Retail Competition on Salop Circle under Linear Demand

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Abstract

In the marketing literature, the Salop circle model of oligopolistic competition, with evenly spaced retailers, uniformly distributed customers, and constant demand has been used to examine retailer price competition and the effect of online retailer entry. In this standard model, distance is a barrier to competition, so an increase in transport cost raises equilibrium price and profits. We modify the standard model by introducing linear demand \((a - bP)\) where \(P\) is the price for the customer including transport costs. We examine the price Nash equilibrium and find:

1. If there are only local retailers:
   - If \(\frac{a}{b}\) is low, retailers are spatial monopolists, the entire market is not served, and higher transport cost lowers demand and profit.
   - Above a critical \(\frac{a}{b}\) value the entire market is served and retailers compete. An increase in transport cost, acting as barrier to competition, raises price and profit.
   - Price, profit and the number of retailers in competitive equilibrium are less than suggested by the standard model with constant demand. The difference is significant if \(\frac{a}{b}\) is low but vanishes asymptotically if \(\frac{a}{b}\) is high.
   - Our model reduces to a spatial monopoly when \(\frac{a}{b}\) is low, and to a constant demand model when \(\frac{a}{b}\) is high.
   - A monopolist manufacturer selling to the retailers should use fewer retailers if \(\frac{a}{b}\) increases.

2. If an online retailer enters the market:
   - The highest possible margin of any retailer drops.
   - Customers pay a lower price, and demand expands, particularly when the online retailer has a strong reputation or cost advantage.
   - Unless the online retailer has a significant reputation or cost advantage, local retailers have non-zero market shares.
   - An increase in either \(\frac{a}{b}\) or transport cost makes both local and online retailers more likely to have non-zero market shares.
   - Our model reduces to a constant demand model when \(\frac{a}{b}\) is high.
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1. Introduction and Literature Review

Researchers in economics and marketing have long been interested in price competition among retailers. A major stream of research in the area focuses on the Bertrand Nash equilibrium in price competition in a duopoly based on the Hotelling (1929) framework where two retailers compete for customers distributed uniformly on a line. The retailers select location and price, and a customer at any location selects the retailer that offers the highest utility, which is determined by price and distance from the retailer. In an extension to the Hotelling framework, Salop (1979) introduced a circular market, the Salop circle, where customers are uniformly distributed on the circle, and retailers are also located on the circle. The Salop circle can accommodate any number of competing retailers. Also, as each retailer has a competitor on both sides, retailers have no incentive to locate close to one another. Thus, it is meaningful to assume that retailers are located at equal distances on the circle (Economides 1989, Kats 1995). Tirole (1988) provides a summary of the Bertrand Nash equilibrium in retailer price competition with the Salop circle when retailers offer the same standardized product, adopt a free on board (FOB) origin pricing policy, and transport cost of a customer is proportional to distance. The results include price at equilibrium, retailer profit, and the number of retailers that can exist under perfect competition.

In the marketing literature, Balasubramanian (1998) applies the Salop model to examine competition between local and online retailers. In Balasubramanian’s model, local retailers are located at equal distances on the Salop circle, and an online retailer is located at the center of the circle, equidistant from all customers. Balasubramanian identifies conditions when an online retailer can enter the market, and examines the role of information in multichannel markets. Jerath, Sajeesh and Zhang (2015) examine competition between unorganized retailers located on the circle and an organized retailer at the center of the circle, in the context of emerging markets.

The modeling literature based on the Salop circle assumes that all customers have high enough reservation prices to purchase the product. (Otherwise, as noted by Salop (1979), retailers become local monopolies and only serve customers nearby.) Demand being constant, focus shifts to price competition among retailers. As transport cost is the only barrier to competition, a higher transport cost allows retailers to set higher prices and generate higher profits.

In contrast, in the literature on spatial pricing by a monopoly, the focus is on demand generation. As transport cost makes it more costly for a customer to patronize a store, a higher transport cost reduces retailer profit. Thus, the effect of transport cost is different in a spatial monopoly from that under perfect competition examined by the Salop model.

In this article, we consider a circular market with customers distributed uniformly on the circle, and retailers located at equal distances on the circle. Each retailer sells the same standardized product and has the same marginal cost. Our model differs from previous research based on the Salop model in that, instead of assuming that demand is constant, we assume that demand density at any location is the linear demand function \( (a - bP) \), where \( P \) is the unit price actually paid by the customers. \( \frac{a}{b} \) is the price ceiling and there is zero demand at price levels
above this ceiling. As the linear demand function corresponds to a uniform distribution of reservation price in \([0, \frac{a}{b}]\), the assumption of linear demand implies that there are always customers with low reservation prices in the market.

We first consider the case where there are only local retailers in the market and derive the Bertrand Nash equilibrium in closed form. We find that when price ceiling is low relative to transport cost, the retailers become spatial monopolies similar to the result of Salop (1979), and some locations are not served by any retailers. Here, an increase in transport cost makes it harder to attract distant customers, thus reducing market area served, demand, and profit.

As price ceiling increases, retailers serve progressively greater market areas until we have actual competition. Still, for lower levels of price ceiling, primary demand generation remains the key issue, and an increase in transport cost reduces retailer price and profit. As price ceiling continues to rise, focus shifts to the generation of selective demand. Transport cost is the only barrier to competition, and an increase in transport cost raises price and profit. Thus, the effect of transport cost on retailer profit changes with price ceiling.

We find that price, demand and profit are always lower than these quantities from the existing literature, and the difference is significant at lower price ceilings. Thus, when price ceiling is low, the market can sustain significantly fewer competing retailers than that suggested by the existing literature. Our results converge to the results from earlier research when price ceiling is high.

We also examine the case where there is an online retailer located at the center of the circular market in addition to the local retailers. Using this framework, we identify the conditions when the local or online retailers can generate non zero market shares. We find that when price ceiling or transport cost is high, both local and online retailers are more likely to have non zero market shares. Also, if an online retailer enters the market, effective prices drop for all customers, expanding demand. The demand expansion is particularly significant when price ceiling is low, and the online retailer has a major reputation or cost advantage over local retailers. When price ceiling is very high, our results converge to the findings of Balasubramanian (1998).

To our knowledge, the Salop circle model has not been examined previously under linear demand. Zhang and Sexton (2001) used a model of supply proportional to price to examine a duopoly of agricultural processors in a Hotelling model who compete to buy produce from farmers uniformly distributed on a line. As they model supply instead of demand, transport cost has the effect of reducing rather than increasing price, and the equilibrium price is different from what we derive. Also, Zhang and Sexton (2001) focus on consumer surpluses generated by FOB origin and uniform delivered pricing plans, and not on how prices and profits are affected by the number of local retailers and the presence or absence of an online retailer as we do.

The rest of the paper is organized as follows. In section 2, we consider competition between local retailers only, derive the Bertrand Nash equilibrium in price, and examine how the results depend on model parameters. We also compare our results with those for a spatial monopoly, and for perfect competition under the assumption of constant demand made by earlier researchers. Implications for a monopolist manufacturer selling to the competing retailers are also discussed.
In section 3, we consider the addition of an online retailer to market, and discuss when the online retailer can get a foothold in the market, and how the equilibrium prices are affected by the presence of the online retailer. Section 4 concludes by summarizing the findings and identifying directions of future research.

2. Competition among Local Retailers

2.1 Framework and Terminology

In this section we consider the case where only local retailers are present and assume:

(1) Customers are uniformly distributed on a circle of unit length. The demand density at any location is \( a - bP \), where \( P \) is the price actually paid by customers. We call \( a \), the highest possible demand, the demand potential, and \( \frac{a}{b} \), the price up to which there is demand, the price ceiling.

(2) There are \( N \) evenly spaced local retailers on the Salop circle with a distance \( \frac{1}{N} \) between adjacent retailers.

(3) To purchase a product unit from a retailer at distance \( x \), a customer incurs transport cost \( tx \), where \( t > 0 \).

(4) Each retailer sells an identical product and has the same unit variable cost \( c \).

(5) \( \frac{a}{b} > c \), which is a necessary condition for a local retailer to participate in the market.

(6) Each retailer employs free on board origin (FOB origin) pricing. If a retailer sets price \( P \) at the origin, a customer located at a distance \( x \) from the retailer pays the delivered price \( (P + tx) \). The customer selects the retailer that offers the lowest delivered price.

We call the locations on the Salop circle where a retailer generates non-zero demand the market served by the retailer.

2.2 Two Benchmarks

Before we address the problem stated above, we establish two benchmarks.

**Benchmark 1. Standard Salop Circle Model:** Since previous research based on the Salop (1979) model assumes that all customers have high enough reservation prices to purchase the product, we call it the Standard Salop Circle model (henceforth SSC). In this model, there are \( N \) evenly spaced retailers on the circle, each retailer employs FOB origin pricing, and the product is always purchased, that is, demand is constant. For consistency with our model assumptions, we assume that the Salop circle has unit length and the total demand is \( a \). Then, there is a symmetric Nash equilibrium in price competition (see, for example, the discussion of the circular city in Tirole (1988), pages 282-285) where:

(1) Retail price \( P_{SSC} = c + \frac{t}{N} \)

(2) Total demand \( Q_{SSC} = a \)
(3) Retailer gross profit $\Pi_{SSC} = \frac{at}{N^2}$

This result holds as long as all customers have reservation prices of at least $c + \frac{t}{N}$. Each retailer serves the market up to a distance of $\frac{1}{2N}$ on each side, the total demand is $a$, and the demand for each retailer is $\frac{a}{N}$. Since the unit gross margin for each retailer is $\frac{t}{N}$, each retailer generates a gross profit $\frac{at}{N^2}$. If the fixed cost of a retailer is $F$, the number of retailers that can exist in competitive equilibrium is given by $F = \frac{at}{N^2}$, that is,

(4) $N_{SSC} = \sqrt{\frac{at}{F}}$

**Benchmark 2. Spatial Monopoly:** In this case (henceforth SM), a single retailer is located on a Salop circle of unit length with demand density $a - bP$. Denoting the profit maximizing retail price by $P_{SM}$, the demand by $Q_{SM}$, and retailer gross profit before considering fixed costs by $\Pi_{SM}$, the results are as follows.

If $\frac{a}{b} - c < \frac{3t}{4}$, then the market served is up to a distance of $x = \frac{2}{3t} (\frac{a}{b} - c) < \frac{1}{2}$ on each side of the retailer, that is, the entire market is not served, and

(5) $P_{SM} = \frac{a}{3b} + \frac{2c}{3} = c + \frac{1}{3} (\frac{a}{b} - c)$, $Q_{SM} = \frac{4b}{9t} (\frac{a}{b} - c)^2$, and $\Pi_{SM} = \frac{4b}{27t} (\frac{a}{b} - c)^3$

If $\frac{a}{b} - c \geq \frac{3t}{4}$, then the whole market is served, and

(6) $P_{SM} = \frac{a}{2b} + \frac{c}{2} - \frac{t}{8} = c + \frac{1}{2} (\frac{a}{b} - c) - \frac{t}{4}$, $Q_{SM} = \frac{b}{2} (\frac{a}{b} - c - \frac{t}{4})$, and $\Pi_{SM} = \frac{b}{4} (\frac{a}{b} - c - \frac{t}{4})^2$

The results for spatial monopoly follow from standard results in spatial pricing (see, for example, Basu, Ingene and Mazumdar 2004); the proof is presented in Appendix A1 for completeness.

### 2.3 Retail Price at Nash Equilibrium

We now examine the problem under consideration where $N$ retailers are located at equal distances on a Salop circle of unit length and the demand density is $a - bP$. As the case of a single retailer is same as spatial monopoly, we assume that $N \geq 2$.

**Special Case. Local Monopoly:** If $\frac{a}{b} - c < \frac{3t}{4N}$, that is, $\frac{2}{3t} (\frac{a}{b} - c) < \frac{1}{2N}$, it follows from the results for the spatial monopoly that each retailer serves a market up to distance $x = \frac{2}{3t} (\frac{a}{b} - c) < \frac{1}{2N}$ on each side. The entire Salop circle is not served, and we have retailer local monopoly. The profit maximizing price, demand and profit of a retailer are given by

(7) $P^* = \frac{a}{3b} + \frac{2c}{3}, Q^* = \frac{4b}{9t} (\frac{a}{b} - c)^2$, and $\Pi^* = \frac{4b}{27t} (\frac{a}{b} - c)^3$.

**Local Competition:** We henceforth assume that $\frac{a}{b} - c \geq \frac{3t}{4N}$, which we call local competition to distinguish it from a local monopoly. With this assumption, $\frac{2}{3t} (\frac{a}{b} - c) \geq \frac{1}{2N}$, that is, the entire market is served.
Consider a retailer with price $P$ at the origin who faces retailer A to the left and retailer B to the right, with prices $P_A$ and $P_B$, respectively. Denoting the distances of the marginal customers at the left and the right by $x_1$ and $x_2$, respectively,

$$P + tx_1 = P_A + t \left( \frac{1}{N} - x_1 \right),$$

that is, $x_1 = \frac{1}{2N} + \frac{P_A - P}{2t}$. Similarly, $x_2 = \frac{1}{2N} + \frac{P_B - P}{2t}$.

Noting that, in the market served, the demand density at distance $y$ on either side of the focal retailer is $a - b(P + ty)$, the demand for the focal retailer is

$$(8) \quad Q = \int_{0}^{x_1} [a - b(P + ty_1)]dy_1 + \int_{0}^{x_2} [a - b(P + ty_2)]dy_2 = b[x_1(\frac{a}{b} - P - \frac{tx_1}{2})] + x_2(\frac{a}{b} - P - \frac{tx_2}{2})]$$

Profit for the focal retailer is

$$(9) \quad \Pi = (P - c)Q = b(P - c)[x_1(\frac{a}{b} - P - \frac{tx_1}{2})] + x_2(\frac{a}{b} - P - \frac{tx_2}{2})]$$

Differentiating (9), noting that $\frac{\partial x_1}{\partial P} = \frac{\partial x_2}{\partial P} = -\frac{1}{2t}$ and simplifying,

$$(10) \quad \frac{\partial \Pi}{\partial P} = b[x_1(\frac{a}{b} - P - \frac{tx_1}{2})] + x_2(\frac{a}{b} - P - \frac{tx_2}{2})] - \frac{b}{2t}(P - c)[(\frac{a}{b} - P - \frac{tx_1}{2}) + (\frac{a}{b} - P - \frac{tx_2}{2})]
- (P - c)(x_1 + x_2)(\frac{3b}{4})$$

If a symmetric Nash equilibrium in pure strategies exists, then $P_A = P_B = P$, that is, $x_1 = x_2 = \frac{1}{2N}$. Substituting in (10) and simplifying,

$$(11) \quad \frac{\partial \Pi}{\partial P} = \frac{b}{t}[(P - c)^2 - (P - c)(\frac{a}{b} - c + \frac{3t}{2N}) + \frac{t}{N}(\frac{a}{b} - c - \frac{t}{4N})]$$

As, at Nash equilibrium, $\frac{\partial \Pi}{\partial P} = 0$, $(P - c)$ is one of the roots of the quadratic equation obtained by equating the right hand side of (11) to zero. Simplifying, these roots are given by

$$(12) \quad P - c = \frac{1}{2} \left\{ \left( \frac{a}{b} - c + \frac{3t}{2N} \right) \pm \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right\}$$

Hence, if a symmetric Nash equilibrium exists, the price $P$ must be one of the following:

$$(13) \quad P_h = \frac{1}{2} \left( \frac{a}{b} + c + \frac{3t}{2N} \right) + \frac{1}{2} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \quad \text{(high solution)}$$

or

$$(14) \quad P_l = \frac{1}{2} \left( \frac{a}{b} + c + \frac{3t}{2N} \right) - \frac{1}{2} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \quad \text{(low solution)}$$

2.4 Price, Demand and Profit at Nash Equilibrium: From (13),

$$P_h > \frac{1}{2} \left( \frac{a}{b} + c + \frac{3t}{2N} \right) + \left( \frac{a}{b} - c - \frac{t}{2N} \right) > \frac{a}{b}$$

Therefore, the high solution generates zero demand and zero profit for all retailers. As shown in Appendix A2, the low solution $P_l$ has the following properties:

- $c < P_l < \frac{a}{b}$. Thus, the low solution can generate demand and a positive contribution.
• $\frac{\partial^2 \Pi}{\partial P^2} < 0$ at $P = P_l$. Thus, profit is maximized at $P_l$.

• $P_l < c + \frac{t}{N}$. Thus, as is the case with the Standard Salop Circle model, neighboring retailers cannot undercut the price of a retailer at its location.

Therefore, $P_l$ is the unique price in a symmetric Nash equilibrium in pure strategies. The Nash equilibrium price and the demand and profit it generates are summarized in Proposition 1. Demand is obtained by substituting $x_1 = x_2 = \frac{1}{2N}$ and the expression for $P_l$ in equation (8). The expression for profit is derived in Appendix A3.

