A Competitive Network Design
Problem with Pricing

by

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A Competitive Network Design Problem with Pricing

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This paper presents a simple model of competition between transport firms that captures the interaction of system design, price setting, and consumer choice. Transport competition is modeled as a noncooperative game where firms first select network designs, then prices for transportation between any two nodes. The goal is to find a Nash equilibrium in prices and system designs for all competing firms. Competition is studied under two alternate assumptions about consumer choice: customers can bundle separately purchased legs and customers cannot. If bundling cannot occur, it is shown that unique Nash equilibrium prices exist and that each firm’s profit can be written as the difference between two minimum cost flow problems. Sufficient conditions for the existence of equilibrium network designs are also developed. If bundling can occur, it is shown that a price equilibrium may not exist, and if it does, the price equilibrium may not be unique. Lack of existence or uniqueness implies that firm profit is not a well-defined function of network designs. This shows that the network design problem with bundling is difficult. With bundling, some results are possible in the case of duopoly competition: an equilibrium in prices always exists but equilibrium prices may not be unique. However, when each firm chooses a network design to maximize the lower bound of its profit, the equilibrium network designs chosen are the same as those chosen when bundling is ignored.

One of the most important decisions made by a transport firm is the design of its service system. The design of a system involves choices such as the cities to be served, the modes of transport and the routings to be used, the frequencies of service and the capacities of vehicles. These decisions affect both the cost of providing service and the attractiveness of that service to potential customers. If customers choose carriers on the basis of the attractiveness of service, these decisions affect the firm’s demand.

Transport firms in many deregulated markets must also decide on prices for services. When firms set prices, customers select transport providers on the basis of service price as well as attractiveness. The demand (and therefore the revenue generated) for any firm’s service is a function of all firms’ system designs and prices. Clearly, a profit maximizing transport firm must consider the tradeoff between the cost of service and the revenue generated when choosing its system design and prices.

The choice of system design and prices is more complex if several competing firms make system design and pricing decisions at roughly the same time. In making its choices, each firm must consider what its competitors’ choices are likely to be.

The purpose of this paper is to present a simple noncooperative game played by transport providing firms that captures the interaction of system design, price setting, competitive firm behavior and consumer choice.

The motivation of this paper is to study how firms choose prices and network designs in competitive, deregulated markets. Both of these issues have been important ones in the United States during the period following deregulation of the trucking, railroad and passenger airline industries. Intense price competition and network design changes have occurred in all three industries. In airlines, hub and spoke systems have become a dominant design choice and many regional carriers have been merged into larger carriers serving more markets. In railroads, many firms have integrated into other modes such as barge and trucking, expanding the reach of their networks. This paper shows that competitive prices and network designs
depend on firm costs and costs imposed on consumers due to the convenience of services offered. Under some restrictive conditions, each firm maximizes its profit by choosing its network design to minimize the sum of firm and customer costs to serve all customers. Two implications occur because of this result: transportation firms choose networks to maximize efficiency, and different transportation sectors demonstrate different design choices due to different firm and customer costs.

From an operations research perspective, this paper can be viewed as a competitive version of the "network design-traffic equilibrium" problem described by Magnanti and Wong. Magnanti and Wong refer to a network design problem subject to the constraint that transport flow is a traffic equilibrium as a "network design-traffic equilibrium" problem. This formulation does not consider price as a decision variable. In contrast, the formulation presented in this paper allows firms to competitively choose prices in addition to network designs. Each firm's objective is to maximize its profit, subject to the constraint that the traffic flow be a traffic equilibrium.

Although consumer choice of transportation services has been extensively studied in the traffic equilibrium literature, the competitive design of transportation systems and competitive price setting on these systems have not. Friesz et al., Harker, Haurie and Marcotte, and LeBlanc et al. all study customers' equilibrium choice of transportation on a common transportation network. Customers choose services to minimize the sum of transport fees and travel time and congestion costs. These models do not study oligopolistic price setting.

Some recent papers address competitive network design. Harker studies Cournot competition between bus companies. Dobson and Lederer model profit maximizing pricing and scheduling of flights in a hub and spoke system and solve for equilibrium decisions. Bittlingmayer studies network economics for hub and spoke systems and entry preventing pricing. Lederer presents a simpler version of the model presented here.

This paper contributes to the literature by integrating the problems of competitive network design and pricing with consumer choice. Transport competition is modeled as a noncooperative game where firms first select network designs, then prices. The goal is to find a Nash equilibrium in prices and system designs. Competition is studied under two different assumptions about consumers' choice: that customers can bundle separately purchased services and that customers cannot. Bundling can lower a customer's total cost when the price of separately purchased legs plus the cost of inconvenience of traveling on these legs is less than the price of a single ticket from a single firm plus the cost of inconvenience of traveling on this routing. Bundling is an option for customers in many transportation markets, such as common carrier transport and passenger airlines. However, a firm can sometimes effectively forbid customers from buying and combining separate services by not providing easy connections between separately purchased paths. For example, a customer bundling common carrier transport must transfer goods itself between segments. If customer inconvenience is high enough customers will never desire to bundle together separate paths and bundling can be ignored. The effect of bundling on prices and transport networks has been little studied. In another context, Katz and Shapiro and Klempener studied the effect of bundling costs on firm's product compatibility decisions for durable goods.

If bundling cannot occur, because customer inconvenience is too great, this paper shows that unique Nash equilibrium prices exist, and that each firm's profit can be written as the difference between two minimum cost flow problems. Sufficient conditions for the existence of equilibrium network designs are also developed. Under a restrictive assumption, each firm maximizes its profit by choosing a network design that minimizes the total cost to all firms and customers of satisfying transportation demand. The global solution to this minimum cost problem is an equilibrium network design.

