

Selective and Aggregate Disclosure*

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Abstract

We study the disclosure strategy of a firm's manager who may privately observe two signals that are informative about the firm's prospects. The two signals correspond to the firm's two projects and are observationally correlated, i.e., the observation of one affects the likelihood of observing the other. If the manager makes a separate disclosure of each signal, we show that, there exists a "selective" disclosure equilibrium where the manager is "falsely modest"; i.e., he sometimes suppresses favorable information. We then demonstrate that this equilibrium may result in lower welfare than an aggregate disclosure regime, where the manager is restricted to disclosing the aggregate of the two signals, even though it garbles information.

1 Introduction

Managers privately observe the performance of multiple business segments; researchers are privately knowledgeable about the potential success or failure of multiple research and development projects; and political leaders are privately informed about the government's performance in multiple sectors of the political economy. This privately-held multi-dimensional information potentially affects the decisions of other less-informed individuals. For example, potential investors condition their stock purchase decision on firms' information releases; senior executives base their project funding decision on information indicating the potential success of R&D projects; and information on a country's political and socioeconomic climate affects the activities of its citizens. The extant information economics literature has largely focused on the disclosure decision of an individual who is privately informed about a single piece of information that is valuable to a group of uninformed individuals.¹ In this paper, we examine information disclosures by a manager who is privy to multiple information signals about his firm's prospects.

A manager possessing multiple information signals may attempt to positively influence investors' valuation by "selectively" disclosing only a subset of his private information; that is he discloses some but not all of the information. Public policy makers are troubled by such selective disclosure because it potentially misleads investors by conveying an unreasonably "rosy" picture of

¹Exceptions include Hayes and Lundholm [1996], Kirschenheiter [1997], Pae [2005] and Shin [1994]. For a comprehensive summary of the disclosure literature see Verrecchia [2001] and Dye [2001].

the firm; favorable information is disclosed, while unfavorable information suppressed. For example, in a July 2005 conference call, Abercrombie & Fitch's managers disclosed that denim sales were better than expected but did not reveal any information about margins. Investors drove up A&F stock price thinking that margins were unchanged only to subsequently realize that the increased sales were accompanied by lower margins and that the managers possessed both pieces of information (Wall Street Journal August 29, 2005).² In addition to examining a manager's disaggregate disclosure policy, we analyze the manager's disclosure strategy if he discloses only the aggregate of his information signals. Finally, we compare the welfare consequences of the two disclosure regimes to characterize conditions under which each is preferred.

We construct a stylized model of a firm with potential investors who are uncertain about its manager's information endowment. Specifically, we model a firm whose value is a simple sum of the individual values of its two projects. Each project's value is determined by both its outcome (either a success or a failure) and the actions taken by the firm's investors. For instance, a pharmaceutical project's value is affected by both the FDA approval decisions (i.e., the outcome) and the investment in its production process (i.e., action). Each project generates a binary signal (again, either a success or a failure) that is informative of its outcome but may or may not be privately observed by the firm's

²As another example, Apple Computers is known to disclose sales revenues for its different product lines (computers, iPods, etc.) but not operating profits. It chooses to disclose operating profits by geographic regions rather than product lines. Some analysts question whether Apple chooses its disclosure strategy to mask the low profitability of certain products (MarketWatch, June 26, 2006.)

manager. In addition, the manager's information endowment is not common knowledge, *i.e.*, whether the manager privately observes both, one or neither of the two signals is unknown to the firm's potential investors. The two signals reflect their respective project's outcome with differing value-relevance, *i.e.*, project 1's signal may be stochastically more informative of project 1's value than project 2's signal is of project 2's value.³ For example, accounting earnings tend to more accurately capture a mature division's fundamentals than a nascent one's.

A salient innovation in our model is that the two signals may be observationally correlated, *i.e.*, the manager's observation of one signal may affect the likelihood that he observes the second signal. This assumption is motivated by the observation a manager may or may obtain all his private information signals at the same time. For example, a firm with multiple pharmaceutical products in the development phase may receive FDA approval for a single drug and get no other information about the status of the others, which would imply that the manager is privy to only one information signal. Alternatively, government approval of one drug may coincide with its approval of another in which case the manager receives both signals concurrently. The observational correlation reflects the extent to which whether a signal is received by the manager is related to the likelihood that the other signal is also received.⁴ Finally, we assume that

³We capture value-relevance by the probability that a project signal correctly reflects that project's outcome.

⁴In another context, a demand shock may only affect sales of a product and not margins if the production function exhibits constant returns to scale, or it may affect both sales and marginal costs. As an example of this scenario, consider Abercrombie & Fitch, where new information about sales increases may or may not be correlated with new information about

the likelihood of observing a signal, its value relevance and the observational correlation between signals are all known to potential investors, and collectively describe a firm's information environment.

We consider two disclosure regimes where disclosures are such that the manager is unable to falsify information but may, nevertheless, withhold information by pretending that he is uninformed. In the first regime, the disaggregate disclosure regime, the manager discloses the two signals separately, which in turn implies that he may choose between not disclosing any information, disclosing either one or both signals. An important feature of disaggregate disclosure is that if the manager discloses a signal, he must also disclose which project it relates to. Under the disaggregate disclosure regime, we show that a unique selective disclosure equilibrium arises: the manager discloses the more value relevant signal if and only if it is favorable and the less value relevant signal is disclosed only if both signals are favorable. This equilibrium is unique when the two signals are sufficiently correlated in observation and different in their value relevance. The intuition behind this result is that a high observational correlation between the two signals makes it difficult for the manager to only disclose the less value-relevant signal as doing so leads investors to infer that the withheld (and more value-relevant) signal indicates unfavorable news. We show that when the observational correlation is sufficiently high, the manager is deterred from disclosing a favorable realization of the less value-relevant signal, unless the more value-relevant signal is also favorable. An implication of this

margin changes.

equilibrium is that the manager practices "false modesty": he sometimes withholds favorable information even if it is the only information available. Thus, in contrast to regulators' concerns about managers painting an unreasonably "rosy" picture of the firm, our model shows that selective disclosure may result in managers conveying an unreasonably "gloomy" one.

We next examine how accounting aggregation affects the manager's disclosure behavior. Under aggregate disclosure, the manager may only disclose how many successes and failures have been observed but provides no other indication of its components. For instance, if the manager only observes one success and discloses that signal as an aggregate, investors are unable to infer from the disclosure as to which of the two signals was actually observed by the manager. We derive equilibrium conditions under which only two pure-strategy disclosure policies exist in the aggregate disclosure setting. The two policies differ in that the manager discloses the aggregate of a favorable and an unfavorable realization in one, whereas he does not disclose it in the other.

Finally, we compare the welfare consequence of disaggregate disclosure with that of aggregate disclosure and show that aggregate disclosure results in higher welfare when the two signals are not "too" highly observationally correlated.⁵ The intuition is as follows. In equilibrium, disaggregate disclosure lowers welfare by suppressing favorable information on the less value relevant signal, i.e.,

⁵In our model, welfare is based on the premise that investors derive some utility from better matching their actions to the outcomes. We assume that investors incur a fixed disutility if there is a mismatch. Thus, the welfare loss under a disclosure regime can be interpreted as the maximum that investors are willing to pay for an information system that fully reveals the manager's private information.

false modesty. In contrast, in equilibrium, aggregate disclosure lowers welfare because investors are unable to discern which signal is favorable and which is unfavorable from the aggregate of a favorable and unfavorable realization, i.e., information garbling. When the signals are highly observationally correlated it is unlikely that the manager observes one signal and not the other. Consequently, the welfare loss from false modesty under disaggregate disclosure is relatively small. On the other hand, since the manager is very likely to have observed both signals, aggregation lowers the information quality through garbling the two signals. When the two signals are not too highly correlated, the manager is more likely to be falsely modest. Aggregate disclosure overcomes the false modesty since investors' inability to discern which of the signals indicates good news reduces the manager's incentive to suppress it. Further, the lower observational correlation implies that the manager is less likely to observe both signals. As a result, the welfare loss from information garbling under aggregation is not too high. Thus, even though aggregation exacerbates information asymmetry through information garbling, it expands the region (or the information set) over which information is disclosed when the signals are not "too" observationally correlated.

