Online Supplement A. Derivation of the distribution of observed earnings in model (1)–(3)

Let $m \in \{0,1\}$ be the earnings management decision ($m = 0$ when earnings are reported "as is" and $m = 1$ when earnings are managed to report a small profit). After integrating out unobservable $EARN^*$ and m, the probability density function of reported earnings is

$$f(EARN|X) = \int_{EARN^*} \sum_{m=0,1} f(EARN|EARN^*, m, X) \Pr(m|EARN^*, X)\, f^*(EARN^*|X) dEARN^* \quad (A1)$$

where $f(EARN|EARN^*, m, X)$ is the density of reported earnings conditional on pre-managed earnings $EARN^*$, the earnings management decision m, and explanatory variables X.

We simplify (A1) for three scenarios on $EARN$.

Case 1: $EARN < -K^-$ or $EARN \geq K^+$, i.e., a large loss or a large profit. The conditional density $f(EARN|EARN^*, m, X)$ is non-zero only for $EARN^* = EARN$ and $m = 0$.[1] Equation (A1) simplifies to

$$f(EARN|X) = \Pr(m = 0|EARN^* = EARN, X)\, f^*(EARN^* = EARN|X) \quad (A2)$$

where $\Pr(m = 0|EARN^* = EARN, X) = 1 - P(EARN^* = EARN, X) = 1$ because $EARN \notin [-K^-, 0)$. Therefore, (A2) simplifies to equation (4a) in Section 2

$$f(EARN|X) = f^*(EARN^* = EARN|X) \quad (4a)$$

Case 2: $EARN \in [-K^-, 0)$, i.e., a small loss. The conditional density $f(EARN|EARN^*, m, X)$ is non-zero only for $EARN^* = EARN$ and $m = 0$.[2] Therefore, (A1) simplifies to (A2), similar to Case 1. For $EARN \in [-K^-, 0)$, $\Pr(m = 0|EARN^* = EARN, X) = 1 - P(EARN^* = EARN, X)$. Therefore, (A2) simplifies to equation (4b) in Section 2

$$f(EARN|X) = [1 - P(EARN^* = EARN, X)] f^*(EARN^* = EARN|X) \quad (4b)$$

Case 3: $EARN \in [0, K^+)$, i.e., a small profit. The conditional density $f(EARN|EARN^*, m, X)$ is non-zero in two situations: (a) $m = 0$ and $EARN^* = EARN$, and (b) $m = 1$ and $EARN^* \in [-K^-, 0)$. Therefore, (A.1) becomes

$$f(EARN|X) = \Pr(m = 0|EARN^* = EARN, X)\, f^*(EARN^* = EARN|X) +$$
$$\int_{-K^-}^{0} f(EARN|EARN^*, m = 1, X) \Pr(m = 1|EARN^*, X)\, f^*(EARN^*|X) dEARN^* \quad (A3)$$

In the first line of (A3), $\Pr(m = 0|EARN^* = EARN, X) = 1$ because $EARN \notin [-K^-, 0)$. In the second line of (A3), $f(EARN|EARN^*, m = 1, X) = g(EARN|EARN^*, X)$ per (2b) and $\Pr(m = 1|EARN^*, X) = P(EARN^*, X)$. Therefore, (A3) simplifies to equation (4c) in Section 2

$$f(EARN|X) = f^*(EARN^* = EARN|X) + G(EARN, X) \quad (4c)$$

where

$$G(EARN, X) = \int_{-K^-}^{0} g(EARN|EARN^*, X) P(EARN^*, X) f^*(EARN^*|X) dEARN^* \quad (A4)$$

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[1] When $m = 1$ (i.e., earnings are managed to report a small profit), the probability of reporting a large loss or a large profit is zero. When $m = 0$ (i.e., no earnings management), reported $EARN$ must equal the pre-managed $EARN^*$. Because $f(EARN|EARN^*, m, X)$ collapses to a mass point with weight 1, it drops out of the computation.

