The $Q$-theoretical link between
stock and investment returns

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Abstract
Investment-based asset pricing uses the equivalence between stock and investment returns to tie expected stock returns with firm characteristics. This equivalence holds even under variable capacity utilization, proportional operating costs, irreversible investment, and dividend constraints. With multiple capital goods, stock return is the value-weighted average of individual investment returns. However, no analytical link is available between the two returns under time-to-build.

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1 Introduction

Traditional asset pricing models determine expected stock returns with the intertemporal rate of substitution of consumers (e.g., Rubinstein (1976), Lucas (1978), and more recently, Campbell and Cochrane (1999)). As first pointed out by Cochrane (1991, 1996), however, expected returns can also be tied with the intertemporal rate of transformation of firms from the $Q$-theory of investment. Restoy and Rockinger (1994) further prove that, with constant return to scale, stock return equals investment return that depends explicitly on firm characteristics. This approach seems promising given the anomalies literature that has documented many empirical associations between firm characteristics and future stock returns in capital markets (e.g., Fama (1998) and Schwert (2003)).

I investigate the structural link between stock and investment returns under more general conditions in the real economy than previously considered. The main finding is that the structural link seems fairly general. Specifically, the equivalence between the two returns subsists even with endogenous capacity utilization, flow operating costs, investment irreversibility, and dividend constraints (that impose a lower bound on dividend payment or an upper bound on external equity financing). With multiple capital goods, the stock return equals the value-weighted average of individual investment returns. However, under time-to-build, no analytical link is available between stock and investment returns. The reason is that the investment return concerns only the tradeoff between marginal benefits and marginal costs of new investment projects. But the stock return is the return to the entire firm that derives its market value not only from the new but also from the incomplete projects.

The newly-derived relations between stock and investment returns also provide transparent intuition about the real determinants of expected returns. Specifically, the degree of the
dividend constraints and the rate of capital depreciation are negatively associated with future stock returns. And the degree of the irreversibility constraint and the structure-to-equipment ratio in a firm’s capital holdings are positively associated with future stock returns.

Besides Cochrane (1991, 1996) and Restoy and Rockinger (1994), this paper is most related to Lettau and Ludvigson (2002), who are among the first to demonstrate that predictive variables for excess stock returns can predict long-horizon fluctuations of aggregate investment growth using the $Q$-framework. An early version of Gomes, Yaron, and Zhang (2004) shows that the investment return is the leverage-weighted average of equity and debt returns when there exists debt financing. More generally, this paper adds to the literature that tries to understand the real determinants of the cross-section of expected returns. Recent examples include, among others, Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004), Cooper (2005), Gourio (2005), and Whited and Wu (2005).

Section 2 establishes the equivalence between stock and investment returns in the baseline $Q$-model. Sections 3–7 then explore the link between the two returns with endogenous capacity utilization and flow operating costs, investment irreversibility, dividend constraints, multiple capital goods, and time-to-build, respectively. Section 8 concludes.

2 The Baseline Model

This section derives the equivalence between stock and investment returns under constant return to scale — the Hayashi (1982) conditions — in the standard $Q$-model. This result is first established by Restoy and Rockinger (1994). My exposition is a simplified version of that in Zhang (2005). The goal is to lay the ground for the generalizations in later sections.

Consider a firm that uses capital and a vector of costlessly adjustable inputs, such as
labor, to produce a perishable output. The firm chooses the levels of these inputs each period to maximize its operating profit, defined as its revenue minus the expenditures on these inputs. Let $\Pi_t = \Pi(K_t, X_t)$ denote the maximized operating profit at time $t$, where $K_t$ is the capital stock at time $t$ and $X_t$ is a vector of exogenous shocks.

Assumption 1 The operating profit function exhibits constant return to scale:

$$\Pi(K_t, X_t) = \Pi_1(K_t, X_t)K_t$$

where the marginal product of capital, denoted $\Pi_1(K_t, X_t)$, is strictly positive, and where subscript $i$ denotes the first-order partial derivative with respect to the $i^{th}$ argument. Assumption 1 applies to firms that are price-takers in output and factor markets.

Taking the operating profit as given, firms then choose optimal investment to maximize their market values. Capital accumulates according to:

$$K_{t+1} = I_t + (1 - \delta)K_t$$

which says that the end-of-period capital equals investment, denoted $I_t$, plus beginning-of-period capital net of depreciation. Capital depreciates at a fixed proportional rate of $\delta$.

Investment involves costs such as purchase-sale costs and convex costs of physical adjustment. Purchase-sales costs occur when firms buy or sell uninstalled capital. When firms disinvest, these costs are negative. For analytical simplicity, I assume that the relative purchase price and relative sale price of capital are equal, and are both normalized to unity. Convex costs of physical adjustment are nonnegative costs that are zero when $I_t = 0$. These costs are continuous, strictly convex in $I_t$, non-increasing in capital $K_t$, and differentiable with respect to $I_t$ and $K_t$ everywhere. Thus, the total cost of investment represents the sum
of purchase-sale costs and convex costs of physical adjustment. And I denote the total as
\( \Phi(I_t, K_t) \) and referred to it as the augmented adjustment-cost function.

**Assumption 2** The augmented adjustment-cost function exhibits constant return to scale in \( I_t \) and \( K_t \), i.e.,

\[
\Phi(I_t, K_t) = \Phi_1(I_t, K_t)I_t + \Phi_2(I_t, K_t)K_t
\]

Moreover, \( \Phi_1(I_t, K_t) > 0 \) and \( \Phi_2(I_t, K_t) \leq 0 \).

A firm’s dynamic value-maximization problem is:

\[
V(K_t, X_t) = \max_{\{I_{t+j}, K_{t+1+j}\}_{j=0}^{\infty}} E_t \left[ \sum_{j=0}^{\infty} M_{t,t+j} (\Pi(K_{t+j}, X_{t+j}) - \Phi(I_{t+j}, K_{t+j})) \right]
\]

where \( V(K_t, X_t) \) is the cum-dividend market value because when \( j = 0 \), \( \Pi(K_t, X_t) - \Phi(I_t, K_t) \)
is included in \( V(K_t, X_t) \). \( M_{t,t+j} > 0 \) is the stochastic discount factor from time \( t \) to \( t+j \).