**Proposition 1:** If $N \geq 2$ and $(\frac{a}{b} - c) \geq \frac{3t}{4N}$, the price at symmetric Nash equilibrium is uniquely given by

$$P^* = \frac{1}{2} \left( \frac{a}{b} + c + \frac{3t}{2N} \right) - \frac{1}{2} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}}$$

The demand for every local retailer $Q^*_R$ and the total demand $Q^*_T$ are given by

$$Q^*_T = NQ^*_R = \frac{b}{2} \left[ \left( \frac{a}{b} - c - \frac{2t}{N} \right) + \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right]$$

The profit for a local retailer is

$$\Pi^* = \frac{bt}{N^2} \left[ \frac{1}{8} \left( \frac{a}{b} - c \right) - \frac{25t}{16N} + \frac{7}{8} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right]$$

### 2.5 Comparative Statics in Local Competition

Propositions 2-5 summarize how price, demand, and retailer profit depend on the retailer’s variable cost $c$, transport cost $t$, price ceiling $\frac{a}{b}$, and the number of retailers $N$. Proposition 2 follows directly from (15), (16) and (17). Proofs of propositions 3-5 are in appendix A3.

**Proposition 2:** If $N \geq 2$ and $(\frac{a}{b} - c) > \frac{3t}{4N}$, then $P^*$ is a strictly increasing function of $c$, and $Q^*_R$ and $\Pi^*$ are strictly decreasing functions of $c$.

**Proposition 3:** If $N \geq 2$ and $(\frac{a}{b} - c) \geq \frac{3t}{4N}$, then:

- $Q^*_R$ and $Q^*_T$ are strictly decreasing functions of $t$.

- The unit gross margin $(P^* - c)$ is a strictly decreasing function of $t$ if $(\frac{a}{b} - c) < 1.049(\frac{t}{N})$, and is a strictly increasing function of $t$ if $(\frac{a}{b} - c) > 1.049(\frac{t}{N})$. If $(\frac{a}{b} - c) \gg \frac{t}{N}$, then $\frac{\partial P^*}{\partial t} \approx \frac{1}{N}$.

- If $t$ increases, $\Pi^*$ may increase or decrease depending on relative sizes of $(\frac{a}{b} - c)$ and $\frac{t}{N}$. If $(\frac{a}{b} - c)$ is close to $\frac{3t}{4N}$, then $\frac{\partial \Pi^*}{\partial t} < 0$. If $(\frac{a}{b} - c) \gg \frac{t}{N}$, then $\frac{\partial \Pi^*}{\partial t} > 0$. If, in addition, $\frac{a}{b} \gg c$, then $\frac{\partial \Pi^*}{\partial t} \approx -\frac{a}{N^2}$.
Proposition 4: If $N \geq 2$ and $(\frac{a}{b} - c) \geq \frac{3t}{4N}$, then:

- The unit gross margin $(P^* - c)$ is a strictly increasing function of $(\frac{a}{b} - c)$ and increases from $\frac{t}{4N}$ when $(\frac{a}{b} - c) = \frac{3t}{4N}$ to the limit of $\frac{t}{N}$ when $(\frac{a}{b} - c) \gg \frac{t}{N}$.

- For a given $b$, $Q^*_R$ and $Q^*_T$ are strictly increasing functions of $(\frac{a}{b} - c)$. $Q^*_T$ increases from $\frac{1}{3}(ab - bc)$ when $(\frac{a}{b} - c) = \frac{3t}{4N}$ to the limit of $(ab - bc)$ when $(\frac{a}{b} - c) \gg \frac{t}{N}$. If, in addition, $\frac{a}{b} \gg c$, then $Q^*_T \to a$ when $\frac{a}{b} \gg \frac{t}{N}$.

- For a given $b$, $\Pi^*$ is a strictly increasing function of $(\frac{a}{b} - c)$. $\Pi^*$ increases from $(\frac{a}{b} - bc)\frac{t}{12N^2}$ when $(\frac{a}{b} - c) = \frac{3t}{4N}$ to the limit of $(\frac{a}{b} - bc)\frac{t}{N^2}$ as $(\frac{a}{b} - c) \gg \frac{t}{N}$. If, in addition, $\frac{a}{b} \gg c$, then $\Pi^* \to a\frac{t}{N^2}$ when $\frac{a}{b} \gg \frac{t}{N}$.

Proposition 5: If $N \geq 2$ and $(\frac{a}{b} - c) \geq \frac{3t}{4N}$, then:

- $(P^* - c)$ is a strictly increasing function of $N$ when $(\frac{a}{b} - c) < 1.049(\frac{t}{N})$, and $(P^* - c)$ is a strictly decreasing function of $N$ when $(\frac{a}{b} - c) > 1.049(\frac{t}{N})$.

- Total demand $Q^*_T$ is a strictly increasing function of $N$.

- $\Pi^*$, the profit of a retailer, is strictly decreasing in $N$ as long as $(\frac{a}{b} - c) > \frac{3t}{4N}$.

- $N\Pi^*$, the total profit of all retailers combined, increases with $N$ when $(\frac{a}{b} - c)$ is small and decreases with $N$ when $(\frac{a}{b} - c)$ is large.

Discussion: Our results show that the competitive equilibrium is shaped by price ceiling, transport cost, number of retailers, and the interactions among these factors. When price ceiling is low relative to transport cost, retailers become local monopolies. Whether two retailers are competitors therefore depends on the ratio of price ceiling to transport cost. For instance, consider two retailers selling office supplies. For a lower priced staple product like writing paper, the ratio is likely to be low, and a retailer can ignore the pricing decisions of the other retailer. However, for a high priced product like laptop computer, the ratio is high, and the same two retailers can become competitors.

Effect of transport cost: We find that under local competition, transport cost has two distinct effects. First, transport cost increases the delivered price for all customers, resulting in lower demand. An increase in transport cost reduces demand even more, generating a downward...
pressure on price as retailers try to counter the loss in demand. This is the dominant effect when price ceiling is low. Then, an increase in transport cost reduces price and profit.

However, transport cost is also a barrier to competition. Thus, an increase in transport cost generates an upward pressure on price. This effect dominates when price ceiling is high and most customers have high reservation prices, or there is a large number of retailers and intense price competition. Then, price and profit increase with transport cost.

**Effect of number of competing retailers:** When \( \frac{a}{b} - c \) is large compared to \( \frac{t}{N} \), there is strong price competition among the retailers. As more retailers enter the market, increased competition leads to a decrease in price. As price drops at all locations, total demand increases.

To explain the increase in price with an increase in \( N \) when \( \frac{a}{b} - c \) is small, consider the case where \( N = 2 \), and \( \frac{a}{b} - c = \frac{3t}{4N} = \frac{3t}{8} \). In this case, the markets served by the two retailers do not overlap, and the retailers have to reduce price to generate demand from distant customers. If a third retailer enters the market, the retailers can actually raise price as they serve a smaller market and do not have to drop price as much to attract distant customers. Even though the prices at the retailer locations increase, the addition of a new retailer reduces transport cost for some customers, and overall demand still increases with \( N \).

**Effect of price ceiling:** When price ceiling is low, demand drops off rapidly with distance and the retailers become local monopolies. While there is no price competition, retailers have to set prices low to generate demand. When price ceiling is high, retailers are constrained by competition from increasing price above \( c + \frac{t}{N} \). As most customers have reservation prices above the retail price, demand approaches the demand potential and our results converge to Standard Salop circle results under the assumption of constant demand.

### 2.6 Comparison with Benchmark Models

**Comparison with Standard Salop Circle Results:** There are two key differences between the present framework and the Standard Salop model. First, in the Standard Salop model, transport cost only serves as a barrier to competition, and an increase in transport cost always increases price and retailer profit. In contrast, in our model, the role of transport cost evolves with price ceiling. When price ceiling is low, transport cost hinders demand generation from customers with low reservation prices and an increase in transport cost reduces profit. If price ceiling is high, most customers purchase the product at all prices under consideration. Then, an increase in transport cost, as barrier to competitive price undercutting, raises price and profit.

Second, for the linear demand density \( a - bP \), there are always customers with low reservation prices. Thus, in our model, the retailers set a lower price than the Standard Salop Circle model and still generate a lower demand and lower profit than the standard model. Note that the price ceiling \( \frac{a}{b} \) is the highest reservation price in the market. When price ceiling is high, most customers have higher reservation prices than prices under consideration, and the results for the present model converge to the results for the Standard Salop Circle model in the limit.
Specifically, consider how the three ratios \( \frac{P^* - c}{P_{SSC} - c} \), \( \frac{Q^*_R}{Q_{SSC}} \), and \( \frac{\Pi^*}{\Pi_{SSC}} \) vary as \( \frac{a}{b} - c \) increases from \( \frac{3t}{4N} \). From Proposition 4:

- The margin ratio \( \frac{P^* - c}{P_{SSC} - c} \) increases from \( \frac{1}{4} \) to 1 as \( \frac{a}{b} - c \) becomes very large.

- The demand ratio \( \frac{Q^*_R}{Q_{SSC}} \) increases from \( \frac{1}{3} (1 - \frac{bc}{a}) \) to 1 as \( \frac{a}{b} - c \) becomes very large. Thus, demand increases from at most one third of the demand potential to the full demand potential when the price ceiling is high.

- The profit ratio \( \frac{\Pi^*}{\Pi_{SSC}} \) increases from \( \frac{1}{12} (1 - \frac{bc}{a}) \) to 1 as \( \frac{a}{b} - c \) becomes very large. Thus, profit increases from at most one twelfth of the Standard Salop Circle profit to the full profit potential when the price ceiling is high. This result also implies that the number of retailers than can exist in equilibrium is less than what is given by the Standard Salop Circle model. For instance, when the price ceiling is close to the lower boundary \( (c + \frac{3t}{4N}) \), the number of retailers in equilibrium is \( \sqrt{\frac{(a - bc)t}{12F}} \), which is less than a third of the number \( \sqrt{at/F} \) from the Standard Salop Circle model.

For \( c = 0 \), Figure 1 plots the four ratios: \( \frac{P^*}{P_{SSC}} \), \( \frac{Q^*_R}{Q_{SSC}} \), \( \frac{\Pi^*}{\Pi_{SSC}} \), and the ratio of the number of retailers in equilibrium in our model to the number of retailers from the standard model, against the ratio \( \frac{(a/b)}{(t/N)} \). Figure 1 shows that while the results from our model converge to the results for the standard model, it is a slow convergence. For instance, the profit from our model becomes 90% of the Standard Salop Circle model profit when \( \frac{a}{b} \approx 19.25 \left( \frac{t}{N} \right) \), and is 95% of the limit when \( \frac{a}{b} \approx 39.25 \left( \frac{t}{N} \right) \).

Comparison with Spatial Monopoly: When price ceiling is low relative to transport cost, a retailer in our model becomes a spatial monopolist, with the same price, demand and profit. Then, the main impact of an increased transport cost is higher delivered price for customers, which reduces profit. As price ceiling increases, results in our model diverge from the results for a spatial monopoly:

- Retail price increases with price ceiling in both our model and a spatial monopoly. However, in our model, price is bounded above by \( (c + \frac{t}{N}) \) regardless of price ceiling. In contrast, in a spatial monopoly, the plot of price against price ceiling is a straight line with slope \( \frac{1}{3} \).
when the whole market is not served, and a line with slope \( \frac{1}{2} \) when the whole market is served and there is no need to keep price lower to expand market.

- Like price, retailer profit also increases with price ceiling in both our model and a spatial monopoly. When the price ceiling is high, the profit for a retailer in a spatial monopoly increases at a much faster rate than a retailer in competition.

- When the price ceiling is high, price and profit increase with transport cost in our model. In a spatial monopoly, an increase in transport cost always reduces profit.

Figure 2 about here

Figure 2 plots the profits for the spatial monopoly, and for \( N = 2 \) and \( N = 3 \) against price ceiling when \( c = 0, b = 1, \) and \( t = 1 \). While the profit for a monopoly increases at an increasing rate, the profits in competition are approximately linear functions of price ceiling. Note that even for a duopoly, the profit diverges rapidly from the monopoly profit as price ceiling increases.

2.7 Implications for Distribution Strategy of Monopolistic Manufacturer

We consider a monopolistic manufacturer that sells its product to the retailers in our model, and examine two decision variables of the manufacturer, the number of retailers to sell to \( (N) \), and the wholesale price \( c \) which we assume is same as the unit variable cost of the retailer. For simplicity, we first ignore any promotion or distribution cost of the manufacturer. Denoting the unit variable cost of the manufacturer by \( c_0 \) and total demand by \( Q_T \), the gross profit of the manufacturer is

\[
\Pi_M(c, N) = (c - c_0)Q_T
\]

In the Standard Salop model, the total demand is always \( a \), that is, \( \Pi_M = (c - c_0)a \), which does not depend on \( N \), and is not bounded above as \( c \) increases. Thus, the Standard Salop model cannot provide guidance regarding the intensity of distribution or wholesale price. We now examine how the manufacturer’s profit depends on \( c \) and \( N \) in our framework. In all cases (spatial monopoly, local monopoly, and local competition), total demand is given by

\[
Q_T = 2Nx^*(a - bP^* - \frac{btx^*}{2}),
\]

where \( P^* \) is the equilibrium retailer price given \( c \) and \( N \), and \( x^* \) is the distance up to which a retailer serves the market on each side of its location. Since \( P^* > c \) and \( 0 < x^* \leq \frac{1}{2N} \), it follows from (18) and (19) that

\[
\Pi_M(c, N) < (c - c_0)(a - bc) \leq \frac{b}{4}(\frac{a}{b} - c_0)^2 = \tilde{\Pi}_M,
\]

where \( \tilde{\Pi}_M \), henceforth “profit potential,” is the maximum of \( (c - c_0)(a - bc) \), attained at \( c = \tilde{c} = \frac{1}{2}(\frac{a}{b} + c_0) \) and is an upper bound to manufacturer profit. By its choice of \( c \) and \( N \), the manufacturer can dictate if there is local monopoly or local competition as well as the price and
profit of each retailer. For a given \( N \), let \( c^* \) denote the profit maximizing wholesale price and \( \Pi_M^* \) the manufacturer’s profit. The next three results show how the manufacturer’s profit depends on \( N \), proofs are provided in Appendix A5.

**Result 1:** If \( N = 1 \):

- If \( \left( \frac{a}{b} - c_0 \right) > \frac{5t}{4} \), then \( c^* = \frac{1}{2} \left( \frac{a}{b} + c_0 - \frac{t}{4} \right) \) and \( \Pi_M^* = \frac{b}{8} (\frac{a}{b} - c_0 - \frac{t}{4})^2 \).
- If \( \frac{9t}{8} \leq \left( \frac{a}{b} - c_0 \right) \leq \frac{5t}{4} \), then \( c^* = \frac{a}{b} - \frac{3t}{4} \) and \( \Pi_M^* = \frac{bt}{4} (\frac{a}{b} - c_0 - \frac{3t}{4}) \).
- If \( \left( \frac{a}{b} - c_0 \right) < \frac{9t}{8} \), then \( c^* = \frac{a}{3b} + \frac{2c_0}{3} \), and \( \Pi_M^* = \frac{16b}{243} (\frac{a}{b} - c_0)^3 \).
- For a given \( t \), \( \frac{\Pi_M^*}{\Pi_M} \) is strictly increasing in \( \left( \frac{a}{b} - c_0 \right) \), and converges to 0.5 when \( \left( \frac{a}{b} - c_0 \right) \gg t \).

**Result 2:** If \( N \geq 2 \) then the manufacturer’s profit is maximized by a retailer local monopoly (that is, \( \frac{a}{b} - c^* < \frac{3t}{4N} \), or, equivalently, \( c^* > \frac{a}{b} - \frac{3t}{4N} \) ) only if \( \frac{a}{b} - c_0 < \frac{9t}{8N} \). If that happens, then \( \frac{\Pi_M^*}{\Pi_M} < \frac{8}{27} \).

**Result 3:** If \( N \geq 2 \) and \( \frac{a}{b} - c_0 \leq \frac{3t}{2N} \), then \( \frac{\Pi_M^*}{\Pi_M} \leq \frac{1}{2} \). If \( N \geq 2 \) and \( \frac{a}{b} - c_0 > \frac{3t}{2N} \), then:

- \( \Pi_M^* \) is a strictly increasing function of \( N \).
- \( \left( 1 - \frac{5t}{4N (\frac{a}{b} - c_0)} \right)^2 < \frac{\Pi_M^*}{\Pi_M} \leq \left( 1 - \frac{t}{2N (\frac{a}{b} - c_0)} \right)^2 \)

The first part of Result 3 follows because if \( \frac{a}{b} - c_0 \leq \frac{3t}{2N} \) then \( \frac{a}{b} - c \leq \frac{3t}{2N} \) for any \( c \geq c_0 \), and, from (16), \( Q_T \leq \frac{1}{2} (\frac{a}{b} - c) \). \( \Pi_M^* \) strictly increases with \( N \) because \( Q_T^* \) is strictly increasing in \( N \) under local competition (from Proposition 5). The bounds on \( \frac{\Pi_M^*}{\Pi_M} \) come from the result that \( c + \frac{t}{4N} \leq P^* < c + \frac{t}{N} \).

From Results 1-3, we find that a manufacturer can always increase gross profit by increasing the intensity of distribution. Intuitively, an increase in \( N \) has two effects. First, retail price \( P^* \), which is bounded above by \( c + \frac{t}{N} \), becomes closer to \( c \). Second, the customers are now located closer to retailers, and demand loss due to transport cost decreases. Combining, we have \( Q_T \to a - bc \), that is, the manufacturer’s problem reduces to the problem of a monopolist facing a demand function \( (a - bc) \) and no transport costs. Note that profit gain from increasing \( N \) is moderated by price ceiling. If price ceiling is low, the manufacturer suffers significant loss of gross profit when \( N \) is small. For example, if \( \frac{a}{b} - c_0 = t \) and \( N = 3 \), \( \Pi_M^* \) is less than 70% of the profit potential \( \hat{\Pi}_M \). However, if price ceiling is high, the marginal gain from having more
retailers is low. For instance, if $\frac{a}{b} - c_0 = 25t$, the manufacturer gets more than 95% of the profit potential with two retailers. Two factors restrain the increase in $N$. First, increasing $N$ may result in greater administration costs of promotion and distribution. Second, retailer profit decreases with $N$. Thus, if $N$ is high, the retailer may not attain the profit level required to enter the market, especially if selling the product requires major investment by the retailer in terms of employee training, post-purchase service, etc.