Albeit a highly stylized setting, our model seeks to capture several features of financial information disclosures. First, a salient characteristic of the accounting disclosures, as well recognized in the literature and recently summarized in Dye (2002), is essentially a process of classification: a firm is either a going concern or is not; an expenditure is either a periodic expense or an investment; an asset

is either short-term or long-term; etc. Consequently, accounting information is often presented in highly categorical terms. While this feature is a result of the specific rules, standards and conventions that govern accounting disclosures, more importantly, it reflects the fact that accounting information can only help resolve some, but not all, of the firm's underlying uncertainty. To capture this feature, we follow the literature and model the disclosure as providing a binary signal that is informative about some underlying true outcomes. Second, we note that accounting disclosures are oftentimes highly aggregate in nature: results of different operating units are often disclosed as a simple aggregate. This implies that investors are not always able to precisely infer the components of the disclosed aggregate, a feature that is captured by our formulation of aggregation in the model. Last but not the least, managers typically have considerable amount of discretion in determining the degree of aggregation in disclosure. Our model captures this by allowing the manager to commit to one of two regimes (aggregate vs. disaggregate) and completely characterizes manager's optimal disclosure decisions as well as regime choices.

Our model extends the prior disclosure research, in particular Dye [1985] and Jung and Kwon [1988], to a multiple-signal setting where the market is uncertain about the extent to which the manager is informed. In Dye's and Jung and Kwon's model, a partial disclosure equilibrium emerges in which the manager always discloses his information if the single signal that he privately observes takes a value above a certain threshold. A direct consequence of this single-signal setup is that the equilibrium partial disclosure policy is unique

because, post-disclosure, all information asymmetry between the investors and manager is resolved. In contrast, extending their model to a multiple-signal setting may result in multiple equilibria because when the manager discloses one signal, investors are free to hold off-equilibrium beliefs regarding the undisclosed signal. Our paper establishes precise conditions for a unique equilibrium to arise under disaggregation.

Our disaggregate disclosure model is similar in spirit to Pae [2005] where a manager potentially observes two signals and the market is uncertain of his information endowment.⁶ He derives a symmetric pure strategy equilibrium in which both signals are disclosed if they are favorable and confirm each other, and only one is disclosed if it is sufficiently more favorable than the other. Our analysis, however, differs from Pae's in two ways. First, we identify conditions under which a unique selective disclosure equilibrium exists in a disaggregate disclosure regime. This is due to the fact that we allow the two signals, observed by the manager, to have differing value relevance (or precision), while the two signals studied in Pae's model are restricted to have equal precision. In our setting, we demonstrate that selective disclosure may result in the suppression of good news. This "false modesty" equilibrium can only arise if the two information signals differ in their value relevance. Second, we demonstrate the role of accounting aggregation in eliciting greater disclosure. Even though aggregation garbles information, it potentially precludes the information suppression

⁶Our model is also related to Kirschenheiter's [1997] setting in which a privately informed manager, facing proprietary costs of disclosure (Verrecchia [1983]), chooses between disclosing both or either of the signals. We depart from this setting by focusing on nonproprietary information and by allowing the manager to withhold information entirely.

endemic in selective disclosure.

The remainder of the paper is organized as follows. In the next section we describe our stylized model. Section 3 derives the equilibrium under disaggregate disclosure and discusses the corresponding comparative statics. Section 4 derives the aggregate disclosure equilibrium and section 5 compares the welfare consequence of selective disclosure with that of aggregate disclosure. Section 6 concludes.

2 The Model

We model a firm whose risk neutral founder-manager (simply referred to as the manager hereafter), for unmodelled liquidity reasons, desires to sell the firm to a group of homogeneous risk neutral investors who competitively bid for its shares. The manager's objective is to maximize the price at which the firm's ownership is transferred. Given our competitive bidding assumption, the selling price must equal investors' expected valuation of the firm conditional on all the information available to them at the time of the transaction. The firm consists of two projects, indexed by a superscript $k = 1$ or 2 . Project k 's preliminary outcome (or, simply, outcome), denoted by i^k , can be either a success (h) or a failure (l), i.e., $i^k \in \{h, l\}$. For simplicity, we assume that $\Pr(i^k = h) = \Pr(i^k = l) = \frac{1}{2}$ ⁷ and the two projects' outcomes are distributed independently. Firm value, denoted by v , equals the sum of the two projects'

⁷In section 5, we will show that our results are not qualitatively changed if this assumption is relaxed.

values, $v = v^1 + v^2$. Each project's value, v^k , is a function of its outcome and the actions taken by investors after they purchase the firm, j^k . The investors' actions are also binary, high or low, i.e., $j^k \in \{h, l\}$. Specifically, the relation between the project outcome, investors' action and project value is

$$v^k \equiv \theta I(i^k = h) - I(j^k \neq i^k)L^k \quad (1)$$

where $i^k, j^k \in \{h, l\}$ and $I(\cdot)$ an indicator function. The first term implies that a successful project increases firm value by θ relative to a unsuccessful project. The second term implies that if investors' subsequent action doesn't match the project's outcome, then L^k is the loss resulting from this mismatch. Our formulation reflects the notion that firm value is affected by both the effectiveness of its business model (or outcome) and its owner's actions that are complementary to that outcome. For example, while firm value is higher from greater demand for its products, profit is maximized only when investment in production is able to fulfill the potential demand. In our setting, disclosure is valuable precisely because it enables investors to better match their actions (e.g., investment) with the preliminary outcome (e.g., demand). We further assume that investors' action mismatches impact the values of the two projects differentially. Specifically, we assume that $L^1 = mL$ and $L^2 = L$, where $m \in [0, 1]$, i.e., the loss from an action mismatch is potentially larger for project two. For instance, project 1 may be in a mature business line where investors have other sources of information in addition to firm's disclosure, while project 2 may be in a new

business line with no alternative information sources.⁸

The manager and investors are asymmetrically informed about the outcomes of the two projects. Formally, each project generates a signal, $e^k, e^k \in \{e_h, e_l\}$, $k = 1, 2$, which is informative about its outcome i^k and may be privately observed only by the manager. Although both signals are informative about i^k 's, they have different precision, with e^1 being more precise. For example, project 1 may be such that the manager has extensive prior experience and thus has an acute sense of its prospect, while project 2 is in an unfamiliar territory for the manager. Without loss of generality we assume e^1 is in fact a perfectly informative signal, i.e., $\Pr(e^1 = e_h | i^1 = h) = \Pr(e^1 = e_l | i^1 = l) = 1$.⁹ In contrast, e^2 is a noisy signal of i^2 : it correctly reflects i^2 with probability $q > \frac{1}{2}$, i.e., $\Pr(e^2 = e_h | i^2 = h) = \Pr(e^2 = e_l | i^2 = l) = q$. We interpret the parameter q as the value relevance of e^2 , the signal for the second project.

Consistent with the prior literature on disclosure, the manager observes e^1 with an unconditional probability $p \in (0, 1)$ (Dye [1985]). In addition, we assume that whether the manager observes e^2 is correlated with whether he observes e^1 : i.e., $\Pr(e^2 \text{ is observed} | e^1 \text{ is observed}) = \Pr(e^2 \text{ is not observed} | e^1 \text{ is not observed}) = t \in [0, 1]$. The parameter t measures the strength of this observational correlation between the two signals: as t increases, observing the first signal increases the likelihood that the manager also observes the second. Con-

⁸Our assumption of a differential loss across projects does not affect the equilibrium disclosure strategy in either of the disclosure regimes. However, it does play an important role in the welfare comparisons across disclosure regimes, as described in section 5.

⁹Assuming e^1 is a noisy signal does not qualitatively change our results.

sequently, the probability parameters, p and t , specify the information system available to the firm as follows,

$$\begin{aligned} \Pr(e^1 \text{ observed}) &= p & \Pr(e^1 \text{ not observed}) &= 1 - p \\ \Pr(e^2 \text{ observed}) &= 1 - (p + t - 2pt) & \Pr(e^2 \text{ not observed}) &= p + t - 2pt \end{aligned} \quad (2)$$

In order to influence investors' assessment of the firm's value, the manager makes strategic disclosures about his private information, e^k 's. Our analysis allows the manager to commit to one of two disclosure regimes, disaggregate or aggregate, to report the two project outcomes. Under the disaggregate disclosure regime, the manager may make two separate disclosures, d_D^1 and d_D^2 , one for each project, where the subscript D denotes disaggregate disclosure and the superscript denotes the project. An important feature of disaggregate disclosure is that if the manager elects to disclose a signal, he not only needs to disclose the signal but must also disclose the project that generated it. Consistent with the prior literature, we assume that the manager is unable to falsify information, although, he may withhold it. That is, if the manager privately observes $e^k = e_l$ (e_h), he can either truthfully disclose his private information, i.e., $d_D^k = e_l$ (e_h) or not disclose anything, $d_D^k = \phi$. We preclude the possibility that the manager misreports by disclosing $d_D^k = e_h$ (e_l). A consequence of this assumption is that the manager's observation of at least one of the two signals is a necessary condition for disclosure, i.e., if the manager does not observe e^k ,

his disclosure must be $d_D^k = \phi$.