\( M_{t,t} = 1 \) and \( M_{t,t+i}M_{t+i+1,t+j} = M_{t,t+j} \) for some integer \( i \) between 0 and \( j \). For notational
simplicity, I use \( M_{t+j} \) to denote \( M_{t,t+j} \) whenever the starting date is \( t \).

Let \( q_t \) be the present-value multiplier associated with capital accumulation equation (2). The Lagrange formulation of the firm value is then:

\[
V(K_t, X_t) = \max_{\{I_{t+j}, K_{t+1+j}\}_{j=0}^{\infty}} E_t \left[ \sum_{j=0}^{\infty} M_{t+j} (\Pi(K_{t+j}, X_{t+j}) - \Phi(I_{t+j}, K_{t+j}) - q_{t+j}[K_{t+j+1} - (1 - \delta)K_{t+j} - I_{t+j}]) \right]
\]

The first-order conditions with respect to \( I_t \) and \( K_{t+1} \) are, respectively,

\[
q_t = \Phi_1(I_t, K_t)
\]

\[
q_t = E_t[M_{t+1}[\Pi_1(K_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}, K_{t+1}) + (1 - \delta)q_{t+1}]]
\]
Solving equation (7) recursively yields an economic interpretation for marginal $q_t$, i.e., the expected present value of marginal benefits of capital:

$$q_t = E_t \left[ \sum_{j=1}^{\infty} M_{t+j}(1 - \delta)^{j-1}(\Pi_1(K_{t+j}, X_{t+j}) - \Phi_2(I_{t+j}, K_{t+j})) \right] > 0 \quad (8)$$

where the inequality follows from $\Pi_1(K_t, X_t) > 0$ and $\Phi_2(I_t, K_t) \leq 0$.

The optimality conditions (6) and (7) say that firms will adjust investment until the marginal cost of investment equals marginal $q_t$, which is the expected present value of marginal benefits of capital or the shadow price of uninstalled capital. Combining the first-order conditions in equations (6) and (7) yields:

$$E_t[M_{t+1}r_{t+1}^I] = 1 \quad (9)$$

where $r_{t+1}^I$ denotes the investment return:

$$r_{t+1}^I = \frac{\Pi_1(K_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}, K_{t+1}) + (1 - \delta)\Phi_1(I_{t+1}, K_{t+1})}{\Phi_1(I_t, K_t)} \quad (10)$$

Intuitively, the investment return is the ratio of the marginal benefit of investment at time $t+1$ divided by the marginal cost of investment at time $t$. The denominator, $\Phi_1(I_t, K_t)$, is the marginal cost of investment. By optimality, it equals the marginal $q_t$, the expected present value of marginal profits of investment. In the numerator of equation (10), $\Pi_1(K_{t+1}, X_{t+1})$ is the extra operating profit from the extra capital at $t+1$; $-\Phi_2(I_{t+1}, K_{t+1})$ captures the effect of extra capital on the augmented adjustment cost; and $(1 - \delta)\Phi_1(I_{t+1}, K_{t+1})$ is the expected present value of marginal profits evaluated at time $t+1$, net of depreciation.

**Proposition 1** Define the ex-dividend firm value, denoted $P_t$, and stock return, denoted
\( r_{t+1}^S \), as, respectively,

\begin{align*}
P_t & \equiv V(K_t, X_t) - \Pi(K_t, X_t) + \Phi(I_t, K_t) \\
r_{t+1}^S & \equiv \frac{P_{t+1} + \Pi(K_{t+1}, X_{t+1}) - \Phi(I_{t+1}, K_{t+1})}{P_t} \tag{12}
\end{align*}

Under Assumptions 1 and 2, \( P_t = q_t K_{t+1} \) and \( r_{t+1}^S = r_{t+1}^I \).

Proof. I first expand the value function (5) as follows:

\[
P_t + \Pi(K_t, X_t) - \Phi(I_t, K_t) = \Pi(K_t, X_t) - \Phi(I_t, K_t) - q_t(K_{t+1} - (1 - \delta)K_t - I_t) \\
+ \mathbb{E}_t[M_{t+1}(\Pi(K_{t+1}, X_{t+1}) - \Phi(I_{t+1}, K_{t+1}) - q_{t+1}(K_{t+2} - (1 - \delta)K_{t+1} - I_{t+1}))) + \ldots]
\]

Recursive substitution using equations (3), (6), and (7) implies that:

\[
P_t + \Pi(K_t, X_t) - \Phi(I_t, K_t) = q_t(1 - \delta)K_t + \Pi(K_t, X_t) - \Phi_2(I_t, K_t)K_t
\]

Therefore, \( P_t = q_t(1 - \delta)K_t + \Phi_1(I_t, K_t)I_t = q_t((1 - \delta)K_t + I_t) = q_t K_{t+1} \). Next, using this equation along with equations (1), (2), and (3) to rewrite equation (12) as

\[
r_{t+1}^S = \frac{q_{t+1}(I_{t+1} + (1 - \delta)K_{t+1}) + \Pi_1(K_{t+1}, X_{t+1})K_{t+1} - \Phi_1(I_{t+1}, K_{t+1})I_{t+1} - \Phi_2(I_{t+1}, K_{t+1})K_{t+1}}{q_t K_{t+1}}
\]

\[
= \frac{\Pi_1(K_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}, K_{t+1}) + (1 - \delta)\Phi_1(I_{t+1}, K_{t+1})}{\Phi_1(I_t, K_t)} = r_{t+1}^I
\]

where the second equality follows from equation (6).

The conditions under which Proposition 1 holds can be extended to the case in which the operating-profit and the adjustment-cost functions are homogeneous of the same degree. Under these conditions, Abel and Eberly (1994) show that marginal \( q \) is proportional to average \( Q \), and Zhang (2005) shows that stock return equals investment return.
Together, equation (10) and Proposition 1 provide a convenient analytical link between
stock returns and firm characteristics.

3 Capacity Utilization and Flow Operating Costs

It is relatively straightforward to model endogenous capacity utilization and flow operating
costs in the $Q$-framework. Cooper, Gerard, and Wu (2005) link utilization rate to return
volatility, beta, and expected returns. I instead focus on demonstrating that the equivalence
between stock and investment returns is robust to these two extensions.