We now use two numerical examples to show how the manufacturer’s profit and wholesale price depend on $N$, and how the optimal $N$ behaves if we assume that the manufacturer incurs an administration cost proportional to the number of retailers.

**Numerical Example 1:** We set $t = 10.0$, $b = 1.0$, and $c_0 = 0$, and use a full factorial design with six levels of $\frac{a}{b}$ (1, 5, 10, 20, 50, and 100) and 100 levels of $N$ (1 to 100, in steps of 1). For each case, $c$ is chosen by grid search to maximize $\Pi_M$ given by (18). Figure 3 plots $\Pi_M^*/\tilde{\Pi}_M$ and Figure 4 plots $c^*/\tilde{c}$ against $N$ for the six levels of $\frac{a}{b}$ examined. From Figures 3 and 4, $c^*$ and $\Pi_M^*$ increase to $\tilde{c}$ and $\tilde{\Pi}$ respectively as $N$ becomes large. In particular, when price ceiling $\frac{a}{b}$ is high, convergence is rapid with strong diminishing marginal returns, suggesting that net profit will be maximized at a relatively low level of $N$ if administration cost is included in the model.

Figures 3 and 4 about here

**Numerical Example 2:** We now assume that the manufacturer’s net profit is the gross profit given by (18) minus an administration cost $\eta N\tilde{\Pi}_M$ where $\eta > 0$. We keep $b$, $t$ and $c_0$ same as in Numerical Example 1, and use a full factorial design with four levels of $\eta$ (.001, .002, .003, and .004) and 100 levels of $\frac{a}{b}$ (1 to 100, in steps of 1). For each case, we vary $N$ from 1 to 1000 in steps of 1, find $c^*$ and $\Pi_M^*$ for that $N$, and then find $N^*$, the value of $N$ that maximizes net profit. Figure 5 plots $N^*$ against $\frac{a}{b}$. From Figure 5, we find that as price ceiling $\frac{a}{b}$ increases, $N^*$ decreases for each level of $\eta$, that is, the manufacturer shifts from an intensive to a selective distribution strategy. The shift is more rapid if $\eta$ is higher.

Figure 5 about here

**Discussion:** We find that before considering administration costs, the manufacturer’s gross profit always increases with $N$. When price ceiling is higher, the increase in gross profit shows stronger diminishing marginal returns with $N$. Thus, once costs of administration are incorporated, the optimal number of retailers decreases with price ceiling. Intuitively, a larger number of retailers reduces transport cost for customers. When price ceiling is high, transport costs become relatively less significant, and intensive distribution is no longer necessary.

Regarding retailer participation, there is a dichotomy. Consider a product, such as farm equipment, that requires frequent service after purchase. If price ceiling is high, the manufacturer can reduce the number of retailers without losing much profit. Then, a selective distribution
strategy with a small number of retailers who provide post-purchase service is optimal. In contrast, if price ceiling is low, the manufacturer suffers major profit loss by reducing the number of retailers. Then, it is optimal for the manufacturer to employ intensive distribution and provide post-purchase service itself to reduce the level of profit a retailer needs to carry the product.

3. Competition among Online and Local Retailers

3.1 Introduction and Framework

We now extend the framework developed in the previous section to include an online retailer in addition to local retailers evenly spaced on the Salop circle. Closely following Balasubramanian (1998) and using the same notations whenever possible, we make six assumptions:

1. The local retailers are located at equal intervals on a Salop circle of unit length, and the online retailer is located at the center of the circle, and is, therefore, equidistant from all customers.

2. The local retailers adopt FOB origin pricing while the online retailer offers the same delivered price to all customers. Each local retailer faces a market of length \( \frac{1}{N} \) on each side. Let \( P_d \) and \( P_r \) denote the prices offered by the online retailer and the local retailer, respectively. For a customer at distance \( x \) from a local retailer, the effective price of purchasing a product unit is \( (P_r + tx) \), where \( t \) is the unit transport cost, and the effective price of purchasing from the online retailer is \( (P_d + \mu) \) where \( \mu \) is a model parameter. The customer selects the retailer that offers the lower effective price.

The parameter \( \mu \) was introduced by Balasubramanian (1998) as the lack of fit between the customer needs and the online retailer. \( \mu \) can also capture effects such as the psychic cost of waiting for product delivery. The sign and size of \( \mu \) are not clear a priori as \( \mu \) can include any reputation advantage the online retailer has over local retailers. When the reputation effect is strong, \( \mu \) may even be negative.

3. The local retailers have identical marginal costs \( c \), and the online retailer has a marginal cost of \( c + T \). The difference in marginal cost, \( T \), captures transport cost of the online retailer (which is same for all customers). If the online retailer has a lower marginal cost at the origin due to proximity to or quantity discounts from the manufacturer, that is also included in the parameter \( T \). Thus, the sign and size of \( T \) are also not clear a priori. If the transport cost is the major component of \( T \), then \( T \) should be positive. It is necessary to include \( T \) in the model because unlike \( \mu \), \( T \) enters the unit margin of the online retailer directly.

4. The demand density at any location on the Salop circle is \( (a - bP) \), where \( P \) is the effective price for the customer.

5. \( \frac{a}{b} - c \geq \frac{3t}{4N} \)

6. \( \frac{a}{b} - \mu > c + T \)

Assumptions 1 and 2 are identical to Balasubramanian (1998). The third assumption differs
from Balasubramanian (1998) who does not include $T$ in the model. The fourth assumption is
departs from previous research as it replaces constant demand $a$ by the linear demand density
$(a - bP)$. The fifth assumption ensures that in the absence of the online retailer, we do not
have local monopolies. Noting that the demand density for the online retailer at a location it
serves is $\{a - b(P_d + \mu)\}$, the sixth assumption is a necessary condition for the online retailer to
participate in the market.

3.2 Benchmark

We first examine the case where all customers have reservation prices above the prices under
consideration so that the demand potential $a$ is always achieved. This is same as making the
first three assumptions above, and assuming constant instead of linear demand. The benchmark
model is, therefore, the Balasubramanian (1998) model extended by the inclusion of $T$. For the
marginal customer, $P_r + tx = P_d + \mu$, that is, $x = \frac{1}{t}(P_d - P_r + \mu)$. Thus, the market share and
profit of the local retailer are

$$(21) \quad s = \frac{2}{t}(P_d - P_r + \mu), \quad \Pi_r = as_r(P_r-c) = \frac{2a}{t}(P_r-c)(P_d-P_r+\mu)$$

The market share and the profit of the online retailer are

$$(22) \quad s_d = 1 - Ns = 1 - \frac{2N}{t}(P_d - P_r + \mu), \quad \Pi_d = a(P_d - c - T)[1 - \frac{2N}{t}(P_d - P_r + \mu)]$$

From (21) and (22),

$$(23) \quad \frac{\partial \Pi_r}{\partial P_r} = \frac{2a}{t}(P_d + \mu + c - 2P_r),$$

and

$$(24) \quad \frac{\partial \Pi_d}{\partial P_d} = a[1 - \frac{2N}{t}(P_d - P_r + \mu) - \frac{2N}{t}(P_d - c - T)] = aN[\frac{1}{N} + \frac{2}{t}(P_r + c + T - \mu) - \frac{4P_d}{t}]$$

At Nash equilibrium,

$$P_d = 2P_r - \mu - c, \quad \frac{4P_d}{t} = \frac{2(P_r + c + T - \mu)}{t} + \frac{1}{N}.$$ Solving,

$$(25) \quad P_r = c + \frac{\mu}{3} + \frac{T}{3} + \frac{t}{6N}, \quad \text{and} \quad P_d = c - \frac{\mu}{3} + \frac{2T}{3} + \frac{t}{3N}$$

Note that the online retailer transfers two thirds of the transport costs to the customer and
absorbs one third of the cost. For the online retailer to serve the market profitably, we must have

$$P_d = c - \frac{\mu}{3} + \frac{2T}{3} + \frac{t}{3N} \geq c + T, \quad \text{that is,} \quad \frac{t}{N} \geq \mu + T.$$ Similarly, for the local retailer to serve the market profitably, we must have

$$P_r - c \geq 0, \quad \text{that is,} \quad (\mu + T) \geq -\frac{t}{2N}$$

This condition is satisfied unless the online retailer has a strong cost or reputation advantage.

**Market shares and profits:** From (21) and (25), the market share and profit of a local retailer
at equilibrium are
\[ s_r = \frac{2}{t} \left[ \frac{T}{3} + \frac{\mu}{3} + \frac{t}{6N} \right] = \frac{2}{3t} \left[ \mu + T + \frac{t}{2N} \right], \text{ and } \Pi_r = \frac{2a}{9t} \left( \mu + T + \frac{t}{2N} \right)^2 \]

From (22) and (25), the profit of the online retailer is
\[ \Pi_d = \frac{2aN}{9t} \left( \frac{t}{N} - \mu - T \right)^2 \]

3.3 Linear Demand

With this benchmark, we now examine the case where demand density is linear in effective price, and make all six assumptions stated in the framework. In this case, a local retailer competes with the online retailer for a market up to a distance of \( \frac{1}{2N} \) on each side. Since we assume that \( \left( \frac{a}{b} - \mu \right) > c + T \), the online retailer can get a positive demand and a positive margin from all locations. Also, since we assume that \( \left( \frac{a}{b} - c \right) \geq \frac{3t}{4N} \), the local retailers serve all locations not served by the online retailer. Thus, our assumptions ensure that the entire market is served.

Nash Equilibrium: We first derive the Nash equilibrium under the premise that both the local and the online retailers have non-zero market shares. Then, we establish when the premise is valid. Let \( P_r \) and \( P_d \) denote the prices of the local and the online retailer, respectively, and \( x_m \) the distance of the marginal customer from the local retailer. Then,
\[ P_r + tx_m = P_d + \mu, \quad x_m = \left( \frac{P_d - P_r}{t} \right) + \frac{\mu}{t}, \quad \frac{\partial x_m}{\partial P_r} = -\frac{1}{t}, \text{ and } \frac{\partial x_m}{\partial P_d} = \frac{1}{t} \]

Local Retailer at Nash Equilibrium: The local retailer serves customers up to \( x_m \leq \frac{1}{2N} \) on each side and generates profit
\[ \Pi_r = 2(P_r - c) \int_0^{x_m} [a - b(P_r + ty)]dy = 2b x_m (P_r - c) \left( \frac{a}{b} - P_r - \frac{tx_m}{2} \right) \]

Differentiating (29), substituting \( \frac{\partial x_m}{\partial P_r} = -\frac{1}{t} \) and simplifying,
\[ \frac{\partial \Pi_r}{\partial P_r} = \frac{2b}{t} \left[ (P_r - c)^2 - (P_r - c) \left( \frac{a}{b} - c + tx_m \right) + tx_m \left( \frac{a}{b} - c - \frac{tx_m}{2} \right) \right] \]

At Nash equilibrium, \( \frac{\partial \Pi_r}{\partial P_r} = 0 \), that is, \( (P_r - c) \) is one of the roots of the quadratic equation obtained by equating the right hand side of (30) to zero. Simplifying, the price of the local retailer at equilibrium must be either
\[ P_{rh} = \frac{1}{2} \left[ \left( \frac{a}{b} + c + tx_m \right) + \sqrt{\left( \frac{a}{b} - c - tx_m \right)^2 + 2t^2x_m^2} \right] \text{ (high solution)} \]

or,
\[ P_{rl} = \frac{1}{2} \left[ \left( \frac{a}{b} + c + tx_m \right) - \sqrt{\left( \frac{a}{b} - c - tx_m \right)^2 + 2t^2x_m^2} \right] \text{ (low solution)} \]

From (31), \( P_{rh} > \frac{a}{b} \), that is, the local retailer generates zero demand and zero profit by setting price \( P_{rh} \). As shown in Appendix B1, the low solution satisfies \( c < P_{rl} < \frac{a}{b} \), and \( \frac{\partial^2 \Pi_r}{\partial P_r^2} < 0 \) at \( P_r = P_{rl} \) as long as \( 0 < x < \frac{1}{2N} \), that is, \( \Pi_r \) is maximized by \( P_{rl} \). Therefore, if a Nash
equilibrium exists with \(0 < x_m < \frac{1}{2N}\) the price for the local retailer is \(P_{rt}\).

**Online Retailer at Nash Equilibrium:** The online retailer serves all market areas not served by the local retailers. For the online retailer, demand density is \(\{a - b(P_d + \mu)\}\) and the profit is

\[
(33) \Pi_d = (P_d - c - T)(1 - 2Nx_m)\{a - b(P_d + \mu)\} = (2bN)(\frac{1}{2N} - x_m)(P_d - c - T)\{(\frac{a}{b} - \mu) - P_d\}
\]

Differentiating, substituting \(\frac{\partial x_m}{\partial P_d} = \frac{1}{l}\), and simplifying,

\[
(34) \frac{\partial \Pi_d}{\partial P_d} = \frac{2bN}{t}[(P_d - c - T)^2 - (P_d - c - T)\{(\frac{a}{b} - \mu - c - T) + 2t(\frac{1}{2N} - x_m)\} + t(\frac{1}{2N} - x_m)(\frac{a}{b} - \mu - c - T)]
\]

At Nash equilibrium \(\frac{\partial \Pi_d}{\partial P_d} = 0\), that is, \((P_d - c - T)\) is one of the roots of the quadratic equation obtained by equating the right hand side of (34) to zero. Simplifying, the price of the online retailer at Nash equilibrium must be one of the following:

\[
(35) P_{dh} = \frac{1}{2}(\frac{a}{b} - \mu + c + T) + t(\frac{1}{2N} - x_m) + \frac{1}{2}\sqrt{(\frac{a}{b} - \mu - c - T)^2 + \left(2t(\frac{1}{2N} - x_m)\right)^2}
\]

or,

\[
(36) P_{dl} = \frac{1}{2}(\frac{a}{b} - \mu + c + T) + t(\frac{1}{2N} - x_m) - \frac{1}{2}\sqrt{(\frac{a}{b} - \mu - c - T)^2 + \left(2t(\frac{1}{2N} - x_m)\right)^2}
\]

From (35), \(P_{dh} > \frac{a}{b} - \mu\), that is, the online retailer generates zero demand and zero profit by setting price \(P_{dh}\). As shown in Appendix B1, \((c + T) < P_{dl} < \left(\frac{a}{b} - \mu\right)\), and \(\frac{\partial^2 \Pi_d}{\partial P_d^2} < 0\) when \(P_d = P_{dl}\), that is, given \(P_r\), the online retailer’s profit is maximized at \(P_{dl}\). Therefore, \(P_{dl}\) is the online retailer’s price at Nash equilibrium. Proposition 6 summarizes the prices if a Nash equilibrium exists with \(0 < x_m < \frac{1}{2N}\). Some bounds on the prices, derived in Appendix B1, are also presented in Proposition 6.

**Proposition 6:** If a Nash equilibrium exists where both the local and online retailers have non-zero market shares, that is, \(0 < x_m < \frac{1}{2N}\), the price of the local retailer \(P_r^*\) and the price of the online retailer \(P_d^*\) are given by:

\[
(37) P_r^* = \frac{1}{2}(\frac{a}{b} + c + tx_m) - \sqrt{(\frac{a}{b} - c - tx_m)^2 + 2t^2x_m^2}, \text{ and}
\]

\[
(38) P_d^* = \frac{1}{2}(\frac{a}{b} - \mu + c + T) + t(\frac{1}{2N} - x_m) - \frac{1}{2}\sqrt{(\frac{a}{b} - \mu - c - T)^2 + \left(2t(\frac{1}{2N} - x_m)\right)^2}
\]

\(P_r^*\) and \(P_d^*\) satisfy the following conditions:

- \(P_r^* < \frac{a}{b}\) and \(P_d^* < \frac{a}{b} - \mu\)
- \(c < P_r^* < c + tx_m < \frac{1}{2N}\)
are three distinct scenarios:

- \( c + T < P^*_d < (c + T) + t\left(\frac{1}{2N} - x_m\right) < (c + T) + \frac{1}{2N} \)
- \( 0 < (P^*_r - c) + (P^*_r - c - T) < \frac{t}{2N} \)

Finding \( x_m \) and Conditions for Retailer Participation: If a Nash equilibrium exists with \( 0 < x_m < \frac{1}{2N} \), we have, from (28),

\[
tx_m - \mu = P^*_d - P^*_r
\]

\[
= \frac{T}{2} - \frac{\mu}{2} + t\left(\frac{1}{2N} - x_m\right) - \frac{tx_m}{2} - \frac{1}{2}\sqrt{\left(\frac{a}{b} - \mu - c - T\right)^2 + \left(2t\left(\frac{1}{2N} - x_m\right)\right)^2 + \frac{1}{2}\left(\frac{a}{b} - c - tx_m\right)^2 + 2t^2x_m^2}
\]

Hence, \( x_m \) is the solution of \( \phi(x) = 0 \) where

\[
(39) \phi(x) = \frac{1}{2}(\mu + T + \frac{t}{N}) - \frac{5tx}{2} - \frac{1}{2}\sqrt{\left(\frac{a}{b} - \mu - c - T\right)^2 + \left(2t\left(\frac{1}{2N} - x\right)\right)^2 + \frac{1}{2}\left(\frac{a}{b} - c - tx\right)^2 + 2t^2x^2}
\]

It is shown in Appendix B2 that \( \frac{\partial\phi(x)}{\partial x} < 0 \), that is, \( \phi(x) = 0 \) can have only one solution. There are three distinct scenarios:

1. If \( \phi\left(\frac{1}{2N}\right) \geq 0 \), the solution of \( \phi(x) = 0 \) occurs at \( x \geq \frac{1}{2N} \), that is, the online retailer has zero market share, and competition reduces to competition among local retailers examined earlier.
2. If \( \phi(0) \leq 0 \), the solution of \( \phi(x) = 0 \) occurs at \( x \leq 0 \), that is, the local retailers have zero market shares. Then, the online retailer captures the entire market.
3. If \( \phi(0) > 0 > \phi\left(\frac{1}{2N}\right) \), the unique solution of \( \phi(x) = 0 \) occurs at \( 0 < x_m < \frac{1}{2N} \), that is, the local and online retailers all have non-zero market shares. We henceforth denote this \( x_m \) by \( x^* \). \( P^*_r \) and \( P^*_d \) are as given in Proposition 6.

