A Temporary Monopolist: Taking Advantage of Information Transparency on the Web

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Abstract: Information displayed on an e-commerce site can be used not just by the intended customers but also by competitors. While retailers enhance service quality by linking inventory systems to Web servers and making stockout information available in real time, that stockout information could also be used by competitors in determining their prices on current stocks. In this paper, we examine the effect of such proactive use of information in the setting of e-commerce retailing where duopoly retailers set their prices of a commodity that is in short supply. We show that when customer reservation value is relatively high and retailers are differentiated in fill
rate, both retailers choose the dynamic pricing strategy in equilibrium. By investing in Web scraping technology, retailers automatically monitor each other's stock status and dynamically adjust prices contingent on rival's stock availability.

**Key words and phrases:** contingent pricing, e-commerce, information transparency, vertical differentiation.

Retailing on the Internet offers unprecedented amounts of information to customers and rivals alike. Many retailers provide detailed product information, such as price, product description, technical specification, product manual, accessories, expert and customer reviews, and product recommendations, on their Web sites. In the early stages of e-commerce, many online merchants did not accurately tie their Web sites to their inventory systems, so that stockout information was not displayed in real time. During the 1999 holiday season, many shoppers ordered popular items online but were notified days later that the item was sold out [11]. To improve customer service quality, nowadays many retailers voluntarily display real-time inventory status on their Web sites [22]. For example, retailers such as Amazon.com, Buy.com, and Drugstore.com clearly state each product's availability on their sites. Some retailers such as Amazon and HSN even indicate the number of units left when inventory is running low. While this degree of information transparency is useful to customers in their product purchase decision, it can also be used by competitors to determine their price and product offerings. This strategic use of a rival's stockout information is the focus of this paper.

The use of information transparency for competitive purposes has been documented and observed in financial markets. The London Stock Exchange's "big bang" in 1986, in which it changed from open outcry to an electronic, screen-based trading system, demonstrates this strategic use of information transparency. The new information system at the Exchange publicizes dealer trades and positions to competing dealers, allowing the latter to take advantage of this information. Clemons and Webber [6, 7] observed that if a dealer had an unusually short position in a stock—that is, if the dealer had an unusually low inventory level in the shares of the stock—the other dealers would raise their prices and thus reduce the profitability of the dealer with the short position. In this fashion, the dealers sustained losses of hundreds of millions of pounds as they used the transparency to compete with each other in unprecedented ways. We think that the increased transparency offered by the Internet to online retailers, combined with the ease with which prices can be changed, leads to a similar profit-reducing phenomenon in electronic retailing.

In this paper, we examine online retailers' proactive information use in a market with shortages. Oftentimes it is inevitable that products are out of stock because it is difficult to accurately predict customer demand, especially for seasonal and fad items. For example, no one can know exactly which toy will take the top toy position until the
prime selling season is over. Because of long production lead times, it is not always possible to fulfill shortages quickly. In the 2006 Christmas season, Tickle Me Elmo (list price $39.99) and Nintendo Wii (list price $539.93) were in such short supply that sellers on eBay were selling the first item for $100 or more and the second item for as much as $3,000 [27].

While the issue of stockout is common in both the traditional and online retail channels, e-retailers have more incentive to maintain lower inventory levels because of their extensive product variety offerings. Although online shoppers will accept some shipping delay, they expect the item to be shipped from stock unless they have been notified otherwise. One frequent complaint from customers is that products are out of stock. According to a site survey of 50 retailers by the Chicago-based e-tailing group Inc., 14 percent had backordered or out-of-stock merchandise [22]. Clay et al. [5] sampled 107 books, including New York Times best sellers and randomly picked Books in Print, over 13 U.S.-based online bookstores and two physical chain stores in April 1999. They found that there were significant price dispersions across stores, and no single bookstore in either channel carried all the sample titles, although every store in the sample carried at least 90 percent of the best sellers.

Even though shortages may be inevitable, online retailers make an effort to communicate stockout information in real time with customers by connecting inventory systems to Web servers and offering some indication of inventory position on their Web sites. This product availability information together with price and detailed product information is accessible to both buyers and sellers through search. We observe that there are some price comparison sites such as Dealtime.com and Nextag.com that offer product availability information across stores. This information, however, is not comprehensive or updated frequently enough. So, even though the buyers have some information about availability, it is far from perfect for items that are in short supply. Hence, a customer using price comparison sites is constrained by the update frequency schedule and item coverage chosen by the product/retailer comparison Web site. To get real-time stock and price information, customers have to search retailers’ Web sites directly.

On the other hand, a retailer who wants real-time information from its rivals’ sites may be less limited. Because competing firms are constantly interacting with each other, they have the incentive to invest in screen scraping or Web scraping technology so as to cost-effectively gather information about the target product from their rivals’ sites. Traditionally, retailers cannot efficiently monitor rivals’ stock information in the physical channel. However, in the online environment, because product stockout information is available at retailers’ sites, a firm can build a Web agent, which is a software program that can navigate through Web sites and automatically monitor key content or extract relevant data. With a one-time investment in Web scraping technology, retailers can significantly reduce the cost of tracking competitors’ stockout information by automating the monitoring process. But for individual customers, benefits from a one-time purchase cannot justify the high fixed cost involved in setting up the Web agent. Because retailers are less constrained than customers in getting real-time stock information, they have more up-to-date product availability information than
that available to customers and are able to exploit this transparent information more strategically in the electronic market.

We know that the costs of price adjustment are high in the physical channel because sales personnel have to manually change the displayed prices for each product in addition to changing the price in the database. In the online channel, firms only need to update the price table once and the price change is reflected on every relevant Web page. With real-time stockout information about rivals and negligible costs involved in price adjustment, online retailers can act strategically and take advantage of this information transparency. Specifically, they can make prompt price adjustments in response to rivals’ inventory status changes. The objective of this paper is to investigate retailers’ optimal dynamic pricing strategies in the online channels, which has not been studied before.

We consider two situations. In the first case, duopoly retailers are opportunistic but are symmetric in customer expectation of availability. Because customers expect equal probability of finding the product in stock at retailers, firms compete only on price. We show that although a pure strategy Nash equilibrium does not exist, an undercut-proof equilibrium does.

In the second case, customers expect different levels of product availability from the two retailers. We show that in a market with shortages, customers will choose which retailer to visit based on both retailers’ prices and their product availability levels. We find that when customer reservation value is relatively high, a unique pricing strategy Nash equilibrium exists. In equilibrium, both retailers choose to adopt the Web agent technology, monitor each other’s inventory status, and exploit the online information transparency by dynamically adjusting prices contingent on rival’s availability.

Literature Review

This paper is related to a few streams of literature that study competition with product stockouts, consumer search, and the competitive use of information. Papers (e.g., [4, 8, 9]) in the economics and marketing literature model firms’ prices when there is a positive probability that customers may be rationed. These papers assume that the customers are homogeneous. In equilibrium in these papers, customers do not search and visit only one firm, and firms do not use rivals’ stockout information strategically.

Another set of papers in the operations literature (notably, Lippman and McCardle [16], Mahajan and van Ryzin [17], and Parlar [20]) have dealt with firms’ competition on product availability while ignoring customers’ cost of searching for a firm that has the item in stock. The focus of these papers is on firms’ inventory decision and not on dynamic pricing.

Bakos [1], like other papers on consumer search (e.g., [10, 21, 26, 30]), studies the firms’ pricing strategies while considering customer search behavior but assumes perfect product availability. Balachander and Farquhar [2] study a duopoly competition where customers consider both price and product availability in choosing between firms. Customers know a firm’s price and product availability level, but they have to incur a search cost to find out if the product is available at the firm. In their model [2],
both firms charge a fixed price and do not adjust prices in case of rival stockouts. But customers can search more than one firm in case of a stockout, depending on their expected utilities. They show that when search costs are low, both firms will choose imperfect availability levels, even if higher availability levels are costless in the long run. Because occasional stockout can soften firms’ price competition, firms can gain more by stocking less. Varian [29] studies a symmetric $n$-firm’s competitive pricing equilibrium. There are informed and uninformed customers in the market. Firms choose mixed strategies on price and occasionally offer sales to price discriminate between informed and uninformed customers. Although this paper allows firms to randomize on price, it does not consider the issue of stockout and its effect on customers’ search behavior.