Under the aggregate disclosure regime, the manager commits to only disclosing how many success signal(s) and failure signal(s) are observed. Notationally, a disclosure under the aggregate regime is characterized by $d_A = (*, *)$, where the subscript A denotes aggregation and the first (second) element in the bracket denotes the number of success (failure) signals e_h (e_l) observed. Consistent with the prior setting, we assume that the manager can either truthfully disclose the aggregate of the two signals or not disclose anything. An important distinction between the two disclosure regimes lies in the fact that with aggregation the manager only discloses the aggregate without indicating the corresponding project. For example, if the manager privately observes only one project's signal, say $e^1 = e_h$, under aggregate disclosure he may either disclose $d_A = (1, 0)$, or $d_A = \phi$. If the manager discloses $d_A = (1, 0)$, investors cannot infer which project generated e_h , the success signal.

The sequence of events is as follows. At date 1, the manager commits to follow one of the two disclosure regimes. At date 2, the manager stochastically observes the information signals e^k 's and makes strategic disclosures consistent with the prevailing disclosure regime. Finally, investors observe the manager's disclosure, price the firm and ownership is transferred at date 3.

3 Equilibria under Disaggregate Disclosure

Our disaggregate disclosure model extends the single-signal disclosure problem studied in the prior literature (e.g., Dye [1985] and Jung and Kwon [1988]) to a multiple-signal setting. However, unlike prior research, our analysis is complicated by the fact that information asymmetry between the manager and investors persists post-disclosure. Consequently, multiple disclosure equilibria may exist in our setting. This complication is absent in Dye [1985] and Jung and Kwon [1988], since in their model, all information asymmetry between the manager and investors is resolved if a manager discloses his private information. In other words, the one-dimensional nature of their model precisely determines all off-equilibrium investor beliefs. This is not the case in our model where a manager may selectively disclose some but not all the information he privately possesses. When there are two signals and a manager discloses only one of the signals, information asymmetry between the manager and investors regarding the withheld signal still persists. Thus, multiple equilibria may arise depending on investors' off-equilibrium beliefs.

Our analysis begins with an intuitive observation that the manager will always disclose (never disclose) e^1 when he privately observes $e^1 = e_h$ ($e^1 = e_l$) since it is perfectly informative of project 1's initial outcome, i^1 . In contrast, since e^2 is only a noisy signal of i^2 and thus its disclosure has a smaller price impact, the manager's strategic behavior is restricted to the disclosure of e^2 . Consequently, we can restrict ourselves to analyzing equilibria that differ only

in the manager's disclosure of e^2 .¹⁰ To illustrate the existence of multiple equilibria and develop the disaggregate disclosure equilibrium, consider the following two candidate disclosure strategies. For expositional ease, we denote a manager's type as (x, y) if he observes $e^1 = x$ and $e^2 = y$. For example, if a manager privately observes $e^1 = e_l$, but is uninformed of e^2 , his type is denoted as (e_l, ϕ) .

Case 1 Type $(e_l, e_l), (e_l, \phi), (\phi, e_l)$ and (ϕ, ϕ) managers disclose $d_D^1 = \phi$ and $d_D^2 = \phi$; type (e_l, e_h) and (ϕ, e_h) managers disclose $d_D^1 = \phi$ and $d_D^2 = e_h$; type (e_h, e_l) and (e_h, ϕ) managers disclose $d_D^1 = e_h$ and $d_D^2 = \phi$; and type (e_h, e_h) manager disclose $d_D^1 = e_h$ and $d_D^2 = e_h$.

Case 2 Type $(e_l, e_h), (\phi, e_h), (e_l, e_l), (e_l, \phi), (\phi, e_l)$ and (ϕ, ϕ) managers disclose $d_D^1 = \phi$ and $d_D^2 = \phi$; type (e_h, e_l) and (e_h, ϕ) managers disclose $d_D^1 = e_h$ and $d_D^2 = \phi$; and type (e_h, e_h) manager disclose $d_D^1 = e_h$ and $d_D^2 = e_h$.

These two cases differ in the manager's disclosure choice with respect to the second signal e^2 . In case 1, a privately informed manager discloses e^2 if and only if it is good news ($e^2 = e_h$). An interpretation of this disclosure strategy is that the manager's disclosure choices for the two projects are independently determined. This case represents an equilibrium only if the type (e_l, e_h) and (ϕ, e_h) managers' payoff is higher than type $(e_l, e_l), (e_l, \phi), (\phi, e_l)$ and (ϕ, ϕ) managers' payoff.

¹⁰The proof of Proposition 1 provides details.

In case 2, however, the manager does not disclose $d_D^2 = e_h$ unless $e^1 = e_h$. This case describes a "false modesty" equilibrium: favorable news is suppressed lest it create a poor impression.¹¹ For this strategy to be an equilibrium, investors must hold a sufficiently pessimistic belief when the manager unexpectedly announces $d_D^1 = \phi$ and $d_D^2 = e_h$. As the manager is unable to falsify information, when the investors observe $d_D^1 = \phi$ and $d_D^2 = e_h$, such a disclosure can only be made by a type (e_l, e_h) or a (ϕ, e_h) manager. As long as the investors' belief puts a sufficient weight on the manager being of type (e_l, e_h) , it can be self-fulfilling: the type (ϕ, e_h) manager is deterred from disclosing $d_D^2 = e_h$. As illustrated in the following proposition, when the two signals are sufficiently observationally correlated (that is, the manager observing e^1 implies a higher likelihood of him also observing e^2), case 2 emerges as a unique pure strategy equilibrium.¹²

Proposition 1 *For L and q sufficiently small, there exists a unique pure strategy equilibrium if $q \neq \frac{L(p-2-mp)+2\theta}{2(2-p)(\theta-L)}$ or $\frac{L[-4t+p(6t-2+m(t-2))]+[4t-p(7t-4)]\theta}{4[2t-p(3t-1)](\theta-L)}$ and $t \in (\hat{t}, 1]$, where \hat{t} is given by*

$$\hat{t} = \frac{4L + 4(1-q)[2q - 1 + p(1-q)]\theta}{[8q - 4 - 4p(4q - 3) + p^2(6q - 7)]\theta}.$$

This unique pure strategy equilibrium is characterized by: Type (e_l, e_h) , (ϕ, e_h) , (e_l, e_l) ,

¹¹The suppression of good news ($e^2 = e_h$) is similar in spirit to a "false modesty" equilibrium in Harbaugh and To [2006] where good news is withheld when the audience already has a sufficiently favorable impression of the person making the disclosure. However, our setup differs from theirs since investors in our model are uncertain about the manager's information endowment, a possibility not allowed in Harbaugh and To [2006].

¹²Following the prior literature, we exclusively focus on pure strategy equilibria.

(e_l, ϕ) , (ϕ, e_l) and (ϕ, ϕ) managers disclose $d_D^1 = \phi$ and $d_D^2 = \phi$; type (e_h, ϕ) and (e_h, e_l) managers disclose $d_D^1 = e_h$ and $d_D^2 = \phi$; and type (e_h, e_h) manager discloses $d_D^1 = e_h$ and $d_D^2 = e_h$.

Proof. (See Appendix) ■

The intuition behind Proposition 1 is based on the fact that case 1 cannot be an equilibrium when t is sufficiently large. To illustrate this, let us suppose case 1 is an equilibrium. When t is large, a manager who discloses e^2 but not e^1 is perceived negatively on the equilibrium path because investors believe that this manager is more likely to have observed (e_l, e_h) ; i.e., having observed $e^1 = e_l$ he decided to withhold his private information about e^1 . Consequently, both type (e_l, e_h) and type (ϕ, e_h) managers would rather withhold e^2 even though it is favorable. Thus, a high observational correlation between signals (sufficiently large t) deters the disclosure of e^2 . Further, proposition 1 also establishes a condition under which a unique pure strategy disclosure equilibrium exists where the manager discloses e^1 only if it is favorable and discloses e^2 only if both signals are favorable.¹³

How does the disclosure equilibrium change with the value relevance of the signal? We present a simple comparative static in the following corollary.