Proposition 7 specifies when these three scenarios occur, proof is presented in Appendix B3.

**Proposition 7:** The online retailer has zero market share if

\[
(40) \mu + T \geq \theta_d = \frac{t}{N} - \frac{1}{2}\sqrt{\left(\frac{a}{b} - c - \frac{t}{2N}\right)^2 + \frac{t^2}{2N^2} + \left(\frac{a}{b} - c - \frac{t}{2N}\right)^2 - \left(\frac{a}{b} - c - \frac{t}{2N}\right)}
\]

\( \theta_d > 0 \) is a strictly increasing function of \( \frac{a}{b} - c \) and of \( \frac{t}{N} \). \( \theta_d = \frac{3t}{4N} \) when \( \frac{a}{b} - c = \frac{3t}{4N} \) and it approaches the limit \( \frac{t}{N} \) when \( \frac{a}{b} - c \gg \frac{t}{N} \).

The local retailers have zero market shares if

\[
(41) \mu + T \leq \theta_r, \text{ where } \theta_r < 0 \text{ is the unique solution of } z - \sqrt{\left(\frac{a}{b} - c - z\right)^2 + \frac{t^2}{N^2} + \left(\frac{a}{b} - c + \frac{t}{N}\right)^2} = 0.
\]

\( \theta_r < 0 \) is a strictly decreasing function of both \( \frac{a}{b} - c \) and \( \frac{t}{N} \). For a given \( \frac{t}{N} \), \( \theta_r \) decreases from \( -\frac{0.3t}{N} \) when \( \frac{a}{b} - c = \frac{3t}{4N} \) to the limit of \( \frac{t}{2N} \) when \( \frac{a}{b} - c \gg \frac{t}{N} \).

Both the local and online retailers have non-zero market shares if

\[
(42) \theta_r < \mu + T < \theta_d
\]
Discussion: From Proposition 7, both the local and online retailers have non-zero market shares only when \((\mu + T)\) falls in a “habitable zone” between \(\theta_r < 0\) and \(\theta_d > 0\). The habitable zone expands in both directions if \(\frac{a}{b}\) or \(t\) increases, and it shrinks from both directions if \(c\) or \(N\) increases. The upper threshold \(\theta_d\) is always less than \(\frac{t}{N}\). As \((\frac{a}{b} - c)\) becomes large, \(\theta_d\) approaches \(\frac{t}{N}\), which is the threshold identified by Balasubramanian (1998).

Since \(\theta_d < \frac{t}{N}\), an online retailer with \(\mu + T > 0\) cannot have a positive market share when \(N\) is large. Intuitively, when \(N\) is large, each customer location is close to a local retailer. Thus, transport cost is low for all customers, and the online retailer cannot compete in price with local retailers.

Finally, since \(\theta_r < -\frac{0.3t}{N}\), the local retailer can only be forced out of the market if \((\mu + T)\) is strongly negative. This can happen, for example, if the online retailer offers unique value such as after sales service or quality assurance.

3.4 Properties of the Nash Equilibrium

We now consider the case where (42) applies, that is, both the online and local retailers have non-zero market shares. The local retailer serves to a distance \(x^*\) on each side where \(x^*\) is the solution of \(\phi(x) = 0\). Propositions 8 and 9 summarize how markets served, prices, unit gross margins and profits depend on model parameters. Proofs are in Appendices B4 and B5.

Proposition 8

- \(\frac{\partial x^*}{\partial (\mu + T)} > 0\), and \(x^* \to 0\) as \(\mu + T \to \theta_r\), and \(x^* \to \frac{1}{2N}\) as \(\mu + T \to \theta_d\).

- \(\frac{\partial x^*}{\partial N} < 0\)

- \(x^*\) is a strictly increasing function of \((\frac{a}{b} - c)\) if \(x < \hat{x}\) and \(x^*\) is a strictly decreasing function of \((\frac{a}{b} - c)\) if \(x > \hat{x}\), where \(0 < \hat{x} < \frac{1}{2N}\) is the solution of \(\sqrt{2(\frac{a}{b} - c - tx)}(\frac{1}{2N} - x) = x(\frac{a}{b} - c - T)\).

- \(x^*\) is strictly increasing in \(t\) when \(x^*\) is small, and is strictly decreasing in \(t\) when \(x^*\) is close to \(\frac{1}{2N}\).

Proposition 9

(1) Ceteris paribus, for \(\theta_r < \mu + T < \theta_d\):

- \(P^*_r\) is a strictly increasing function of \((\mu + T)\). \(P^*_r \to c\) as \(\mu + T \to \theta_r\), and \(P^*_r \to P^*_{r\text{max}} = (c + \theta_d - \frac{t}{2N})\) as \(\mu + T \to \theta_d\).

- \((P^*_d - c - T)\) is a strictly decreasing function of \((\mu + T)\). \(P^*_d - c - T \to -\theta_r\) as \(\mu + T \to \theta_r\), and \(P^*_d - c - T \to 0\) as \(\mu + T \to \theta_d\).
• Effective price for the customer at any location, given by \( P^* r + tx \) if the local retailer is selected and by \( P^*_d + \mu = P^* r + tx^* \) if the online retailer is selected, strictly increases with \((\mu + T)\). Effective price at all locations approaches \( c \) when \( \mu + T \to \theta_r \). When \((\mu + T) \to \theta_d \), effective price increases with \( x \) from \((c + \theta_d - \frac{t}{2N})\) at \( x = 0 \) to \((c + \theta_d)\) when \( x = \frac{1}{2N} \).

• The local retailer’s profit \( \Pi^*_r \to 0 \) as \( \mu + T \to \theta_r \), and the online retailer’s profit \( \Pi^*_d \to 0 \) as \( \mu + T \to \theta_d \).

(2) Ceteris paribus, if \( \frac{a}{b} \) increases:

• \( P^*_{r_{\text{max}}} \) strictly increases from \((c + \frac{t}{4N})\) when \( \frac{a}{b} - c = \frac{3t}{4N} \) to the limit of \((c + \frac{t}{2N})\) when \( \frac{a}{b} - c \gg \frac{t}{N} \).

• Denoting the equilibrium price when there are \( N \) local retailers only by \( P^*_e \), \( (P^* - P^*_{r_{\text{max}}}) \) strictly increases from zero when \( \frac{a}{b} - c = \frac{3t}{4N} \) to the limit of \( \frac{t}{2N} \) when \( \frac{a}{b} - c \gg \frac{t}{N} \).

(3) \( P^* > P^*_e \), and effective price at any location is less if both local and online retailers have non-zero shares than if there is no online retailer.

Discussion: We find that as long as an online retailer gets a non-zero market share, effective price for the customer drops at all locations. Thus total demand, which is determined by effective price, increases anytime an online retailer enters the market successfully. This happens even when \((\mu + T) > 0 \), that is, the online retailer is at a disadvantage due to lack of fit or transport cost. Everything else being same, suppose \((\mu + T)\) decreases, that is, the online retailer has greater reputation or cost advantage. Then, the local retailer reduces price at the origin and still serves a smaller market, effective price decreases at all locations, and total demand increases. In the limit where the local retailer has a very small but still non-zero market share, effective price approaches \( c \), the marginal cost of the local retailer, and total demand approaches \((a - bc)\).

The unit gross margin of the local retailer \( (P^*_r - c) \) or the unit gross margin of the online retailer \( (P^*_d - c - T) \) is always less than \( \frac{t}{2N} \). In contrast, if there is no online retailer, the price charged by the local retailer can approach \((c + \frac{t}{N})\). Thus the online retailer, competing with all the \( N \) local retailers simultaneously, has an effect on price similar to that of doubling the number of local retailers. The gap in price is significant when price ceiling is high as \( P^* \) does approach \((c + \frac{t}{N})\) in that situation.

Finally, as \((\mu + T) \to \theta_d \), the unit margin of the local retailer \( (P^*_r - c) \) approaches \( \frac{t}{2N} \) when price ceiling is high. Similarly, the unit margin of the online retailer approaches \( \frac{t}{2N} \) when \((\mu + T) \to \theta_r \) and price ceiling is high. In either case, the margin of the other retailer approaches zero because the sum of the two margins is less than \( \frac{t}{2N} \).
Numerical Example: To explore how demand expansion due to online retailer entry depends on price ceiling and \((\mu + T)\), we examine the following parameter values:

- \(b = 1\), \(t = 10\), \(c = 0\), and \(N = 5\).
- \(\frac{a}{b}\) varies from \(.75\left(\frac{t}{N}\right)\) to \(10.65\left(\frac{t}{N}\right)\) in steps of \(\frac{1}{N}\).
- \((\mu + T)\) is \(\theta_r\), \(.5\theta_r\), \(0\), \(.5\theta_d\), or \(\theta_d\).

For each combination of parameter values, Figure 6 plots the percentage increase in demand compared to the case where there is the same number of local retailers but no online retailer.

From Figure 6, we find that demand expansion is most significant when price ceiling is low and the online retailer has a significant reputation or cost advantage over local retailers. When price ceiling is very high, demand expansion declines to zero as the demands in all cases converge to the demand ceiling \(a\).

3.5 Results in the limit of high price ceiling: From (37) and (38), it is easy to show that when \((\frac{a}{b} - c)\) becomes large compared to \(t\), \(P^*_r\) and \(P^*_d\) converge to the limits \((c + tx^*)\) and 
\[c + T + t\left(\frac{1}{2N} - x^*\right)\], respectively. Also, if \((\frac{a}{b} - c) \gg \frac{t}{2N}\),
\[\phi(x) \approx \frac{1}{2}(\mu + T + \frac{t}{N}) - \frac{5tx}{2} - \frac{1}{2}\left(\frac{a}{b} - \mu - c - T\right) + \frac{1}{2}\left(\frac{a}{b} - c - tx\right) = \mu + T + \frac{t}{2N} - 3tx,\]
that is, \(x^* \approx \frac{1}{6N} + \frac{1}{3t}(\mu + T)\).

Hence, when \((\frac{a}{b} - c)\) is large compared to \(t\),
\[P^*_r \approx c + tx^* \approx c + \frac{\mu}{3} + \frac{T}{3} + \frac{t}{6N}, \text{ and } P^*_d \approx c + T + t\left(\frac{1}{2N} - x\right) \approx c + \frac{2T}{3} + \frac{t}{3N} - \frac{\mu}{3}\]
These are same as the results for the benchmark case where all customers have high reservation prices and demand is constant. If \((\mu + T) = 0\), \(x^* \approx \frac{1}{6N}\), which is Balasubramanian’s result: in the limit the local retailer serves one third of the market and cedes the other two thirds of the market to the online retailer.

4. Conclusion

4.1 Summary of Findings

In the extant literature on retail competition using the Salop circle, all customers are assumed to have high enough reservation prices to buy the product at all prices under consideration. Demand being constant, focus turns to price competition among retailers. As transport cost is a barrier to competition, a higher transport cost raises prices and retailer profits.
Similar to the existing literature in this area, we examine evenly spaced retailers on the Salop circle with identical marginal costs, all offering the same, standardized product. Unlike the previous literature, we assume that the demand density at any location on the circle is \((a - bP)\), where \(P\) is the price the customer at the location actually pays. Thus, our model incorporates two aspects of retailer pricing, demand generation from customers, and price competition with other retailers. The key difference between our model and the extant literature on retail competition is the presence of customers with low reservation prices that the previous literature ignored.

We first examine competition among local retailers only, and derive the Bertrand Nash equilibrium price in closed form. In this case, price, demand and profit are determined by four factors, price ceiling \(\frac{a}{b}\), retailer marginal cost \(c\), unit transport cost \(t\), and the number of competitors \(N\).

Given \(c\), \(t\) and \(N\), we find that retailer price and profit both increase with price ceiling. When price ceiling is low, each retailer behaves as a spatial monopoly and is not affected by the pricing policies of other retailers. The market served by each retailer is limited, and there are unserved gaps in the market. The retailer’s focus is on demand generation from distant customers, which generates a downward pressure on price. If transport cost increases, market served, demand and retailer profit decrease.

As price ceiling increases, the market served by each retailer increases until the entire circular market is served. As price ceiling increases further, retailers start competing with each other, that is, we have local competition instead of local monopolies. This transition occurs as \(\left(\frac{a}{b} - c\right)\) exceeds \(\frac{3t}{4N}\), a threshold that depends on both unit transport cost and the number of competing retailers. Under local competition, price grows at a diminishing rate as price ceiling increases and is bounded above by \(\left(c + \frac{t}{N}\right)\). This is in contrast to a monopoly where price increases linearly with price ceiling with slope \(\frac{1}{2}\). Profit also increases with price ceiling, but at a much slower rate than for a monopoly: for a monopoly profit increases at an increasing rate with price ceiling while in local competition profit is approximately linear in price ceiling.

Under local competition, an increase in transport cost reduces price and profit when price ceiling is relatively low, as demand generation is more important than price competition. When price ceiling is high, price competition among retailers become significant and, as transport cost shields a retailer from competition, an increase in transport cost raises price and profit.

We also find that unless price ceiling is very high, the retail price, profit and demand are all much less than what the existing models with assumption of high reservation price obtain. Consequently, the number of retailers that can exist in equilibrium can be much lower than what the existing literature proposes.

Finally, we find that a monopolist manufacturer who sells to the competing retailers can always expand demand by adding more retailers. However, the marginal gain from adding more retailers diminishes when price ceiling. After factoring in cost of administration, numerical results suggest that the optimal number of retailers drops as price ceiling increases.

We next examine what happens when, in addition to the local retailers already considered,
an online retailer, equidistant from all customers on the circle, is introduced. We find that the presence of the online retailer reduces the effective price for all customers and increases demand. This is particularly significant when the online retailer has a strong reputation or cost advantage.

The unit gross margin of the local or the online retailer, or the sum of the two margins, can never exceed $\frac{t}{2N}$. This is half of the margin $\frac{t}{N}$ a retailer can achieve when the online retailer is absent. Thus, the presence of the online retailer cuts the unit margin the local retailer can achieve by at least half.

We find that the local retailer can get zero market share, that is, be forced out of the market, only when the online retailer has a significant cost or reputation advantage, and that when price ceiling or transport cost is high, both the local and online retailers are more likely to have non-zero market shares.

Similar to Balasubramanian (1998) who models competition between local and online retailers under constant demand, we find that when the number of local retailers is large, the online retailer is less likely to generate a positive market share. In the limit when price ceiling is high, our results converge to those of Balasubramanian (1998).

### 4.2 Managerial implications, Limitations, and Directions for Future Research

The proposed framework offers areas of exploration in both theoretical and empirical research. Our results suggest that the market boundary of a retailer depends on the price ceiling of the product. When price ceiling is low, the market area served by the retailer is small, and the retailer often is a local monopoly. In contrast, when price ceiling is high, market area expands, and a retailer is more likely to face competition. While an increase in price ceiling allows the retailer to raise price, the increase in price is much smaller in competition compared to a spatial monopoly. For instance, with store level sales data, one can test if price changes by a competitor have greater effect on demand for a more expensive product category with higher price ceiling than for a low priced product category.

Our results also suggest that in an area of low wealth where price ceiling is lower, retailer profit will also be lower. As a result, the area will have fewer retailers in equilibrium, which may actually lead to higher retail prices compared to a wealthy area. This effect should be more pronounced for luxury items for which there should be less demand in a lower income area. This can be tested both within the national boundary and internationally.

On the theoretical dimension, one can make small modifications in the proposed model to explore how factors such as local taxes affect competition between local and online retailers. As the model allows us to find retail prices in both local only as well as local versus online competitions, it is possible to find how outcomes of policy interest such as the number of retailers in equilibrium and consumer surplus depend on the these factors.

The proposed framework, which yields price, demand and profit at the retail level, can be incorporated into a model of channel of distribution. Our results suggest that a manufacturer should distribute through fewer retailers if price ceiling is higher, and should provide post-purchase service itself if price ceiling is low. Future research can examine how alternative channel...
organizations such as adding an online retailer, or direct selling to customers, affect manufacturer profit and consumer surplus.