If firms cannot prevent bundling by imposing high customer bundling costs, firms must set prices considering customer's bundling possibilities. If bundling occurs, it is shown that a price equilibrium may not exist, and if it does, it may not be unique. Without existence or uniqueness of a price equilibrium, firm profit is not a well defined function of network designs. This shows that the network design problem with bundling is difficult for firms operating in networks: prediction of market prices may not be possible. With bundling, some results are possible in the case of duopoly competition. First, equilibrium prices always exist but may not be unique. Second, when each firm chooses a network design to maximize the lower bound of its profit, the equilibrium network designs chosen are the same as those chosen when bundling is ignored.

The paper is organized as follows. Section 1 presents the model of the firms and the market and Section 2 introduces competition. Sections 3 and 4 characterize the price equilibrium and the network
design equilibrium. Section 5 examines price equilibria under bundling and when only two firms compete. Section 6 briefly summarizes and suggests extensions.

1. THE MODEL

We study competitive choice of network designs and prices as a two-staged process. Firms first choose network designs, then prices. This sequential choice can be justified in two ways. First, the time to implement any network is long compared with the time to set prices. Second, the price decision can be changed with short notice and small cost relative to the design decision.

Firms must make network design and price decisions considering what their competitors are likely to choose and considering how consumers choose between competing services. The competitive process will be modeled as a noncooperative game in Section 2. First the market, the firms and their decisions are described.

1.1. The Network

Let $G(N, A)$ be a directed graph where $N$ is the finite set of nodes and $A$ is the finite set of directed arcs. For simplicity, assume that there is a directed arc connecting each pair of nodes. Let a generic element of $N$ be $n$, and a generic element of $A$ be $a$. Each arc in $A$ has a well defined length, $L(a)$. Assume that there is a demand for transport between each pair of nodes. Let $M$ denote the set of all ordered pairs of $N$. An element of $M$ is referred to as a “market” and is denoted by $m$. An important assumption is that customer demand for each $m$ is inelastic with respect to price and equal to $ho(m)$ in units of volume. There is a finite set of firms, $I$, each of which seeks to provide transport services between nodes. A generic element of $I$ is denoted by $i$ or $j$.

1.2. Network Design and Pricing Decisions of Firms

A firm's network design is specified by the frequency of trips on each arc of $G$. This frequency specification captures essential aspects of the design of transportation networks. Fixed costs associated with maintaining terminals or equipment increase with frequency of service. Dispatch, loading and unloading costs increase as well. Customer cost declines with more frequent departures because of more convenient departure times, shorter waiting times for departure and shorter waiting times for connecting trips. Congestion of commonly used facilities with capacity constraints increase with frequency, resulting in longer customer waiting times. For many transport industries, such as passenger airlines, frequency related congestion seems to be more important than passenger volume related congestion.

A firm's network design is given by $t_i$ with $i \in R^{|A|}$. The components of $t_i$ correspond to arcs in $A$. The component corresponding to arc $a$ is $t_i(a)$ which is a scalar and interpreted as the frequency of service on arc $a$ by firm $i$ for $t_i$. The set of all possible network designs by firm $i$ is denoted by $T_i$, which is assumed to be closed compact subset of $R^{|A|}$ that contains the origin. $T_i$ corresponds to the technologically feasible set of network designs. The fact that $0$ belongs to $T_i$ means that firm $i$ may decide to have zero frequency on all arcs, that is the firm does not to provide any service to any market. Let $T$ be the space of the cross product of the $T_i$'s and let an element of $T$ be $t. We write $t = (t_1, \ldots, t_I)$.

A firm provides transportation using a path consisting of arcs from its network. Denote the set of all paths in $G$ by $P$ and a generic path by $p$. Paths are assumed to be simple, that is contain no cycles. A path is a connected set of arcs in $A$: $p = (a_1, a_2, \ldots, a_n)$. For $m = (n, n')$, let $P(m)$ be the set of all paths that begin at $n$ and end at $n'$. $P_i$ are firm $i$'s paths.

Firm $i$'s price for market $m$ depends on the path used by customers. It is assumed that each firm sets a price for every $p \in P_i$. Firm $i$'s price on path $p$ is written $\pi_i(p)$. The vector of firm $i$'s prices for all paths is denoted $\pi_i$. We refer to $\pi_i$ as a firm $i$'s “price schedule.”

Firms incur fixed and variable costs when providing services. Fixed costs are independent of the firm's volume and include the capital cost of building the network and all fixed operating expenses associated with network design $t_i$. Firm $i$'s fixed cost for network design $t_i$ is denoted $F_i(t_i)$ and is continuous on $T_i$. Variable costs include all costs that are incurred as function of traffic volume. It is assumed that the per unit variable cost for each $p$ is scale independent, but dependent upon a firm's network design. We write firm $i$'s variable cost per unit transported on path $p$ as $v_i(t_i, p)$. Assume that $v_i(t_i, p)$ is continuous on $T_i$ and that $v_i(0, p) > 0$ for all $p$.

Different firms may have different costs and therefore different functions $F_i(t_i)$ or $v_i(t_i, p)$. This is because firms may use different modes of transport, resulting in very different fixed and variable costs. Firms that use the same mode may have access to network technologies having different fixed and variable costs. The fixed cost function reflects incremental fixed cost investments. A firm
with an existing network may have lower fixed cost than other firms for networks that utilize parts of its existing design. Similarly, an existing firm could have lower variable cost than other firms for some network designs due to operating experience or other factors. We assume $F_i(0) < 0$; firms not in the market incur no fixed cost; $F_i(0) < 0$ is possible if an existing firm $i$ can recover part of its past investment.