Our paper is also related to literature that studies competitive use of information in financial markets. Clemons and Webber [6] observed that following the London Stock Exchange’s “big bang,” the demise of the trading floor became obvious during the first week of screen-based trading. The screen-based system increases information transparency on the price and the quantity available to buy or sell. However, despite the benefits to the Exchange in terms of increased liquidity and efficient price setting, Exchange members lost money because they were able to use the transparency to compete in unprecedented ways. A later study [7] also shows that the simple screen-based trading system does not reveal subtle information, such as why customers are trading, what they know, and how much more they have to trade, which experienced traders could infer in the old floor trading. In the absence of information needed for effective price negotiation, market makers are cross-subsidizing some unprofitable trades with informed traders. Thus, the naive designs of electronic markets place intermediaries at risk of opportunistic new entrants (off-exchange trading systems) that target attractive accounts.

We incorporate new features of online retailing in our model and investigate the impact of information transparency on retailers’ price competition. We study a market with shortages and consider customers’ optimal search behavior. In our model, retailers can invest in Web agent technology with which they can cheaply monitor their rival’s stock availability. Similarly, as in Clemons and Webber [6, 7], we find that retailers do not always benefit by taking advantage of such information transparency, but the social welfare is improved because search costs are reduced in the new pricing equilibrium.

The Model

Consider two online retailers, indexed 1 and 2, selling a seasonal product. Customer arrival at each retailer is a random process. Because of demand uncertainty, from time to time retailers experience stockouts. Customers choose which retailer to visit based on their expectation of retailers’ prices and product availability levels. Note that product availability is a commonly used performance measurement for both internal and external evaluations. Usually, customers cannot observe retailers’ inventory level, even in the online environment. But they do form an expectation of a retailer’s service
level or order fill rate in terms of the probability that the product is in stock, based on their past experience, word of mouth, or advertising.

We assume that both retailers have access to the Web scraping technology and are financially capable of adopting the technology and developing a software agent that automatically tracks its rival’s stock status. We describe the decision timeline in Figure 1. First, each firm decides whether or not to make its prices contingent on its rival’s stock position. With the help of the software agent technology, retailers can choose a dynamic pricing policy because of the negligible online menu cost. Then they pick their prices. Firm $i$ ($i = 1, 2$) will charge $p_i$ when its rival has the product in stock and $q_i$ when its rival is out of stock. Intuitively, $p_i \leq q_i$. So if firm $i$ chooses a static pricing strategy, it will not adjust its price when its rival is out of stock—that is, $p_i = q_i$.

Having observed retailers’ prices and fill rates, customers will choose which retailer to visit based on their optimal search strategies, and they may visit the second retailer if the first one is out of stock.

Next, consider customers. Customers have a common reservation value $r$ for the product, and each customer has a unit demand for the product. As stated earlier, customers form a common expectation on retailers’ prices and fill rates. However, because they cannot observe retailers’ inventory capacity and do not know the demand fulfillment at the retailers’ sites, at the time of purchase, customers have to search at a retailer’s site to learn about its inventory status. In addition, customers incur a search cost $k$ when shopping online, and $k$ is different among customers. This assumption of heterogeneous search cost is consistent with the results of empirical studies [13, 14]. We assume that search cost $k$ is uniformly distributed in the interval of $[0, 1]$ among the population. There is a rich stream of literature on consumer search; see Bakos [1], Diamond [10], Reinganum [21], Stahl [26], and Wolinsky [30], among others. In those models, customers either search for the best price or for the best fit product. As in Balachander and Farquhar [2], customers search to discover product availability in our model. A customer’s utility from a search is defined as

$$u = \begin{cases} r - p - k & \text{if products are in stock} \\ -k & \text{if products are out of stock} \end{cases}$$

In case of a stockout at the first retailer, a customer can continue searching at the second retailer or choose a third option, which could be consuming other products, placing a backorder, or purchasing in the physical channel. To better focus on retailers’ price competition game, we normalize customers’ utility from a third option to zero. Although it is possible that no customer may search or even buy the product if $r$ is small relative to $k$, we focus our study on the more interesting case where all customers search for the product. We assume that the customer reservation value is high enough so that they prefer search over the third option.

Let $s_1$ and $s_2$ be the order fill rates of retailers 1 and 2, respectively. We study two cases: symmetric fill rate when $s_1 = s_2 = s$ and asymmetric fill rate when we take $s_1 < s_2$, without loss of generality.
Each firm decides whether or not to make its prices contingent on rival's stock position

Each firm picks prices

- \( p_i \) if rival has it in stock
- \( q_i \) if rival is out of stock

\( p_i \leq q_i \)

Customers decide which store to visit first and they may visit second store

Figure 1. The Decision Time Line

Case 1: Symmetric Fill Rate \((s_1 = s_2 = s)\)

Consider the case in which customers expect an equal probability of finding the product in stock at retailers, assuming \(1/2 < s_1 = s_2 = s < 1\). If both retailers charge the same price, then customers will randomly choose a retailer to visit. If one retailer charges a lower price, for example \( p_1 < p_2 \), then all customers prefer to visit the low-price retailer 1 first. Only when they find retailer 1 is out of stock will they search retailer 2 for the product. This model setting differs from the standard Bertrand competition because the firm charging a lower price cannot satisfy the total market demand. So marginal cost pricing is not an equilibrium result, and the high-price firm will gain a positive profit. To see this, consider the case in which both the firms charge at marginal cost. If one of them raises its price, then all of the customers will first visit the other firm. However, they will come back to the firm with the high price when the lower-price firm has a stockout. Hence the firm that raises its price from the marginal price gets a positive profit. So marginal cost pricing is not an equilibrium. In fact, we can show the stronger result that there is no Nash equilibrium in pure strategy, an idea that goes back to Frances Edgeworth.

\textit{Lemma 1: When retailers have an equal fill rate, there is no Nash equilibrium in pure strategies.}

\textit{The proofs are in the Appendix.}\(^5\)

Morgan and Shy [19] and Shy [23, 24] introduce an equilibrium concept called \textit{undercut-proof} equilibrium. Under this equilibrium concept, each firm chooses its price so as to maximize profit while ensuring that its rival will not undercut its price. That is, a firm picks a price that is sufficiently low so that its rival would not find it profitable to set an even lower price so as to grab all of the first firm's customers.

Here we exhibit an undercut-proof equilibrium in which a retailer sets a price \( \hat{p} \) such that its rival has no incentive to undercut it.

\textit{Proposition 1: When retailers are symmetric in fill rate \((s_1 = s_2 = s)\), a unique undercut-proof equilibrium exists in this retailer price competition game in which retailer i charges \( \hat{p} = w + (r - 1/s - w)(1 - s) \) and retailer j charges \( r - 1/s \). The high-price retailer j sells only when the low-price retailer i is out of stock.}
This is similar to the randomized-rationing case as analyzed in Tirole [28] of the capacity-constrained Bertrand competition model. In this undercut-proof equilibrium, retailers have equal expected profits. Because retailers are symmetric in fill rate, customers will pool on the low-price retailer first when retailers charge different prices. The high-price retailer will serve customers only when the first retailer is out of stock. In this case, retailers will not invest in Web scraping technology, and they will not dynamically adjust prices either.

Case 2: Asymmetric Fill Rate \((s_1 < s_2)\)

In this case, we assume that retailers are differentiated in order fill rate, that is, retailers have established their reputation on order fill rates \((s_1, s_2)\) in the market, and the fill rates satisfy \(1/2 < s_1 < s_2 < 1\), which are exogenously given in our model. This assumption says that customers all believe that they have a higher probability of finding the product at retailer 2, the high service quality retailer. We focus our analysis on the fulfilled expectation equilibrium. Thus, we assume that retailers will have corresponding inventory levels such that customers’ expectations on retailers’ prices and stock availabilities are fulfilled in equilibrium. Also, the high fill rate retailer has a higher marginal cost, that is, \(w_2 > w_1\), because of longer product shelf time.