[2] When $m = 1$, the probability of reporting a small loss is zero. When $m = 0$, reported $EARN$ must equal the pre-managed $EARN^*$.

Online Supplement B. Implementation details of maximum likelihood (ML) estimation

For each firm-year observation i, t, the log-likelihood is $\ln f(EARN_{i,t}|X_{i,t})$ as defined in (4a)–(6b), where $EARN_{i,t}$ and $X_{i,t}$ are from the data, and the coefficient vector comprises $\alpha_{0,0} \dots \alpha_{P,M}$ and $\pi_0 \dots \pi_M$. Maximum likelihood estimation finds the coefficients $\alpha_{0,0} \dots \alpha_{P,M}$ and $\pi_0 \dots \pi_M$ that maximize the total log-likelihood for the sample.[3] We code the log-likelihood as a user-defined Stata function. The *ml* command in Stata takes this function as an input and handles the numerical optimization and the computation of the standard errors.

We estimate the model for the subsample with earnings in a relatively narrow interval $[-R, R)$ around zero (e.g., $EARN \in [-0.04, 0.04)$ for our main definitions), as illustrated in Panel C of Figure 3. Although this approach involves selection on the dependent variable, we show that it yields the same parameter estimates as conventional maximum likelihood estimation on the full sample. The conventional ML approach solves

$$\max_{\theta} \sum_{i=1\dots N, t=1\dots T} \ln f(EARN_{i,t}|X_{i,t}) =$$

$$= \max_{\theta} \left(\sum_{i,t:\, EARN_{i,t} \in [-R,R)} \ln f(EARN_{i,t}|X_{i,t}) + \sum_{i,t:\, EARN_{i,t} \notin [-R,R)} \ln f(EARN_{i,t}|X_{i,t}) \right) \qquad (B1)$$

where θ is the full parameter vector, which comprises $\alpha_{0,0} \dots \alpha_{P,M}$, $\pi_0 \dots \pi_M$, and additional parameters that determine the earnings distribution outside $[-R, R)$; $f(EARN_{i,t}|X_{i,t})$ is the probability density function of reported earnings from equations (4a)–(4c), evaluated at the parameter values θ; and $[-R, R)$ is the earnings interval used in our subsample-specific estimation.

Suppose that a researcher uses a separate subset of parameters $\theta_{outside}$ for the pre-managed earnings distribution outside the estimation interval $[-R, R)$. From equations (4a)–(4c) in Section 2, all other components of θ (i.e., the pre-managed distribution parameters $\alpha_{0,0} \dots \alpha_{P,M}$ for the interval $[-R, R)$ and the earnings management parameters $\pi_0 \dots \pi_M$) affect the likelihood only for observations with reported earnings inside $[-R, R)$. Therefore, (B1) can be rewritten as

$$\max_{\pi_0 \dots \pi_M,\, \alpha_{0,0} \dots \alpha_{P,M}} \sum_{i,t:\, EARN_{i,t} \in [-R,R)} \ln f(EARN_{i,t}|X_{i,t}) + \max_{\theta_{outside}} \sum_{i,t:\, EARN_{i,t} \notin [-R,R)} \ln f(EARN_{i,t}|X_{i,t}) \quad (B2)$$

The first maximization in this expression is equivalent to our maximum likelihood estimation procedure for the subsample with earnings in the interval $[-R, R)$. It fully determines all of the parameters that we are interested in, i.e., $\alpha_{0,0} \dots \alpha_{P,M}$ and $\pi_0 \dots \pi_M$. Therefore, our estimation of $\alpha_{0,0} \dots \alpha_{P,M}$ and $\pi_0 \dots \pi_M$ on the restricted sample is equivalent to conventional maximum likelihood estimation on the full sample with an additional parameter vector $\theta_{outside}$.