1.3. Customer Costs

Customers pay fares for service and also incur costs associated with the relative convenience of service. The latter costs are referred to as "customer costs." Customer costs for a path are caused by factors such as the distance traveled, the number of nodes crossed, time lost due to waiting for flights, and congestion. The customer cost per unit transported using firm $i$'s path $p$ is $e_i(t, p); e_i(t, p)$ is assumed to be continuous on $T$. An example of how $e$ can be defined follows. Suppose a path $p \in P$ is made up of directed arcs $a_1 = (n_1, n_2), a_2 = (n_2, n_3), \ldots, a_\eta = (n_\eta, n_{\eta+1})$. Each customer traveling on $p$ experiences a distance related customer cost of

$$d(p) = \sum_{k=1}^{\eta} L(a_k). \tag{1-1}$$

Each customer experiences a positive node crossing related customer cost, $g(t_i, n), \text{ if node } n \text{ is traversed. If firm } i \text{ 's network design is } t_i, \text{ the node crossing cost is}$

$$g(t_i, p) = \sum_{k=1}^{n} g(t_i, n_k). \tag{1-2}$$

Each customer experiences a frequency related customer cost of $u_i(a, t_i(a)) > 0$ when arc $a$ is traversed, due to waiting time required for connections; the frequency related customer cost on path $p$ is

$$w_i(t_i, p) = \sum_{k=1}^{\eta} u_i(a_k, t_i(a_k)). \tag{1-3}$$

Due to congestion caused by all firms employing arc $a$, each customer experiences a congestion related cost of $c(\sum_{i \in T} t_i(a))$ when arc $a$ is traversed. $c$ is assumed to be convex and increasing function of its argument. Along path $p$, the total customer congestion cost is the sum of these costs for each arc in the path,

$$c(t, p) = \sum_{k=1}^{\eta} c(\sum_{i \in T} t_i(a_k)). \tag{1-4}$$

The customer cost for path $p$ provided by firm $i$ is $e_i(t, p) = d(p) + g(t_i, p) + w_i(t_i, p) + c(t, p)$.

1.4. Consumer Choice

Consumer choice of paths is described using the following definitions.

**Definition 1.1.** If firms choose network designs $t$ and firm $i$ chooses price policy $\pi_i$, the "total price for path $p$ provided by firm $i"$ is the sum of the price and the customer cost, $\pi_i(p) + e_i(t, p)$.

**Definition 1.2.** If firms choose designs $t$ and firm $i$ chooses price policy $\pi_i$, the "total cost for path $p$ provided by firm $i"$ is the sum of the variable cost to the firm and customer cost, $v_i(t_i, p) + e_i(t, p)$.

For each market customers' total demand is inelastic with respect to total cost and customers choose the least total price path. Another assumption is that a customer in market $m = (n, n')$ buys transportation from a single firm, and does not bundle together separately purchased paths connecting $n$ to $n'$. In Section 5 this assumption is relaxed and its effect on competitive behavior is studied.

Given network designs $t$ and price schedules for each firm, $\pi_i$, the paths with the least total price for market $m$ provided by firm $i$ are $P_i(m) = \arg\min_{p \in P(m)} [\pi_i(p) + e_i(t, p)]$. $P_i(m)$ may not be a singleton. It is assumed that firm $i$ can direct customers onto paths in $P_i(m)$ that are most profitable. These paths are $P^*_i(m) = \arg\max_{p \in P_i(m)} [\pi_i(p) - v_i(t, p)]$.

For each $m$ the firm with the least total price serves. But, it must be specified how a market is split when there are two or more least total price paths. A "market share rule," explains how sharing occurs.

**Definition 1.3.** A market share rule is a function $r_i(\cdot)$, which specifies the share of market $m$ captured by firm $i$'s paths as a function of the network designs and price schedules of all firms. That is, $r_i(\cdot) > 0$ and $\sum_{i \in T} \sum_{p \in P_i(m)} r_i(\cdot) = 1$, for all $m$.

Market share rules do not award any service to firms pricing above the least total price. This property captures an important aspect of transportation price competition: a customer's choice of firm is sensitive to total price. In many markets, a firm that sets a higher total price than competitors experiences little demand. This situation occurs in passenger airline markets: customer sensitivity to price causes firms to set identical prices for services with similar customer costs.

Assuming market share rule $r$ has been chosen, the profit for firm $i$ can now be written as a function of the network designs and prices of all firms:

$$Z_i(t_1, \pi_1, \ldots, t_N, \pi_N)$$

$$= \sum_{m \in M} \sum_{p \in P(m)} [\pi_i(p) - v_i(t_i, p)] \rho(m) r_i(\cdot) - F_i(t_i). \tag{1-5}$$
2. MODELING COMPETITION

COMPETITION is modeled as a two-staged game. First, firms simultaneously and privately choose network designs, and these are revealed; then, firms privately and simultaneously choose prices, and these are revealed. Next, customers choose the most desirable path. Finally, payoffs are made in terms of profit as given by (1-5). This model captures important aspects of competition: prices depend on all firms’ network designs and prices are flexible even after network designs are chosen. Simultaneous choice of network designs is an appropriate assumption if the time frame and cost required to set or change networks are the same for all firms. Under these time/cost conditions, no firm can, a priori, be forecast to lead the others.

A firm’s price schedule strategy is a function of all firms’ networks because firms choose their price schedules after all firms’ network designs are chosen. With network designs $t$, firm $i$’s price schedule choice is written $\pi_i(t)$. Given $\pi_i(t)$, firm $i$’s price for path $p$ is written $\pi_i(t, p)$. Firm $i$’s price strategy specifies a choice of $\pi_i(t)$ for all $t \in T$ and is written $\overline{\pi}_i$. The set of all $\overline{\pi}_i$’s is denoted $\overline{\Pi}_i$.