Corollary 1 *When L is sufficiently small, the threshold \hat{t} strictly increases in q .*

¹³Although Pae [2005] looks at a similar setting, the two signals studied in his model are equally informative. Observe that when $q = 1$, $\hat{t} = 1$, i.e., the two segments' signals are equally informative, disclosing e^2 whenever it is high always constitutes an equilibrium.

Proof. Taking derivative of $\hat{t} = \frac{4L+4(1-q)[2q-1+p(1-q)]\theta}{[8q-4-4p(4q-3)+p^2(6q-7)]\theta}$ with respect to q generates $\frac{4(2-p)[L(-4+6p)+p(2-3p+p^2)]\theta}{[8q-4-4p(4q-3)+p^2(6q-7)]^2\theta}$, which is strictly positive when L is small.

■

To understand Corollary 1, when q is large, the precision of e^2 as a signal of i^2 increases. Consequently, the penalty for withholding e^2 increases, motivating the manager to disclose $e^2 = e_h$ and shrinking the region where case 2 is the unique pure strategy equilibrium. Thus, greater value relevance (represented by a higher q) reduces selective disclosure.

4 Equilibria under Aggregate Disclosure

Having characterized the equilibria under disaggregate disclosure, we now analyze the manager's strategic behavior when he can only disclose aggregate information. Since the manager is unable to falsify information, under the aggregate disclosure regime the manager may only disclose the aggregate of the two signals, if he chooses to disclose anything. The following proposition characterized equilibria under the aggregate disclosure regime.

Proposition 2 *Under aggregate disclosure, when L and q are sufficiently small, there exist only two pure strategy equilibria.*

- **Equilibrium 1:** type (e_h, e_h) manager discloses $d_A = (2, 0)$; type $(e_h, 0)$ and $(0, e_h)$ managers disclose $d_A = (1, 0)$; type (e_h, e_l) and (e_l, e_h) managers disclose $d_A = (1, 1)$; and the remaining types disclose $d_A = \phi$.
- **Equilibrium 2:** type (e_h, e_h) manager discloses $d_A = (2, 0)$; type (e_h, ϕ)

and (ϕ, e_h) managers disclose $d_A = (1, 0)$; and the remaining types disclose $d_A = \phi$.

Proof. (See Appendix) ■

The two equilibria differ from each other only in terms of the type (e_h, e_l) and (e_l, e_h) managers' behavior. In the first equilibrium, both types disclose $d_A = (1, 1)$, while in the second they chooses to report $d_A = \phi$. Both equilibria are possible because investors may hold different beliefs about the manager's observed information. For instance, if the manager discloses $d_A = (1, 1)$ investors may believe him to be a type (e_l, e_h) thereby dissuading him from making any disclosure. Alternatively, investors may believe that such a disclosure comes from either a type (e_h, e_l) manager or a type (e_l, e_h) manager, which provides an incentive for the manager to disclose. Consequently, investors' beliefs sustain both equilibria.

Although only the aggregate of the two signals can be disclosed, the problem still cannot be reduced to the single-signal disclosure model studied in the prior research. This is because investors cannot always infer the manager's observed information set from the disclosure and residual information asymmetry between the investors and manager persists (post-disclosure). Hence, depending on the investors' off-equilibrium beliefs, there always exist multiple equilibria under aggregation.¹⁴ However, the following lemma shows that the two equilibria are identical in terms of efficiency; i.e. the investors' expected loss from mismatching

¹⁴Observe that under aggregation Equilibrium 1 and 2 exist for all values of t , where there is a unique pure strategy equilibrium under selective disclosure for large t .

their action to the state of nature is the same in both equilibria.

Lemma 1 *The two equilibria in Proposition 2 generate the same level of welfare.*

Proof. *The Lemma is established by noting that the expected loss from investors' actions mismatching the states of nature is the same in both equilibria.*

Q.E.D. ■

This efficiency equivalence of the two equilibria is a direct consequence of our assumption that both states of nature are equally likely, i.e. $\Pr(i^k = h) = \Pr(i^k = l) = \frac{1}{2}$, $k = 1$ or 2 . Given that the "high" state is as likely as the "low" one, on observing $d_A = (1, 1)$ investors infer that the manager is equally likely to be of either type (e_h, e_l) or (e_l, e_h) . Consequently, they are indifferent between taking a "high" action, h , or a "low" action, l . Whether the disclosed information is $d_A = (1, 1)$ or $d_A = \phi$ has no bearing on this indifference. An implication of this is that we can focus on either equilibrium in comparing the welfare consequences of aggregate and disaggregate disclosure.¹⁵

5 Welfare Comparison: Disaggregate versus Aggregate Disclosure

In this section, we compare the welfare implications of disaggregate and aggregate disclosure to identify the conditions under which one regime is preferred

¹⁵While Lemma 1 is obtained as a special case when $\Pr(i^k = h) = \frac{1}{2}$, the welfare comparison analysis in the next section is not qualitatively changed when the ex ante probability of success is not equal to that of failure. See the discussion toward the end of section 5.

over the other. In our model, disclosures have two distinct effects: a distributional effect and an efficiency effect. The distributional effect arises because disclosures determine the price at which the firm is sold and thus the wealth transfer between the manager and investors. As such, this distributional effect solely determines how the wealth is divided between different generations of owners and does not affect total welfare. In contrast, the efficiency effect of disclosure influences welfare because the disseminated information affects the investors' actions, j^k , and these actions must be consistent with initial outcomes, i^k , to maximize value (otherwise investors incur a loss, L^k). By providing information on i^k , disclosures enable investors to better match their actions with the random outcomes. Although disclosure improves welfare via the efficiency effect, it potentially reduces the manager's wealth via its distributional effect. Consequently, the manager may find it optimal to strategically withhold information in order to maximize his own benefit at the expense of reduced welfare.

To compare the welfare consequences of the two disclosure regimes we begin with the efficiency effect. Relative to disaggregate disclosure, aggregate disclosure masks information because when investors observe the manager's disclosure $d_A = (1,1)$ they are unable to infer which of the projects generated the success signal. Because aggregate disclosure reduces the information quality of disclosures, it seems that it should result in lower welfare than disaggregation. However, this intuition fails to recognize the fact that the disclosure regime affects the manager's disclosure incentives through the distributional effect.

To elaborate, note that there are the two countervailing effects on welfare

under aggregate disclosure. The first effect of aggregation, indeed, masks information because investors are unable to infer the manager's private information set from the disclosed aggregate number. This potentially lowers the efficiency of the investors' action. In our model, this manifests itself as a type (e_l, e_h) manager mimicking a type (e_h, e_l) manager. An implication of this is that aggregation worsens efficiency by inducing investors to mismatch their action with the state of nature on one of the two types.

In contrast, the second effect of aggregation is to improve welfare by eliciting greater disclosure by the manager. To illustrate this, consider the manager's strategy under disaggregate disclosure. Under disaggregation, a type (ϕ, e_h) manager is discouraged from disclosing his private information because doing so leads investors to infer that he may be a type (e_l, e_h) manager and results in a lower valuation. Under aggregate disclosure, however, he discloses $d_A = (1, 0)$, thereby pooling with a type (e_h, ϕ) manager who issues the same disclosure in equilibrium. This results in a higher firm valuation and motivates greater disclosure. The fact that investors are unable to distinguish the type (ϕ, e_h) and (e_h, ϕ) managers provides incentives for the former to divulge his private information which improves the efficiency of investors' action.

The following proposition illustrates the welfare consequences of the two disclosure regimes, confining attention to the region over which there exists a unique pure strategy disaggregate disclosure equilibrium.¹⁶

¹⁶To sharpen our results, we refrain from comparing the two regimes when there are multiple selective disclosure equilibria. Following Proposition 1 this occurs when the correlation between observations of the two signals is sufficiently small.

Proposition 3 *Assuming all parameter values are specified as in Proposition 1 and 2, the aggregate disclosure results in higher total welfare than disaggregate disclosure, if m is sufficiently small and $t \in (\hat{t}, \tilde{t})$, where $\hat{t} = \frac{4L+4(1-q)[2q-1+p(1-q)]^\theta}{[8q-4-4p(4q-3)+p^2(6q-7)]^\theta}$, and $\tilde{t} = \frac{2(1-p)(2q-1)}{(4q-2)+p(2+m-4q)}$.*

Proof. (See appendix). ■

Proposition 3 establishes that the trade-off between the two effects of aggregation is a function of t , the correlation parameter. Note, in our model, aggregation encourages greater disclosure when the manager privately observes $e^2 = e_h$ (the less informative signal) only. When t is large, however, the probability of such an event (i.e., a type (ϕ, e_h) manager) is small, because not observing one signal increases the likelihood that the second is also not observed by the manager. In fact, aggregation increases the chance of a mismatched action and reduces welfare by enabling a type (e_l, e_h) manager to mimic a type (e_h, e_l) manager, a likely outcome when the signals are highly observationally correlated, i.e. large t . Thus, when t is sufficiently close to 1, aggregate disclosure lowers welfare, and selective disclosure dominates. In contrast, if t is relatively small, the manager is less likely to privately observe both signals. This is precisely the circumstance where the benefit of aggregate disclosure in enabling greater disclosure is greater than its cost from garbling information. Consequently, aggregation results in higher welfare when the two signals are not "too" highly correlated.