Clearly, our model makes several simplifying assumptions which future research can relax. We assume that customers may only differ in location and reservation price. In reality some customers may be loyal to local retailers while others are loyal to the online retailers. Intuition suggests that the presence of loyal customers allows a retailer to raise price, which allows competing retailers to raise price also. It will be interesting to check if that really happens.

We assume that unit transport cost is same for all customers. If, instead, transport cost varies across customers, retailers may incorporate such variation in pricing strategy. For example, if the local retailers offer “free delivery” as part of a menu, that may affect local competition as well as competition between local and online retailers.

We assume that retailers sell a single, standardized product. It will be interesting to explore what happens when the retailer sells multiple products with different price ceilings to the same market or applies loss leader pricing.

Like Balasubramanian (1998), we assume that the online retailer offers a uniform price to all customers. It will be interesting to explore how results change if the online retailer charges “in between” customers distant from local retailers a higher price.

We only examine pricing strategy under linear demand. Future research may consider other forms such as the constant elasticity demand function. Also, as model parameters such as price ceiling depend on promotional expenditure, it will be interesting to explore how much retailers spend on promotion in equilibrium as the number of retailers varies. We hope future research will explore these issues.
References


Appendix A

Appendix A1

Results for Spatial Monopoly: If the retailer sets a price $P$ at the origin, the delivered price for a customer at a distance $x$ from the store is $(P + tx)$, and there is a non zero demand at $x$ if and only if $a - b(P + tx) > 0$, that is, $x < \frac{1}{t}(\frac{a}{b} - P)$. Since the Salop circle has unit length, the retailer generates non zero demand up to a distance of $x = \min[\frac{1}{t}(\frac{a}{b} - P), \frac{1}{2}]$ on each side. Demand $Q$ and profit $\Pi$ are given by

\begin{equation}
Q = 2 \int_{0}^{x} (a - bP - bx)dx = 2bx(\frac{a}{b} - P - \frac{tx}{2}), \quad \Pi = (P - c)Q = 2bx(P - c)(\frac{a}{b} - P - \frac{tx}{2})
\end{equation}

If $P > \frac{a}{b} - \frac{t}{2}$, then $x = \frac{1}{t}(\frac{a}{b} - P) < \frac{1}{2}$. Substituting $x$ into (A1) and simplifying,

(A2) $\Pi(P) = \frac{b}{t}(P - c)(\frac{a}{b} - P)^2$

If $P \leq \frac{a}{b} - \frac{t}{2}$ then $\frac{1}{t}(\frac{a}{b} - P) \geq \frac{1}{2}$, that is, $x = \frac{1}{2}$. Substituting into (A1),

(A3) $\Pi = b(P - c)(\frac{a}{b} - P - \frac{t}{4})$

From (A2) and (A3), it is easy to verify that $\Pi(P)$ is a continuous function of $P$ and is differentiable for $P > \frac{a}{b} - \frac{t}{2}$ and for $P < \frac{a}{b} - \frac{t}{2}$.

If $P > \frac{a}{b} - \frac{t}{2}$, then

\begin{equation}
\frac{\partial \Pi}{\partial P} = \frac{b}{t}(\frac{a}{b} - P)(\frac{a}{b} + 2c - 3P) \quad \text{and} \quad \lim_{P \to (\frac{a}{b} - \frac{t}{2})^+} \frac{\partial \Pi}{\partial P} = b\{\frac{3t}{4} - (\frac{a}{b} - c)\}
\end{equation}

If $P < \frac{a}{b} - \frac{t}{2}$, then

\begin{equation}
\frac{\partial \Pi}{\partial P} = b[\frac{a}{b} + c - \frac{t}{4} - 2P] \quad \text{and} \quad \lim_{P \to (\frac{a}{b} - \frac{t}{2})^-} \frac{\partial \Pi}{\partial P} = b\{\frac{3t}{4} - (\frac{a}{b} - c)\}
\end{equation}

Case 1. $\frac{a}{b} - c \geq \frac{3t}{4}$. If $\frac{a}{b} - c = \frac{3t}{4}$, then $\frac{\partial \Pi}{\partial P} > 0$ if $P < \frac{a}{b} - \frac{t}{2}$ and $\frac{\partial \Pi}{\partial P} < 0$ if $P > \frac{a}{b} - \frac{t}{2}$.

Hence, the retailer’s profit maximizing price is $P^* = \frac{a}{b} - \frac{t}{2}$, which also equals $(\frac{a}{2b} + \frac{c}{2} - \frac{t}{8})$ here.

If $\frac{a}{b} - c > \frac{3t}{4}$, then $\frac{\partial \Pi}{\partial P} < 0$ if $P > \frac{a}{b} - \frac{t}{2}$ and $\lim_{P \to (\frac{a}{b} - \frac{t}{2})^-} \frac{\partial \Pi}{\partial P} < 0$. Hence, $P^* < \frac{a}{b} - \frac{t}{2}$, and,

from (A5), is given by $(\frac{a}{2b} + \frac{c}{2} - \frac{t}{8})$. Demand and profit follow from (A1) and (A3), respectively.

Case 2. $\frac{a}{b} - c < \frac{3t}{4}$. Then, $\frac{\partial \Pi}{\partial P} > 0$ if $P < \frac{a}{b} - \frac{t}{2}$, and $\lim_{P \to (\frac{a}{b} - \frac{t}{2})^+} \frac{\partial \Pi}{\partial P} > 0$. Hence, $P^* > \frac{a}{b} - \frac{t}{2}$.

From (A4), $P^* = \frac{1}{3}(\frac{a}{b} + 2c)$. Demand and profit follow from (A1) and (A2), respectively.

Appendix A2

The low solution $P_l$ has the following properties:
(1) \( c < P_t < \frac{a}{b} \) (2) \( \frac{\partial^2 \Pi}{\partial P^2} < 0 \) at \( P = P_t \), and (3) \( P_t < c + \frac{t}{N} \).

**Proof:** From (14),

(A6) \( P_t = \frac{1}{2}(\frac{a}{b} + c + \frac{3t}{2N}) - \frac{1}{2}\sqrt{(\frac{a}{b} - c - \frac{t}{2N})^2 + \frac{3t^2}{N^2}} \)

**Property 1:** From (A6),

\[ \frac{a}{b} - P_t = \frac{1}{2}(\frac{a}{b} - c - \frac{3t}{2N}) + \sqrt{(\frac{a}{b} - c - \frac{t}{2N})^2 + \frac{3t^2}{N^2}} \geq \frac{1}{2}[\frac{3t}{4N} + \sqrt{(\frac{3t}{4N} - \frac{t}{2N})^2 + \frac{3t^2}{N^2}}] = \frac{t}{2N} > 0, \]

from the assumption that \( \frac{a}{b} - c \geq \frac{3t}{4N} \). Also, from (A6)

\[ P_t - c = \frac{1}{2}(\frac{a}{b} - c + \frac{3t}{2N}) - \sqrt{(\frac{a}{b} - c - \frac{t}{2N})^2 + \frac{3t^2}{N^2}} \]

Since \( \frac{a}{b} - c - \frac{t}{2N} \geq \frac{3t}{4N} - \frac{t}{2N} = \frac{t}{4N} > 0 \) and \( \frac{t\sqrt{3}}{N} > 0 \),

\[ \sqrt{(\frac{a}{b} - c - \frac{t}{2N})^2 + \frac{3t^2}{N^2}} < (\frac{a}{b} - c - \frac{t}{2N}) + \frac{t\sqrt{3}}{N} \]

Hence,

\[ P_t - c > \frac{1}{2}(\frac{a}{b} - c + \frac{3t}{2N}) - \left( (\frac{a}{b} - c - \frac{t}{2N}) + \frac{t\sqrt{3}}{N} \right) = \left(\frac{2 - \sqrt{3}}{2N}\right)t > 0 \]

**Property 2:** From (10),

(A7) \( \frac{\partial \Pi}{\partial P} = \{ x_1[a - bP - \frac{bt x_1}{2}] + x_2[a - bP - \frac{bt x_2}{2}] \} - \frac{1}{2t}(P - c)[(a - bP - \frac{bt x_1}{2}) + (a - bP - \frac{bt x_2}{2})] \)

\[-(P - c)(x_1 + x_2)(\frac{3b}{4})\]

Since, for \( i = 1 \) and \( 2 \), \( \frac{\partial x_i}{\partial P} = -\frac{1}{2t} \) and \( \frac{\partial}{\partial P}(a - bP - \frac{bt x_i}{2}) = -b - \frac{bt}{2}(-\frac{1}{2t}) = -\frac{3b}{4} \),

\[ \frac{\partial^2 \Pi}{\partial P^2} = \left( -\frac{1}{2t} \right)[(a - bP - \frac{bt x_1}{2}) + (a - bP - \frac{bt x_2}{2})] + \{ x_1(-\frac{3b}{4}) + x_2(-\frac{3b}{4}) \} \]

\[ -\frac{1}{2t}[(a - bP - \frac{bt x_1}{2}) + (a - bP - \frac{bt x_2}{2})] - \frac{1}{2t}(P - c)(2)(-\frac{3b}{4}) \]

\[ -(x_1 + x_2)(\frac{3b}{4}) - 2(P - c)(\frac{3b}{4})(-\frac{1}{2t}) \]

\[ = \frac{-1}{t}[(a - bP - \frac{bt x_1}{2}) + (a - bP - \frac{bt x_2}{2})] - (x_1 + x_2)(\frac{3b}{2}) + \frac{3b}{2}(P - c) \]

\[ = \frac{-2b}{t}[(\frac{a}{b} - P) + \frac{t}{2}(x_1 + x_2) - (\frac{3}{4})(P - c)] \]

Substituting \( x_1 = x_2 = \frac{1}{2N} \),

(A8) \( \frac{\partial^2 \Pi}{\partial P^2} = \frac{-2b}{t}[(\frac{a}{b} - P) + \frac{t}{2N} - (\frac{3}{4})(P - c)] \)

From (A6),

\[ 26 \]
From equations (15)-(17),

\[
\frac{a}{b} - P_l = \frac{1}{2} \left( \frac{a}{b} - c - \frac{3t}{2N} \right) + \frac{1}{2} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}},
\]

and

\[
P_l - c = \frac{1}{2} \left( \frac{a}{b} - c + \frac{3t}{2N} \right) - \frac{1}{2} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}}.
\]

Hence at \( P = P_l \),

\[
\frac{\partial^2 \Pi}{\partial P^2} = -\frac{2b}{t} \left[ \left( \frac{1}{2} - \frac{3}{8} \right) \left( \frac{a}{b} - c \right) - \left( \frac{1}{2} + \frac{3}{8} \right) \left( \frac{3t}{2N} \right) + \frac{t}{2N} + \left( \frac{1}{2} + \frac{3}{8} \right) \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right]
\]

\[
= -\frac{2b}{t} \left[ \frac{1}{8} \left( \frac{a}{b} - c \right) - \left( \frac{7}{8} \right) \left( \frac{3t}{2N} \right) + \frac{t}{2N} + \left( \frac{7}{8} \right) \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right] < 0
\]

since \( \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \geq \sqrt{\frac{3t^2}{N^2}} = \frac{t\sqrt{3}}{N} > \frac{3t}{2N} \).

**Property 3:** Rewriting (A6),

\[
P_l = \left( c + \frac{t}{N} \right) - \frac{1}{2} \sqrt{ \left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2} } - \left( \frac{a}{b} - c - \frac{t}{2N} \right) < \left( c + \frac{t}{N} \right)
\]

**Appendix A3**

**Derivation of Expression for \( \Pi^* \):** Substituting \( x_1 = x_2 = \frac{1}{2N} \) in (8), the profit of a retailer is

\[
(A9) \quad \Pi = b(P - c) \left\{ x_1 \left( \frac{a}{b} - P - \frac{tx_1}{2} \right) + x_2 \left( \frac{a}{b} - P - \frac{tx_2}{2} \right) \right\} = \frac{b}{N} (P - c) \left( \frac{a}{b} - P - \frac{t}{2N} \right)
\]

\[
= \frac{b}{N} \left[ \left( \frac{a}{b} - c - \frac{t}{4N} \right) (P - c) - (P - c)^2 \right]
\]

From (11), at Nash equilibrium,

\[
(P - c)^2 - (P - c) \left( \frac{a}{b} - c + \frac{3t}{2N} \right) + \frac{t}{N} \left( \frac{a}{b} - c - \frac{t}{4N} \right) = 0,
\]

that is,

\[
(A10) \quad \left( \frac{a}{b} - c - \frac{t}{4N} \right) (P - c) - (P - c)^2 = \frac{t}{N} \left( \frac{a}{b} - c - \frac{t}{4N} \right) - \frac{7t}{4N} (P - c).
\]

Combining (A9) and (A10), at Nash equilibrium,

\[
(A11) \quad \Pi = \frac{b}{N} \left[ \frac{t}{N} \left( \frac{a}{b} - c - \frac{t}{4N} \right) - \frac{7t}{4N} (P - c) \right] = \frac{bt}{N^2} \left[ \left( \frac{a}{b} - c - \frac{t}{4N} \right) - \frac{7(P - c)}{4} \right]
\]

Substituting \( P^* \) from (15) and simplifying, the profit of a retailer at Nash equilibrium is

\[
(A12) \quad \Pi^* = \frac{bt}{N^2} \left[ \frac{1}{8} \left( \frac{a}{b} - c \right) - \frac{25t}{16N} + \frac{7}{8} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right]
\]

**Preliminary Results for Propositions 3-5:** From equations (15)-(17),

\[
(A13) \quad P^* = \frac{1}{2} \left( \frac{a}{b} + c + \frac{3t}{2N} \right) - \frac{1}{2} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}}, \text{ that is,}
\]

\[
(A14) \quad P^* - c = \frac{1}{2} \left( \frac{a}{b} - c + \frac{3t}{2N} \right) - \frac{1}{2} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}}
\]

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\[
\frac{t}{N} - \frac{1}{2} \left[ \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} - \left( \frac{a}{b} - c - \frac{t}{2N} \right) \right]
\]

(A15) \( Q_T^* = NQ_R^* = b \left[ \left( \frac{a}{b} - c - \frac{2t}{N} \right) + \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right] \)

(A16) \( \Pi^* = \frac{bt}{N^2} \left[ \left( \frac{a}{b} - c \right) - \frac{25t}{16N} + \frac{7}{8} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right] \)

From (A14)-(A16),

(A17) \( \frac{\partial Q_R^*}{\partial t} = \frac{b}{2N} \left[ -\frac{2}{N} + \frac{1}{2} \left( \frac{6t}{N^2} - \frac{1}{N} \left( \frac{a}{b} - c - \frac{t}{2N} \right) \right) \right] = -\frac{b}{4N^2} \left[ 4 + \frac{u - 6}{\sqrt{u^2 + 3}} \right] \)

(A18) \( \frac{\partial(P^* - c)}{\partial(t/N)} = \frac{1}{2} \left( \frac{6t}{N^2} - \frac{1}{N} \left( \frac{a}{b} - c - \frac{t}{2N} \right) \right) \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \)

(A19) \( \frac{\partial \Pi^*}{\partial t} = \frac{b}{N^2} \left[ \left( \frac{a}{b} - c \right) - \frac{25t}{16N} + \frac{7}{8} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right] \)

\[+ \frac{bt}{N^2} \left[ -\frac{25}{16N} + \frac{7}{16} \left( \frac{6t}{N^2} - \frac{1}{N} \left( \frac{a}{b} - c - \frac{t}{2N} \right) \right) \right] \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \]

\[= \frac{bt}{N^2} \left[ \left( \frac{a}{b} - c \right) - \frac{25t}{16N} + \frac{7}{8} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right] \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \]

\[= \frac{bt}{N^2} \left[ \left( \frac{a}{b} - c \right) - \frac{25t}{16N} + \frac{7}{8} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right] \]

\[= \frac{bt}{N^2} \left[ \left( \frac{a}{b} - c \right) - \frac{25t}{16N} + \frac{7}{8} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right] \]

\[+ \frac{bt}{N^2} \left[ -\frac{25}{16N} + \frac{7}{16} \left( \frac{6t}{N^2} - \frac{1}{N} \left( \frac{a}{b} - c - \frac{t}{2N} \right) \right) \right] \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \]

From (A17)-(A19),

(A20) \( \frac{\partial Q_T^*}{\partial N} = \frac{b}{2N} \left[ \left( \frac{2t}{N^2} + \frac{1}{2} \left( \frac{6t}{N^2} \right) \right) \right] \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \)

\[= \frac{b}{2N} \left[ \left( \frac{2t}{N^2} + \frac{1}{2} \left( \frac{6t}{N^2} \right) \right) \right] \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \]

\[= \frac{b}{2N} \left[ \left( \frac{2t}{N^2} + \frac{1}{2} \left( \frac{6t}{N^2} \right) \right) \right] \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \]

where \( u = \frac{a}{b} - c - \frac{t}{2N} \). Regarding the function \( \frac{u - 6}{\sqrt{u^2 + 3}} \), we note that

- \( \frac{\partial}{\partial u} \left[ \frac{u - 6}{\sqrt{u^2 + 3}} \right] = \frac{1}{(u^2 + 3)^{3/2}} \left( \frac{u}{(u^2 + 3)^{3/2}} \right) > 0 \), that is, for a given \( t \), \( \frac{u - 6}{\sqrt{u^2 + 3}} \) is a strictly increasing function of \( \left( \frac{a}{b} - c \right) \).