Specifically, I follow Abel and Eberly (1998) and Cooper, Gerard, and Wu (2005) and
assume that the production function with variable capacity utilization is given by:

$$F(K_t, U_t, X_t) = f(X_t)(K_tU_t) - mU_t^\eta K_t - cK_t$$

(13)

where $U_t$ denotes the rate of capacity utilization, $m$ is the operating costs per unit of utilized
capital, and $\eta > 1$ so that the variable operating costs increase with the utilization rate. I
also follow Cooper et al. and Carlson, Fisher, and Giammarino (2004) and assume that firms
face operating costs proportional to capital, $cK_t$ where $c > 0$.

Maximizing equation (13) with respect to $U_t$ yields the optimal rate of capacity
utilization: $U_t^* = \left[ \frac{f(X_t)}{m\eta} \right]^{\frac{1}{\eta-1}}$. Plugging $U_t^*$ back into equation (13) yields the operating-
profit function:

$$\Pi(K_t, X_t) = f(X_t) \left[ \frac{f(X_t)}{m\eta} \right]^{\frac{1}{\eta-1}} - m \left[ \frac{f(X_t)}{m\eta} \right]^{\frac{\eta}{\eta-1}} - c \right) K_t$$

which is of constant return to scale. Because incorporating endogenous capacity utilization
and proportional operating costs does not affect Assumption 1, the equivalence between
stock and investment returns in Proposition 1 continues to hold.

4 Irreversible Investment

A large portion of the investment literature studies the implications of irreversible investment, which says that investment can never be negative, i.e.,

\[ I_t \geq 0 \] (14)


This section shows that, perhaps surprisingly, incorporating irreversibility does not affect the equivalence between stock and investment returns. Let \( \nu_t \geq 0 \) denote the Lagrange multiplier associated with equation (14). The value function is then:

\[
\max_{\{I_{t+1}, K_{t+1}\}_{j=0}^{\infty}} E_t \left[ \sum_{j=0}^{\infty} M_{t+j} (\Pi(K_{t+j}, X_{t+j}) - \Phi(I_{t+j}, K_{t+j}) - q_{t+j} [K_{t+j+1} - (1 - \delta) K_{t+j} - I_{t+j}] + \nu_{t+j} I_{t+j}) \right]
\]

The first-order conditions with respect to \( I_t \) and \( K_{t+1} \) are, respectively,

\[
q_t = \Phi_1(I_t, K_t) - \nu_t \] (15)

\[
q_t = E_t [M_{t+1} [\Pi_1(K_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}, K_{t+1}) + (1 - \delta) q_{t+1}]] \] (16)

Moreover, the Kuhn-Tucker condition is

\[ \nu_t I_t = 0 \] (17)

Equation (15) says that, when the irreversibility constraint is binding, i.e., \( \nu_t > 0 \) at the point where \( I_t = 0 \). The marginal \( q \) is lower than the sum of the marginal purchase-sale costs
plus the marginal physical adjustment costs of investment, \( \Phi_1(I_t, K_t) \). The difference is given by the multiplier associated with the irreversibility constraint, \( \nu_t \). Intuitively, irreversibility prevents firms from disinvesting. To see this, from the linear homogeneity of \( \Phi \), I can write 
\[
\Phi(I_t, K_t) = G(I_t / K_t)K_t
\]
for some \( G(\cdot) \) with \( G' > 0 \) and \( G'' > 0 \). Thus, \( \Phi_1(I_t, K_t) = G'(I_t / K_t) \).
Equation (15) then implies that \( I_t / K_t = \frac{G^{-1}(q_t + \nu_t)}{G_0^{-1}(q_t + \nu_t)} \), where \( G^{-1}(\cdot) \) is the inverse function of \( G'(\cdot) \), and is an increasing function because \( G'' > 0 \). It follows that the zero rate of investment when the irreversibility constraint is binding, \( G^{-1}(q_t + \nu_t) = 0 \), is higher than the rate of investment without irreversibility, \( G^{-1}(q_t) \), which would involve disinvesting.

Combining equations (15) and (16) yields the investment return:

\[
r^I_{t+1} = \frac{\Pi_t(K_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}, K_{t+1}) + (1 - \delta)(\Phi_1(I_{t+1}, K_{t+1}) - \nu_{t+1})}{\Phi_1(I_t, K_t) - \nu_t}
\] (18)

Thus the investment return is again the marginal benefits of investment at time \( t+1 \) divided by the marginal cost of investment. But with irreversibility, the marginal cost of investment equals the marginal purchase-sale and adjustment costs minus the multiplier associated with the irreversibility constraint when the irreversibility constraint is binding.

Note that this generalized marginal cost of investment, \( \Phi_1(I_t, K_t) - \nu_t \), is strictly positive. The reason is that it equals the marginal \( q \) from equation (15). And because equation (16) with irreversibility is the same as equation (7) in the baseline model, equation (8) continues to hold with irreversibility. In particular, \( q_t > 0 \).

The following proposition says that incorporating irreversibility does not affect the equivalence between stock and investment returns.

**Proposition 2** Under the irreversibility constraint (14) and Assumptions 1 and 2, \( P_t = q_t K_{t+1} \) and \( r^S_{t+1} = r^I_{t+1} \).
Proof. I first expand the value function with irreversibility:

\[ P_t + \Pi(K_t, X_t) - \Phi(I_t, K_t) = \Pi(K_t, X_t) - \Phi(I_t, K_t) - q_t(K_{t+1} - (1 - \delta)K_t - I_t) + \nu_tI_t \]

\[ + E_t[M_{t+1}(\Pi(K_{t+1}, X_{t+1}) - \Phi(I_{t+1}, K_{t+1}) - q_{t+1}(K_{t+2} - (1 - \delta)K_{t+1} - I_{t+1}) + \nu_{t+1}I_{t+1}) + \ldots] \]

Recursive substitution using equations (3), (15), (16), and (17) implies that:

\[ P_t + \Pi(K_t, X_t) - \Phi(I_t, K_t) = q_t(1 - \delta)K_t + (q_t + \nu_t)I_t + \Pi(K_t, X_t) - \Phi_2(I_t, K_t)K_t \]

Therefore, \( P_t = q_t(1 - \delta)K_t + (\Phi_1(I_t, K_t) - \nu_t)I_t = q_t((1 - \delta)K_t + I_t) = q_tK_{t+1}. \) Next, using this equation along with equations (1), (2), and (3) to rewrite equation (12) as

\[ r_{t+1}^S = \frac{q_t(1 - \delta)K_t}{K_{t+1}} + \Pi_1(K_{t+1}, X_{t+1})K_{t+1} - \Phi_1(I_{t+1}, K_{t+1})I_{t+1} - \Phi_2(I_{t+1}, K_{t+1})K_{t+1} \]

\[ = \frac{\Pi_1(K_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}, K_{t+1}) + (1 - \delta)(\Phi_1(I_{t+1}, K_{t+1}) - \nu_{t+1})}{\Phi_1(I_t, K_t) - \nu_t} = r_{t+1}^I \]

where the second equality follows from equation (15).