To summarize, in this section, we have compared the welfare consequence

of the two regimes and explicitly identified conditions under which aggregation dominates disaggregation in terms of welfare. Four points are worth noting. First, given our assumption that investors competitively bid for the firm's shares, the selling price exactly equals the investors' expected future payoff from ownership. This implies that the manager captures the entire surplus. As such, the manager's optimal disclosure regime choices also maximize the total welfare. Thus, in our model, there is no role for mandatory (aggregate or disaggregate) disclosure rules, since the manager maximizes efficiency. In fact, our result suggests that requiring firms to always make disaggregate disclosures could actually lower welfare as it encourages "false modesty".

Second, for aggregation to improve efficiency, it is crucial that the manager is able to commit ex ante to such a regime before he may be privately informed. When such commitment is absent, the two aggregate disclosure equilibria identified in Proposition 2 are unsustainable. To see this, note both equilibria have a type (ϕ, e_h) manager pool with a type (e_h, ϕ) manager. But if the (e_h, ϕ) manager is able to renege, he will strictly prefer to issue disaggregate disclosures ex post in order to distinguish himself from the (ϕ, e_h) manager. Albeit a strong assumption, such commitment can be justified using a repeated game argument: if the manager expects to repeatedly make disclosures, he must consider the consequence of current behavior on his future payoff. For instance, a type (e_h, ϕ) manager this period has to consider the possibility of becoming a type (ϕ, e_h) manager next period. Thus, renegeing on his commitment to aggregation may jeopardize his ability to commit in the future. To the extent

that a manager is sufficiently concerned about the long term, a type (e_h, ϕ) may still find it optimal to issue aggregate disclosures.

Third, in our formulation of aggregation, the manager is restricted to either truthfully disclosing the aggregate or not disclosing anything. An alternative way of modelling aggregation is to allow the manager to "selectively aggregate", i.e., he also chooses which signals are aggregated. For example, a type (e_l, e_h) manager may disclose $d_{SA} = (1, 1)$, $(0, 1)$, $(1, 0)$, or ϕ , where the subscript SA denotes "selective aggregation". It is easy to verify the existence of a "selective aggregation" equilibrium, for all parameter values, where type (e_h, e_h) manager discloses $d_{SA} = (1, 1)$; type (e_h, ϕ) , (ϕ, e_h) , (e_h, e_l) , and (e_l, e_h) managers disclose $d_{SA} = (1, 0)$; and the remaining types disclose $d_{SA} = \phi$. Clearly, this equilibrium is welfare equivalent to the two equilibria in Proposition 2. Thus, even "selective aggregation" could improve efficiency relative to disaggregate disclosure.¹⁷ The fact that our results are robust to "selective aggregation" shows that the welfare improvement under aggregation is not driven by restricting the manager's reporting space but instead is driven by investors' inability to perfectly back out individual signals from the disclosed aggregate.

Lastly, our assumption that $\Pr(i^k = h) = \Pr(i^k = l) = \frac{1}{2}$ may appear overly restrictive. Indeed, once it is relaxed, Lemma 1 ceases to hold: Equilibrium 2 identified in Proposition 2 generates strictly less welfare than Equilibrium

¹⁷There are of course other equilibria under selective aggregation, some of which may be dominated by disaggregation in terms of efficiency. But at a minimum selective aggregation offers a possibility of welfare improvement over disaggregation. Furthermore, we don't believe these other equilibria are reasonable in the sense that they are pareto dominated (the manager is strictly better off and investors are indifferent) by the equilibrium identified in the text.

1, as the former is a garbling of the latter. However, even if we bias against aggregation by only comparing Equilibrium 2 with the unique equilibrium under disaggregation, the general intuition behind Proposition 3 is not qualitatively changed. This is because the welfare function is continuous in the ex ante probability of success, i.e., $\Pr(i^k = h)$, and thus Proposition 3 qualitatively holds as long as this probability is in the neighborhood of $\frac{1}{2}$. The following corollary illustrates this point.

Corollary 2 *When m is sufficiently small and $\Pr(i^k = h)$ is sufficiently close to $1/2$, there exists a cutoff \bar{t} such that Equilibrium 2 of Proposition 2 generates strictly higher welfare than the disaggregate disclosure equilibrium of Proposition 1 if and only if $t < \bar{t}$.*

6 Conclusion

The extant disclosure literature has focused on both the incentives of privately informed agents to reveal their private information and the welfare consequences of their actions.¹⁸ The problems that have been studied largely relate to an agent's observation of a single signal of firm prospects that investors use to value the firm's assets. We extend this literature by examining an agent's disclosure strategy when he privately observes two signals, with differing value relevance. We demonstrate the existence of a pure strategy equilibrium if the agent is allowed to disclose in a disaggregate fashion. If the observation of the two

¹⁸See Verrecchia [2001] for a summary of the theoretical literature on disclosure.

signals is sufficiently correlated, i.e. privately observing one signal increases the likelihood of observing the other, we show that it is optimal for the manager to be "falsely modest": to withhold favorable information about the less value-relevant signal even if it is the only information that he possesses.

We then consider the situation where the agent can only disclose the aggregate of the two observed signals if he elects to disclose. Aggregation reduces the information quality of the disclosure because investors may not be able to infer the manager's observed information from the disclosed aggregate. However, we show that welfare is higher under aggregate disclosure than under disaggregate disclosure when the observational correlation between the two signals is not "too high." This occurs because, even though aggregation reduces the information content of the disclosure, it expands disclosure by precluding false modesty. In contrast, when the two signals are highly observationally correlated the manager is unlikely to be "falsely modest" under disaggregate disclosure, so aggregate disclosure reduces welfare by garbling the released information. Our results provide a rationale for accounting aggregation by identifying conditions under which it potentially improves disclosure efficiency.

7 Appendix

Proof of Proposition 1. For notational ease, denote $V\{*(j^1, j^2)\}$ as the investors' valuation of the firm when the manager is believed to be a type in set $\{*\}$ and actions j^1 and j^2 are chosen. For example, $V\{(e_h, e_l)|(h, l)\} \equiv (2-q)\theta - (1-q)L$, and $V\{(e_h, e_l), (e_h, \phi)|(h, l)\} \equiv \Pr(\text{manager is type } (e_h, e_l)|\text{manager is either type } (e_h, e_l) \text{ or } (e_h, \phi))V\{(e_h, e_l)|(h, l)\} + \Pr(\text{manager is type } (e_h, \phi)|\text{manager is either type } (e_h, e_l) \text{ or } (e_h, \phi))V\{(e_h, \phi)|(h, l)\}$. The proof then proceeds with seven claims.

Claim 1 When $t > \frac{4L+4(1-q)[2q-1+p(1-q)]\theta}{[8q-4-4p(4q-3)+p^2(6q-7)]\theta}$,

$$V\{(e_l, e_l), (e_l, \phi), (\phi, e_l), (\phi, \phi)|(l, l)\} > V\{(e_l, e_h), (\phi, e_h)|(l, h)\}.$$

Proof of Claim 1. Easy to show that

$$V\{(e_l, e_l), (e_l, \phi), (\phi, e_l), (\phi, \phi)|(l, l)\} - V\{(e_l, e_h), (\phi, e_h)|(l, h)\} =$$

$$\frac{AL - B\theta}{C}, \text{ where}$$

$$\begin{aligned} A = & -2p - 2mp + 2mp^2 + 2p^2(1 - 2q) + 4pq + [-4 + 4mp - 4mp^2 - 9p^2(1 - 2q) \\ & + 8q - 12(2q - 1)]t + [4 + 9p^2(1 - 2q) - 8q + 12p(2q - 1)]t^2 \end{aligned}$$

$$B = 4(2q - 1)(t - 1) - 4p[2 - 3t + q(4t - 3)] + p^2[4 - 7t + q(6t - 4)];$$

$$C = [(3p - 2)t - 2][2 - 2t + p(3t - 2)].$$

Easy to see that $C < 0$. If we can show $AL - B\theta < 0$, then we are done. As the coefficient before t^2 is negative and that before t is strictly positive,

$$\begin{aligned} A &< -2p - 2mp + 2mp^2 + 2p^2(1 - 2q) + 4pq + \\ &\quad [-4 + 4mp - 4mp^2 - 9p^2(1 - 2q) + 8q - 12(2q - 1)] \times 1 \\ &\quad + [4 + 9p^2(1 - 2q) - 8q + 12p(2q - 1)] \times 0 \\ &= -4 + 2p(5 + m - 10q) + 8q + p^2(-7 - 2m + 14q) \end{aligned}$$