- If \( \left( \frac{a}{b} - c \right) = \frac{3t}{4N} \) then \( u = \frac{1}{4} \) and \( \frac{u - 6}{\sqrt{u^2 + 3}} = -\frac{23}{7} \). Also, \( \lim_{u \to \infty} \frac{u - 6}{\sqrt{u^2 + 3}} = 1 \). Hence, for a given \( t \), \( \frac{u - 6}{\sqrt{u^2 + 3}} \) increases from \( -\frac{23}{7} \) at \( \left( \frac{a}{b} - c \right) = \frac{3t}{4N} \) to the limit of 1 as \( \left( \frac{a}{b} - c \right) \to \infty \).

- Numerically, we find that \( \frac{u - 6}{\sqrt{u^2 + 3}} = -3 \) when \( u = 0.549 \), that is, \( \frac{a}{b} - c = 1.049(\frac{t}{N}) \).

Proof of Proposition 3: Since \( 4 + \frac{u - 6}{\sqrt{u^2 + 3}} \geq 4 - \frac{23}{7} > 0 \), it follows from (A17) that \( \frac{\partial Q_R^*}{\partial t} < 0 \). Hence, \( Q_T^* = NQ_R^* \) is also a strictly decreasing function of \( t \).

From (A18), \( \frac{\partial(P^* - c)}{\partial(t/N)} < 0 \) if \( \left( \frac{a}{b} - c \right) < 1.049(\frac{t}{N}) \) and \( \frac{\partial(P^* - c)}{\partial(t/N)} > 0 \) if \( \left( \frac{a}{b} - c \right) > 1.049(\frac{t}{N}) \). Hence, for a given \( N \), \( P^* - c \) is strictly decreasing in \( t \) if \( \left( \frac{a}{b} - c \right) < 1.049(\frac{t}{N}) \) and strictly increasing in \( t \) if \( \left( \frac{a}{b} - c \right) > 1.049(\frac{t}{N}) \).
Substituting in (A19), we find that \((a/b - c) = 3t/4N\), that is, \(u = 1/4\), then \(\frac{\partial \Pi^*}{\partial t} = -\frac{bt}{16N^3} < 0\). As, for a fixed \(t/N\), \((a/b - c) \) increases, \(u\) and \(\sqrt{u^2 + 3}\) are unbounded above while \(\frac{u - b}{\sqrt{u^2 + 3}}\) is bounded above by 1. Hence, from (A19), \(\frac{\partial \Pi^*}{\partial t} > 0\) when \((a/b - c)\) is large.

If \((a/b - c) \gg \frac{t}{N}\) and \(a/b \gg c\), then \(a/b \approx \frac{tu}{N}\), and, from (A19),

\[
\frac{\partial \Pi^*}{\partial t} \approx \frac{btu}{N^3} = (\frac{b}{N^2})(\frac{tu}{N}) \approx \frac{a}{N^2}
\]

**Proof of Proposition 4:** From (A14),

\[
\frac{\partial (P^* - c)}{\partial (a/b - c)} = \frac{1}{2}[1 - \frac{a/c - \frac{t}{2N}}{\sqrt{(\frac{a}{b} - c - \frac{t}{2N})^2 + \frac{3t^2}{N^2}}}] > 0
\]

It follows directly from (A14) that \(P^* - c = \frac{t}{4N}\) when \(a/b - c = \frac{3t}{4N}\), and \(P^* - c \to \frac{t}{N}\) when \(a/b - c \gg \frac{t}{N}\).

It follows directly from (A15) and (A16) that:

- Given \(b, Q^*_R, Q^*_T, \) and \(\Pi^*\) are strictly increasing functions of \((a/b - c)\).
- If \((a/b - c) = \frac{3t}{4N}\), then \(Q^*_T = \frac{bt}{4N} = \frac{1}{3}(a - bc)\) and \(\Pi^* = \frac{bt^2}{16N^3} = \frac{(a - bc)t}{12N^2}\).

If \((a/b - c) \gg \frac{t}{N}\), then \(\sqrt{(\frac{a}{b} - c - \frac{t}{2N})^2 + \frac{3t^2}{N^2}} \approx \frac{a}{b} - c\). Hence, from (A15), \(Q^*_T \approx (a - bc)\), and, from (A16), \(\Pi^* \approx \frac{(a - bc)t}{N^2}\). If, in addition, \(a/b \gg c\), then \(Q^*_T \approx a\) and \(\Pi^* \approx \frac{at}{N^2}\).

**Proof of Proposition 5**

**Effect of \(N\) on unit gross margin:** Since \(\frac{\partial (P^* - c)}{\partial (t/N)} < (>) 0\) if \((a/b - c) < (>) 1.049(\frac{t}{N})\), it follows that given \(t\), \(\frac{\partial (P^* - c)}{\partial N} > 0\) when \((a/b - c) < 1.049(\frac{t}{N})\), and \(\frac{\partial (P^* - c)}{\partial N} < 0\) when \((a/b - c) > 1.049(\frac{t}{N})\).

**Effect of \(N\) on Demand:** From (A20),

\[
\frac{\partial Q^*_T}{\partial N} = (\frac{bt}{4N^2})[4 + \frac{u - 6}{\sqrt{u^2 + 3}}] \geq (\frac{bt}{4N^2})[4 - \frac{23}{7}] > 0
\]

**Effect of \(N\) on \(\Pi^*\):** We can write (A16) as

\[
(a/b - c) = \frac{b}{t}f(v, w), \text{ where } f(v, w) = w^2\left\{\frac{v}{8} - \frac{25w}{16} + \frac{7}{8}\sqrt{(v - \frac{w}{2})^2 + 3w^2}\right\}, v = \frac{a}{b} - c, w = \frac{t}{N}.
\]

(A21) \[\frac{\partial f}{\partial w} = 2w\left\{\frac{v}{8} - \frac{25w}{16} + \frac{7}{8}\sqrt{(v - \frac{w}{2})^2 + 3w^2}\right\} + w^2\left\{\frac{25}{16} + \frac{7}{16}\frac{6w - (v - \frac{w}{2})}{\sqrt{(v - \frac{w}{2})^2 + 3w^2}}\right\} \]
\[
\frac{w}{16}[4v - 75w + 28\sqrt{(v - \frac{w}{2})^2 + 3w^2} + \frac{42w^2 - 7w(v - \frac{w}{2})}{\sqrt{(v - \frac{w}{2})^2 + 3w^2}}]
\]

Now,
\[
\frac{\partial}{\partial v}[4v - \frac{7w(v - \frac{w}{2})}{\sqrt{(v - \frac{w}{2})^2 + 3w^2}}] = 4 - \frac{21w^3}{(v - \frac{w}{2})^2 + 3w^2} \geq 4 - \frac{21w^3}{(49w^2/16)^{3/2}} = 4 - \frac{21 \times 64}{343} > 0,
\]

since \(v = \frac{a}{b} - c \geq \frac{3t}{4N} = \frac{3w}{4N} \). Also,
\[
\frac{\partial}{\partial v}[28\sqrt{(v - \frac{w}{2})^2 + 3w^2} + \frac{42w^2}{\sqrt{(v - \frac{w}{2})^2 + 3w^2}}] = \frac{28(v - \frac{w}{2})}{\sqrt{(v - \frac{w}{2})^2 + 3w^2}} - \frac{42w^2(v - \frac{w}{2})}{(v - \frac{w}{2})^2 + 3w^2}^{3/2}
= \{\frac{v - \frac{w}{2}}{(v - \frac{w}{2})^2 + 3w^2}^{3/2}\}[28\{(v - \frac{w}{2})^2 + 3w^2\} - 42w^2] > 0
\]

Therefore, from (A22), \(\frac{\partial}{\partial v}(\frac{\partial f}{\partial w}) > 0\). If \(v = \frac{3w}{4}\), (A22) gives \(\frac{\partial f}{\partial w} = 0\). Therefore, if \(v > \frac{3w}{4}\), \(\frac{\partial f}{\partial w} > 0\), that is, if \((\frac{a}{b} - c) > \frac{3t}{4N}\) and \(t\) is fixed, \(\Pi^*\) is strictly increasing in \(\frac{t}{N}\), that is, strictly decreasing in \(N\).

**Effect of \(N\) on \(\Pi^*\):** Using the same notations, the total profit of the \(N\) retailers is
\[
(A23) \ \Pi^* = bw\{v - \frac{25w}{16} + \frac{7}{8}\sqrt{(v - \frac{w}{2})^2 + 3w^2}\} = \frac{bw}{16}[2v - 25w + 14\sqrt{(v - \frac{w}{2})^2 + 3w^2}]
\]
\[
(A24) \ \frac{\partial(\Pi^*)}{\partial w} = \frac{b}{16}[\{2v - 25w + 14\sqrt{(v - \frac{w}{2})^2 + 3w^2}\} - 25 + 7\frac{6w - (v - \frac{w}{2})}{\sqrt{(v - \frac{w}{2})^2 + 3w^2}}]
= \frac{b}{16}[2v - 50w + 14\sqrt{(v - \frac{w}{2})^2 + 3w^2} + (7w)\{\frac{6w - (v - \frac{w}{2})}{\sqrt{(v - \frac{w}{2})^2 + 3w^2}}\}]
\]

Since \(\frac{v - \frac{w}{2}}{\sqrt{(v - \frac{w}{2})^2 + 3w^2}} \leq 1\), it follows that \(\Pi^*\) is strictly increasing in \(w = \frac{t}{N}\), that is, strictly decreasing in \(N\) when \(v = (\frac{a}{b} - c) \gg w = \frac{t}{N}\). Also, if \(\frac{a}{b} - c = \frac{3t}{4N}\), that is, \(v = \frac{3w}{4}\), substitution in (A24) shows that \(\frac{\partial \Pi^*}{\partial w} < 0\), that is, \(\frac{\partial \Pi^*}{\partial N} > 0\). By continuity, \(\Pi^*\) is increasing in \(N\) when \((\frac{a}{b} - c)\) is small.

**Appendix A5**

**Proof of Result 1:** If \(c > \frac{a}{b} - \frac{3t}{4}\), that is, \(\frac{a}{b} - c < \frac{3t}{4}\), then \(Q = \frac{4b}{9t}(\frac{a}{b} - c)^2\). Hence,
\[
(A25) \ \Pi_M = \frac{4b}{9t}(c - c_0)(\frac{a}{b} - c)^2 \rightarrow \frac{\partial \Pi_M}{\partial c} = \frac{4b}{9t}(\frac{a}{b} - c)(\frac{a}{b} + 2c_0 - 3c), \text{ and}
\]
\[
(A26) \ \lim_{c \rightarrow \frac{(t - \frac{a}{b})}{3}} \frac{\partial \Pi_M}{\partial c} = \frac{2b}{3}(\frac{9t}{8} - (\frac{a}{b} - c_0))
\]

If \(c < \frac{a}{b} - \frac{3t}{4}\), that is, \(\frac{a}{b} - c > \frac{3t}{4}\), then \(Q = \frac{b}{2}(\frac{a}{b} - c - \frac{t}{4})\). Hence,
(A27) $\Pi_M = \frac{b}{2}(c - c_0)\left(\frac{a}{b} - c - \frac{t}{4}\right)$ $\rightarrow \frac{\partial \Pi_M}{\partial c} = \frac{b}{2}\left(\frac{a}{b} + c_0 - \frac{t}{4} - 2c\right)$, and

\[
(A28) \lim_{c \to \left(\frac{t}{4} - \frac{b}{a}\right)} \frac{\partial \Pi_M}{\partial c} = \frac{b}{2}\left(\frac{5t}{4} - \left(\frac{a}{b} - c_0\right)\right)
\]

From (A25)-(A28):

If $\frac{a}{b} - c_0 > \frac{5t}{4}$, then $c^* = \left(\frac{1}{2}\left(\frac{a}{b} + c_0 - \frac{t}{4}\right)\right)$, $\Pi_M = \frac{b}{8}\left(\frac{a}{b} - c_0 - \frac{t}{4}\right)^2$, and $\frac{\Pi_{M}^*}{\Pi_M} = \frac{1}{2}\left(1 - \frac{t}{4(a/b - c_0)}\right)^2$.

If $\frac{9t}{8} \leq \frac{a}{b} - c_0 \leq \frac{5t}{4}$ then $c^* = \frac{a}{b} - \frac{3t}{4}$, $\Pi_M^* = \frac{bt}{4}\left(\frac{a}{b} - c_0 - \frac{3t}{4}\right)$, and $\frac{\Pi_{M}^*}{\Pi_M} = \frac{243}{27}\left(\frac{a}{b} - c_0\right)$.

If $\frac{a}{b} - c_0 < \frac{9t}{8}$, then $c^* = \frac{a}{3b} + \frac{2c_0}{3}$, $\Pi_M = \frac{16b}{243t}\left(\frac{a}{b} - c_0\right)^3$, and $\frac{\Pi_{M}^*}{\Pi_M} = \frac{64}{243t}\left(\frac{a}{b} - c_0\right)$.

It is easy to verify that $\frac{\Pi_{M}^*}{\Pi_M}$ is strictly increasing in $(\frac{a}{b} - c_0)$ and converges to 0.5 when $\frac{a}{b} - c_0 \gg t$.

**Proof of Result 2:** The manufacturer’s profit is maximized by a retailer local monopoly if and only if $\frac{a}{b} - c^* < \frac{3t}{4N}$, that is, $c^* = \frac{a}{b} - \frac{3t}{4N}$. The retailers are local monopolies when $\frac{a}{b} - c < \frac{3t}{4N}$, that is, $c > \frac{a}{b} - \frac{3t}{4N}$. Then, from (7), total demand is $Q_T = \frac{4bN}{9t}\left(\frac{a}{b} - c\right)^2$, and

\[
(A29) \Pi_M(c) = (c - c_0)Q_T = \frac{4bN}{9t}f(c), \text{ where } f(c) = (c - c_0)\left(\frac{a}{b} - c\right)^2
\]

\[
(A30) f'(c) = (\frac{a}{b} - c)^2 - 2(\frac{a}{b} - c)(c - c_0) = (\frac{a}{b} - c)(\frac{a}{b} + 2c_0 - 3c)
\]

For $c < \frac{a}{b}$, $f(c)$ is maximized at $\hat{c} = \frac{a}{3b} + \frac{2c_0}{3}$, and $f(\hat{c}) = \frac{4}{27}(\frac{a}{b} - c_0)^3$. Hence,

\[
(A31) \Pi_M(c) = \frac{4bN}{9t}f(\hat{c}) = \frac{16bN}{243t}\left(\frac{a}{b} - c_0\right)^3 = \hat{\Pi}_M \text{ (say)}
\]

Suppose $\frac{a}{b} - c_0 \geq \frac{9t}{8N}$, that is, $\hat{c} = \frac{a}{3b} + \frac{2c_0}{3} = \frac{a}{b} - \frac{2}{3}\left(\frac{a}{b} - c_0\right) \leq \frac{a}{b} - \frac{3t}{4N}$. Then, $\forall c > \frac{a}{b} - \frac{3t}{4N}$, $\frac{\partial \Pi_M}{\partial c} < 0$, which implies local competition.

Therefore, the manufacturer’s profit is maximized by a retailer local monopoly only if

$\frac{a}{3b} + \frac{2c_0}{3} > \frac{a}{b} - \frac{3t}{4N}$, that is, $\frac{a}{b} - c_0 < \frac{9t}{8N}$. If the manufacturer’s profit is maximized by a retailer local monopoly, it follows from (A31) that

\[
\frac{\Pi_{M}^*}{\Pi} \leq \frac{\hat{\Pi}_M}{\Pi} = \frac{64N}{243t}\left(\frac{a}{b} - c_0\right) < \left(\frac{64N}{243t}\right)\left(\frac{9t}{8N}\right) = \frac{8}{27}
\]

**Proof of Result 3**

**Case 1.** $\frac{a}{b} - c_0 \leq \frac{3t}{2N}$: We already established that if the manufacturer’s profit is maximized under local retailer monopolies, then $\frac{\Pi_{M}^*}{\Pi_M} < \frac{8}{27}$. Suppose now there is local competition. Since
\[
\frac{a}{b} - c_0 \leq \frac{3t}{2N}, \\
\text{we have } \frac{a}{b} - c \leq \frac{3t}{2N} \text{ for any } c \geq c_0. \text{ Since } \frac{a}{b} - c \leq \frac{3t}{2N},
\]

\[
\Pi_M = (c - c_0)Q_T^* = \frac{b}{2}(c - c_0)\left[\frac{a}{b} - c - \frac{2t}{N} + \sqrt{(\frac{a}{b} - c - \frac{t}{2N})^2 + \frac{3t^2}{N^2}}\right]
\leq \frac{b}{2}(c - c_0)\left[\frac{a}{b} - c - \frac{2t}{N} + \frac{3t^2}{N^2}\right] = \frac{b}{2}(c - c_0)(\frac{a}{b} - c) \leq \frac{b}{8}(\frac{a}{b} - c_0)^2 = \frac{1}{2} \tilde{\Pi}_M
\]

Therefore, \( \frac{\Pi_M^*}{\Pi_M} \leq \frac{1}{2} \).