A firm’s strategy also specifies a network design, $t_i$. A complete strategy for firm $i$ is a pair $(t_i, \overline{\pi}_i)$. If firms adopt strategies $(t_1, \overline{\pi}_1, \ldots, t_I, \overline{\pi}_I)$, the firms’ network designs are $t = (t_1, \ldots, t_I)$ and their price schedules are $(\pi_1(t), \ldots, \pi_I(t))$.

Given market share rule $r$, if firms adopt strategies $(t_1, \overline{\pi}_1, \ldots, t_I, \overline{\pi}_I)$, profit for firm $i$ is

$$Z_i(t_i, \overline{\pi}_1, \ldots, t_I, \overline{\pi}_I) = \sum_{m \in M} \sum_{p \in P(m)} [\pi_i(t_i, p(m)) - v_i(t_i, p(m))] \rho(m) r_i(m) - F_i(t_i). \tag{2-1}$$

The firms’ strategies for the firms, and payoff functions (2-1) define an $I$-person strategic form game. We seek a Nash solution to this game, which is a vector of strategies for the firms such that each firm’s choice of $t_i$ and $\overline{\pi}_i$ is a best response to the other firms’ choices. Because the game has two stages, the game is analyzed recursively, the last stage first. In the last stage, given firms’ network designs, each firm chooses its price schedule to be an optimal response to the other firms’ price schedules. An important assumption we make is that each firm chooses its price schedules to be an optimum response to the other firms’ prices, no matter what network designs are chosen by the firms.

Then, the first stage is analyzed. The network designs that solve the game have the property that each firm’s network design is an optimal response to the others’ network designs which considers how the network design will affect price strategies in the second stage. In game theoretic terms, we seek a subgame perfect Nash equilibrium.

**Definition 2.1.** A subgame perfect Nash equilibrium is a vector of network designs and price schedule strategies for all firms: $(t_i^*, \overline{\pi}_i^*), \ldots, (t_I^*, \overline{\pi}_I^*)$, such that for all $i \in I$:

$$Z_i(t_i^*, \overline{\pi}_1^*, \ldots, t_I^*, \overline{\pi}_I^*) \geq Z_i(t_i^*, \overline{\pi}_1^*, \ldots, t_i^*, \overline{\pi}_i^*, \overline{\pi}_1^*, \ldots, \overline{\pi}_I^*)$$

for all $t_i \in T_i$ and all $\overline{\pi}_i \in \overline{\Pi}_i$. \tag{2-2}

and for all $i \in I$ and all $t \in T$:

$$Z_i(t_1^*, \overline{\pi}_1^*, \ldots, t_i^*, \overline{\pi}_i^*) \geq Z_i(t_1^*, \overline{\pi}_1^*, \ldots, t_i^*, \overline{\pi}_i^*, \overline{\pi}_1^*, \ldots, \overline{\pi}_I^*)$$

for all $\overline{\pi}_i \in \overline{\Pi}_i$. \tag{2-3}

Condition (2-2) guarantees that each firms’ network design and price schedule is an optimal response to the others’ choices. Condition (2-3) guarantees that no matter what network design is chosen, each firm’s price schedule is an optimal response to the others’ price schedules.

Any $\overline{\pi}^* = (\overline{\pi}_1^*, \ldots, \overline{\pi}_I^*)$ satisfying (2-3) for all firms $i$ and all network designs $t$ is referred to as an “equilibrium price schedule.” Any $t^*$ satisfying (2-2) is referred to as an “equilibrium network design.”

3. PRICE EQUILIBRIA CONSIDERING CONSUMER CHOICE

Next, the problem of pricing assuming fixed network designs is studied. We need to present and discuss two assumptions:

**Assumption 3.1.** Firms price at or above variable cost on every path.

This is the expected behavior on paths served by a firm, because otherwise it could raise its price and increase its profit. However, some firm, say $i$, might price below its variable cost but above some competitor’s variable cost if it were confident that some other firm, say $j$, would undercut its price and serve the market. Such behavior is of no direct benefit to firm $i$, but acts to force a lower price on firm $j$.

**Assumption 3.2.** If two firms have the same least total price, the firm with the least total cost will serve all of the demand.
The second assumption concerns market share rules. No Nash equilibrium in price schedules can exist unless the market share rule has the property that if two firms have the same total price, the firm with the least total cost will serve all of the demand. Suppose a market share rule does not obey this property. If both firms have the same total price and serve a market, the low total cost firm could cut its price slightly, capture all demand and increase its profit. The profit would rise because the low total cost firm’s price is greater than or equal to its variable cost by the first assumption.

Next we characterize equilibrium price schedules:

**Theorem 3.1.** Suppose firms choose network designs \( t \). The firms’ equilibrium price schedules are characterized as follows:

1. For each market \( m \), the firm \( i^* \) which is the minimizer of
   \[
   \min_{i \in I} \min_{p \in P(m)} [v_i(t_i, p) + e_i(t, p)]
   \]  
   serves customers using path \( p^* \) which minimizes (3.1).
2. Firm \( i^* \) sets its price in market \( m \) for path \( p^* \) at
   \[
   \min_{j \in \{1, \ldots, t_i\}} \min_{p \in P(m)} [v_j(t_j, p) + e_j(t, p)]
   \]  
   \[- e_i(t, p^*).\]
3. A firm \( j^* \) that minimizes (3.2) with path \( p_j \), prices at its variable cost: \( v_j(t_j, p^*_j) \).
4. All other path prices for all firms are equal to, or above, variable cost.