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[3] For each observation i, t, we normalize the probability density function (6a) by imposing the standard restriction that the total mass $\int f(EARN|X_{i,t}) dEARN$ of the conditional earnings distribution must equal 1. Without this normalization, the ML estimation procedure would artificially drive the log-likelihood to infinity by increasing the coefficients α to infinity. This issue is specific to ML estimation. In our two-stage method, the estimates minimize the distance between predicted and actual bin frequencies, and thus the actual bin frequencies directly determine the scale of the density parameters, removing the need for an explicit normalization.

Online Supplement C. Implementation details of the two-stage estimation method

We omit the firm and year indexes for brevity. To prepare the data, we restrict the sample to observations with scaled earnings $EARN$ in the estimation interval (Panel C of Figure 3) and convert each of these firm-year observations into B firm-year-bin observations with a dummy dependent variable Y_b ($b = 1 \dots B$) that equals 1 if $EARN$ is in bin b and 0 otherwise. Because our estimation equations pool all relevant bins, the bin grid can have many bins in the small-loss and small-profit intervals (versus just one bin for each interval in the standard tests). In practice, the maximum viable number of bins is restricted only by computation time and available memory.

Stage 1 uses all firm-years in the estimation sample; for each firm-year, we only include firm-year-bins outside the small-loss and small-profit intervals, as illustrated in Panel A of Figure 4.[4] We estimate a discrete version of equation (6a) for a pooled sample of these bins

$$Y_b = \alpha_0(X) + \alpha_1(X) \times z_b + \alpha_2(X) \times z_b^2 + \cdots + \alpha_P(X) \times z_b^P + \varepsilon_b \qquad (C1)$$

where the polynomial coefficients $\alpha_p(X) = \alpha_{p,0} + \alpha_{p,1}X_1 + \cdots + \alpha_{p,M}X_M$ follow (6b), $X = X_1 \dots X_M$ are the explanatory variables for the firm-year, and z_b is the midpoint of earnings bin b. Because the dependent variable Y_b is binary, the predicted value in regression (C1) captures the bin probability $Pr(Y_b = 1|X, z_b)$ conditional on explanatory variables X.[5] This probability interpretation follows the linear probability model (e.g., Wooldridge, 2002, Ch. 15), and the pooling of data for all relevant bins imposes the smooth polynomial structure (C1) across the bins.

Stage 2 uses all firm-years in the estimation sample; for each firm-year, we only include firm-year-bins inside the small-loss and small-profit intervals (Panel B of Figure 4). The regression model is

$$Y_b - \hat{Y}_b = (\pi_0 + \pi_1 X_1 + \cdots + \pi_M X_M) \times W_b + u_b \qquad (C2)$$

where $\hat{Y}_b$ is the predicted probability of bin b for the firm-year based on the pre-managed earnings distribution (C1) from stage 1, $Y_b - \hat{Y}_b$ is the deviation from the pre-managed distribution, $\pi_0 \dots \pi_M$ are the earnings management parameters, and W_b is a synthetic explanatory variable shown in Panel C of Figure 4 that embeds the earnings management definitions of Models I and II.

For the small-loss bins, W_b is defined as $-\hat{Y}_b$ in Model I and $-q(z_b) \times \hat{Y}_b$ in Model II, where $q(z_b)$ is the triangular interaction term from (5b). These definitions implement the dip in the density of small losses due to earnings management, represented by $-f^*(EARN|X)P(EARN, X)$ in equation (4b). In Model I, $P(EARN, X)$ is flat with respect to the size of the small loss $EARN$ (Panel A of Figure 3), and thus the dip is proportional to the pre-managed earnings density $f^*(EARN|X)$, approximated by $\hat{Y}_b$. In Model II, the earnings management probability is triangular with respect to $EARN$ (Panel B of Figure 3), and thus the dip is proportional to the triangular interaction term $q(z_b)$ times the pre-managed density approximated by $\hat{Y}_b$.

