is a random variable approximated by $\tilde{R}_{rn} = R_n - \tilde{\pi}$, where $\tilde{\pi}$ is the stochastic inflation rate over the period. The CAPM indicates that in equilibrium:

$$E(\tilde{R}_{rn}) = R_f + \frac{E(\tilde{R}_m) - R_f}{\sigma(\tilde{R}_m)} \cdot \frac{\text{cov}(\tilde{R}_m, \tilde{R}_{rn})}{\sigma(\tilde{R}_m)} \quad (1)$$

Since $R_{rn} = R_n - \tilde{\pi}$, we have

$$R_n - \bar{\pi} = R_f + \lambda \text{cov}(\tilde{R}_m, \tilde{R}_{rn}) = R_f - \lambda \text{cov}(\tilde{R}_m, \tilde{\pi}), \quad (2)$$

8. It may be possible for nominal assets to affect the return on real assets. Let us assume that the riskless rate of return is negative, and the government maintains a constant quantity of government fiat currency in existence. If the price level (the reciprocal of the price of money in terms of commodities) is uncorrelated with the random commodity return, then the expected return on money is zero in a stationary economy. No one would invest in the production process, and the commodity stock would dwindle to zero. Only a trivial equilibrium exists. If we consider the production process endogenous, however, depending on the input of labor as well as commodities, such that the production process has decreasing marginal returns to the commodity input given the labor input, then a decreasing stock of commodities will lead to a rise in the rate of return of commodity production, a rate which would rise until the own real rate of return on commodities becomes zero. In this case the government can determine the endogenous real rate of interest but still cannot necessarily create a riskless real asset. In fact, by destroying money at rate θ , the government can push the real return on the production process up to that value, simply because money's expected return is now θ .

9. Since production occurs over a set of real inputs and outputs, it is a puzzle why contractual payments are so often nominally denominated instead of indexed. In a world of perfect capital markets, only real securities might exist. While Siegel (1974) does not show why there are *no* indexed securities in the economy, he does demonstrate that a demand for some nominally denominated contracts could arise if various capital market imperfections exist, especially those concerning the nondiversifiability of human capital.

where $\lambda = (E(\tilde{R}_m) - R_f) / \sigma^2(\tilde{R}_m)$ and $\bar{\pi} = E(\tilde{\pi})$. Therefore,

$$R_n \begin{matrix} \geq \\ < \end{matrix} R_f + \bar{\pi} \quad \text{iff } \text{cov}(\tilde{R}_m, \tilde{\pi}) \begin{matrix} \leq \\ > \end{matrix} 0. \quad (3)$$

The nominal rate of interest will be less (greater) than the real rate plus the expected value of the anticipated rate of inflation if prices and *real* market returns are positively (negatively) correlated.

The intuition behind this result is clear. If unexpected increases in the price level are positively associated with real asset value changes, nominal contracts are a *hedge* in the sense that their real value fluctuates counter-cyclically to the market. Any asset which has this counter-cyclical property will be priced "favorably" by the market, in the sense of having a lower expected return than the market portfolio. Only in the case where the monetary sector of the economy is completely dichotomized from the real sector, such that the price of money and the stochastic returns on production are uncorrelated, will the real risk-free rate of interest be equal to the nominally stated rate minus the expected rate of inflation.

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