**Proof.** Fix market \( m \). The least total cost firm (\( i^* \)) and the next lowest total cost firm (\( j^* \)) set prices equal to (3.2), and \( v_j(t_j, p_j) \), respectively, at \( m \). Suppose not. If firm \( i^* \) prices above (3.2), firm \( j^* \) can undercut its price, and profitably serve \( m \); firm \( i^* \)'s profit falls and this cannot occur in equilibrium. If firm \( i^* \) prices below (3.2), it could raise its price to (3.2) and continue to serve the market alone; its profit rises—again, this cannot happen in equilibrium. If firm \( j^* \) prices above \( v_j(t_j, p_j) \), firm \( i^* \) can price so that its total price is equal to \( j^* \)'s. The market rule share assigns the entire market to \( i^* \), which induces \( j^* \) to reduce its price to capture the market and raise its profit. Firm \( j^* \) cannot price above \( v_j(t_j, p_j) \). Any firm \( j \), including \( j^* \), is forbidden from pricing below its variable cost. Therefore, equilibrium prices for \( m \) are as stated. Q.E.D.

Firms compete in price for each market, undercutting competitor’s prices until the firm with the least total cost path has a total price equal to the next most efficient competitor’s least total cost path. The firm and the competitor set the same total price, which is equal to the competitor’s total cost. Note that for each market, at least two firms offer the same least total price but only one serves.

Next, an example of equilibrium price schedules given fixed network designs is presented.

**Example 3.1.** Consider the three node network depicted in Figure 1; the directed arcs of the network are written \( a_1 = (n_1, n_2) \), \( a_2 = (n_1, n_3) \), \( a_3 = (n_3, n_2) \) and the length of the arcs are \( L(a_1) = 1.5 \), \( L(a_2) = 1 \), and \( L(a_3) = 1 \). Denote the markets \( m_1 = (n_1, n_2) \), \( m_2 = (n_1, n_3) \), \( m_3 = (n_3, n_2) \) and suppose that \( \rho(m_1) = 1, \rho(m_2) = 1, \rho(m_3) = 1 \) are the only nonzero transport demands.

Let three firms, \( I = \{1, 2, 3\} \), with identical variable and fixed cost functions compete in this market. The firms all have variable cost functions \( v_i(t_i, p) = d(p) \), and fixed cost functions \( F_i(t_i) = \sum_{a_i} t_i(a_i) \). Customer cost equals \( e_i(t, p) = d(p) + g(t, p) + w_i(t_i, p) + c(t, p) \), with components defined by (1.1)-(1.4). Let congestion costs on each arc be \( 1 - \sum_{j \in \{1, 2\}} c(t_i, a_i) \) (if the denominator is non-positive, let congestion be considered infinite).

Node crossing and waiting costs are respectively:
\[
\begin{align*}
   g(t_i, n) &= 1, \\
   w_i(t_i, p) &= \sum_{a_i} 5(1 - t_i(a_i)).
\end{align*}
\]

If \( t_i = (t_i(a_1), t_i(a_2), t_i(a_3)) \), and \( t_i = (1/2, 0, 0) \), \( t_2 = (0, 1/2, 0) \), and \( t_3 = (0, 0, 1/2) \), then the firms’ equilibrium price schedules are: \( \pi^{(t)}(t, a_1) = 10; \pi^{(t)}(t, a_2) = 16.5; \pi^{(t)}(t, a_3) = 9 \); and \( \pi^{(t)}(t, a_3) = 9 \), for all firms \( i \). Market \( m_1 \) is served by firm 1, market \( m_2 \) is served by firm 2, and market \( m_3 \) is served by firm 3, using single arc paths. Firms’ profits are identical and \( Z_i = 2.0 \).

With fixed network designs and equilibrium price schedules, each market is served by the firms with the least total cost, \( I(m) \).

\[
I(m) = \{h \in I | \min_{p \in P(m)} [v_h(t_h, p) + e_h(t, p)] \\
   \leq \min_{j \in I} \min_{p \in P(m)} [v_j(t_j, p) + e_j(t, p)]\}.
\]  

The set of markets that firm \( i \) serves alone is \( S_i(t_1, \ldots, t_i) \), where
\[
S_i(t_1, \ldots, t_i) = \{m \in M | I(m) = i\}.
\]

The set of markets that are served by more than one firm is \( S_0(t_1, \ldots, t_i) \), where
\[
S_0(t_1, \ldots, t_i) = \{m \in M | \text{cardinality of } I(m) > 1\}.
\]

(3.5)

It is important to note that if \( m \in S_0(t) \), each firm serving \( m \) sets price at its variable cost on its least total cost path. It earns zero contribution.
can be expressed as

\[ K(t_1, \ldots, t_I) = \sum_{m \in M} \min_{j \in I} \min_{p \in P_j(m)} \left[ v_j(t_j, p) + e_j(t, p) \right] \rho(m) + \sum_{j \in I} F_j(t_j) \] (4-1)

Social cost is the value of the minimum total cost flow that uses all firms’ networks and satisfies customer demand. By adding and subtracting

\[ \sum_{m \in M} \min_{j \in I} \min_{p \in P(m)} v_j(t_j, p) + e_j(t, p) \rho(m) + \sum_{j \in I} F_j(t_j) \] (4-2)

to the right hand side of (3-6) and rearranging the terms, firm i’s profit can be written

\[ Z_i(t_1, \bar{\pi}_1, \ldots, t_I, \bar{\pi}_I) = \sum_{m \in M} \min_{j \in I} \min_{p \in P_j(m)} \left[ v_j(t_j, p) + e_j(t, p) \right] \rho(m) + \sum_{j \in I} F_j(t_j) - K(t_1, \ldots, t_I). \] (4-3)

The sum of the last two terms of (4-3) is the value of the minimum total cost flow satisfying customer demand that does not utilize firm i’s network. An important conclusion is:

**Lemma 4.1.** Each firm increases its profit by choosing its network design to maximize its contribution to lowering the total cost of satisfying all customer demand.