Some intuition can be obtained regarding the effects of irreversibility on future stock returns. In a two-period model, equation (18) reduces to:

\[ r_{t+1}^I = \frac{\Pi_1(K_{t+1}, X_{t+1}) + (1 - \delta)}{\Phi_1(I_t, K_t) - \nu_t} \quad (19) \]

In this world, firms exist for only two periods. In period \( t \), firms pay the marginal cost of investment, \( \Phi_1(I_t, K_t) \), when the irreversibility constraint is not binding, and pay the marginal cost of \( \Phi_1(I_t, K_t) - \nu_t \) when the constraint is binding. During the period \( t+1 \), firms use the extra unit of capital to produce the marginal product of capital, \( \Pi_1(K_{t+1}, X_{t+1}) \), and sell the capital net of depreciation at the end of period \( t+1 \) for the amount of \( 1 - \delta \). This two-period simplification ignores the dynamic effects captured by \( -\Phi_2(I_{t+1}, K_{t+1}) \) and \( \Phi_1(I_{t+1}, K_{t+1}) - \)
\( \nu_{t+1} \) in equation (18), but it provides a convenient vehicle to fix the basic intuition.

From equation (19) and Proposition 2, all else equal, more constrained firms with higher \( \nu_t \) are associated with higher future stock returns. Although this two-period setup ignores the dynamic effects of constrained investment, this basic intuition is consistent with the simulation results from the dynamic model of Kogan (2004).

5 Dividend Constraints

This section shows that the equivalence between stock and investment returns holds with dividend constraints. The constraints impose a lower bound on firm dividends or an upper bound on external equity. Several studies have used the dividend constraints to capture financial frictions in dynamic Q models (e.g., Cooper and Ejarque (2003), Gomes, Yaron, and Zhang (2004), and Whited and Wu (2004)).

Specifically, I assume

\[
\Pi(K_t, X_t) - \Phi(I_t, K_t) \geq 0
\]

(20)

Allowing a lower bound other than zero does not change the results. And using the zero lower bound simplifies somewhat the derivations.

Let \( \mu_t \) denote the Lagrange multiplier associated with the inequality constraint (20). \( \mu_t \geq 0 \) is then the shadow price of external funds, and its magnitude measures the degree of financial constraints. The Lagrange formulation of the value function is now

\[
\max_{\{I_{t+j}, K_{t+j+1} \}} \mathbb{E}_t \left[ \sum_{j=0}^{\infty} M_{t+j} \left( (1 + \mu_{t+j})[\Pi(K_{t+j}, X_{t+j}) - \Phi(I_{t+j}, K_{t+j})] - q_{t+j}[K_{t+j+1} - (1 - \delta)K_t - I_t] \right) \right]
\]
The first-order conditions with respect to $I_t$ and $K_{t+1}$ are, respectively

$$q_t = (1 + \mu_t)\Phi_1(I_t, K_t)$$ (21)

$$q_t = E_t[M_{t+1}[(1 + \mu_{t+1})(\Pi_1(K_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}, K_{t+1})) + (1 - \delta)q_{t+1}]]$$ (22)

Moreover, the Kuhn-Tucker condition is:

$$\mu_t(\Pi(K_t, X_t) - \Phi(I_t, K_t)) = 0$$ (23)

From equation (21), the marginal cost of investment equals the ratio of the marginal $q_t$ divided by $1 + \mu_t$, and is less than the marginal $q_t$ when the dividend constraint is binding. Intuitively, binding dividend constraints lower the firms’ rates of investment. To see this, recall Assumption 2 implies that $\Phi_1(I_t, K_t) = G'(I_t/K_t)$ for some function $G$ with $G', G'' > 0$. Equation (21) then implies that the rate of optimal investment when the dividend constraint is binding, i.e., $I_t/K_t = G''^{-1}(q_t/(1 + \mu_t))$, is lower than $I_t/K_t = G''^{-1}(q_t)$, the rate of investment when the constraint is not binding.

Combining the first two equations yields the investment return:

$$r^{'I}_{t+1} = \frac{(1 + \mu_{t+1})(\Pi_1(K_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}, K_{t+1}) + (1 - \delta)\Phi_1(I_{t+1}, K_{t+1}))}{(1 + \mu_t)\Phi_1(I_t, K_t)}$$ (24)

Thus the investment return is again the marginal benefits of investment at period $t+1$ divided by the marginal cost of investment at period $t$. But with the dividend constraint, the price of one unit of available resources is not unity, but one plus the shadow price of the external funds, $\mu_t$. This observation implies that both the marginal benefits and marginal costs of investment must be rescaled with the appropriate prices of resources. In the special case when the dividend constraint is never binding, i.e., $\mu_t = \mu_{t+1} = 0$, equation (24) reduces to
the investment return in the standard setup as in equation (10).

The following proposition says that incorporating the dividend constraint does not affect the equivalence between stock and investment returns.

**Proposition 3** Under the dividend constraint (20) and Assumptions 1 and 2, \( P_t = q_t K_{t+1} \) and \( r_{t+1}^S = r_{t+1}^I \).