Because each retailer has two price strategies to choose from—static pricing or dynamic pricing—four scenarios are under consideration, as shown in Table 1. We obtain the retailers’ equilibrium pricing strategies in each scenario separately, assuming that all customers search in case of a stockout. Hereafter, we use superscripts to refer to scenarios. For example, \(\pi_i^{DP}\) refers to the profit of retailer \(i\) in the dynamic pricing game.

Each customer will decide which retailer to visit first depending on his or her search cost and retailers’ prices and fill rates. Also, when a customer first visits a retailer and finds the product is in stock, he or she may purchase the product or continue to search the other retailer. We use \(S_{ij}\) to represent a customer’s contingent search strategy, which is to visit retailer \(i\) first and continue searching retailer \(j\) in case retailer \(i\) is out of stock. If we suppose that a customer chooses to purchase the product without searching further when it is in stock, then his or her expected utilities under search strategies \(S_{12}\) and \(S_{21}\) are given by

\[
\begin{align*}
    u_{12} & = s_1 [s_2 (r - p_j) + (1 - s_j)(r - q_j)] - k + (1 - s_j)[s_2 (r - q_j) - k] \quad (1) \\
    u_{21} & = s_2 [s_1 (r - p_j) + (1 - s_j)(r - q_j)] - k + (1 - s_j)[s_1 (r - q_j) - k]. \quad (2)
\end{align*}
\]

Rearranging terms, we can express a customer’s expected utility under the contingent search strategy \(S_{ij}\) as \(R_{ij} - P_{ij} - K_{ij}\), where \(R_{ij} = (s_i + s_j - s_{ij})r\) is the customer’s expected value under this search strategy, \(P_{ij} = s_is_jp_i + s(1 - s_j)q_j + (1 - s_j)s_j q_j\) is the expected payment under this search strategy, and \(K_{ij} = (2 - s_j)k\) is the expected search cost under this search strategy. Customers will choose a search strategy that maximizes their expected utilities. In choosing the optimal search strategy, customers have to trade off between expected payments and expected search costs.
Table 1. Retailers’ Pricing Strategy Matrix

<table>
<thead>
<tr>
<th>Retailer 1</th>
<th>Scenario 1: (dynamic, dynamic)</th>
<th>Scenario 2: (dynamic, static)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retailer 2</td>
<td>Scenario 3: (static, dynamic)</td>
<td>Scenario 4: (static, static)</td>
</tr>
</tbody>
</table>

Lemma 2: (a) There exists a customer with search cost \( k_0 \) such that all customers with search cost \( k < k_0 \) prefer to visit retailer 1 first and those with search cost \( k > k_0 \) prefer to visit retailer 2 first. (b) If a customer finds the product is in stock, it is optimal for him or her to purchase the product instead of continuing to search at the other retailer.

We describe the relationship between customers’ search costs and their expected utilities under the two contingent search strategies in Figure 2, which shows that a customer makes a higher expected payment and incurs a lower expected search cost under the contingent search strategy \( S_{21} \) than the strategy \( S_{12} \).

Lemma 2(a) indicates that if \( 0 < k_0 < 1 \), then customers with lower search costs will visit the low fill rate retailer first because of the lower expected payment and customers with higher search costs will visit the high fill rate retailer first because of the lower expected search costs. So \( k_0 \) is the type of customer who is indifferent between the two contingent search strategies. When \( k_0 = 0 \) or \( k_0 = 1 \), customers will pool on retailer 2 or retailer 1, respectively. Hereafter, we focus our study on the customer-separating equilibrium—that is, \( 0 < k_0 < 1 \). Lemma 2(b) implies that when customers follow their optimal search strategy, they will purchase the product if they find the product in their first search and will not check the second retailer because of imperfect product availability and the existence of search costs.

Static Price Competition

We begin with a benchmark case, the static pricing game, which mimics the traditional retail price competition. In the static price competition game, retailers charge one price throughout the period. Their expected profit functions are given in Equations (3) and (4):

\[
\pi_1^{ss} = s_1 [k_0 + (1 - k_0)(1 - s_2)](p_1^{ss} - w_1) \tag{3}
\]

\[
\pi_2^{ss} = s_2 [1 - k_0 + k_0(1 - s_1)](p_2^{ss} - w_2). \tag{4}
\]

Proposition 2: When parameters satisfy \( C^{ss} > w_2 - w_1 \), there is a unique pricing equilibrium

\[
p_1^{ss*} = \frac{(s_2 - s_1)(2s_1 + s_2 - 2s_1s_2)}{3s_1^2s_2^2} + \frac{2w_1 + w_2}{3}
\]

and
Figure 2. Customers' Utilities Under Contingent Search Strategies $S_{12}$ and $S_{21}$, Where $P_{12}$ and $P_{21}$ Are the Corresponding Expected Prices and $k_0$ Is the Indifferent Customer Type in Terms of Search Cost

\[ P_{21}^{ss} = \frac{(s_2 - s_1)(s_1 + 2s_2 - s_1s_2)}{3s_1^2s_2^2} + \frac{w_1 + 2w_2}{3} \]

in the static pricing game, and the indifferent customer type is given by

\[ k_0^{ss} = \frac{1}{3} + \frac{s_2 - s_1}{3s_1s_2} + \frac{s_1s_2(w_2 - w_1)}{3(s_2 - s_1)} < 1, \]

where

\[ C^{ss} = \frac{(s_2 - s_1)(2s_1s_2 - s_2 + s_1)}{s_1^2s_2^2}. \]

We focus our study on the case where the two retailers are active in the market at the same time—that is, $0 < k_0^{ss} < 1$—so that some customers prefer to visit retailer 1 first and others prefer to visit retailer 2 first. This happens when the parameter condition $C^{ss} > w_2 - w_1$ is satisfied. At the proposed equilibrium prices, customers will separate with more than one-third of them preferring to visit the low fill rate retailer first. As can be seen in Figures 3a and 3b, both retailers' prices are decreasing in the low fill rate $s_2$ and increasing in the high fill rate $s_1$. This implies that when the retailers are more differentiated in terms of fill rate or service level, the price competition is relaxed and each retailer can charge a higher price. Also, the constraint term $C^{ss}$ is increasing in $s_2$ and decreasing in $s_1$. This indicates that we are likely to have a customer-separating equilibrium when retailers are more differentiated in fill rate, given a fixed marginal cost difference.

It is interesting to see that, depending on the parameter values, the profit level of the high fill rate retailer may be lower than that of the low fill rate retailer (see Figures 4a and 4b). This is because, even though it has a lower probability of satisfying demand,
the low-price retailer may have a higher expected demand and inventory level. Specifically, the low service retailer will enjoy a higher profit when parameters satisfy

$$\frac{(s_1 - s_2)(s_1 s_2 + s_2 - s_1)}{2s_1 s_2} < w_2 - w_1.$$ 

Because parameters have the relationship

$$\frac{(s_2 - s_1)(s_1 s_2 + s_2 - s_1)}{2s_1 s_2} < c^{ss},$$
Figure 4a. Optimal Profits for the Static Price Competition at Parameter Values $r = 4$, $w_1 = 1$, $w_2 = 1.3$, $s_2 = 0.9$, and $s_1 \in [0.6, 0.75]$

![Graph showing the optimal profits for the Static Price Competition.]

Figure 4b. Optimal Profits for the Static Price Competition at Parameter Values $r = 4$, $w_1 = 1$, $w_2 = 1.3$, $s_1 = 0.6$, and $s_2 \in [0.7, 0.9]$

![Graph showing the optimal profits for the Static Price Competition.]