**Case 2.** \( \frac{a}{b} - c_0 > \frac{3t}{2N} \): From Result 2, the manufacturer’s profit is maximized by retailer local competition here, and we only consider \( c \leq \frac{a}{b} - \frac{3t}{4N} \), that is, \( \frac{a}{b} - c \geq \frac{3t}{4N} \). For a given \( N \), suppose wholesale price \( c^* \) maximizes the manufacturer’s profit. If \( c \) is kept fixed at \( c^* \) and \( N \) increases, the condition for local competition is maintained and \( Q_T^* \) increases (Proposition 5), that is, \( \Pi_M^* \) increases.

**Bounds on** \( \frac{\Pi_M^*}{\Pi_M} \): Since \( c + \frac{t}{4N} \leq P^* < c + \frac{t}{N} \),

\[
f_1(c) = b(c - c_0)(\frac{a}{b} - c - \frac{5t}{4N}) < \Pi_M = b(c - c_0)(\frac{a}{b} - P^* - \frac{t}{4N}) \leq b(c - c_0)(\frac{a}{b} - c - \frac{t}{2N}) = f_2(c)
\]

Since \( \frac{a}{b} - c_0 > \frac{3t}{2N} > \frac{5t}{4N} \), \( f_1(c) \) is maximized at \( \tilde{c} = \frac{1}{2}(\frac{a}{b} + c_0 + \frac{5t}{4N}) > c_0 \), and the maximum is \( f_1(\tilde{c}) = b\left(\frac{a}{b} - c_0 - \frac{5t}{4N}\right)^2 \). Also, since \( \frac{a}{b} - c_0 > \frac{3t}{2N} \), \( \frac{a}{b} - \tilde{c} = \frac{1}{2}(\frac{a}{b} + c_0 + \frac{5t}{4N}) > \frac{3t}{4N} \), that is, we have local retailer competition if \( c = \tilde{c} \). Hence,

\[
\Pi_M^* \geq \Pi_M(\tilde{c}) > f_1(\tilde{c}) = \frac{b}{4}\left(\frac{a}{b} - c_0 - \frac{5t}{4N}\right)^2 \Rightarrow \frac{\Pi_M^*}{\Pi_M} > \left\{1 - \frac{5t}{4N(\frac{a}{b} - c_0)}\right\}^2
\]

Since \( \frac{a}{b} - c_0 > \frac{3t}{2N} > \frac{t}{2N} \), \( f_2(c) \) is maximized at \( \overline{c} = \frac{1}{2}(\frac{a}{b} + c_0 - \frac{t}{2N}) > c_0 \), and the maximum is \( f_2(\overline{c}) = \frac{b}{4}\left(\frac{a}{b} - c_0 - \frac{t}{2N}\right)^2 \). Hence,

\[
\Pi_M^* \leq \frac{b}{4}\left(\frac{a}{b} - c_0 - \frac{t}{2N}\right)^2 \Rightarrow \frac{\Pi_M^*}{\Pi_M} \leq \left\{1 - \frac{t}{2N(\frac{a}{b} - c_0)}\right\}^2
\]
Technical Appendix B

Note: If the manuscript is accepted for publication, Technical Appendix B will be available as an online supplement.

Appendix B1

Properties of $P_{rl}$: $c < P_{rl} < \frac{a}{b}$, and $\frac{\partial^2 \Pi_r}{\partial P_r^2} < 0$ if $P_r = P_{rl}$.

Proof: From (32),

$$\frac{a}{b} - P_{rl} = \frac{1}{2}\left[\left(\frac{a}{b} - c - tx_m\right) + \sqrt{\left(\frac{a}{b} - c - tx\right)^2 + 2t^2x_m^2}\right] > 0,$$

since $\left(\frac{a}{b} - c\right) \geq \frac{3t}{4N} \geq \frac{3tx_m}{2} > tx_m$.

$$P_{rl} - c = \frac{1}{2}\left[\left(\frac{a}{b} - c + tx_m\right) - \sqrt{\left(\frac{a}{b} - c - tx_m\right)^2 + 2t^2x_m^2}\right]$$

$$\geq \frac{1}{2}\left[\left(\frac{a}{b} - c + tx_m\right) - \{\left(\frac{a}{b} - c - tx_m\right) + tx_m\sqrt{2}\}\right] = \frac{tx_m}{2}(2 - \sqrt{2}) > 0,$$

since $\left(\frac{a}{b} - c - tx_m\right) > 0$, and $tx_m\sqrt{2} \geq 0$.

From (30),

(B1) $\frac{\partial \Pi_r}{\partial P_r} = \frac{2b}{t}\left[(P_r - c)^2 - \left(\frac{a}{b} - c + tx_m\right)(P_r - c) + tx_m\left(\frac{a}{b} - c - \frac{tx_m}{2}\right)\right]$

Since $\frac{\partial x_m}{\partial P_r} = -\frac{1}{t}$, $\frac{\partial}{\partial P_r}\left(\frac{a}{b} - c + tx_m\right) = t(-\frac{1}{t}) = -1$, and $\frac{\partial}{\partial P_r}\left(\frac{a}{b} - c - \frac{tx_m}{2}\right) = \frac{1}{2}$,

(B2) $\frac{\partial^2 \Pi_r}{\partial P_r^2} = \frac{2b}{t}\left[2(P_r - c) - (-1)(P_r - c) - \left(\frac{a}{b} - c + tx_m\right) - \left(\frac{a}{b} - c - \frac{tx_m}{2}\right) + \frac{tx_m}{2}\right]$

$$= \frac{2b}{t}\left[3(P_r - c) - 2\left(\frac{a}{b} - c\right)\right]$$

Substituting $P_r = P_{rl}$,

$$\frac{\partial^2 \Pi_r}{\partial P_r^2} = \frac{2b}{t}\left[3\left(\frac{a}{b} - c + tx_m\right) - \sqrt{\left(\frac{a}{b} - c - tx_m\right)^2 + 2t^2x_m^2}\right] - 2\left(\frac{a}{b} - c\right)$$

$$= \frac{2b}{t}\left[3tx_m - \frac{1}{2}\left(\frac{a}{b} - c\right) - \frac{3}{2}\sqrt{\left(\frac{a}{b} - c - tx_m\right)^2 + 2t^2x_m^2}\right] < 0$$

Properties of $P_{dl}$: $c + T < P_{dl} < \frac{a}{b} - \mu$, and $\frac{\partial^2 \Pi_d}{\partial P_d^2} < 0$ if $P_d = P_{dl}$.

Proof: From (36),

$$\left(\frac{a}{b} - \mu\right) - P_{dl} = \frac{1}{2}\left(\frac{a}{b} - \mu - c - T\right) + \frac{1}{2}\left[\left(\frac{a}{b} - \mu - c - T\right)^2 + \left(\frac{a}{b} - \mu - C - T\right)^2 + 2t\left(\frac{1}{2N} - x_m\right)^2 - 2t\left(\frac{1}{2N} - x_m\right)^2\right]$$

$$> \frac{1}{2}\left(\frac{a}{b} - \mu - c - T\right) > 0 \text{ since } \left(\frac{a}{b} - \mu\right) > c + T.$$
since \((\frac{a}{b} - \mu - c - T) > 0\), and \(2t(\frac{1}{2N} - x_m) > 0\).

Since \(\frac{\partial x_m}{\partial P_d} = \frac{1}{t}\), we have, from (34),

\[
(B3) \quad \frac{\partial^2 \Pi_d}{\partial P_d^2} = \frac{2bN}{t} \left[2(P_d - c - T) - (\frac{a}{b} - \mu - c - T) - 2t(\frac{1}{2N} - x_m) + 2(P_d - c - T) - (\frac{a}{b} - \mu - c - T)\right]
\]

\[
= \frac{2bN}{t} \left[4(P_d - c - T) - 2(\frac{a}{b} - \mu - c - T) - 2t(\frac{1}{2N} - x_m)\right]
\]

If \(P_d = P_d^*\),

\[
(B4) \quad \frac{\partial^2 \Pi_d}{\partial P_d^2} = \frac{2bN}{t} \left[2t(\frac{1}{2N} - x_m) - 2\sqrt{(\frac{a}{b} - \mu - c - T)^2 + \{2t(\frac{1}{2N} - x_m)\}^2}\right] < 0
\]

**Derivations of bounds in Proposition 6:** We already established that \(c < P_r^* < \frac{a}{b}\), and \((c + T) < P_d^* < (\frac{b}{N} - \mu)\). Rewriting (37) and (38),

\[(B5) \quad P_r^* = (c + tx_m) - \frac{1}{2} \left[\sqrt{(\frac{a}{b} - c - tx_m)^2 + 2t^2x_m^2} \right] < c + tx_m < c + \frac{t}{2N},\]

and

\[(B6) \quad P_d^* = [(c + T) + t(\frac{1}{2N} - x_m)] - \frac{1}{2} \left[\sqrt{(\frac{a}{b} - \mu - c - T)^2 + \{2t(\frac{1}{2N} - x_m)\}^2} - (\frac{a}{b} - \mu - c - T)\right]
\]

\[
< (c + T) + t(\frac{1}{2N} - x_m) < (c + T) + \frac{t}{2N},
\]

since \(x_m < \frac{1}{2N}\). Since \(c < P_r^* < c + tx_m\), and \(c + T < P_d^* < c + T + t(\frac{1}{2N} - x_m)\),

\[
0 < (P_r^* - c) + (P_d^* - c - T) < tx_m + t(\frac{1}{2N} - x_m) = \frac{t}{2N}
\]

**Appendix B2**

\(\phi(x)\) is a strictly decreasing function of \(x\).

**Proof:** From (39),

\[
\phi(x) = \frac{1}{2}(\mu + T + \frac{t}{N}) - \frac{5tx}{2} - \frac{1}{2} \sqrt{(\frac{a}{b} - \mu - c - T)^2 + \{2t(\frac{1}{2N} - x)\}^2} + \frac{1}{2} \sqrt{(\frac{a}{b} - c - tx)^2 + 2t^2x^2}
\]

Differentiating,

\[
(B7) \quad \phi'(x) = -\frac{5t}{2} - \frac{1}{2} \frac{\partial}{\partial x} \sqrt{(a/b - \mu - c - T)^2 + \{2t(1/2N - x)\}^2} + \frac{1}{2} \frac{\partial}{\partial x} \sqrt{(a/b - c - tx)^2 + 2t^2x^2}
\]

\[
= -\frac{5t}{2} - \frac{K_1}{2} + \frac{K_2}{2}
\]

where

\[
(B8) \quad K_1 = \frac{\partial}{\partial x} \sqrt{(a/b - \mu - c - T)^2 + \{2t(1/2N - x)\}^2} = -\frac{4t^2(1/2N - x)}{\sqrt{(a/b - \mu - c - T)^2 + \{2t(1/2N - x)\}^2}},
\]

and
(B9) \( K_2 = \frac{\partial}{\partial x} \sqrt{\frac{a}{b} - (c - tx)^2 + 2t^2x^2} = \frac{1}{2} \frac{4t^2x - 2t(\frac{a}{b} - c - tx)}{\left(\frac{a}{b} - (c - tx)^2 + 2t^2x^2\right)^{3/2}} \)

Since \( |K_1| \leq \frac{4t^2(\frac{a}{b} - x)}{2t(\frac{a}{b} - x)^2} = 2t \), and \( K_2 \leq \frac{2t^2x}{\sqrt{2t^2x^2}} = \sqrt{2} t \),

\( \phi'(x) \leq -\frac{5t}{2} + \frac{|K_1|}{2} + \frac{K_2}{2} \leq -\frac{5t}{2} + t + \frac{t}{\sqrt{2}} < 0 \)

Appendix B3

Proof of Proposition 7: From (39),

(B10) \( \phi(x) = \frac{1}{2} (\mu + T + \frac{t}{N} - \frac{5tx}{2}) - \frac{1}{2} \sqrt{\frac{a}{b} - \frac{\mu - c - T}{2N} + \left\{ \frac{2t(\frac{1}{2N} - x)}{2} \right\}^2 + \frac{1}{2} \sqrt{\frac{a}{b} - (c - tx)^2 + 2t^2x^2} \)

Online Retailer: If \( \phi\left(\frac{1}{2N}\right) \geq 0 \), the solution of \( \phi(x) = 0 \) occurs at \( x \geq \frac{1}{2N} \), that is, the online marketer does not have any market share. From (B9), this condition can be stated as

\( \phi\left(\frac{1}{2N}\right) = \frac{1}{2} (\mu + T + \frac{t}{N} - \frac{5tx}{2}) - \frac{1}{2} \sqrt{\frac{a}{b} - \frac{\mu - c - T}{2N} + \left\{ \frac{2t(\frac{1}{2N} - \frac{1}{2N})}{2} \right\}^2 + \frac{1}{2} \sqrt{\frac{a}{b} - (c - tx)^2 + 2t^2x^2} \)

that is,

(B11) \( \mu + T \geq \theta_d = \frac{t}{N} - \frac{1}{2} \sqrt{\frac{a}{b} - c - \frac{t}{2N} + \frac{t^2}{2N^2} - \left( \frac{a}{b} - c - \frac{t}{2N} \right)} \)

Properties of \( \theta_d \)

(1) Since \( \left( \frac{a}{b} - c - \frac{t}{2N} \right) \geq \left( \frac{3t}{4N} - \frac{t}{2N} \right) > 0 \)

\( \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{t^2}{2N^2} < \left( \frac{a}{b} - c - \frac{t}{2N} \right) + \sqrt{\frac{t^2}{2N^2} = (a/b - c - t/2N) + \frac{1}{2} \sqrt{t/N} \). \)

Hence,

\( \theta_d = \frac{t}{N} - \frac{1}{2} \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{t^2}{2N^2} - \left( \frac{a}{b} - c - \frac{t}{2N} \right) + \frac{1}{2} \sqrt{t/N}} > \frac{t}{N} - \frac{1}{2} \sqrt{\frac{t}{N}} > 0 \)

(2) \( \frac{\partial \theta_d}{\partial (a/b - c)} = \frac{1}{2} - \frac{1}{2} \left( \frac{1}{\sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{t^2}{2N^2}} \right) > 0 \), that is, \( \theta_d \) strictly increases with \( (a/b - c) \).

(3) \( \frac{\partial \theta_d}{\partial (t/N)} = \frac{3}{4} - \frac{1}{4} \left( \frac{1}{\sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{t^2}{2N^2}} \right) \geq \frac{3}{4} - \frac{1}{4} \left( \frac{1}{\sqrt{\left( \frac{3t}{4N} - \frac{t}{2N} \right)^2 + \frac{t^2}{2N^2}} \right) \]

\( = \frac{3}{4} - \frac{1}{4} \left( \frac{3t/4N}{\sqrt{\frac{3t}{4N} - c - \frac{t}{2N} + \frac{t^2}{2N^2}} \right) \geq \frac{3}{4} - \frac{1}{4} \left( \frac{3t/4N}{\sqrt{\frac{3t}{4N} - \frac{t}{2N} + \frac{t^2}{2N^2}} \right) \]

\( = \frac{3}{4} - \frac{1}{4} \left( \frac{3t/4N}{\sqrt{\left( \frac{3t}{4N} - \frac{t}{2N} \right)^2 + \frac{t^2}{2N^2}} \right) \geq \frac{3}{4} - \frac{1}{4} \left( \frac{3t/4N}{\sqrt{\frac{3t}{4N} - \frac{t}{2N} + \frac{t^2}{2N^2}} \right) \]

since we assume that \( (a/b - c) \geq \frac{3t}{4N} \). Therefore, \( \theta_d \) is a strictly increasing function of \( t/N \).

(4) It follows directly from (B11) that \( \theta_d \rightarrow \frac{t}{N} \) when \( (a/b - c) \gg \frac{t}{N} \) and that \( \theta_d = \frac{3t}{4N} \) when
\[
\left(\frac{a}{b} - c\right) = \frac{3t}{4N}.
\]

**Local Retailer:** If \( \phi(0) \leq 0 \), the local retailer has no market share. From (B10),

\[
\phi(0) = \frac{1}{2}(\mu + T + \frac{t}{N}) - \frac{1}{2}\sqrt{\left(\frac{a}{b} - c - T\right)^2 + \frac{t^2}{N^2} + \frac{1}{2}\left(\frac{a}{b} - c\right)}
\]

Hence, \( \phi(0) \leq 0 \) if and only if

(B12) \( \xi(z) = z - \sqrt{\left(\frac{a}{b} - c - z\right)^2 + \frac{t^2}{N^2} + \left(\frac{a}{b} - c + \frac{t}{N}\right)} \leq 0, \)

where \( z = \mu + T \). From (B12),

\[
\frac{\partial \xi}{\partial z} = 1 + \frac{\frac{a}{b} - c - z}{\sqrt{\left(\frac{a}{b} - c - z\right)^2 + \frac{t^2}{N^2}}} > 0 \text{ since we assume that } \left(\frac{a}{b} - c\right) > z = \mu + T. \]

Also, \( \xi(z) < 0 \) if \( z = -\left(\frac{a}{b} - c + \frac{t}{N}\right) \), and \( \xi(z) > 0 \) as \( z \) approaches \( \left(\frac{a}{b} - c\right) \). Hence, \( \xi(z) = 0 \) has a unique solution \( \theta_r \), and \( \phi(0) \leq 0 \) if and only if \( z = \mu + T \leq \theta_r \).

**Properties of \( \theta_r \)**

(1) If \( z \geq 0, \xi(z) \geq \xi(0) = \left(\frac{a}{b} - c\right) + \frac{t}{N} - \sqrt{\left(\frac{a}{b} - c\right)^2 + \frac{t^2}{N^2}} > 0 \text{ since } \left(\frac{a}{b} - c\right) > 0 \text{ and } \frac{t}{N} > 0. \)

Therefore, \( \theta_r < 0 \).