**Proof.** I first expand the value function with the dividend constraint:

\[
P_t + (1 + \mu_t)(\Pi(K_t, X_t) - \Phi(I_t, K_t)) = (1 + \mu_t)(\Pi(K_t, X_t) - \Phi(I_t, K_t)) - q_t(K_{t+1} - (1 - \delta)K_t - I_t) \\
+ E_t[M_{t+1} \left((1 + \mu_{t+1})(\Pi(K_{t+1}, X_{t+1}) - \Phi(I_{t+1}, K_{t+1})) - q_{t+1}(K_{t+2} - (1 - \delta)K_{t+1} - I_{t+1})\right) + \ldots]
\]

Recursive substitution using equations (3), (21), and (22) implies that:

\[
P_t + (1 + \mu_t)(\Pi(K_t, X_t) - \Phi(I_t, K_t)) = q_t(1 - \delta)K_t + (1 + \mu_t)\Pi(K_t, X_t) - (1 + \mu_t)\Phi_2(I_t, K_t)K_t
\]

Therefore, \( P_t = q_t(1 - \delta)K_t + (1 + \mu_t)\Phi_1(I_t, K_t)I_t = q_t((1 - \delta)K_t + I_t) = q_tK_{t+1} \). Next, using this equation along with equations (1), (2), and (3) to rewrite equation (12) as

\[
r_{t+1}^S = \frac{q_t(I_{t+1} + (1 - \delta)K_{t+1}) + (1 + \mu_t)(\Pi_1(K_{t+1}, X_{t+1})K_{t+1} - \Phi_1(I_{t+1}, K_{t+1})I_{t+1} - \Phi_2(I_{t+1}, K_{t+1})K_{t+1})}{q_tK_{t+1}} \\
= \frac{1 + \mu_{t+1}}{1 + \mu_t} \times \frac{\Pi_1(K_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}, K_{t+1}) + (1 - \delta)\Phi_1(I_{t+1}, K_{t+1})}{\Phi_1(I_t, K_t)} = r_{t+1}^I
\]

where the second equality follows from equation (21).

Some intuition can be obtained regarding the relation between the degree of the dividend constraint and future stock returns. In a two-period world, equation (24) reduces to:

\[
r_{t+1}^I = \frac{\Pi_1(K_{t+1}, X_{t+1}) + (1 - \delta)}{(1 + \mu_t)\Phi_1(I_t, K_t)} \tag{25}
\]
Combined with Proposition 3, equation (25) says that, all else equal, more constrained firms with higher $\mu_t$ are associated with lower future stock returns. Although this two-period setup ignores dynamic effects of constrained investment, the basic intuition is consistent with the simulation results from the dynamic model of Livdan, Sapriza, and Zhang (2005).

6 Multiple Capital Goods

Many different capital goods are used as inputs in firms’ production process, although most Q-theoretical models assume homogeneous capital. Wildasin (1984) extends the Q-theory of investment to the case with multiple capital goods. Tuzel (2005) studies the link between the composition of firms’ capital holdings and their future stock returns. And Tuzel finds that, because investment is irreversible and real estates depreciate more slowly than other types of capital, firms with high percentages of real estates holdings are riskier and earn higher expected returns than firms with low percentages of real estates holdings.

Building on the previous results in Wildasin (1984) and Tuzel (2005), I derive in this section the link between stock and investment returns with multiple capital goods. For simplicity, I consider only two capital goods, structure and equipment, denoted by $S$ and $E$, respectively. Extension to the case with more than two types of capital is straightforward, but with much more notational complexity.

Assumption 3 The operating-profit and the augmented adjustment-cost functions with two capital goods are both linear homogeneous, i.e.,

$$\Pi_t \equiv \Pi(S_t, E_t, X_t) = \Pi_1(S_t, E_t, X_t)S_t + \Pi_2(S_t, E_t, X_t)E_t \quad (26)$$

$$\Phi_t \equiv \Phi(I^S_t, S_t, I^E_t, E_t) = \Phi_{1}I^S_t + \Phi_{2}S_t + \Phi_{3}I^E_t + \Phi_{4}E_t \quad (27)$$
where $I_t^S$ and $I_t^E$ denote investment on structure and equipment, respectively.

Structure and equipment evolve according to, respectively,

\[
S_{t+1} = I_t^S + (1 - \delta_S)S_t
\]

(28)

\[
E_{t+1} = I_t^E + (1 - \delta_E)E_t
\]

(29)

where $\delta_S$ and $\delta_E$ are the depreciation rates for structure and equipment, respectively.

Let $q_{t}^S$ and $q_{t}^E$ be the present-value multipliers associated with structure and equipment accumulation equations (28) and (29), respectively. The Lagrange formulation of the value function is then:

\[
V(S_t, E_t, X_t) = \max_{\{I_t^S, S_{t+1}, I_t^E, E_{t+1}\}} E_t \sum_{j=0}^{\infty} M_{t+j} (\Pi(S_{t+j}, E_{t+j}, X_{t+j}) - \Phi(I_{t+j}^S, S_{t+j}, I_{t+j}^E, E_{t+j}))
\]

(30)

- $q_{t}^S [S_{t+1} - (1 - \delta_S)S_t - I_{t+1}^S] - q_{t+1}^E [E_{t+1} - (1 - \delta_E)E_t - I_{t+1}^E])$

The first-order conditions with respect to $I_t^S$, $S_{t+1}$, $I_t^E$, and $E_{t+1}$ are, respectively,

\[
q_{t}^S = \Phi_1(I_t^S, S_t, I_t^E, E_t)
\]

(31)

\[
q_{t}^E = E_t[M_{t+1} (\Pi_1(S_{t+1}, E_{t+1}, X_{t+1}) - \Phi_2(I_{t+1}^S, S_{t+1}, I_{t+1}^E, E_{t+1}) + (1 - \delta_S)q_{t+1}^S)]
\]

(32)

\[
q_{t}^E = \Phi_3(I_t^E, S_t, I_t^E, E_t)
\]

(33)

\[
q_{t}^E = E_t[M_{t+1} (\Pi_2(S_{t+1}, E_{t+1}, X_{t+1}) - \Phi_4(I_{t+1}^S, S_{t+1}, I_{t+1}^E, E_{t+1}) + (1 - \delta_E)q_{t+1}^E)]
\]

(34)

Solving equations (32) and (34) forward yields the economic interpretation for $q_{t}^S$ and $q_{t}^E$ as
the expected present value of marginal benefits of structure and equipment, respectively,

\[ q_t^S = E_t \left( \sum_{j=1}^{\infty} M_{t+j}(1 - \delta_S)^{j-1}\left( \Pi_1(S_{t+j}, E_{t+j}, X_{t+j}) - \phi_2(I_{t+j+1}^S, S_{t+j+1}, I_{t+j+1}^E, E_{t+j+1}) \right) \right) \]

\[ q_t^E = E_t \left( \sum_{j=1}^{\infty} M_{t+j}(1 - \delta_E)^{j-1}\left( \Pi_2(S_{t+j}, E_{t+j}, X_{t+j}) - \phi_4(I_{t+j+1}^S, S_{t+j+1}, I_{t+j+1}^E, E_{t+j+1}) \right) \right) \]

The interpretation of the optimality conditions (31) and (33) is exactly the same as that in the baseline model. Intuitively, firms adjust structure and equipment capital stocks until the marginal costs of their respective investments equal their respective marginal marginal q.