In the static pricing game, an equilibrium exists where the high fill rate retailer enjoys a higher profit than its rival only when

$$w_2 - w_1 < \frac{(s_2 - s_1)(s_1s_2 + s_2 - s_1)}{2s_1^2s_2^2}.$$

When

$$\frac{(s_2 - s_1)(s_1s_2 + s_2 - s_1)}{2s_1^2s_2^2} < w_2 - w_1 < C_{SS},$$

being the high fill rate retailer does not pay off.
Dynamic Price Competition

When both retailers adopt the Web scraping technology, they can closely monitor each other’s stock status and engage in a dynamic price competition to take advantage of the online information transparency. Each retailer chooses one low price and one high price and adjusts its prices contingent on its rival’s inventory status. We depict the search process of a random customer with search cost $k$ in Figure 5. Because search cost $k$ is uniformly distributed over $[0, 1]$ and customers with $k < k_0$ visit retailer 1 first, $k_0$ is the probability that a random customer will visit retailer 1 first.

**Proposition 3:** When parameters satisfy $C_{DD} > w_2 - w_1$, there is a unique pricing equilibrium

$$p_1^{DD*} = \frac{2w_1 + w_2}{3}$$

$$q_1^{DD*} = r - \frac{1}{s_1}$$

and

$$p_2^{DD*} = \frac{w_1 + 2w_2}{3}$$

$$q_2^{DD*} = r - \frac{3(1 - s_2) - s_1s_2(w_2 - w_1)}{3s_2(1 - s_1)}$$

in the dynamic pricing game, and the indifferent customer type is given by

$$k_0^{DD} = \frac{s_1s_2(w_2 - w_1)}{3(s_2 - s_1)} < 1,$$

where

$$C^{DD} = \frac{3(s_2 - s_1)}{s_1s_2}.$$  

In the dynamic pricing game, because all customers search in case of a stockout, a retailer’s expected profit is increasing in its high price $q_i$. In equilibrium, the low price $p_i^{DD}$ that a retailer charges when they both have products in stock is independent of its fill rates.

Compared with the static pricing game, fewer customers choose to visit the low fill rate retailer first in the dynamic pricing game than in the static pricing game because when they both have products in stock, the price difference between retailers is lower in the dynamic pricing game. Also, $p_i^{DD}$ is lower than $p_i^{SS}$ whereas $q_i^{DD}$ is greater than $p_i^{SS}$. That is, customers pay a lower price when both firms have the product in stock but a higher price when only one firm has the product in stock. For retailer 2, its low price is even lower than its marginal cost, but because it has a lower chance of stockouts,
retailer 2 is compensated with a high margin when its rival retailer 1 is out of stock. The constraint term has the relationship $C^{DO} > C^{SS}$, and $C^{DO}$ is also increasing in $s_2$ and decreasing in $s_1$. Therefore, at a given set of parameters, if a customer-separating price equilibrium exists in the static price game as in Proposition 2, then a customer-separating price equilibrium exists in the dynamic price game as in Proposition 3.

Dynamic Versus Static Price Competition

When only one retailer adopts the Web scraping technology, it will choose a dynamic pricing strategy and adjust prices contingent on its rival’s availability, and its rival will choose a static pricing strategy.

**Proposition 4(a):** If retailer 1 chooses a dynamic pricing strategy and retailer 2 chooses a static pricing strategy, when parameters satisfy $C^{OS} > w_2 - w_1$, there is a unique pricing equilibrium

$$p_1^{DS^*} = \frac{s_2 - s_1}{3s_1^2s_2} + \frac{2w_1 + w_2}{3}$$

$$q_1^{DS^*} = \frac{r}{s_1}$$

and

$$p_2^{DS^*} = \frac{2(s_2 - s_1)}{3s_1^2s_2} + \frac{w_1 + 2w_2}{3},$$

and the indifferent customer type is given by

$$k_0^{DS} = \frac{1}{3s_1} + \frac{s_1s_2(w_2 - w_1)}{3(s_2 - s_1)} < 1.$$
where
\[
C^{DS} = \frac{(s_2 - s_1)(3s_1 - 1)}{s_1^2 s_2}.
\]

Proposition 4(b): If retailer 2 chooses a dynamic pricing strategy and retailer 1 chooses a static pricing strategy, when parameters satisfy \(C^{SD} > w_2 - w_1 > 0\), there is a unique pricing equilibrium

\[
p_1^{SD*} = \frac{2(s_2 - s_1)(1-s_2) + 2w_1 + w_2}{3s_1 s_2^2}
\]

and

\[
p_2^{SD*} = \frac{(s_2 - s_1)(1-s_2) + w_1 + 2w_2}{3s_1 s_2^2}
\]

\[
d_2^{SD*} = \frac{(2s_2 + s_1)(1-s_2) + s_1 + s_1 s_2^2 (w_2 - w_1)}{3s_2^2 (1-s_1)}.
\]

and the indifferent customer type is given by

\[
0 < k_0^{SD} = \frac{s_2 - 1}{3s_2} + \frac{s_1 s_2 (w_2 - w_1)}{3(s_2 - s_1)} < 1,
\]

where

\[
C^{SD} = \frac{(2s_2 + 1)(s_2 - s_1)}{s_1 s_2^2}
\]

and

\[
C^{SS} = \frac{(1-s_2)(s_2 - s_1)}{s_1 s_2^2}.
\]

Because the retailer who invests in the Web scraping technology has its rival’s stock-out information, it will exploit this information transparency and exert a temporary monopolistic power when its rival is out of stock. In response, the retailer without the Web scraping technology has to reduce its price in order to effectively compete with its rival when they both have products in stock. Figure 6a depicts the corresponding prices in different scenarios at given parameter values. We see that the prices have the following relationships: \(p_1^{DD} < p_1^{SD} < p_1^{SS}\) and \(p_2^{DD} < p_2^{DS} < p_2^{SS}\). Whereas \(p_1^{SD}\) is much lower than \(p_1^{SS}\), \(p_2^{DS}\) is very close to \(p_2^{SS}\). This is because retailer 2 knows that it has a better chance of having the product in stock. In this (dynamic, static) case, even though retailer 2 does not know when its rival is out of stock, by charging a relatively high
Figure 6a. Comparison of Optimal Prices in Different Scenarios at Parameter Values $r = 4$, $w_1 = 1$, $w_2 = 1.3$, $s_2 = 0.9$, and $s_i \in [0.6, 0.75]$.

Figure 6b. Comparison of the Indifferent Customer Type $k_0^*$ in Different Scenarios at Parameter Values $r = 4$, $w_1 = 1$, $w_2 = 1.3$, $s_2 = 0.9$, and $s_i \in [0.6, 0.75]$.

Static price, it can ensure better profits when its rival runs out. On the other hand, the retailer who chooses the dynamic pricing strategy can also charge a higher price when they both have products in stock—that is, $p_{1D}^{SD} > p_{2D}^{DS}$ and $p_{1D}^{DS} > p_{1D}^{DD}$.

Figure 6b illustrates the relationship of the indifferent customer type in the four scenarios with $1 > k_0^{DS} > k_0^{SS} > k_0^{DD} > k_0^{SD} > 0$. Note that the expected demands of retailers are given by $D_1 = k_0 + (1 - k_0)(1 - s_2)$ and $D_2 = 1 - k_0 + k_0(1 - s_i)$, which are increasing and decreasing in $k_0^*$ respectively. Thus a retailer has the highest expected demand when it chooses the dynamic pricing strategy and its rival chooses the static pricing strategy, and it has the lowest expected demand when it chooses the static pricing strategy and its rival chooses the dynamic pricing strategy.
Pricing Equilibrium Analysis

Lemma 3: When the customer reservation value satisfies $r > \max\{2/s_i + w_i, 2/s_z + w_z\}$, it is optimal for both retailers to choose a pricing strategy such that all customers choose contingent search strategies in each scenario.

It is possible that firms may choose to only serve customers with lower search costs. Lemma 3 shows that when customers have a high reservation value, such as for the holiday shopping season, it is optimal for firms to serve all customer types, and all customers prefer to search in case of a stockout.