For the small-profit bins, W_b is defined as $-\frac{K^-}{K^+}W_{mean}$ in Model I and $-q(z_b)\frac{K^-}{K^+}W_{mean}$ in Model II, where the ratio K^-/K^+ adjusts for the relative widths of the small-loss and small-profit intervals, W_{mean} is the mean of W_b across the small-loss bins for the firm-year, and $q(z_b)$ is the triangular interaction term for small profits. These definitions implement the bump in the density

[4] In both stages, this bin selection is unrelated to reported earnings for the year. For example, firm-years with small (moderately large) profits are included in stage 1 (stage 2), but all of the included bin dummies equal zero.

[5] The discrete bin probability (C1) approximates the integral of the continuous density function (6a) for the bin using the function value at the bin midpoint. Therefore, the empirical coefficients α in (C1) are not entirely equivalent to the theoretical α in (6a). Because the α-s are not individually interpretable (as parts of a polynomial), and their only job is to approximate the underlying smooth distribution, we slightly abuse the notation and reuse α in (C1) for brevity.

of small profits due to earnings management, represented by $G(EARN, X)$ in equation (4c). For each firm-year, the total mass of the bump for small profits must equal the total mass of the dip for small losses (Panel C of Figure 1). In both models, this restriction is implemented through $\frac{K^-}{K^+} W_{mean}$. In Model I, $G(EARN, X)$ is flat with respect to the size of the small profit $EARN$ (Panel A of Figure 3), and thus W_b does not require any further adjustments. In Model II, $G(EARN, X)$ is triangular (Panel B of Figure 3), and thus W_b incorporates the triangular interaction term $q(z_b)$.

Standard errors of the two-stage estimates

Because the explanatory variables in the second-stage regression (C2) are constructed based on the first-stage estimates $\hat{\alpha} = (\hat{\alpha}_{0,0} \dots \hat{\alpha}_{P,M})'$ from (C1), the standard errors of $\hat{\pi} = (\hat{\pi}_0 \dots \hat{\pi}_M)'$ in the second stage should be adjusted for the first-stage estimation noise. The usual OLS standard errors (with appropriate clustering) do not incorporate this adjustment and should not be used.[6]

Using the method of moments representation of OLS (e.g., Wooldridge, 2002, Ch. 14), the regression estimates $\hat{\alpha}$ and $\hat{\pi}$ in the two stages (C1) and (C2) are defined by the moment conditions

$$\bar{h}(\hat{\alpha}) = \frac{1}{N} \sum_{\substack{i,t,b: \\ b \notin small\ loss/profit\ bins}} \left(Y_{i,t,b} - \hat{\alpha}' P_{i,t,b}\right) P_{i,t,b} = 0 \qquad (C3)$$

$$\bar{g}(\hat{\alpha}, \hat{\pi}) = \frac{1}{N} \sum_{\substack{i,t,b: \\ b \in small\ loss/profit\ bins}} \left(Y_{i,t,b} - \hat{Y}_{i,t,b}(\hat{\alpha}) - \hat{\pi}' Q_{i,t,b}(\hat{\alpha})\right) Q_{i,t,b}(\hat{\alpha}) = 0 \qquad (C4)$$

where $P_{i,t,b}$ is the full vector of explanatory variables in stage 1 (i.e., $1, z_b, z_b^2 \dots z_b^P$ and its interactions with $X_{i,t,1} \dots X_{i,t,M}$), $\hat{Y}_{i,t,b}(\hat{\alpha})$ is the predicted value from stage 1, and $Q_{i,t,b}(\hat{\alpha})$ is the full vector of explanatory variables in stage 2 (i.e., $W_{i,t,b}$ and its interactions with $X_{i,t,1} \dots X_{i,t,M}$). The estimation noise in $\hat{\alpha}$ affects the second-stage standard errors through both $\hat{Y}_{i,t,b}(\hat{\alpha})$ and $Q_{i,t,b}(\hat{\alpha})$ in (C4).