Combining equations (31)–(34) yields:

\[ E_t[M_{t+1}r_{St+1}^I] = 1 \]

\[ E_t[M_{t+1}r_{Et+1}^I] = 1 \]

where the structure-investment return, \( r_{St+1}^I \), and the equipment-investment return, \( r_{Et+1}^I \), are given by, respectively,

\[ r_{St+1}^I = \frac{\Pi_1(S_{t+1}, E_{t+1}, X_{t+1}) - \phi_2(I_{t+1}^S, S_{t+1}, I_{t+1}^E, E_{t+1}) + (1 - \delta_S)\phi_1(I_{t+1}^S, S_{t+1}, I_{t+1}^E, E_{t+1})}{\phi_1(I_{t+1}^S, S_{t+1}, I_{t+1}^E, E_{t+1})} \] (35)

\[ r_{Et+1}^I = \frac{\Pi_2(S_{t+1}, E_{t+1}, X_{t+1}) - \phi_4(I_{t+1}^S, S_{t+1}, I_{t+1}^E, E_{t+1}) + (1 - \delta_E)\phi_3(I_{t+1}^S, S_{t+1}, I_{t+1}^E, E_{t+1})}{\phi_3(I_{t+1}^S, S_{t+1}, I_{t+1}^E, E_{t+1})} \] (36)

The interpretation of the investment returns is parallel to that in the baseline model. The structure-investment return in equation (35) is the ratio of the marginal benefits of structure-investment at period \( t+1 \) divided by the marginal cost of structure-investment at period \( t \). In the numerator of equation (35), \( \Pi_1(S_{t+1}, E_{t+1}, X_{t+1}) \) is the extra operating profit produced by the extra unit of structure at the beginning of period \( t+1 \), \( -\phi_2(I_{t+1}^S, S_{t+1}, I_{t+1}^E, E_{t+1}) \) captures the benefits of the extra unit of structure on adjusting both structure and equipment, and
(1 − δS)Φ1(I^S\text{t+1}, S_{t+1}, I^E_t, E_{t+1}) is the expected present value of marginal benefits of the extra unit of structure at the end of period t+1, net of depreciation. The interpretation of the equipment-investment return in equation (36) is similar.

The following proposition says that, with multiple capital goods, the stock return equals the value-weighted average of individual investment returns.

**Proposition 4** Define the ex-dividend firm value as:

\[ P_t = V(S_t, E_t, X_t) − Π(S_t, E_t, X_t) + Φ(I^S_t, S_t, I^E_t, E_t) \quad (37) \]

Then under Assumption 3, \( P_t = q^S_t S_{t+1} + q^E_t E_{t+1} \). Moreover,

\[ r^S_t = \frac{q^S_t S_{t+1} + q^E_t E_{t+1}}{P_t} \quad (38) \]

**Proof.** I first expand the value function (30):

\[ P_t + Π(S_t, E_t, X_t) − Φ(I^S_t, S_t, I^E_t, E_t) = Π(S_t, E_t, X_t) − Φ(I^S_t, S_t, I^E_t, E_t) − q^S_t (S_{t+1} − (1 − δS)S_t − I^S_t) \]
\[ − q^E_t (E_{t+1} − (1 − δE)E_t − I^E_t) + E_t[M_{t+1}(Π(S_{t+1}, E_{t+1}, X_{t+1}) − Φ(I^S_{t+1}, S_{t+1}, I^E_{t+1}, E_{t+1})) \]
\[ − q^S_{t+1} (S_{t+2} − (1 − δS)S_{t+1} − I^S_{t+1}) − q^E_{t+1} (E_{t+2} − (1 − δE)E_{t+1} − I^E_{t+1})) + \ldots] \]

Recursive substitution using equations (27), (31), (32), (33), and (34) implies that:

\[ P_t + Π_t − Φ_t = Π_t + q^S_t (1 − δS)S_t − Φ_{2t} S_t + q^E_t (1 − δE)E_t − Φ_{4t} E_t \]

Therefore, \( P_t = q^S_t (1 − δS)S_t + Φ_{tt} I^S_t + q^E_t (1 − δE)E_t + Φ_{tt} I^E_t = q^S_t S_{t+1} + q^E_t E_{t+1} \). Next, using
equations (26)–(29), (35), and (36) to rewrite the right-hand side of equation (38) as

\[
\frac{\Phi_{1t} S_{t+1} \Pi_{1t+1} - \Phi_{2t+1} + (1 - \delta_S) \Phi_{1t+1}}{P_t} = \frac{\Phi_{3t} E_{t+1} \Pi_{2t+1} - \Phi_{4t+1} + (1 - \delta_E) \Phi_{3t+1}}{P_t}
\]

where the third equality follows from \( P_t = q^S_t S_{t+1} + q^E_t E_{t+1} \).

Equations (35), (36), and (38) are useful to glean some intuition on the relation between firms’ capital composition and future stock returns. In a two-period world, equations (35) and (36) reduce to, respectively,

\[
\begin{align*}
  r^I_{St+1} &= \frac{\Pi_1(S_{t+1}, E_{t+1}, X_{t+1}) + (1 - \delta_S)}{\Phi_1(I^S_t, S_t, I^E_t, E_t)} \\
  r^I_{Et+1} &= \frac{\Pi_2(S_{t+1}, E_{t+1}, X_{t+1}) + (1 - \delta_E)}{\Phi_1(I^E_t, S_t, I^E_t, E_t)}
\end{align*}
\]

The effects of depreciation rates on expected returns highlighted in Tuzel (2005) are clear from the above two equations. Because structure depreciates more slowly than equipment, i.e., \( \delta_S < \delta_E \), equations (39) and (40) imply that, all else equal, the expected structure-investment return must be higher than the expected equipment-investment return.

And from equation (38), firms with higher percentages of structure holdings have stock returns with higher weights on the relatively high structure-investment returns. These firms then must earn higher expected returns than firms with lower percentages of structure holdings. Although the two-period example ignores the dynamic effects of investment, the basic intuition is consistent with the simulation results from the dynamic model of Tuzel (2005).
7 Time-to-Build

Several studies emphasize the role of time-to-build in driving business cycle fluctuations (e.g., Kydland and Prescott (1982), Altug (1993), and Christiano and Todd (1996)). The time-to-build technology assumes that multiple periods are required to build new capital projects, as opposed to the one-period convention embedded in the standard capital accumulation equation (2). This section incorporates time-to-build into the $Q$-framework and derives the relation between the market value of the firm and its marginal $q$. But no analytical link is available between stock and investment returns.