Under the parameter condition $s_i > 1/2$, we can show that $C^{OS} > C^{SD}$. If the parameters satisfy $C^{OS} = w_2 - w_1 > C^{SD}$, then the parameter conditions in Propositions 2, 3, and 4 are all satisfied, so that a unique pricing equilibrium exists in each scenario. Also, under this parameter condition, we can show that $2/s_1 + w_1 > 2/s_2 + w_2$. The retailers’ profit functions in each scenario are summarized in Table 2. Based on retailers’ payoffs in each scenario, we can derive the unique pricing equilibrium in this retailer competition game.

Proposition 5: When the parameters satisfy $r > 2/s_i + w_i$ and $C^{OS} > w_2 - w_1 > C^{SD}$, the unique pricing equilibrium of this duopoly price competition game is that each retailer chooses the dynamic pricing strategy—that is, its price is contingent on its rival’s inventory status.

Because retailing on the Internet offers an unprecedented amount of information, especially to rival firms, retailers will act strategically and exploit this online information transparency with the help of Web scraping technology. Proposition 5 shows that in our simple duopoly setting, there exists a unique pricing equilibrium in which retailers dynamically adjust their prices based on their rival’s inventory status. Under this dynamic pricing strategy, price competition is intensified when both retailers have the product in stock. However, when one retailer is out of stock, the other retailer with product in stock will promptly increase its price and exert a temporary monopoly power. Unlike the case in the static price game, where the high fill rate retailer sometimes has a lower profit than the low fill rate retailer, in the dynamic price equilibrium, the high fill rate retailer always enjoys a higher profit than the low fill rate retailer.

Proposition 6: Compared with the static price game, in the unique pricing equilibrium as stated in Proposition 5: (a) the social welfare is improved; (b) all customer types have a higher expected payment and lower expected utility; (c) the high fill rate retailer enjoys a higher profit; (d) when $R_2 > r > 2/s_i + w_i$, the low fill rate retailer has a lower profit; only when $r > \max\{R_2, 2/s_i + w_i\}$ will the low fill rate retailer be better off than in the static pricing scheme.

Proposition 6 compares the welfare effects of the equilibrium dynamic price game with the benchmark case, which is the traditional static price game. We show in Proposition 5 that when the customer reservation value is high, the equilibrium outcome is that both retailers adopt the Web scraping technology and engage in a dynamic price game. Because customers incur lower search costs in the dynamic price game than
Table 2. Retailers' Payoff Functions in Each Scenario

<table>
<thead>
<tr>
<th>Retailer 1</th>
<th>Retailer 2</th>
<th>Static</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>Dynamic</td>
<td>Static</td>
</tr>
<tr>
<td>( \pi_1^{DD} = (1-s_2)(s_1r-s_1w_1-1) + \frac{s_1^2 s_2^2 (w_2-w_1)^2}{9(s_2-s_1)} )</td>
<td>( \pi_2^{DD} = s_2 - s_2 w_2 - 1 + s_2 r (1-s_1) )</td>
<td>( \pi_1^{DS} = \frac{(s_2-s_1)}{9s_1^2} + (1-s_2)(s_1 r-1) + s_s w_1 )</td>
</tr>
<tr>
<td>( \pi_2^{DD} = s_2 - s_2 w_2 - 1 + s_2 r (1-s_1) )</td>
<td>( \pi_2^{DD} = \frac{s_1 s_2 (w_2-w_1)^2 + 7w_1 - 2w_2}{9(s_2-s_1)} + \frac{9s_1^2 w_1}{9(s_2-s_1)} )</td>
<td>( \pi_2^{DS} = \frac{2(s_2-s_1) - s_1^2 s_2 (w_2-w_1)^2}{9s_1^2 (s_2-s_1)} )</td>
</tr>
<tr>
<td>Static</td>
<td>Static</td>
<td>Static</td>
</tr>
<tr>
<td>( \pi_1^{SD} = \frac{2(s_1-s_2)(1-s_2) - s_1 s_2^2 (w_2-w_1)}{9s_2^2 (s_2-s_1)} )</td>
<td>( \pi_2^{SD} = \frac{s_2 - s_1 - 4s_2 s_2 - 5s_2^2 (1-s_1) + 4s_2^3 + 9s_2^3 r (1-s_1)}{9s_2^2} )</td>
<td>( \pi_1^{SS} = \frac{(s_2-s_1)(2s_1 + s_2 - 2s_s s_2) + s_s^2 s_2^2 (w_2-w_1)^2}{9s_1^2 s_2^2 (s_2-s_1)} )</td>
</tr>
<tr>
<td>( \pi_2^{SD} = \frac{2s_2^2 (w_2-w_1) + s_1^2 s_2^2 (w_2-w_1)^2 + s_1 s_2 (7w_2 + 2w_1)}{9(s_2-s_1)} )</td>
<td>( \pi_2^{SD} = \frac{s_1 s_2 (s_2-s_1)(4w_1 + 5w_2) - 9s_2^2 w_2}{9(s_2-s_1)} )</td>
<td>( \pi_2^{SS} = \frac{(s_2-s_1)(s_1 + 2s_2 - 2s_s s_2) - s_1^2 s_2^2 (w_2-w_1)^2}{9s_1^2 s_2^2 (s_2-s_1)} )</td>
</tr>
</tbody>
</table>
in the static price game, the social welfare is improved when retailers exploit the information transparency in the online channel. But because the expected payment is higher in the dynamic price game, all customer types are worse off in the dynamic price game.

A further analysis on retailers' profit shows that the high fill rate retailer always prefers to develop a Web agent to monitor its rival's stock status and adjust its price contingent on its rival's availability so as to extract more profits from customers in case of shortages. But for the low fill rate retailer, transparent information about its rival's stock status does not always generate more profits because, after all, the high fill rate retailer has a better chance of having the product in stock. Only when the customer reservation value is high will the low fill rate retailer also have a higher profit under the dynamic pricing strategy. Thus, we find similar results as in Clemons and Webber [6, 7] that increased information transparency offered by the Internet to online retailers may lead to a profit-reducing phenomenon in electronic retailing.

Concluding Remarks

The objective of this paper is to study strategic information use in online retailing. We incorporate new features of online retailing in our model and analytically study retailers' contingent pricing strategy in response to stockouts while taking into account customer search behavior. We show that in a market with shortages, when the customer reservation value is relatively high and retailers are differentiated in fill rates, the Nash equilibrium outcome of this duopoly price competition game is that both retailers choose to adopt the Web scraping technology, automatically monitor each other's stock availability, and apply the dynamic pricing strategy. When doing so, each retailer makes its pricing decision contingent on its rival's inventory status. When they both have inventories, retailers charge lower prices, compared with the prices under the static pricing scheme. But when one retailer is out of stock, the other retailer will promptly adjust its price and behave as a monopolist extracting more surplus from its customers. Because more customers choose to visit the high-availability retailer first under the dynamic pricing scheme than under the static pricing scheme, the total search cost is reduced. However, all customer types are worse off under the dynamic pricing scheme than under the static pricing scheme because the expected payment is higher. We show that the high-availability retailer is always better off under the dynamic pricing scheme, for its rival has a higher probability of running out earlier, and it can extract more surplus from its customers when its rival is out of stock.

In our model, customers incur different search costs while shopping online. Even though online search costs are lower than those in the traditional channel, we do not expect that they are trivially small. A customer still has to spend time and effort to search through and interact with a retailer's Web site in order to locate the desired product and find out its price and availability information. Also, customers are heterogeneous in search costs because they are different in terms of familiarity with retailers' Web sites, online experience, speed of Internet access, and opportunity costs. Empirical studies [13, 14, 18] of customer online behavior indicate that different customers incur
different search costs while searching for the same product at the same retailer's site and support our assumption on customer search cost.

Because customers incur search costs, with imperfect product availability, retailers charge different prices and target different customer segments. Brynjolfsson and Smith [3] and Clay et al. [5] show that there is a high degree of price dispersion in the online market, and retailers with the lowest prices do not receive the most sales. Our model shows that being a high fill rate retailer does pay off under the dynamic pricing scheme because it can charge higher prices and enjoy a higher profit level in equilibrium. One implication for click-and-mortar retailers is that simply duplicating their physical channel pricing schemes in the online world may not work well. As retailers dynamically adjust their prices contingent on their rivals' stock status, our model also provides one possible explanation for price dispersion within a firm and among retailers.