The Taylor expansion of (C3) and (C4) around the true values α^* and π^* is

$$\begin{bmatrix} \bar{h}(\hat{\alpha}) \\ \bar{g}(\hat{\alpha}, \hat{\pi}) \end{bmatrix} \approx \begin{bmatrix} \bar{h}(\alpha^*) \\ \bar{g}(\alpha^*, \pi^*) \end{bmatrix} + \begin{bmatrix} \nabla_\alpha \bar{h} & 0 \\ \nabla_\alpha \bar{g} & \nabla_\pi \bar{g} \end{bmatrix} \begin{bmatrix} \hat{\alpha} - \alpha^* \\ \hat{\pi} - \pi^* \end{bmatrix} \qquad (C5)$$

After combining (C5) with (C3) and (C4), we have

$$\begin{bmatrix} \hat{\alpha} - \alpha^* \\ \hat{\pi} - \pi^* \end{bmatrix} \approx - \begin{bmatrix} \nabla_\alpha \bar{h} & 0 \\ \nabla_\alpha \bar{g} & \nabla_\pi \bar{g} \end{bmatrix}^{-1} \begin{bmatrix} \bar{h}(\alpha^*) \\ \bar{g}(\alpha^*, \pi^*) \end{bmatrix} \qquad (C6)$$

From (C6), the asymptotic covariance matrix of the estimates $\hat{\alpha}$ and $\hat{\pi}$ is

$$Cov\left(\sqrt{N} \begin{bmatrix} \hat{\alpha} \\ \hat{\pi} \end{bmatrix}\right) = \Gamma' \Omega \Gamma \qquad (C7a)$$

where

$$\Gamma = \left(-plim \begin{bmatrix} \nabla_\alpha \bar{h} & 0 \\ \nabla_\alpha \bar{g} & \nabla_\pi \bar{g} \end{bmatrix}\right)^{-1} \qquad (C7b)$$

$$\Omega = Cov\left(\sqrt{N} \begin{bmatrix} \bar{h}(\alpha^*) \\ \bar{g}(\alpha^*, \pi^*) \end{bmatrix}\right) \qquad (C7c)$$

[6] In untabulated simulations, conventional single-stage clustered standard errors are biased downward slightly, as expected, which leads to moderate over-rejection in hypothesis tests. Our adjustment resolves this bias. Because the bin dummies are mutually exclusive, the bin-level observations are correlated within each firm-year. Therefore, our adjustment must be combined with clustering to address the within-firm-year correlation.

Because $\hat{\alpha}$ and $\hat{\pi}$ converge in probability to the (unknown) true values α^* and π^*, the matrices Γ and Ω can be evaluated at $\hat{\alpha}$ and $\hat{\pi}$ instead of α^* and π^*. The covariance matrix Ω of the moment conditions is clustered as needed. Our Stata command incorporates these computations.

Online Supplement D. Type-I error and power simulations for basic distribution discontinuity tests without explanatory variables

We generate 1,000 artificial samples of pre-managed earnings $EARN^*$ on the estimation interval $[-0.04, 0.04)$, using the earnings distribution parameters from the main empirical specification (column 4 in Panel A of Table 2), for sample size $N = 5{,}000$ (less than 1/6 of our main sample) and $N = 30{,}000$ (slightly less than the main sample). In Type-I error simulations, the null hypothesis of no earnings management is true, and therefore we do not manage simulated earnings. In test power simulations, we convert some of the small losses into small profits per Model II with the true earnings management probability $\pi_0^{true} = 0.025$ and 0.05 (i.e., 2.5% or 5% of small losses are managed on average). For each simulated sample, we test for earnings discontinuity using Burgstahler and Dichev (1997) standardized difference tests, the main two-stage version of our method, and the ML version of our method as an asymptotically efficient benchmark (Wooldridge, 2002).