Following Kydland and Prescott (1982), I assume new projects require a total of $L \geq 1$ period to complete. Let $O^L_t$ be the total units of a firm’s projects that are $j$ periods from completion at time $t$ for $l=1,\ldots,L$. $O^L_t$ is thus the new projects initiated at time $t$. There is no depreciation of capital before projects enter the production process. All capital projects are homogeneous. Finally, once initiated, projects can never be abandoned or suspended. The recursive representation of capital accumulation is then:

$$
K_{t+1} = (1 - \delta)K_t + O^1_t
$$

$$
O^l_{t+1} = O^{l+1}_t
$$

where $K_t$ denotes the capital stock at the beginning of time $t$ and $\delta$ is the depreciation rate of productive capital. Recursive substitution from the two equations above implies that:

$$
K_{t+L} = (1 - \delta)K_{t+L-1} + O^L_t
$$

Thus the new projects initiated in period $t$ can only become productive in period $t+L$.

Let $\omega_l$ for $l=1,\ldots,L$ denote the fraction of resources allocated to the investment projects
at the $l^{th}$ stage. This definition implies that $\omega_l \geq 0$ and $\sum_{l=1}^{L} \omega_l = 1$. Then the total capital expenditure of the firm all all ongoing projects at time $t$ is:

$$I_t = \sum_{l=1}^{L} \omega_l O_t^l$$

(42)

The standard formulation of time-to-build from Kydland and Prescott (1982) chooses investment weights which imply that the resource costs of a project are distributed evenly throughout the $L$ stages, i.e., $\omega_l = 1/L$. But the parametrization can be altered to capture time-to-plan features of investment projects. For example, to capture one-period planning lag, Christiano and Todd (1996) choose $\omega_1 = 0$ and $\omega_l = 1/(L-1)$ for $l = 2, \ldots, L$.

Let $q_t$ denote the present-value multiplier associated with equation (41). The Lagrange formulation of the value function is then:

$$V(K_t, X_t) = \max_{\{O_{t+j}, K_{t+L+j}\}_{j=0}^{\infty}} E_t \left[ \sum_{j=0}^{\infty} M_{t+j} \times \left( \Pi(K_{t+j}, X_{t+j}) - \Phi \left( \sum_{l=1}^{L} \omega_l O_{t+j}^l, K_{t+j} \right) - q_{t+j} \left[ K_{t+L+j} - (1 - \delta)K_{t+L-1+j} - O_{t+j}^l \right] \right) \right]$$

(43)

The first-order conditions with respect to $O_{t+L}$ and $K_{t+L}$ are, respectively,

$$q_t = E_t \left[ \sum_{l=0}^{L-1} M_{t+l} \Phi_1(I_{t+l}, K_{t+l}) \omega_{L-l} \right]$$

(44)

$$q_t = E_t [M_{t+1}(1 - \delta)q_{t+1} + M_{t+L}[\Pi_1(K_{t+L}, X_{t+L}) - \Phi_2(I_{t+L}, K_{t+L})]]$$

(45)

In the special case of one-period delivery lag, i.e., $L = 1$, the two equations reduce to the standard case in equations (6) and (7), respectively.

Combining the two first-order conditions yields $1 = E_t[M_{t+1}r_{t+1}^L]$ where the investment
return under time-to-build is given by:

\[
\begin{align*}
I_{t+1}^t &= \frac{M_{t+1,t+L} [\Pi_1(K_{t+L}, X_{t+L}) - \Phi_2(I_{t+L}, K_{t+L})] + (1 - \delta) \left[ \sum_{l=0}^{L-1} M_{t+1,t+l+1} \Phi_1(I_{t+l+1}, K_{t+l+1}) \omega_{L-l} \right]}{E_t \left[ \sum_{l=0}^{L-1} M_{t+1} \Phi_1(I_{t+l}, K_{t+l}) \omega_{L-l} \right]}
\end{align*}
\]

which reduces to equation (10) with one-period delivery lag.

Although somewhat complicated, equation (46) is straightforward to interpret. The investment return is again the ratio of marginal benefits of investment evaluated at time \(t+1\) divided by marginal cost of investment at time \(t\). Because projects can never be abandoned or suspended, initiating a new project today means commitment of resources both in the current period and in the next \(L - 1\) periods. The marginal cost in the current period is \(\Phi_1(I_t, K_t)\omega_L\). But the marginal costs in the next one to \(L - 1\) periods have to be discounted using the stochastic discount factor. The total marginal cost of initiating an investment project is given by the denominator of the investment-return equation (46). After \(L\) periods at time \(t+L\), the new project initiated at time \(t\) becomes productive. \(\Pi_1(K_{t+L}, X_{t+L})\) is the extra operating profit from the extra capital at \(t+L\), and \(-\Phi_2(I_{t+L}, K_{t+L})\) captures the effect of extra capital on saving adjustment costs for the firm. Finally, at the end of period \(t+L\), the firm is left with \((1 - \delta)\) unit of extra capital that has a unit price equals the marginal costs of initiating an investment project at time \(t+1\).

The follow proposition gives the relation between the market value of the firm and its marginal \(q_t\) under time-to-build:

**Proposition 5** Under constant return to scale and time-to-build, the ex-dividend value of
the firm satisfies:

\[ P_t = q_t K_{t+L} - E_t \left[ \sum_{i=1}^{L-1} M_{t+i} \Phi_{1t+i} \omega_{L-i} O_{t+i}^L \right] + E_t \left[ \sum_{\tau=1}^{L-1} M_{t+\tau} \left( \Pi_{t+\tau} - \Phi_{2t+\tau} K_{t+\tau} - \sum_{i=1}^{L-1-\tau} \Phi_{1t+i} \omega_{i} O_{t+i}^L \right) \right] \]

(47)