Our model naturally applies to holiday season sales and sales of seasonal or fad items. In such cases, retailers usually face long production lead time, expensive overstock costs, and highly stochastic demands. Therefore a demand-unsatisfied market is quite common. As for customers, because the product is needed in a timely manner, either because it is a limited offering or a promised gift, they are willing to continue searching in case of a stockout. Another conceptual application of our model is for the hotel and airline industries, as both have limited capacity and are very active in the online market. In fact, we often observe that firms in these industries adjust their prices almost simultaneously. This is an indication that they closely monitor rivals' price information. Because the travel industry deals with perishable products and firms often apply complicated revenue management techniques, our model may not directly apply to this industry. In this paper, we emphasize that the information about a rival being sold out does affect a firm's pricing decision for its current stock. Such proactive use of information is not limited to perishable goods. It can be also applied to any markets with shortages. Our model illustrates how firms can act strategically when information about rivals becomes transparent because of the new Web technology.

There are some limitations to our model. First, we focus our study on a duopoly case. Just as in the traditional market, the number of main players in the online retail market is also limited. Survey results and empirical studies show that most customers only purchase from a small number of favorite sites [12, 15]. Although our duopoly model is a simplified case, it still sheds light on oligopoly competition.

Second, retailers are differentiated by product availability level only. While other service factors such as return policies or online support may also affect customers' choices, we omit the effects of these factors to simplify our model analysis. Smith and Brynjolfsson [25] find that while shopbot users in general are very price sensitive, frequent shopbot users are also sensitive to the average delivery time. This finding indirectly supports our assumption that customers choose retailers based on both price and product availability information.

While shopbots make it easy for a customer to acquire price information across retailers, not all products can easily work with shopbots. As we stated early, customers are often constrained by update frequency schedule and product coverage chosen by the
product comparison Web site. Thus, even a shopbot user may not find firms’ accurate product stock information without searching through retailers’ Web sites. And, above all, panel data from Media Metrix [18] show that most customers choose to visit retailers’ sites directly instead of using a shopbot. For example, from July 2001 through May 2002, only 5.7 percent of home Internet users visited a shopbot whereas the rest visited a retailer directly [18]. The study in Montgomery et al. [18] also shows that customers incur costs from using a shopbot, and directly visiting a favorite retailer’s site may yield higher utility than using a shopbot that searches all stores and presents all offers. But for competing firms, one-time investment in Web scraping technology enables them to cost-effectively collect relevant information about rivals. Thus, online retailers possess more information about the market than do consumers.

We assume that fill rates satisfy the condition $s_2 > s_1 > 1/2$. This is a reasonable assumption because retailers with poor order fulfillment will be fined by the Federal Trade Commission. The requirement on customers’ reservation value is also not that strong. Our proposed pricing equilibrium will exist when retailers are well differentiated in fill rate at given marginal cost parameters $(w_1, w_2)$. When retailers are too close to each other in terms of fill rate, they engage in a Bertrand-type competition, and the equilibrium nonexistence problem may surface (see Lemma 4 in the Appendix). In such cases, an undercut-proof equilibrium may exist, as was shown in the case of equal order fill rate.

NOTES

1. The cost of adopting the technology is treated as a fixed cost, which is not included in the model. Furthermore, we assume that customers will not make the same technology investment because they are just making a one-time purchase.

2. For simplicity, we assume that a customer incurs the same search cost at each retailer’s site.

3. A customer who chooses to purchase the product at one of the retailers indicates that (1) he or she prefers this product over other products; (2) he or she prefers shopping at the retailer over shopping in the physical channel; or (3) he or she prefers purchasing the product now over placing a backorder, which often involves long waiting time and more uncertainty, especially when we consider holiday season shopping or sales of seasonal items. Also, there is a cost involved in choosing any of the possible third options.

4. We do not differentiate among the third options. It is possible that because of long lead time, customers prefer purchasing in the physical channel or ordering a different product over placing a backorder. Also, we allow customers to have different third options. Our focus here is to compare the continuing-to-search strategy with a third option when there is a stockout.

5. The nonexistence of a pure strategy Bertrand–Nash equilibrium because of the emergence of price cycle was first identified by Edgeworth for the case where firms have capacity constraints.

6. For products that are in short supply, it is not uncommon for a manufacturer to discriminate between retailers and assign stock based on past relationships and advertising level, and so on. Thus, we focus on studying the case where one retailer has a higher fill rate than the other and not on investigating why this is the case.

7. Similar parameter conditions are considered in Propositions 3 and 4 to ensure that $0 < k_0 < 1$.

8. Let $D_1$ and $D_2$ be the expected demand of retailer 1 and 2, respectively. Then the corresponding expected profits for retailers are given by $\pi_1 = s_1 D_1(p_1 - w_1)$ and $\pi_2 = s_2 D_2(p_2 - w_2)$. So $\pi_2 < \pi_1$ when $(p_2 - w_2)(p_1 - w_1) < s_1 D_1(s_2 D_2)$. 

9. For products that are in short supply, it is not uncommon for a manufacturer to discriminate between retailers and assign stock based on past relationships and advertising level, and so on. Thus, we focus on studying the case where one retailer has a higher fill rate than the other and not on investigating why this is the case.
9. Here,
\[ R^2 = \frac{s_1^3 + s_2^3 (3 - 4s_1) + s_1^2 s_2^2 (1 - s_2) - 4s_1^3 (1 - s_2)^2}{9s_1^3 (1 - s_2) s_2^2} + \frac{2s_2 (w_2 - w_1) + s_1 (1 - s_2) (5w_1 + 4w_2)}{9s_1 (1 - s_2)}. \]

10. We assume that if the customer chooses to continue searching at retailer 1, he or she will again incur a search cost \( k \). But, if the customer finds the product is out of stock at retailer 1, he or she can return to retailer 2’s site and purchase the product without incurring the search cost \( k \) again.

11. It is easy to show that when customers pool on retailer 1, at the given \( r \) condition, it is optimal for retailers to serve all customer types, so that all customers will search in case of a stockout.

12. Here,
\[ R3 = \frac{2 \left( s_1^2 (1 - s_2) (s_1 - s_2 (1 - s_1)) - s_2^2 (1 - s_1) \right) + s_1 \left( -1 + 2s_2 \right) \left( 2w_1 + w_2 \right) - s_2 \left( w_1 + 2w_2 \right)}{3 \left( 2s_1 s_2 - s_1 - s_2 \right)}. \]

REFERENCES


Appendix

Proof of Lemma 1

If we assume that customer reservation value is high, then it is optimal for retailers to serve all customer types (as illustrated next) and all customers search in case of a stockout. Retailers’ profit functions are given in (A1) and (A2).

\[
\pi_1 = \begin{cases} 
    s(p_1 - w_1) & \text{if } p_1 < p_2 \\
    s \left[ \frac{1}{2} + \frac{1}{2} (1-s) \right] (p_1 - w_1) & \text{if } p_1 = p_2 \\
    s(1-s)(p_1 - w_1) & \text{if } p_1 > p_2 
\end{cases} 
\]  \hspace{1cm} (A1)

\[
\pi_2 = \begin{cases} 
    s(1-s)(p_2 - w_2) & \text{if } p_1 < p_2 \\
    s \left[ \frac{1}{2} + \frac{1}{2} (1-s) \right] (p_2 - w_2) & \text{if } p_1 = p_2 \\
    s(p_2 - w_2) & \text{if } p_1 > p_2 
\end{cases} 
\]  \hspace{1cm} (A2)

Next, we show that when customer reservation value is high, it is optimal for retailers to serve all customer types. Suppose retailers charge different prices, for example, \( p_1 < p_2 \). At an equal fill rate \( s_1 = s_2 = s \), all customers prefer to visit retailer 1 first. When retailer 1 is out of stock, a customer will continue searching retailer 2 only if \( s(r - p_2) - k \geq 0 \)—that is, \( k \leq s(r - p_2) \). At a price \( p_2 = r - m/s > p_1 \), retailer 2’s profit function is given by
When \( r > w_2 + 2/s \)—that is, customer reservation value is high—it is optimal for retailer 2 to serve all customer types, and in this case, \( p_2 \) will take the boundary solution \( p_2 = r - 1/s \). Similarly, when customer reservation value is high, retailer 1 will not charge a price above \( r - 1/s \). Thus retailers will serve all customers in the market when \( r \) is high.