(2) \( \frac{\partial \xi}{\partial (t/N)} = 1 - \frac{(t/N)}{\sqrt{\left(\frac{a}{b} - c - z\right)^2 + \frac{t^2}{N^2}}} > 0, \) that is, \( \frac{\partial \theta_r}{\partial (t/N)} = -\frac{\partial \xi}{\partial \xi} < 0, \)

that is, \( \theta_r \) is strictly decreasing in \( \frac{t}{N} \). Similarly, \( \theta_r \) is strictly decreasing in \( \left(\frac{a}{b} - c\right) \).

(3) If \( \left(\frac{a}{b} - c\right) = \frac{3t}{4N}, \) we find numerically that \( \theta_r = -\frac{0.3t}{N}. \) Noting that \( \theta_r < 0 \), it follows from (B11) that as \( \left(\frac{a}{b} - c\right) \gg \frac{t}{N}, \)

\[
0 = \xi(\theta_r) \approx \theta_r - \left(\frac{a}{b} - c - \theta_r\right) + \left(\frac{a}{b} - c + \frac{t}{N}\right) = 2\theta_r + \frac{t}{N}, \]

that is, \( \theta_r \approx -\frac{t}{2N}. \)

**Habitable Zone:** If \( \theta_r \leq \mu + T \leq \theta_d \), then \( \phi(0) > 0 \) and \( \phi\left(\frac{1}{2N}\right) < 0, \) that is, \( 0 < x^* < \frac{1}{2N}. \)

Then, both the local and online retailers have non-zero market shares.

**Appendix B4**

**Proof of Proposition 8:** From (39), \( x^* \) is the unique solution of \( \phi(x) = 0, \) where

(B13) \( \phi(x) = \frac{1}{2}(\mu + T + \frac{t}{N}) - \frac{5tx}{2} - \frac{1}{2}\sqrt{\left(\frac{a}{b} - \mu - c - T\right)^2 + \{2t\left(\frac{1}{2N} - x\right)\}^2} + \frac{1}{2}\sqrt{\left(\frac{a}{b} - c - tx\right)^2 + 2t^2x^2} \)

For any model parameter \( \lambda, \frac{dx^*}{d\lambda} = -\frac{\partial \phi}{\partial x} \) has the same sign as \( \frac{\partial \phi}{\partial \lambda} \) since \( \frac{\partial \phi}{\partial x} < 0. \)

\[
\frac{\partial \phi}{\partial (\mu + T)} = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\left(\frac{a}{b} - \mu - c - T\right)^2 + \{2t\left(\frac{1}{2N} - x\right)\}^2}{\left(\frac{a}{b} - c - T\right)^2 + \{2t\left(\frac{1}{2N} - x\right)\}^2}} > 0 \rightarrow \frac{\partial x^*}{\partial (\mu + T)} > 0
\]

The limits of \( x^* \) follow from the definitions of \( \theta_r \) and \( \theta_d. \)
\[ \frac{\partial \phi}{\partial N} = \frac{t}{2N^2} \left[ 1 - \frac{2t(\frac{1}{2N} - x)}{\sqrt{(\frac{a}{b} - \mu - c - T)^2 + \{2t(\frac{1}{2N} - x)\}^2}} \right] < 0 \rightarrow \frac{\partial x^*}{\partial N} < 0 \]

**Effect of \((\frac{a}{b} - c)\):** Denoting \((\frac{a}{b} - c)\) by \(v\),

\[ \phi = \frac{1}{2}(\mu + T + \frac{t}{N}) - \frac{5tx}{2} - \frac{1}{2}\sqrt{(v - \mu - T)^2 + \{2t(\frac{1}{2N} - x)\}^2} + \frac{1}{2}\sqrt{(v - tx)^2 + 2tx^2} \]

\[ \frac{\partial \phi}{\partial v} = \frac{v - tx}{\sqrt{(v - tx)^2 + 2tx^2}} - \frac{v - \mu - T}{\sqrt{(v - \mu - T)^2 + \{2t(\frac{1}{2N} - x)\}^2}} \]

Since, by model assumptions, \((\frac{a}{b} - \mu - c - T) > 0\) and \((\frac{a}{b} - c - tx) \geq (\frac{a}{b} - c - \frac{t}{2N}) > 0\), \(\frac{\partial \phi}{\partial v} > 0\) if and only if \(\frac{v - tx}{\sqrt{(v - tx)^2 + 2tx^2}} > \frac{v - \mu - T}{\sqrt{(v - \mu - T)^2 + \{2t(\frac{1}{2N} - x)\}^2}}\)

if and only if \((v - tx)^2[(v - \mu - T)^2 + \{2t(\frac{1}{2N} - x)\}^2] > (v - \mu - T)^2[(v - tx)^2 + 2tx^2]\)

if and only if

\[ (B14) \sqrt{2}(v - tx)(\frac{1}{2N} - x) > x(v - \mu - T) \]

The LHS of (B14) is greater than zero at \(x = 0\), and it strictly decreases to 0 as \(x\) increases to \(\frac{1}{2N}\). The RHS of (B14) is zero at \(x = 0\) and is strictly increases with \(x\). Hence, there is a unique \(\hat{x} \in (0, \frac{1}{2N})\) where the LHS and the RHS are equal, and the LHS is greater than the RHS when \(x < \hat{x}\). Thus, \(\frac{\partial x^*}{\partial v} > 0\) if \(x < \hat{x}\). Similarly, \(\frac{\partial x^*}{\partial v} < 0\) if \(x > \hat{x}\).

**Effect of \(t\):** From (B13),

\[ (B15) \frac{\partial \phi}{\partial t} = \frac{1}{2N} - \frac{5x}{2} - \frac{2t(N - x)}{\sqrt{(\frac{a}{b} - \mu - c - T)^2 + \{2t\frac{1}{2N} - x\}^2}} + \frac{tx^2 - \frac{5}{2}(\frac{a}{b} - c - tx)}{\sqrt{(\frac{a}{b} - c - tx)^2 + 2tx^2}} \]

If \(x = 0\), the RHS of (B15) is

\[ \frac{1}{2N} - \frac{(t/2N^2)}{\sqrt{(\frac{a}{b} - \mu - c - T)^2 + \frac{t^2}{N^2}}} > \frac{1}{2N} - \frac{(t/2N^2)}{(t/N)} = 0 \]

By continuity, when \(x^*\) is small, \(\frac{\partial x^*}{\partial t} > 0\).

If \(x = \frac{1}{2N}\), the RHS of (B15) is

\[ \frac{1}{2N} - \frac{5}{4N} + \frac{\frac{t}{N^2} - \left(\frac{1}{2N}\right)(\frac{a}{b} - c - \frac{t}{2N})}{\sqrt{(\frac{a}{b} - c - \frac{t}{2N})^2 + \frac{t^2}{N^2}}} \leq \frac{1}{2N} - \frac{5}{4N} + \frac{\frac{t}{N^2} - \left(\frac{1}{2N}\right)(\frac{3t}{4N} - \frac{t}{2N})}{\sqrt{(\frac{a}{b} - c - \frac{3t}{4N})^2 + \frac{t^2}{N^2}}} \]

\[ = \frac{1}{2N} - \frac{5}{4N} + \frac{\frac{3t}{4N^2}}{\sqrt{(\frac{a}{b} - c - \frac{t}{2N})^2 + \frac{t^2}{N^2}}} \leq \frac{1}{2N} - \frac{5}{4N} + \frac{\frac{3t}{4N^2}}{\sqrt{(\frac{3t}{4N} - \frac{t}{2N})^2 + \frac{t^2}{N^2}}} = \frac{1}{2N} - \frac{5}{4N} + \frac{1}{4N} < 0, \]

since \((\frac{a}{b} - c) \geq \frac{3t}{4N}\). By continuity, \(\frac{\partial x^*}{\partial t} < 0\) when \(x^*\) is close to \(\frac{1}{2N}\).
Appendix B5: Proof of Proposition 9

1. Effects of \((\mu + T)\)

**Effect on \(P^*_r\):** From (37),

\[
\frac{\partial P^*_r}{\partial (\mu + T)} = \frac{\partial P^*_x}{\partial (\mu + T)} = \frac{\partial P^*_x}{\partial x} \frac{\partial x}{\partial (\mu + T)} = \frac{\partial P^*_x}{\partial x} \frac{\partial x}{\partial (\mu + T)}
\]

\[
\frac{\partial P^*_r}{\partial x_m |_{\mu + T}} = \frac{1}{2} \left[ t - \frac{2t^2 x_m - t\left( \frac{a}{b} - c - t x_m \right)}{\sqrt{\left( \frac{a}{b} - c - t x_m \right)^2 + 2t^2 x_m^2}} \right] > \frac{1}{2} \left[ t - \frac{2t^2 x - t\left( \frac{a}{b} - c - t x \right)}{\sqrt{\left( \frac{a}{b} - c - t x \right)^2 + 2t^2 x^2}} \right]
\]

since \(\frac{a}{b} - c \geq \frac{3t}{4N} > \frac{3tx_m}{2}\). Therefore, \(\frac{\partial P^*_r}{\partial (\mu + T)} > 0\) since \(\frac{\partial x_m}{\partial (\mu + T)} > 0\).

**Effect on \(P^*_d - c - T\):** From (38),

\[
\frac{\partial}{\partial (\mu + T)} (P^*_d - c - T) |_{x_m} = \frac{1}{2} \left[ \frac{\left( \frac{a}{b} - c - \mu - T \right)}{\sqrt{\left( \frac{a}{b} - c - \mu - T \right)^2 + 2t^2 x_m^2}} \right] - 1 < 0
\]

Similarly, \(\frac{\partial}{\partial x_m} (P^*_d - c - T) |_{\mu + T} < 0\). Therefore,

\[
\frac{\partial}{\partial (\mu + T)} (P^*_d - c - T) |_{x_m} + \frac{\partial}{\partial x_m} (P^*_d - c - T) |_{(\mu + T)} \frac{\partial x_m}{\partial (\mu + T)} < 0,
\]

since \(\frac{\partial x_m}{\partial (\mu + T)} > 0\).

**Limit as \((\mu + T) \to \theta_r\):** As \((\mu + T) \to \theta_r, x^* \to 0\), that is, \(c + tx^* \to c\). Since \(c < P^*_r < c + tx^*\), \(P^*_r \to c\). Also, denoting demand for the local retailer by \(Q^*_r\), \(\Pi^*_r = (P^*_r - c)Q^*_r \to 0\) since \(P^*_r \to c\), and \(Q^*_r \leq ax^* \to 0\). Finally, from (28), \(P^*_d = P^*_r + tx^* - \mu \to c - \mu = (c + T) - (\mu + T) = (c + T) - \theta_r\).

**Limit as \((\mu + T) \to \theta_d\):** As \((\mu + T) \to \theta_d, x^* \to \frac{1}{2N}\), that is, \((c + T) + t\left( \frac{1}{2N} - x^* \right) \to (c + T)\). Therefore, \(P^*_d \to (c + T)\) and \(P^*_d + tx^* \to c + T + \mu = c + \theta_d\), that is, \(P^*_r \to P^*_\text{max} = c + \theta_d - \frac{t}{2N}\).

Denoting the demand for the online retailer by \(Q^*_d\), \(\Pi^*_d = (P^*_d - c - T)Q^*_d \to 0\) since \(P^*_d \to (c + T)\), and \(Q^*_d \leq a(1 - 2Nx^*) \to 0\).

**Results on effective price:** Suppose currently the local retailer serves up to \(x^*_1\) on each side with price at origin \(P^*_{r1}\). With an increase in \((\mu + T)\), the local retailer serves up to \(x^*_2 > x^*_1\) with price at origin \(P^*_{r2} > P^*_{r1}\) as both \(P^*_r\) and \(x^*\) increase.

- For \(0 < x \leq x^*_1\), the new effective price = \(P^*_{r2} + tx > P^*_{r1} + tx\), the old effective price.
• For \( x_1^* < x \leq x_2^* \), the new effective price is \( P_{r_2}^* + tx = (P_{r_1}^* + tx_1^*) + (P_{r_2}^* - P_{r_1}^*) + t(x - x_1^*) > P_{r_1}^* + tx_1^* \), the old effective price.

• For \( x > x_2^* \), the new effective price is \( P_{r_2}^* + tx_2^* = (P_{r_1}^* + tx_1^*) + (P_{r_2}^* - P_{r_1}^*) + t(x_2^* - x_1^*) > P_{r_1}^* + tx_1^* \), the old effective price.

Therefore, effective price at any given location is a strictly increasing function of \((\mu + T)\).

As \((\mu + T) \to \theta_d\), \( P_r^* \to c \) and \( x^* \to 0 \), that is, \( P_d^* + \mu = P_r^* + tx^* \to c \). Thus, effective price at all locations approach \( c \).

As \((\mu + T) \to \theta_d\), \( P_d^* \to c + T \), that is, \( P_d^* + \mu \to c + (\mu + T) = c + \theta_d \), \( x^* \to \frac{1}{2N} \), and \( P_r^* + tx^* = P_d^* + \mu \to c + \theta_d \). Therefore, \( P_r^* \to c + \theta_d - \frac{t}{2N} \). Combining, effective price increases with \( x \) from \( c + \theta_d - \frac{t}{2N} \) at \( x = 0 \) to \( (c + \theta_d) \) at \( x = \frac{1}{2N} \).

2. Effect of \( \frac{a}{b} \): Since \( \theta_d \) strictly increases with \( \frac{a}{b} \), \( P_{r_{\max}}^* = (c + \theta_d - \frac{t}{2N}) \) is also strictly increasing in \( \frac{a}{b} \). Since \( \theta_d = \frac{3t}{4N} \) when \( \frac{a}{b} - c = \frac{3t}{4N} \) and \( \theta_d \to \frac{t}{N} \) when \( \frac{a}{b} - c \gg \frac{t}{N} \), \( P_{r_{\max}}^* \) increases from \((c + \frac{t}{2N})\) to the limit of \((c + \frac{t}{2N})\) as \((\frac{a}{b} - c)\) increases from \(\frac{3t}{4N}\).

Comparison of \( P^* \) and \( P_{r_{\max}}^* \): From (B11),

\[ P_{r_{\max}}^* = c + \theta_d - \frac{t}{2N} = \frac{1}{2} \left[ \left( \frac{a}{b} + c + \frac{t}{2N} \right) - \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{t^2}{N^2}} \right] \]

From (15),

\[ P^* = \frac{1}{2} \left( \frac{a}{b} + c + \frac{3t}{2N} \right) - \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \]

Combining,

\[ P^* - P_{r_{\max}}^* = \frac{1}{2} \left[ \frac{t}{N} + \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{t^2}{2N^2}} - \sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}} \right] \]

\[ \frac{\partial}{\partial (a/b)} (P^* - P_{r_{\max}}^*) = \frac{1}{2} \left[ \frac{a}{b} - c - \frac{t}{2N} \right] - \frac{a}{b} - c - \frac{t}{2N} \overline{\sqrt{\left( \frac{a}{b} - c - \frac{t}{2N} \right)^2 + \frac{3t^2}{N^2}}} > 0 \]

From (B20), \((P^* - P_{r_{\max}}^*) = 0\) when \(\frac{a}{b} - c = \frac{3t}{4N}\), \((P^* - P_{r_{\max}}^*) \to \frac{t}{2N}\) when \(\frac{a}{b} - c \gg \frac{t}{2N}\).

3. Since \( \mu + T < \theta_d \), \( P_r^* < P_{r_{\max}}^* \leq P^* \).

• If \( 0 \leq x \leq x^* \), effective price with online retailer is \( P_r^* + tx < P^* + tx = \) effective price without online retailer.

• If \( x > x^* \), effective price with online retailer is \( P_r^* + tx^* < P^* + tx = \) effective price without online retailer.
C = 0

Margin Ratio = Margin for Proposed Model / Margin for SSC

Demand Ratio = Demand for Proposed Model / Demand for SSC

Profit Ratio = Profit for Proposed Model / Profit for SSC

Number Ratio = Number of stores in equilibrium for Proposed Model / Number for SSC
Figure 2
Profit Comparison with Spatial Monopoly

Plot for $B = 1$, $T = 1$, and $C = 0$
Figure 3
Fraction of Profit Potential Achieved versus Number of Retailers

$T = 10$
$C_0 = 0$

PIR1-PIR100 correspond to $A/B = 1.0, 5.0, 10.0, 20.0, 50.0,$ and $100.0$
$T = 10$

$C_0 = 0$

CR1-CR100 correspond to $A/B = 1.0, 5.0, 10.0, 20.0, 50.0, \text{ and } 100.0$
$C_0=0$, $B = 1.0$, $T = 10$

Administration Cost = ETA*Profit Potential*Number of Retailers where ETA=.001, .002, .003, and .004
Figure 6
Demand Increase due to Online Retailer

Legend

Y Axis: (Demand with Online Retailer – Demand without Online Retailer)/Demand without online retailer
C = 0

Thetar $\Rightarrow$ $\mu + T = \theta_r$, .5*Thetar $\Rightarrow$ $\mu + T = .5 \theta_r$, Zero $\Rightarrow$ $\mu + T = 0$

.5*Thetad $\Rightarrow$ $\mu + T = \theta_d$, Thetad $\Rightarrow$ $\mu + T = \theta_d$