Table D1 presents the simulated rejection rates. Columns 1 and 4 reflect Type-I error. The left standardized difference test in column 4 has a slightly elevated Type-I error of 6.6%, while all other Type-I errors for all tests are in line with the nominal level.[7]

Consistent with Burgstahler and Chuk's (2014) simulations, the standardized difference test successfully detects earnings management. For example, when $\pi_0^{true} = 0.025$, earnings management affects just 13 observations on average out of $N = 5{,}000$ and 80 out of $N = 30{,}000$.[8] The rejection rates based on the left (right) standardized difference are 22.0% and 59.3% (15.9% and 42.4%) for $N = 5{,}000$ and $30{,}000$, respectively. When $\pi_0^{true} = 0.05$, the rejection rates are 44.3% and 97.7% (36.5% and 92.3%), respectively.[9]

Our main two-stage tests improve these rejection rates by 1.3–29.3 percentage points, as represented by the grey bars in Figure D1. For example, for $\pi_0^{true} = 0.05$ and $N = 5{,}000$ in column 3, the rejection rates improve from 36.5–44.3% in the standardized difference tests to 52.2–57.5% in our method. Notably, the improvement is larger when we use a finer bin grid (the darker grey bars in Figure D1), and it gradually approaches the upper bound of potential test performance, approximated by the ML results (the black bars). For example, when the bin width is 0.005, the power improvement for our two-stage tests is on average 78% as large as that for ML; when the bin width is reduced to 0.0025 and 0.001, the ratio increases to 92% and 98%, respectively.[10] Thus, our main method performs almost as well as ML without any of ML's numerical complications.

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[7] When the rejection rate is 5%, the total number of rejections in 1,000 simulations is a binomial random variable with $n = 1{,}000$ and $p = 0.05$. The corresponding 95% confidence interval for the rejection rate is $[3.6\%, 6.4\%]$.

[8] Small pre-managed losses are approximately 10.7% of the N earnings observations in the interval $[-0.04, 0.04)$. Therefore, when $\pi_0^{true} = 0.025$, the expected number of managed small losses per sample is approximately $0.107 \times 0.025 \times 5{,}000 = 13.38$ for $N = 5{,}000$ and $0.107 \times 0.025 \times 30{,}000 = 80.25$ for $N = 30{,}000$.

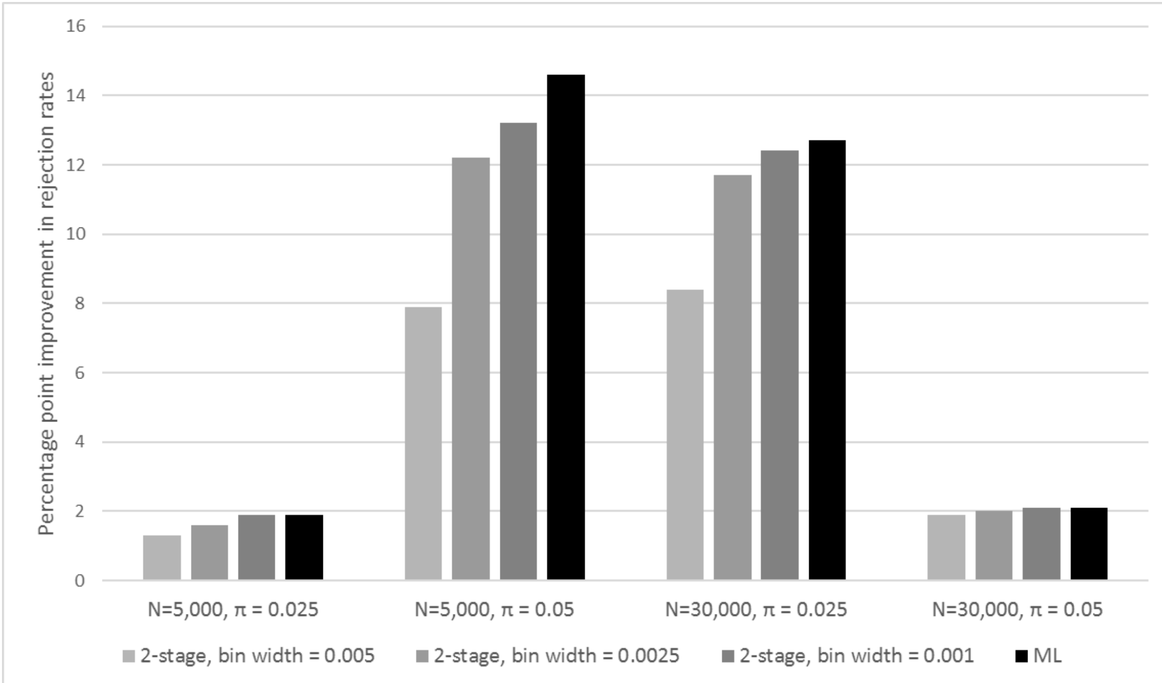
[9] The discrepancy between the left and right standardized differences is likely driven by the curvature of the earnings distribution. The computation of the standardized differences is based on linear interpolation, while the simulated distribution of pre-managed earnings is convex, following the empirical estimates shown in Figure 5. When linear interpolation is applied to a convex distribution, it overstates the missing earnings density below zero and understates the excess earnings density above zero. These interpolation biases cause over-rejection in the left standardized difference test and under-rejection in the right standardized difference test in this simulation. Our method is less sensitive to bad approximation quality, even in the pathological examples shown in Figure 7, because the approximation biases below and above zero partly offset each other in the combined test statistic.