The last expression is further bounded above by setting $m = 1$ and $q = 1$, which gives $A < 4$. Thus a sufficient condition for $AL - B\theta < 0$ is $B\theta > 4L$, which yields

$$4L - [4(2q - 1)(t - 1) - 4p(2 - 3t + q(4t - 3)) + p^2(4 - 7t + q(6t - 4))]\theta < 0.$$

Note that the LHS of the inequality is an decreasing function of t when q is close to $1/2$ and equals 0 at $\hat{t} = \frac{4L + 4(1 - q)[2q - 1 + p(1 - q)]\theta}{[8q - 4 - 4p(4q - 3) + p^2(6q - 7)]\theta}$. Q.E.D. ■

Claim 2 In any equilibrium, type (e_h, e_h) manager will disclose all his private information, i.e., $d_D^1 = e_h$ and $d_D^2 = e_h$.

Proof of Claim 2. Suppose otherwise. Type (e_h, e_h) manager will have to pool with other types whose valuation is strictly less, and thus have incentives to deviate. A contradiction. Q.E.D. ■

Claim 3 *In any equilibrium, with L sufficiently small, type (e_l, e_l) manager will not disclose all his private information.*

Proof of Claim 3. *Suppose otherwise. Type (e_l, e_l) manager will receive the lowest possible payoff when L is sufficiently small and thus have incentives to deviate. A contradiction. Q.E.D. ■*

Claim 4 *With L sufficiently small, there cannot exist an equilibrium in which some manager(s) among type (e_l, e_l) , (e_l, ϕ) and (ϕ, e_l) discloses $e^1 = e_l$ while some manager(s) among type (e_l, e_l) , (e_l, ϕ) and (ϕ, e_l) discloses $e^2 = e_l$.*

Proof of Claim 4. *Suppose otherwise. First, observe that type (e_l, e_l) cannot be the only type among the three that discloses $e^1 = e_l$ or $e^2 = e_l$ because no other types are willing to pool with him and he can do strictly better by pooling with type (ϕ, ϕ) when L sufficiently small. Hence, a different equilibrium could only exist under the following scenarios.*

Scenario 1 Type (e_l, e_l) manager discloses $d_D^1 = \phi$ and $d_D^2 = \phi$, type (ϕ, e_l) discloses $d_D^1 = \phi$ and $d_D^2 = e_l$, and type (e_l, ϕ) manager discloses $d_D^1 = e_l$ and $d_D^2 = \phi$. Clearly, no other types are willing to pool with (ϕ, e_l) and (e_l, ϕ) , as they can do strictly better disclosing all their private information when L is small. For this scenario to become an equilibrium, it must be that

$$V\{*, (e_l, e_l), (\phi, \phi)|(j^1, j^2)\} = V\{(\phi, e_l)|(j^1, l)\} = V\{(e_l, \phi)|(j^2, l)\},$$

where $V\{, (e_l, e_l), (\phi, \phi)|(l, l)\}$ is the payoff type (e_l, e_l) and (e_l, e_l) receive, and $*$ stands for those types that might pool with (e_l, e_l) and (ϕ, ϕ) .*

Clearly, this is not possible, as $V\{(\phi, e_l)|(j^1, l)\} > V\{(e_l, \phi)|(j^2, l)\}$, $\forall q < 1$ and L sufficiently small. A contradiction.

Scenario 2 Type (ϕ, ϕ) manager discloses $d_D^1 = \phi$ and $d_D^2 = \phi$, type (e_l, e_l) and (ϕ, e_l) disclose $d_D^1 = \phi$ and $d_D^2 = e_l$, and type (e_l, ϕ) manager discloses $d_D^1 = e_l$ and $d_D^2 = \phi$. Clearly, no other types are willing to pool with (e_l, ϕ) , as they can do strictly better disclosing all their private information, when L is small. For this scenario to constitute an equilibrium, it must be that

$$V\{*, (\phi, \phi)|(l, l)\} < V\{(e_l, \phi)|(l, l)\},$$

which cannot be true when L is small. A contradiction.

Scenario 3 Type (ϕ, e_l) manager discloses $d_D^1 = \phi$ and $d_D^2 = e_l$, and type (e_l, e_l) and (e_l, ϕ) managers disclose $d_D^1 = e_l$ and $d_D^2 = \phi$. Clearly, no other types are willing to pool with (e_l, e_l) and (e_l, ϕ) , as they can do strictly better disclosing all their private information when L is small. For this scenario to constitute an equilibrium, it must be that

$$V\{*, (\phi, \phi)|(j^1, j^2)\} < V\{(e_l, e_l), (e_l, \phi)|(l, l)\},$$

which cannot be true when L is small. A contradiction. Q.E.D.

■

Claim 5 Under the condition specified in the proposition, in any equilibrium, type (e_l, e_l) , (e_l, ϕ) and (e, e_l) managers do not disclose any private information,

i.e., $d_D^1 = \phi$ and $d_D^2 = \phi$.

Proof of Claim 5. *Suppose Otherwise. Following Claim 4, a different equilibrium could only exist under the following scenarios.*

Scenario 1 Type (e_l, e_l) and (ϕ, e_l) managers disclose $d_D^1 = \phi$ and $d_D^2 = \phi$, and type (e_l, ϕ) manager discloses $d_D^1 = e_l$ and $d_D^2 = \phi$. Clearly, no other types are willing to pool with (e_l, ϕ) , as they can do strictly better disclosing all their private information when L is small. Furthermore, observe that type (e_l, ϕ) can always disclose $d_D^1 = \phi$ and $d_D^2 = \phi$. This implies that type (e_l, e_l) and (ϕ, e_l) managers' equilibrium payoff must be the same as type (e_l, ϕ) 's. Finally, no type other than (ϕ, ϕ) is willing to pool with type (e_l, e_l) and (e_l, ϕ) , either, as all the remaining types can choose a different disclosure strategy and get a payoff strictly higher than $V\{(e_l, \phi)|(l, l)\}$ when L is small. Thus, for scenario 1 to be an equilibrium, $V\{(e_l, \phi)|(l, j^2)\}$ must equal to $V\{(e_l, e_l), (e_l, \phi), (\phi, \phi)|(l, l)\}$. This, in turn, implies

$$V\{(e_l, e_l), (e_l, \phi), (\phi, e_l), (\phi, \phi)|(l, l)\} < V\{(e_l, e_h), (\phi, e_h)|(l, h)\}.$$

A contradiction to Claim 1.

Scenario 2 Type (e_l, e_l) and (e_l, ϕ) managers disclose $d_D^1 = \phi$ and $d_D^2 = \phi$, and type (ϕ, e_l) manager discloses $d_D^1 = \phi$ and $d_D^2 = e_l$. Clearly, no other types are willing to pool with (ϕ, e_l) , as they can do strictly better disclosing all their private information when L is small. Furthermore, observe that type

(ϕ, e_l) can always disclose $d_D^1 = \phi$ and $d_D^2 = \phi$. This implies type (e_l, e_l) and (e_l, ϕ) managers' equilibrium payoff must be the same as type (ϕ, e_l) 's. Note that neither type (e_h, e_l) or (e_h, ϕ) is willing to pool with type (e_l, e_l) , (e_l, ϕ) and (ϕ, ϕ) , as they can do strictly better by disclosing their private information when L is small. Hence, it must be either that only type (e_l, e_l) , (e_l, ϕ) , (ϕ, ϕ) , (e_l, e_h) and (ϕ, e_h) disclose $d_D^1 = \phi$ and $d_D^2 = \phi$, or only type (e_l, e_l) , (e_l, ϕ) and (ϕ, ϕ) disclose $d_D^1 = \phi$ and $d_D^2 = \phi$. For the former to be an equilibrium, it must be that

$$V\{(e_l, e_l), (e_l, \phi), (\phi, \phi), (e_l, e_h), (\phi, e_h)|(l, l)\} = V\{(\phi, e_l)|(j^1, l)\},$$

which implies $q = \frac{L(p-2-mp)+2\theta}{2(2-p)(\theta-L)}$. For the latter to be an equilibrium, it must be that

$$V\{(e_l, e_l), (e_l, \phi), (\phi, \phi)|(l, l)\} = V\{(\phi, e_l)|(j^1, l)\},$$

$$\text{which implies } q = \frac{L[-4t+p(6t-2+m(t-2))]+[4t-p(7t-4)]\theta}{4[2t-p(3t-1)](\theta-L)}.$$