Let \( p \) be the price such that \((F-w)s = (r-1/s-w)s(1-s)\). Suppose \( w_1 = w_2 = w \) and \( p_1 = p_2 = p \), where \( \tilde{p} < p \leq r - 1/s \); then the retailers’ profits are \( \pi_1 = \pi_2 = (p-w)s(1-s/2) \). In this case, each retailer has an incentive to undercut its rival by a small amount \( \epsilon \) to attract all customers and gain a higher profit \( \pi_i = (p - \epsilon - w)s \) because we assume that retailers can fulfill customers’ expectation on fill rate.

If \( p_i = \tilde{p} \), then retailer \( j \) will not undercut \( i \) but instead will charge a high price \( p_j = r - 1/s \). This price pair \((\tilde{p}, r - 1/s)\), however, cannot be a Nash equilibrium because the low-price retailer has an incentive to increase its price. Thus, a pure strategy Nash equilibrium does not exist in this game. Q.E.D.

Proof of Proposition 1

As shown in Lemma 1, when the customer reservation value is high, retailers will set prices in the interval of \([\tilde{p}, r - 1/s]\). Following Morgan and Shy [19] and Shy [23, 24], in an undercut-proof equilibrium environment, firms are sophisticated in that they are always ready to reduce prices and grab rival’s customers whenever undercutting is profitable. Given \( p_j = r - 1/s \), retailer \( i \) will set \( p_i = \tilde{p} \) such that retailer \( j \) finds it is not profitable to undercut price below \( \tilde{p} \). We know that when \( p_i < p_j \), retailer \( j \) sells only when retailer \( i \) is out of stock. If \( p_i = \tilde{p} \), for any price \( \tilde{p} < p_j \leq r - 1/s \), customers all prefer to visit the low-price retailer \( i \) first, and they will visit retailer \( j \) only if retailer \( i \) is out of stock. Hence retailer \( j \) will charge \( r - 1/s \) because \( \pi_j(r - 1/s) > \pi_j(p_j) \) for all \( \tilde{p} \leq p_j < r - 1/s \), and retailer \( j \) has no incentive to undercut price below \( \tilde{p} \). Thus, the unique undercut-proof equilibrium is that one retailer charges \( \tilde{p} \) and the other charges \( r - 1/s \). Q.E.D.

Proof of Lemma 2

(a) Recall that we use \( S_i \) to represent a customer’s contingent search strategy, which is to visit retailer \( i \) first and then continue by searching retailer \( j \) in case of stockout at \( i \). When all customers search in case of a stockout, they will compare their utilities from the two contingent search strategies to decide their best search strategy. Compare \( u_{12} \) and \( u_{21} \), given in Equations (1) and (2). It is straightforward to derive that \( u_{21} \geq u_{12} \) when

\[
k \geq k^* = \frac{s_1 s_2 (p_2 - p_1)}{s_2 - s_1}.
\]
Thus, customers with search cost \( k > k_0 \) prefer to visit the high fill rate retailer first whereas customers with search cost \( k < k_0 \) prefer to visit the low fill rate retailer first.

(b) If we suppose \( 0 < k_0 < 1 \), which implies that \( p_2 > p_1 \), then customers will separate, with low search cost customers searching retailer 1 first and high search cost customers searching retailer 2 first. Consider the static pricing game. When a customer finds the product is in stock in his or her first search, because \( p_2 > p_1 \), only the high search cost customer who visits retailer 2 first will consider searching at the other retailer before purchasing. When a customer with search cost \( k > k_0 \) finds the product is available at retailer 2, he or she will compare his or her utility from purchasing at retailer 2 \( u_2 = r - p_2 \) with that from continuing to search at retailer 1 \( u_1 = s_1(r - p_1) + (1 - s_1)(r - p_2) - k \). Because \( u_2 - u_1 = k - s_1(p_2 - p_1) > k_0 - s_1(p_2 - p_1) = s_1(p_2 - p_1)[s_2/(s_2 - s_1) - 1] > 0 \), it is optimal for high search cost customers to purchase the product if they find it is in stock at retailer 2.

Under the dynamic pricing strategy, a retailer will charge \( p_i \) when they both have products in stock and charge \( q_i \) when its rival is out of stock. When \( 0 < k_0 < 1 \), if a customer with \( k > k_0 \) finds the price \( p_2 \) at retailer 2, he or she can either purchase the product gaining a surplus \( u_2 = r - p_2 \), or he or she can continue searching at retailer 1 gaining a surplus \( u_1 = s_1(r - p_1) + (1 - s_1)(r - p_2) - k \). Comparing these two utilities, we have \( u_2 - u_1 = k - (p_2 - p_1) > k_0 - (p_2 - p_1) = (p_2 - p_1)[s_1s_2/(s_2 - s_1) - 1] > 0 \) because \( s_1s_2 > s_2 - s_1 \) when \( s_2 > s_1 > 1/2 \).

Similarly, we can show that when \( 0 < k_0 < 1 \), in the other two scenarios, a customer will purchase the product in his or her first search if it is in stock.

Suppose \( k_0 = 0 \). This implies that \( p_2 = p_1 \). Because retailer 2 has a higher fill rate than retailer 1 but charges the same price as retailer 1, all customers prefer to visit retailer 2 first and purchase the product if it is in stock. Suppose \( k_0 = 1 \). We have customers pool on retailer 1 and \( p_1 < p_2 \). If a customer finds the product is in stock at retailer 1, he or she will purchase it because \( p_1 < p_2 \). Hence we have shown that it is optimal for customers to purchase without searching further if the product is in stock in their first search. Q.E.D.

Proof of Proposition 3

We omit the superscript to simplify notations. Suppose customer reservation value is high such that retailers optimally serve all customers, and all customers search in case of a stockout. Customers with search cost \( k \in [0, k_0] \) prefer search strategy \( S_{12} \), and customers with search cost \( k \in (k_0, 1] \) prefer search strategy \( S_{21} \). If all customers search in case of a stockout, a retailer's expected profit is increasing in its high price.
In equilibrium, the high price will take its boundary solution. Retailer 1 will set \( q_1 = r - 1/s_1 \), which is the highest price to ensure all high search cost customers search 1 in case of 2 run-out. Retailer 2 will set \( q_2 = r - v_2/s_2 \) with \( v_2 = (1-s_2 + s_2 (p_2 - p_1))/(1-s_1) \) such that customers with \( k \in (k_0, 1) \) prefer visiting retailer 2 first—that is, \( u_{r_2} > u_{r_1} \) and \( u_{r_2} > u_1 \).

The expected demands of retailers \( (D_1 = k_0 + (1-k_0)(1-s_2) \) and \( D_2 = (1-k_0) + k_0(1-s_1) \)\) depend only on the low prices they charge when they both have products in stock. Given retailers’ expected profit functions (A3) and (A4), we can show that a retailer’s profit function is concave in its low price because

\[
\frac{\partial^2 \pi_1}{\partial p_1^2} = \frac{-2s_2^2}{s_2 - s_1} < 0
\]

and

\[
\frac{\partial^2 \pi_2}{\partial p_2^2} = \frac{-2s_2^2}{s_2 - s_1} < 0.
\]

By solving the two first-order conditions jointly, we get the claimed results.

\[
\pi_1 = k_0 s_1 s_2 (p_1 - w_1) + s_1 (1-s_2)(q_1 - w_1)
\]

\[
\pi_2 = (1-k_0) s_1 s_2 (p_2 - w_2) + s_2 (1-s_1)(q_2 - w_2).
\]

Customer separation requires that \( k_0 < 1 \) or \( w_2 - w_1 < 3(s_2 - s_1)/s_2 s_2 \). Conditions \( p_1 < q_1 \) and \( p_2 < q_2 \) imply that \( r > R1 = (1/s_1) + ((2w_1 + w_2)/3) \). Therefore, we have shown that when the customer reservation value is relatively high, at the proposed pricing strategy and fill rate condition, a customer-separating equilibrium exists. (A simple comparison shows that \( 2/s_1 + w_1 > R1 \).)