[10] Bin width = 0.001 can increase the estimation time considerably for reasons explained in the appendix for our estimation command at the end of the paper.

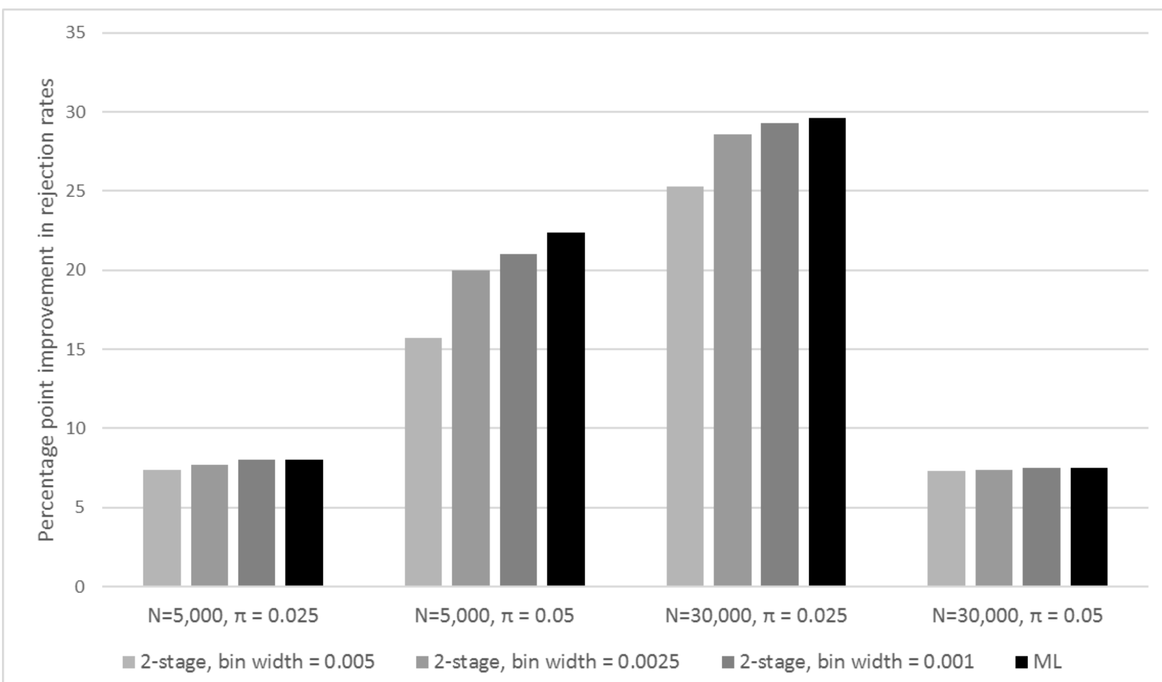
In untabulated simulations, we examine earnings management scenarios with asymmetric small-loss and small-profit intervals ($K^- = 0.01$, $K^+ = 0.005$; and $K^- = 0.005$, $K^+ = 0.01$). Even when our main method uses incorrect interval definitions $K^- = K^+ = 0.01$ in estimation, it outperforms the standardized difference tests in most cases. With correct asymmetric interval definitions, it dominates in all cases.