Scenario 3 Type (ϕ, e_l) manager discloses $d_D^1 = \phi$ and $d_D^2 = \phi$, and type (e_l, e_l) and (e_l, ϕ) managers disclose $d_D^1 = e_l$ and $d_D^2 = \phi$. Clearly, no other types will pool with type (e_l, e_l) and (e_l, ϕ) , as they can do strictly better by disclosing both signals when L is small. Furthermore, observe that when L is small, $V\{(\phi, e_l), (\phi, \phi)|(j^1, l)\} > V\{(e_l, e_l), (e_l, \phi)|(l, l)\}$, and any other types who can possibly pool with type (ϕ, e_l) and (ϕ, ϕ) has a strictly higher payoff

than $V\{(e_l, e_l), (e_l, \phi)|(l, l)\}$. This contradicts the assumption that such an equilibrium exists, as type (e_l, e_l) strictly better off by deviating and choosing $d_D^1 = \phi$ and $d_D^2 = \phi$.

Scenario 4 Type (e_l, ϕ) manager discloses $d_D^1 = \phi$ and $d_D^2 = \phi$, and type (e_l, e_l) and (ϕ, e_l) managers disclose $d_D^1 = \phi$ and $d_D^2 = e_l$. It is easy to verify that $V\{(e_l, e_h), (\phi, e_h)|(l, h)\} > V\{(e_l, e_l), (\phi, e_l)|(l, l)\}$ when L is small. From Claim 1,

$$V\{(e_l, e_l), (\phi, e_l), (e_l, \phi), (\phi, \phi)|(l, l)\} > V\{(e_l, e_h), (\phi, e_h)|(l, h)\},$$

it must be $V\{(e_l, \phi), (\phi, \phi)|(l, l)\} > V\{(e_l, e_l), (\phi, e_l)|(l, l)\}$. Finally, observe type (e_h, e_l) has no incentive to pool with type (e_l, e_l) and (ϕ, e_l) under any equilibrium, as he can do strictly better by simply disclosing both signals. This implies that scenario 4 cannot constitute an equilibrium, because type (e_l, e_l) and (ϕ, e_l) strictly better off by deviating and choosing $d_D^1 = \phi$ and $d_D^2 = \phi$. A contradiction.

In summary, under the condition specified in the Lemma, any equilibrium in pure strategy will have type (e_l, e_l) , (e_l, ϕ) and (ϕ, e_l) managers choose not to disclose, i.e., $d_D^1 = \phi$ and $d_D^2 = \phi$. *Q.E.D.* ■

Claim 6 With parameter values specified in the proposition, in any equilibrium, type (e_h, e_l) and (e_h, ϕ) managers will use the same disclosure strategy, i.e., $d_D^1 = e_h$ and $d_D^2 = \phi$.

Proof of Claim 6. Suppose otherwise. There are three possibilities. First,

type (e_h, e_l) disclose $d_1^D = e_h$ and $d_2^D = \phi$, and type (e_h, ϕ) discloses $d_D^1 = \phi$ and $d_D^1 = \phi$. Observe that when L is sufficiently small,

$$V\{(e_h, \phi), (\phi, e_h), (e_l, e_l), (e_l, \phi), (\phi, e_l), (e_l, e_h), (\phi, \phi)|(l, l)\} < V\{(e_h, e_l)|(h, l)\}.$$

Note the LHS is the highest payoff type (e_h, ϕ) can possibly get. Thus, he has incentives to deviate. A contradiction. Second, type (e_h, ϕ) disclose $d_1^D = e_h$ and $d_2^D = \phi$, and type (e_h, e_l) discloses $d_D^1 = \phi$ and $d_D^1 = \phi$. Observe, when L is small,

$$V\{(e_h, e_l), (\phi, e_h), (e_l, e_l), (e_l, \phi), (\phi, e_l), (e_l, e_h), (\phi, \phi)|(l, l)\} < V\{(e_h, \phi)|(h, l)\}.$$

Hence, type (e_h, e_l) has incentives to deviate. A contradiction. Third, type (e_h, e_l) , (e_h, ϕ) , (e_l, e_h) , (ϕ, e_h) , (e_l, e_l) , (e_l, ϕ) , (ϕ, e_l) and (ϕ, ϕ) managers choose not to disclose any information, i.e., $d_D^1 = \phi$ and $d_D^2 = \phi$. This scenario cannot constitute an equilibrium when L is small, as

$$V\{(e_h, e_l), (e_h, \phi), (e_l, e_h), (\phi, e_h), (e_l, e_l), (e_l, \phi), (\phi, e_l), (\phi, \phi)|(l, l)\} < V\{(e_h, e_l)|(h, l)\}.$$

Hence, type (e_h, e_l) have incentives to deviate. A contradiction. *Q.E.D.* ■

Claim 7 *With parameter values specified in the Lemma, in any equilibrium, type (e_l, e_h) and (ϕ, e_h) managers will use the same disclosure strategy.*

Proof of Claim 7. *Suppose otherwise. There are two possibilities. First, only type (ϕ, e_h) , (e_l, e_l) , (e_l, ϕ) , (ϕ, e_l) and (ϕ, ϕ) managers disclose $d_D^1 = \phi$*

and $d_D^2 = \phi$; only type (e_l, e_h) discloses $d_D^1 = \phi$ and $d_D^2 = e_h$; only type (e_h, e_l) and (e_h, ϕ) managers disclose $d_D^1 = e_h$ and $d_D^2 = \phi$. For this scenario to be an equilibrium, it must be the case that

$$V\{(\phi, e_h), (e_l, e_l), (e_l, \phi), (\phi, e_l), (\phi, \phi)|(l, l)\} = V\{(e_l, e_h)|(l, h)\}.$$

This implies

$$V\{(e_l, e_l), (e_l, \phi), (\phi, e_l), (\phi, \phi)|(l, l)\} < V\{(e_l, e_h), (\phi, e_h)|(l, h)\},$$

when L is small. A contradiction. Second, type (e_l, e_h) manager discloses $d_D^1 = \phi$ and $d_D^2 = \phi$ and pool with other types, and type (ϕ, e_h) manager discloses $d_D^1 = \phi$ and $d_D^2 = e_h$. For this to constitute an equilibrium, these two types must receive the same equilibrium payoff. However, it is easy to see that when L is small

$$V\{(e_l, e_h), (e_l, \phi), (e_l, e_l), (\phi, \phi), (\phi, e_l)|(l, l)\} < V\{(\phi, e_h)|(j^1, h)\}.$$

A contradiction. Q.E.D. ■

Now, the only possible equilibria left is case 2 identified in the text. Q.E.D.

■

Proof of Proposition 2. Following the proof to Proposition 1, let's denote $V\{*(j^1, j^2)\}$ as the investors' valuation of the firm when the manager is believed to be a type in set $\{*\}$ and actions j^1 and j^2 are chosen. Easy to see that in any

equilibrium type (e_h, e_h) manager discloses $d_A = (1, 1)$, as otherwise he would receive a payoff that is strictly less. The proof consists of four claims.