Similarly, we can prove Proposition 2 and Proposition 4. Q.E.D.

Proof of Lemma 3

The proof of Lemma 3 is straightforward, so we only sketch the proof in scenario 4 here. Suppose retailers choose a pricing strategy such that not all customers search in case of a stockout. Let \( v_i \) be a customer’s expected valuation at retailer \( i \) except for search cost. Then \( v_i = (r - p_i)s_i \). Customers with search cost \( k > \max\{v_1, v_2\} \) will not participate, and customers with search cost \( k < \min\{v_1, v_2\} \) will continue searching in case of stockouts regardless of which retailer they visited first. The expected profit functions for retailers are given in Equations (A5) and (A6). Here, we only consider the case \( v_2 > v_1 \), because if \( v_1 \geq v_2 \), then customers will pool on retailer 1.\(^{11}\)

\[
\pi_i = s_i \left[ k_0 + (\min\{v_1, 1\} - k_0)(1-s_2) \right](p_1 - w_1)
\]

(A5)
\[ \pi_2 = s_2 \left( \min \{v_2, 1\} - k_0 + k_0 \left( 1 - s_1 \right) \right) (p_2 - w_2). \]  

(A6)

We can show that when \( r \geq \max \{2/s_1 + w_1, 2/s_2 + w_2\} \), it is optimal for retailers to set prices such that both \( v_1 > 1 \) and \( v_2 > 1 \); and given \( v_2 > 1 \), it is optimal for retailer 1 to set a price such that \( v_1 > 1 \). Therefore we have shown that at the given reservation value condition, it is optimal for retailers to choose a pricing strategy such that all customers search in case of a stockout. Q.E.D.

**Proof of Proposition 5**

From Lemma 4 (shown below) we know that at the given reservation value condition, there is no pricing equilibrium that supports the customer pooling equilibrium, so we focus on retailers’ pricing strategy that supports the customer-separating equilibrium.

Starting from the static pricing quadrant in Table 2, when \( r > 2/s_1 + w_1 \), given that retailer 2 chooses the static pricing strategy, it is optimal for retailer 1 to choose the dynamic pricing strategy. Given that retailer 1 chooses the dynamic pricing strategy, it is optimal for retailer 2 to choose the dynamic pricing strategy. When retailers are in the dynamic pricing quadrant, no one has incentive to deviate. Therefore, the proposed pricing strategy is a unique Nash equilibrium strategy.

Under the parameter conditions of Proposition 5, it is straightforward to show that the high fill rate retailer obtains a higher profit than the low fill rate retailer. Q.E.D.

**Proof of Proposition 6**

Because customers with search costs in the interval of \([k_0^{DD}, k_0^{SS}]\) choose search strategy \( S_{12} \) under the static pricing scheme and search strategy \( S_{21} \) under the dynamic pricing scheme, they incur a lower expected search cost in the new pricing equilibrium. Because other customers incur the same expected search costs as before, the total search costs are reduced. Therefore, the social welfare is improved under the dynamic pricing equilibrium.

A customer’s expected payment is increasing in his or her reservation value \( r \) under the dynamic pricing scheme. When \( r > R3 \), all customers make a higher expected payment in the dynamic pricing scheme. Because \( 2/s_1 + w_1 > R3 \), we have the first part of Proposition 6(b).

Because customers with lower search cost \((k \in [0, k_0^{DD}])\) and customers with higher search cost \((k \in [k_0^{SS}, 1])\) incur the same expected search costs but make higher expected payments in the dynamic pricing equilibrium, they obtain a lower expected utility than before. For customers with \( k \in [k_0^{DD}, k_0^{SS}] \), at the given reservation value condition, savings from reduced search costs cannot cover the increase in expected payments, so that they, too, are worse off in the dynamic pricing equilibrium.

Because retailers’ expected profit functions are increasing in \( r \) under the dynamic pricing scheme, it is straightforward to show Proposition 6(c) and 6(d). Q.E.D.
Existence Problem When the Fill Rate Difference Is Low

Lemma 4: When customer reservation value satisfies \( r > \max\{2/s_1, 2/s_2 + w_2, 2/s_1 + w_1\} \), there is no pure strategy pricing equilibrium that supports a customer-searching equilibrium in which customers are pooling on the low fill rate retailer.

Proof of Lemma 4: Suppose there is a pricing equilibrium \((p_1^*, p_2^*)\) under which customers are pooling on retailer 1; then retailer 2 only serves customers when retailer 1 is out of stock. Let \( v_2 = s_2(r - p_2)\). The expected profit function for retailer 2 is given by

\[
\pi_2 = \begin{cases} 
\frac{s_2 v_2 (1-s_1) (p_2 - w_2)}{s_2 (1-s_1) (p_2 - w_2)} & \text{when } v_2 < 1 \\
\frac{s_2 (1-s_1) (p_2 - w_2)}{s_2 (1-s_1) (p_2 - w_2)} & \text{otherwise.}
\end{cases}
\]

The optimal price is \( p_2^* = (r + w_2)/2 \) when \( r < 2/s_2 + w_2 \), which implies \( v_2 < 1 \). When \( r \geq 2/s_2 + w_2 \), it is optimal for retailer 2 to serve all customer types. By \( r > \max\{w_1 + 2/s_1, w_2 + 2/s_2\} \), it must be that \( p_2^* = r - 1/s_1 \) at the equilibrium. Given this \( p_2^* \), it is optimal for retailer 1 to charge \( p_1^* = r - 1/s_1 \). Under this \((p_1^*, p_2^*)\), \( k_0 = 1 \) and all customers prefer visiting retailer 1 first and continuing to search retailer 2 only when 1 is out of stock.

However, this proposed pricing strategy cannot be an equilibrium pricing strategy because retailer 2 has incentives to deviate. If retailer 2 charges \( p_2 = p_2^* - \epsilon \), then customers with search cost \( k > k_0 = \frac{s_1 s_2 (p_2^* - p_1^*)}{s_2 - s_1} \) will prefer to visit 2 first. Retailer 2’s profit function is \( \pi_2 = s_2 (1 - k_0 + k_0 (1 - s_1))(p_2 - w_2) \). Differentiating \( \pi_2 \) with respect to \( \epsilon \), we have

\[
\frac{\partial \pi_2}{\partial \epsilon} \bigg|_{\epsilon=0} = \frac{(s_1 (1 + s_2) - s_2 - 2s_1^2 + s_1 s_2 (r - w_2)) s_2}{s_2 - s_1} > 0
\]

by the condition of \( r \). Given \( p_2 = p_2^* - \epsilon \), we have \( \pi_1(p_1^*, p_2^*) < \pi_1(p_1^* - \epsilon, p_2^*) \), so that retailer 1 has an incentive to charge \( p_1 = p_1^* - \epsilon \) such that customers pool on retailer 1. But, as we showed earlier, it is optimal for retailer 2 to charge \( p_2^* \) when customers pool on retailer 1; therefore the proposed \((p_1^*, p_2^*)\) cannot sustain a customer-pooling equilibrium. Further analysis shows that retailer 1 will not choose a pricing strategy such that customers pool on retailer 2. Hence, under the given customer reservation value condition, a customer-pooling price equilibrium does not exist.

Also, when retailers’ fill rates are too close to each other such that the constraint on fill rates in Proposition 5 is not satisfied, there is no pricing equilibrium that supports the customer-separating equilibrium in this retailer pricing game. Thus, no pricing equilibrium may exist when retailers are too close to each other in terms of fill rate and customer reservation value is relatively high. Q.E.D.