In additional simulations, we examine whether the cubic polynomial approximation used in our main two-stage test has an advantage over a simpler linear interpolation in the same two-stage test. Linear interpolation over the main estimation interval $[-0.04, 0.04)$ is highly vulnerable to distribution non-linearities and often has excessive Type-I errors. Linear interpolation over a narrower interval such as $[-0.015, 0.015)$ has acceptable Type-I errors but lower power because it uses less data. In all cases, the polynomial approximation performs at least as well as linear interpolation, and often it performs considerably better (e.g., when the discontinuity is near the peak of an asymmetric distribution with high curvature), without any additional implementation costs. In additional tests, we combine the cubic polynomial approximation with dummies for each of the small-loss and small-profit bins, following Chetty et al. (2011). Statistical power decreases considerably because this approach does not impose the restriction that the mass of missing small losses must equal the mass of excess small profits (Panel C of Figure 1).

In summary, our method offers a sizable power improvement relative to the standardized difference tests. Thus, even a researcher who only wants to conduct standard Burgstahler and Dichev (1997) discontinuity analysis without any explanatory variables could benefit from our method, especially when the sample size or effect size is small.



Panel A: Improvement relative to the left standardized difference test



Panel B: Improvement relative to the right standardized difference test

Fig D1. The percentage point improvement in test power for our two-stage and ML estimates, relative to the standardized difference tests, based on the simulation results in Table D1

Table D1. Rejection rates in simulated distribution discontinuity tests without explanatory variables

	Simulated sample comprises 5,000 observations in the interval [-0.04, 0.04)			Simulated sample comprises 30,000 observations in the interval [-0.04, 0.04)		
	true earnings management probability π_0^{true} is					
	0%	2.5%	5%	0%	2.5%	5%
	(1)	(2)	(3)	(4)	(5)	(6)
Burgstahler and Dichev (1997) standardized difference test						
left difference	6.2	22.0	44.3	6.6	59.3	97.7
right difference	4.6	15.9	36.5	3.7	42.4	92.3
Significance test for the earnings management probability π_0 in our main Model II with $K^- = K^+ = 0.01$						
Main two-stage estimation with						
bin width = 0.005	5.2	23.3	52.2	5.3	67.7	99.6
bin width = 0.0025	5.0	23.6	56.5	5.2	71.0	99.7
bin width = 0.001	5.6	23.9	57.5	5.4	71.7	99.8
ML estimation	5.1	23.9	58.9	5.6	72.0	99.8

The table presents the rejection rates in one-tailed tests with a 5% nominal significance level in 1,000 simulated samples. For the Type-I errors in columns 1 and 4, the 95% confidence interval is 3.6% to 6.4%. The simulated distribution of pre-managed earnings follows the estimates from column 4 of Panel A in Table 2, and the simulated earnings management process follows Model II with the true earnings management probability π_0^{true} set to 0, 0.025, or 0.05. In estimation for the simulated data, the estimation interval is [-0.04, 0.04), the small-loss and small-profit interval width is 0.01, and the bin width for earnings discretization in the two-stage method varies from 0.005 to 0.001. The code fragment for the two-stage estimation is:

```
kinkyX simNI, binwidth(0.005) est_bins(8) em_bins(2) em_type(ii) degree(3) cluster(gvkey)
kinkyX simNI, binwidth(0.0025) est_bins(16) em_bins(4) em_type(ii) degree(3) cluster(gvkey)
kinkyX simNI, binwidth(0.001) est_bins(40) em_bins(10) em_type(ii) degree(3) cluster(gvkey)
```