**Proof.** Using the linear homogeneity of \( \Phi \), i.e., \( \Phi_t = \sum_{i=1}^{L} \Phi_{1t+i} O_i^L + \Phi_{2t} K_t \), to expand the value function (43) as follows:

\[
P_t - \Pi_t - \Phi_t = \Pi_t - \sum_{i=1}^{L} \Phi_{1t+i} O_{i+1}^L - \Phi_{2t+1} K_{t+1} - q_t (K_{t+L} - (1 - \delta) K_{t+L-1} - O_{t}^L) \\
+ E_t \left[ M_{t+1} \left( \Pi_{t+1} - \sum_{i=1}^{L} \Phi_{1t+i+1} \omega_{i} O_{i+1}^L - \Phi_{2t+1} K_{t+1} - q_{t+1} (K_{t+L+1} - (1 - \delta) K_{t+L-1} - O_{t+1}^L) \right) \\
+ M_{t+2} \left( \Pi_{t+2} - \sum_{i=1}^{L} \Phi_{1t+i+2} \omega_{i} O_{i+2}^L - \Phi_{2t+2} K_{t+2} - q_{t+2} (K_{t+L+2} - (1 - \delta) K_{t+L+1} - O_{t+2}^L) \right) \\
+ \ldots + M_{t+L} \left( \Pi_{t+L} - \sum_{i=1}^{L} \Phi_{1t+i+L} \omega_{i} O_{i+L}^L - \Phi_{2t+L} K_{t+L} - q_{t+L} (K_{t+L+L} - (1 - \delta) K_{t+L-1} - O_{t+L}^L) \right) + \ldots \right]
\]

Recursively substituting equations (44) and (45) and simplifying yield:

\[
P_t = \Phi_t - \sum_{i=1}^{L-1} \Phi_{1t+i} O_i^L - \Phi_{2t} K_t + q_t (1 - \delta) K_{t+L-1} + E_t \left[ M_{t+1} \left( \Pi_{t+1} - \sum_{i=1}^{L-2} \Phi_{1t+i+1} \omega_{i} O_{i+1}^L - \Phi_{2t+1} K_{t+1} \right) \\
+ M_{t+2} \left( \Pi_{t+2} - \sum_{i=1}^{L-3} \Phi_{1t+i+2} \omega_{i} O_{i+2}^L - \Phi_{2t+2} K_{t+2} \right) + \ldots + M_{t+L-1} \left( \Pi_{t+L-1} - \Phi_{2t+L-1} K_{t+L-1} - \sum_{i=1}^{L-1} \Phi_{1t+i+L} \omega_{i} O_{i+L}^L \right) + \ldots \right] \\
\]

\[
= q_t (1 - \delta) K_{t+L-1} + \Phi_{1t} \omega_{L} O_t^L + E_t \left[ \sum_{\tau=1}^{L-1} M_{t+\tau} \left( \Pi_{t+\tau} - \Phi_{2t+\tau} K_{t+\tau} - \sum_{i=1}^{L-1-\tau} \Phi_{1t+i+\tau} \omega_{i} O_{i+\tau}^L \right) \right]
\]

where the last equality follows from \( \Phi_t = \sum_{i=1}^{L} \Phi_{1t+i} O_i^L + \Phi_{2t} K_t \). Now equation (47) follows by using equations (41) and (44) to rewrite the first two terms in the last equation. ■

In the special case of one-period delivery lag, equation (47) implies that \( P_t = q_t (1 - \delta) K_t + \Phi_{1t} I_t = q_t K_{t+1} \), as in Proposition 1. But in the general case of \( L \geq 2 \), the marginal \( q \) no longer equals the average \( q \) defined as \( P_t / K_{t+1} \). The difference arises because, according to equation
the market value of the firm depends not only on the value of its productive capital stock but also on the value attributed to the incomplete projects. Specifically, once a firm initiates a new project $O_t^L$, the capital stock at time $t+L$ is known at time $t$, and its market value equals $q_t K_{t+L}$. But because of time-to-build, at period $t$ the firm only pays $\Phi_{1t} \omega_{L} O_t^L$, which is only a part of the total project costs. Therefore, the present value of the remaining costs to be paid in the next one to $L-1$ periods, given by $E_t \left[ \sum_{l=1}^{L-1} M_{l+t} \Phi_{1t+l} \omega_{L-l} O_t^L \right]$, should be deducted from the total market value. Moreover, there are also incomplete projects initiated in the last one to $L-1$ periods, projects that can add to the productive capital stock in the next one to $L-1$ periods. Their market values therefore should be included in the total market value of the firm, as done in the last term of equation (47).

Because the marginal $q$ no longer equals the average $q$ with time-to-build, no analytical link is available between stock and investment returns. Intuitively, equation (46) says that the investment return from period $t$ to $t+1$ concerns the tradeoff between marginal benefits and marginal costs of new investment projects initiated at period $t$. But from equation (47), the stock return is the return to the entire firm that includes not only the new but also the old and incomplete projects. As a result, no analytical link between stock and investment returns is available with time-to-build.

8 Conclusion

Previous studies in investment-based asset pricing use the equivalence between stock and investment returns to tie expected returns with firm characteristics. Specifically, the investment return, defined as the ratio of the marginal profits of investment divided by the marginal costs of investment, is an explicit function of firm characteristics. This paper explores the structural link between stock and investment returns under more general
conditions in the real economy than previously considered.

The main finding is that the structural link seems fairly general. The equivalence between stock and investment returns holds even with endogenous capacity utilization, flow operating costs, investment irreversibility, and dividend constraints. With multiple capital goods, the stock return is the value-weighted average of individual investment returns. Under time-to-build, I derive the relation between the market value of a firm and its marginal $q$. But no analytical link is available between stock and investment returns. The intuition is that the investment return quantifies the tradeoff between marginal benefits and marginal costs of new investment projects. But the stock return is the return to the entire firm that derives its market value not only from the new but also from the incomplete projects.

Moreover, the newly-derived analytical relations between stock and investment returns provide transparent intuition on the real determinants of the cross section of returns. The model predicts that the degree of the dividend constraint and the rate of capital depreciation should be negatively associated with future stock returns. And the degree of the investment irreversibility constraint and the structure-to-equipment ratio in a firm’s capital holdings should be positively associated with future stock returns.

These results help understand what the real determinants of expected returns are. This seems an important question. As summarized in Cochrane (2005, p. 4): “[T]he program of understanding the real, macroeconomic risks that drive asset prices (or the proof that they do not do so at all) is not some weird branch of finance; it is the trunk of the tree. As frustratingly slow as progress is, this is the only way to answer the central questions of financial economics, and a crucial and unavoidable set of uncomfortable measurements and predictions for macroeconomics.”
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