Unlike other network design models, a firm does not maximize its profit by designing its system to minimize its own cost, but by maximizing its contribution to lowering total firm and customer cost. Next, an example of equilibrium network designs is given.

**Example 4.1.** The network designs chosen by the firms in Example 3.1 are equilibrium network designs. Each firm serves one market, choosing a frequency of \( \lambda_0 \) for an appropriate arc. The second most efficient firm chooses a zero frequency on that arc. \( K \) is minimized by this design, and each firm maximizes its contribution to minimizing the total cost flow. Note that other equilibrium network designs exist. Any \( t \) such that \( t_i(a_1) = \lambda/2 \) for one firm \( i \), and \( t_j(a_1) = 0 \) for the other firms, \( t_i(a_2) = \lambda \) for one firm \( i \) and \( t_j(a_2) = 0 \) for the other firms and \( t_i(a_3) = \lambda/2 \) for one firm and \( t_j(a_3) = 0 \) for the other firms, are equilibrium network designs. In particular, a single firm could serve all markets in equilibrium. In this example, neither the network designs nor the firm profits are unique in equilibrium.

Lemma 4.1 can be used to find sufficient conditions for an equilibrium network design where only
a single firm provides transport at non zero frequency.

Lemma 4.2. Suppose \( K(t) \) has a minimum on \( T \), say \( t^* \), such that for some firm \( i \), \( t_i^* = 0 \) for all \( j \neq i \). Network design \( t^* \) is an equilibrium network design.

If \( t^* \) is as described, each firm maximizes its contribution to reducing the total cost of meeting customer demand. This shows that \( t^* \) is an equilibrium network design. Therefore, if a single firm's network design can minimize social cost, this design is an equilibrium network design.

The relationship between demand, firms' costs and customers' costs determines properties and configurations of equilibrium network designs. Lemmas 4.1 and 4.2 show how these parameters determine equilibrium network designs, and why these choices are different in different industries.

Although the computation of a equilibrium network design is generally not easy, some simple statements are possible because of Lemma 4.1. With all other factors held constant,

(i) the higher a firm's fixed or variable cost on a path, the lower the firm's service frequency,
(ii) the higher a firm's node crossing cost, the higher the service frequency on direct paths,
(iii) the higher the distance related customer cost, the higher frequency of service on firms' shortest paths, and
(iv) the higher customer congestion cost for all paths, the lower the frequency of service on congested arcs.

The next example exploits Lemma 4.2 to find equilibrium network designs to demonstrate these effects.

Example 4.2. Consider the network of Figure 2 with five regularly spaced nodes located 1 unit from a centrally located sixth node. Suppose that the flow between each pair of nodes is 1. Assume that the hypothesis of Lemma 4.2 holds; that is, a single firm \( i \) has a network design that minimizes \( K \) on \( T \).

The firm's fixed cost on any arc is \( f_i(a) \), with \( f > 0 \); the total fixed cost is \( f \sum_{a \in A} t_i(a) \). Variable cost is zero. Customer cost is given by the sum of distance, node crossing, congestion and waiting costs. A customer's distance cost is just the Euclidean distance of its path; the node crossing cost is \( s \) for each node crossed, congestion cost is \( c_i(a) \) for each arc of the path, and wait cost is the sum of \( w(b - t_i(a)) \) for each arc on the path.

Traffic demand is symmetric, the cost structure is symmetric and \( K \) is concave; therefore the firm's equilibrium network design will be symmetric. Denote an arc from the central node to any other by \( a_1 \), and arc between one of the five other nodes to its closest neighbor by \( a_2 \), and an arc from one of the five nodes to a second closest neighbor by \( a_3 \). A network design is written \( t = (t(a_1), t(a_2), t(a_3)) \); if \( t(a_i) = x \), all arcs of type \( a_i \) have frequency \( x \).

Figure 3b depicts the equilibrium network design that minimizes \( K \) when \( f = 5 \), \( c = 1 \), \( s = 1 \), \( w = 5 \), and \( b = 1 \). Only arcs having non zero frequency are shown. The equilibrium network design is \( t^* = (1, 1, 0) \). Markets are served directly if a single arc connects the nodes. Other markets are served by transport through the hub.

Figure 3a shows the equilibrium network configuration if \( f \) is raised (\( f = 10 \)), or \( c \) is lowered (\( c = 0 \)), or \( s \) is lowered (\( s = 0 \)) compared to the base case. The equilibrium network design is \( t^* = (1, 0, 0) \). All service now takes place via the hub.

Figure 3c shows the equilibrium network configuration if \( f \) is lowered (\( f = 1 \)), or \( c \) is raised (\( c = 2 \)), or \( s \) is raised (\( s = 2 \)) compared to the base case. The equilibrium network design is \( t^* = (1, 1, 1) \). All service now takes place via direct routes.

This example is descriptive of how different firm and customer cost structures induce different network design choices. In some industries, these patterns are adopted by most firms. For example, in passenger airlines, many firms have adopted hub and spoke systems. The example shows that this pattern can be induced by low customer switching and congestion costs or high fixed costs of service.

Existence of an equilibrium network design is difficult to prove without other restrictions. The existence of equilibrium network designs can be guaranteed under the following assumption, however.

Assumption 4.1. For all \( i \in I \), \( e_i(t, p) \) is independent of \( t_j \) for all \( j \neq i \).
still exist (as shown in Example 4.1), but existence cannot be guaranteed. It follows that:

**Theorem 4.1.** If Assumption 4.1 holds, $t^*$ are equilibrium network designs iff for all $i \in I$,

$$K(t_i^*, \ldots, t_j^*) \leq K(t_i^*, \ldots, t_i, \ldots, t_j^*)$$

for all $t_i \in T_i$.