Claim 8 *In any equilibrium, with L sufficiently small, type (e_l, e_l) manager discloses $d_A = \phi$.*

Proof of Claim 8. *Suppose otherwise. It must be the case that type (e_l, e_l) discloses $d_A = (0, 2)$, receiving the worst possible valuation when L is small. Thus, he strictly betters off by pooling with type (ϕ, ϕ) manager. A contradiction. Q.E.D. ■*

Claim 9 *In any equilibrium, with L and q sufficiently small, type (e_h, e_l) and (e_l, e_h) managers use the same disclosure strategy.*

Proof of Claim 9. *Suppose otherwise. There exist two possibilities. First, type (e_h, e_l) discloses $d_A = (1, 1)$, and type (e_l, e_h) discloses $d_A = \phi$. For this to constitute an equilibrium, it must be that the expect payoff for the two types must be the same. Clearly, type (e_h, ϕ) and (ϕ, e_h) managers do not pool with type (e_l, e_h) , as they can achieve strictly better payoff by disclosing $d_A = (1, 0)$. It is also easy to see that type (e_l, ϕ) and (ϕ, e_l) will pool with type (e_l, e_h) as they can only get a strictly less payoff by disclosing $d_A = (0, 1)$. However, with L sufficiently small,*

$$V\{(e_l, e_h), (e_l, \phi), (e_l, e_l), (\phi, e_l), (\phi, \phi)|(l, l)\} < V\{(e_h, e_l)|(h, h)\}.$$

Thus, type (e_l, e_h) manager will have strict incentives to deviate from the equilibrium. A contradiction. Second, type (e_l, e_h) discloses $d_A = (1, 1)$, and

type (e_h, e_l) discloses $d_A = \phi$. Similarly, observe, when q and L are small, $V\{(e_l, e_h)|(l, h)\}$ is close to $\frac{\theta}{2}$, the worst possible payoff. Hence, type (e_h, e_l) manager will have strict incentives to deviate and pool with type (ϕ, ϕ) . Again, a contradiction. *Q.E.D.* ■

Claim 10 *In any equilibrium, with L small, type (e_h, ϕ) and (ϕ, e_h) managers disclose $d_A = (1, 0)$.*

Proof of Claim 10. *Suppose otherwise. There are three possibilities. First, type (e_h, ϕ) discloses $d_A = (1, 0)$, and type (ϕ, e_h) discloses $d_A = \phi$. For this to constitute an equilibrium, it must be that the expect payoff for the two types must be the same. Easy to see that type $(e_l, e_h), (e_h, e_l), (e_l, \phi)$ and (ϕ, e_l) managers will also disclose $d_A = \phi$ as they can only get a strictly less payoff by disclosing their private information. However, when L is small enough,*

$$V\{(\phi, e_h), (e_l, e_h), (e_h, e_l), (e_l, \phi), (e_l, e_l), (\phi, e_l), (\phi, \phi)|(l, l)\} < V\{(e_h, \phi)|(h, j^2)\}.$$

Thus, type (ϕ, e_h) manager will have strict incentives to deviate from the equilibrium. A contradiction. In the second candidate scenario, type (ϕ, e_h) discloses $d_A = e_h$, and type (e_h, ϕ) discloses $d_A = \phi$. Following a similar line of reasoning as above, type $(e_l, e_h), (e_h, e_l), (e_l, \phi)$ and (ϕ, e_l) managers will also disclose $d_A = \phi$. However, When L is small enough,

$$V\{(e_h, \phi), (e_l, e_h), (e_h, e_l), (e_l, \phi), (e_l, e_l), (\phi, e_l), (\phi, \phi)|(l, l)\} < V\{(\phi, e_h)|(j^1, h)\}.$$

Thus, type (e_h, ϕ) manager will have strict incentives to deviate from the equilibrium. A contradiction. Third, type (e_h, ϕ) and (ϕ, e_h) disclose $d_A = \phi$. For this to constitute an equilibrium, disclosing $d_A = \phi$ must bring a payoff larger than $V\{(\phi, e_h)|(j^1, h)\}$ which is the worst possible payoff the manager can get by disclosing $d_A = (1, 0)$. Thus, type $(e_l, e_h), (e_h, e_l), (e_l, \phi)$ and (ϕ, e_l) managers will also disclose $d_A = \phi$. However, when L is small enough,

$$V\{(e_h, \phi), (\phi, e_h), (e_l, e_h), (e_h, e_l), (e_l, \phi), (e_l, e_l), (\phi, e_l), (\phi, \phi)|(l, l)\} < V\{(\phi, e_h)|(j^1, h)\}.$$

A contradiction. Q.E.D. ■

Claim 11 In any equilibrium, with L and q sufficiently small, type (e_l, ϕ) and (ϕ, e_l) managers disclose $d_A = \phi$.

Proof of Claim 11. There are only four scenarios to consider. First, type (e_l, ϕ) pools with type (e_l, e_l) and (ϕ, ϕ) managers by disclosing $d_A = \phi$, while type (ϕ, e_l) discloses $d_A = (0, 1)$. For this to constitute an equilibrium, it must be the case that

$$V\{(e_l, e_l), (\phi, \phi), (e_l, \phi)|(l, l)\} = V\{(\phi, e_l)|(j^1, l)\},$$

which cannot be true when q is small. A contradiction. Second, type (e_l, ϕ) pools with type $(e_l, e_l), (e_h, e_l), (e_l, e_h)$ and (ϕ, ϕ) managers by disclosing $d_A = \phi$, while type (ϕ, e_l) discloses $d_A = (0, 1)$. For this to constitute an equilibrium,

it must be the case that

$$V\{(e_l, e_l), (\phi, \phi), (e_l, \phi), (e_l, e_h), (e_h, e_l)|(l, l)\} = V\{(\phi, e_l)|(j^1, l)\},$$

which cannot be true when q is small. A contradiction. Third, type (e_l, ϕ) and (ϕ, e_l) disclose $d_A = (0, 1)$. Observe that when L and q are small,

$$V\{(e_l, e_l), (\phi, \phi)|(l, l)\} > V\{(\phi, e_l), (e_l, \phi)|(l, l)\}.$$

Note, LHS represents the worst possible payoff for disclosing $d_A = \phi$. Thus, type (e_l, ϕ) and (ϕ, e_l) have incentives to deviate. A contradiction. Fourth, type (e_l, ϕ) discloses $d_A = (1, 0)$, while type (ϕ, e_l) discloses $d_A = \phi$. When L and q are small, clearly no other types are willing to pool with type (e_l, ϕ) . Hence, he obtains a strictly smaller payoff than announcing $d_A = \phi$. A contradiction.

Q.E.D.

Now, the only possible equilibria left are the two identified in the proposition.

Q.E.D. ■

■

Proof of Proposition 3. Under the disaggregate (aggregate) disclosure regime, denote the expected loss from chosen actions in equilibrium not matching the initial outcome as M_D (M_A). Simple algebra yields

$$M_D - M_A > 0 \Rightarrow \frac{1}{4}L[2(2q - 1)(1 - t) + p(2 + 4q(t - 1) - (m + 2)t)] > 0$$

The proposition is proved by noting that the LHS of the last inequality is a decreasing function in t and reaches 0 when $t = \frac{2(1-p)(2q-1)}{(4q-2)+p(2+m-4q)}$. Q.E.D. ■

Proof of Corollary 1. Define $s \equiv \Pr(i^k = h)$. Under the disaggregate (aggregate) disclosure regime, denote the expected loss from chosen actions in equilibrium not matching the initial outcome as M_D (M_A). When s is sufficiently close to $1/2$ and m sufficiently close to 0, it is easy to show that $M_D - M_A$ is a decreasing function in t and reaches 0 at

$$\bar{t} = \frac{(p-1)[q+s-1+m(2s-1)(1-s+q(2s-1))]}{(p-1)(q+s-1)+m[(1-2s)(1-q-s+2qs)+p(q-1+3s-5qs-3s^2+6qs^2)]}.$$

Q.E.D. ■

References

- [1] Dye, Ronald A, 1985. Disclosure of nonproprietary information. *Journal of Accounting Research* 23, 123-145.
- [2] Dye, R. A. "Classifications Manipulation and Nash Accounting Standards." *Journal of Accounting Research* (September 2002): 1125-1162.
- [3] Harbaugh, Rick and Theodore To, 2006. False modesty: when disclosing good news looks bad. Working paper, Indiana University.
- [4] Hayes, Rachel M. and Russell J. Lundholm, 1996. Segment reporting to a capital market in the presence of a competitor. *Journal of Accounting Research* 34, 261-279.
- [5] Jung, Woon-Oh and Young K. Kwon, 1988. Disclosure when the market is unsure of information endowment of managers. *Journal of Accounting Research* 26, 146-153.
- [6] Kirschenheiter, M., 1997. Information quality and correlated signals. *Journal of Accounting Research* 35, 43-59.
- [7] Milgrom, Paul, 1981. Good news and bad news: representation theorems and applications. *Bell Journal of Economics* 12, 380-391.
- [8] Pae, Suil, 2005. Selective disclosures in the presence of uncertainty about information endowment. *Journal of Accounting and Economics* 39, 383-409.
- [9] Shin, H. S., 1994. News management and the value of firms. *RAND Journal of Economics* 25, 58-71.

- [10] Verrecchia, Robert E., 1983, Discretionary disclosure, *Journal of Accounting and Economics* 5, 179-194.
- [11] Verrecchia, Robert E., 2001, Essays on disclosure, *Journal of Accounting and Economics* 32, 97-180.