If Assumption 4.1 holds, each firm maximizes its profit by minimizing social cost, holding the other firms' network designs fixed. A firm's network design is an optimal response to the other's network design if and only if it minimizes social cost with the other firms' network designs held fixed. As modeled, the problem of minimizing social cost is an uncapacitated network design problem with linear firm and customer flow costs. Because $K$ is the sum of the minimum of continuous functions, $K$ is continuous and attains a minimum on the compact set $T$. If $t_0$ is a global minimum of $K$ on $T$, $t_0$ satisfies (2-2) for all firms, and thus $t_0$ is an equilibrium network design.

**Theorem 4.2.** If Assumption 4.1 holds, equilibrium network designs always exist. Further, the network designs that minimize $K(t)$ for $t \in T$ are equilibrium network designs.

Although $t_0$ is an equilibrium network design, equilibrium network designs need not minimize $K$, just minimize $K$ in each component $t_i$ with the other components held fixed. Also, Example 4.1 shows that equilibrium network designs are not necessarily unique.

This section has shown that an equilibrium network design exists under restrictive conditions. An equilibrium network design can be found by designing firms' networks to minimize firm and customer cost subject to satisfying customer demand. This shows that a version of the competitive network design-traffic equilibrium problem with prices can be solved as a network design problem that ignores the traffic equilibrium constraint and pricing. Also, the competitive network design problem with pricing can be solved as a minimum cost flow problem.

## 5. EQUILIBRIUM PRICE SCHEDULES ALLOWING BUNDLING

Up to this point it was assumed that a customer buys transport service from a single firm, and does not wish to, or cannot, bundle together several separately purchased paths.

However, if permitted, customers may find it advantageous to bundle separately purchased paths of a single firm or several firms. When bundling is
not allowed, we say that "bundling is ignored," otherwise, we say that "bundling is allowed." Next, we present some formal notation necessary to analyze equilibrium network designs and prices when bundling is allowed.

A bundle is formally defined as a connected set of separately purchased paths provided by different firms. If \( m = (n, n') \), a bundle providing \( m \) is a connected set of paths \((p_1, \ldots, p_n)\) beginning at \( n \) and terminating at \( n' \). Let \( C \) be the set of all simple bundles (bundles where the routing generated by the paths does not have cycles), and let \( c \) be a generic element of \( C \). If \( m \in M, C(m) \) is the set of all simple bundles providing transport \( m \). If \( c \in C \), let \( c_i \) be the paths provided by firm \( i \) in bundle \( c \). Given any \( c \in C \), let \( v_i(t, c_i) = \sum_{p_i \in c_i} \nu_i(t, p_i) \) and \( e_i(t, c) = \sum_{p_i \in c_i} c_i(t, p_i) \). These are firm \( i \)'s variable cost and customer cost, respectively, for its paths in \( c \). If \((\pi_1, \ldots, \pi_I)\) is a vector of firm price schedules, \( \pi_i(c) = \sum_{p_i \in c_i} \pi_i(p_i) \) is the sum of the prices for firm \( i \)'s paths in \( c \); \( \pi(c) = \sum_{i \in I} \pi_i(c) \) is \( c \)'s price.

Customers incur costs every time they switch between two separately purchased paths in a bundle: a customer traveling on \( c \) = \((p_1, \ldots, p_n)\) experiences a cost of \( s(t, p_k, p_{k+1}) \) at the \( k \)th switch from path \( p_k \) to \( p_{k+1} \); \( s(t, c) = \sum_{k=1}^{n-1} s(t, p_k, p_{k+1}) \) is the total cost of bundling among the paths of bundle \( c \). Then, \( e(t, c) = \sum_{i \in I} e_i(t, c_i) + s(t, c) \) is the customer cost along \( c \). Customers choose the bundle with the least total price: \( \pi(c) + e(t, c) \). To explain how demand is shared between the two bundles that have the same least total price, the definition of a market share rule is generalized.

A market share rule allowing bundling, \( \bar{r}(\cdot) \), specifies the share of traffic for market \( m \) served by each bundle \( c \) as a function of network designs and price schedules of all firms. It is assumed that market shares are always non-negative, and for each market, shares for all bundles sum to 1. If a bundle does not have the least total price, it does not serve the market. We also assume that if a bundle has the least total price for a market, but its total cost is greater than some other least total price bundle, it does not serve the market. The rationale for these assumptions follows those of market share rules.

Given a market share rule \( r \), the profit for firm \( i \) can now be written as a function of the network designs and price schedules of all firms:

\[
Z_i'(t_1, \Pi_1, \ldots, t_I, \Pi_I) = \sum_{m \in M} \sum_{c \in C(m)} [\pi_i(c) - v_i(t_i, c)]\rho(m)\bar{r}(\cdot) - F_i(t_i).
\]

If firms adopt strategies \((t_1, \Pi_1, \ldots, t_I, \Pi_I)\), profit for firm \( i \) is

\[
Z_i'(t_1, \Pi_1, \ldots, t_I, \Pi_I) = \sum_{m \in M} \sum_{c \in C(m)} [\pi_i(t, c) - v_i(t_i, c)]\rho(m)\bar{r}(\cdot) - F_i(t_i).
\]

The solution to the competitive situation is now a vector of network designs and price schedule strategies satisfying (2-2) and (2-3), using payoff functions (5-2).

If bundling costs are so high that for any origin and destination pair, there exists a single firm with a path connecting these nodes that has a lower total cost than any bundle, then bundling never occurs in equilibrium. However, if this is not the case, bundling can occur and firm profit is a discontinuous, non concave function of price schedules. Then the uniqueness or even existence of equilibrium price schedules cannot be assured. This is demonstrated through examples in the next two sections.

If equilibrium price schedules do not exist or are not unique, firm profit is not a well defined function of network designs. Therefore, firms' equilibrium choices of network designs are undefined, since each firm is unsure of its profits. Thus, analysis of network design and price competition allowing bundling is fundamentally different from network design problems that ignore bundling.

Although equilibrium price schedules allowing bundling may not be unique or even exist, Section 5.3 shows in duopoly competition, equilibrium price schedules allowing bundling always exist, but may not be unique. If duopolists choose network designs anticipating that the least favorable equilibrium price schedule will occur, then a new game is defined. The equilibrium network design for this game is any equilibrium network design of the original game which ignores bundling, as described in Section 4.

5.1. An Equilibrium Price Schedule Allowing Bundling May Not Exist

First, it is shown that an equilibrium price schedule may not exist if bundling is allowed. Lack of existence occurs when for some market, a firm must price below the equilibrium price that ignores bundling to prevent a competitor from undercutting its price and inducing bundling. When this happens the competitor serves other markets and prices above the amount needed to induce bundling. These prices cannot form an equilibrium price
schedule: the firm can raise its prices for the market and increase its profit, contradicting the assumption of a price equilibrium. The following example demonstrates this phenomenon.

**Example 5.1.** Consider the three node network of Figure 4; denote the arcs and markets as in Example 3.1, and suppose $\rho(m_1) = 100$, $\rho(m_2) = 10$, and $\rho(m_3) = 10$ are the only nonzero transport demands.

Let three firms, $I = \{1, 2, 3\}$, compete in this market. Denote the paths $p_1 = \{a_1\}$, $p_2 = \{a_2\}$, and $p_3 = \{a_3\}$. Fix the firms network designs and assume that the firms have the following variable costs:

<table>
<thead>
<tr>
<th>Firm</th>
<th>$v_1(t_i, p_1)$</th>
<th>$v_1(t_i, p_2)$</th>
<th>$v_1(t_i, p_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.0</td>
<td>4.8</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>2.0</td>
<td>1.9</td>
<td>2.9</td>
</tr>
<tr>
<td>3</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Assume omitted variable costs are higher than the others in each column and may be ignored. Assume that costs on path $\{a_2, a_3\}$ are also high enough to be ignored. For ease, assume that $e_i(t, p_j) = 0$ for all firms $i$ and all paths $p_j$ and that bundling costs are zero. Figure 4 indicates the total cost for relevant paths.

If the possibility of bundling is ignored, then market $m_1$ is served by firm 1, which prices at 4.8 on $p_1$; market $m_2$ is served by firm 2, which prices at 2 on $p_2$; and market $m_3$ is served by firm 3, which prices at 2.9 on $p_3$. Firm profits are $Z_1 = 180$, $Z_2 = 10$, and $Z_3 = 9$. However, this is not a price equilibrium if bundling is allowed.

With these prices, firm 3 has incentive to lower its price on $p_3$ to induce consumers to switch from $m_1$, directing traffic onto the bundle $(p_2, p_3)$ where $p_2$ is provided by firm 1 and $p_3$ is provided by firm 2. For example, if firm 3 prices at 4.8 - $0.8 = 2.0$ on $p_3$, bundling occurs for $\varepsilon > 0$. Firms 3's profit increases to $110(0.8 - \varepsilon) = 88 - 110\varepsilon$, which is greater than its previous profit for small enough $\varepsilon$. Understanding this, firm 1 can serve market $m_1$ by pricing low enough on $p_1$ so that firm 3 does not have incentive to undercut firm 1's price. The highest price firm 1 can set and not give firm 3 incentive to undercut is to price at $\pi_1(t_1, p_1) = 2.0 + \pi_3$, where $10(2.9 - 2.0) = 110(\pi_3 - 2.0)$, or $\pi_3 = 2(4/10)$. If $\pi_1(t_1, p_1) = 4(9/10)$, firms 2 and 3 have no incentive to induce bundling and will price to serve $m_2$ and $m_3$, respectively: firm 2 prices at 2.0 on $p_2$ and firm 3 prices at 2.9 on $p_3$.

Firm 1's price, $\pi_1(t_1, p_1)$, is not the optimal price against firm 2 and 3's prices. Firm 1 prefers to raise its price on $p_1$ to 4.8. However, if $\pi_1(t_1, p_1) > 4(9/10)$, firm 3 prefers to reduce its price on $p_3$ to induce bundling of $m_1$'s demand onto the bundle. Thus, no equilibrium exists.

**5.2. An Equilibrium Price Schedule May Exist but May Not Be Unique**

The next example demonstrates that even if equilibrium prices exist, there may not be a unique equilibrium price schedule allowing bundling.

**Example 5.2.** Again consider the three node network and network notation of Example 5.1. Let two firms, $I = \{1, 2\}$, compete in this market. Assume that the firms have the following variable costs:

<table>
<thead>
<tr>
<th>Firm</th>
<th>$v_1(t_i, p_1)$</th>
<th>$v_1(t_i, p_2)$</th>
<th>$v_1(t_i, p_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>2</td>
<td>4.8</td>
<td>1.9</td>
<td>2.9</td>
</tr>
</tbody>
</table>

For ease, assume that $e_i(t, p) = 0$ for all firms $i$ and all paths $p$, and that bundling costs are zero. Figure 5 indicates the total costs for relevant paths. If bundling is ignored, equilibrium price schedules are $\pi(t_1, p_1) = 4.8$, $\pi(t_1, p_2) = 2$, and $\pi(t_1, p_3) = 2.9$ for both firms. Firms earn $Z_1 = 89$ and $Z_2 = 1$.

If bundling is allowed, these prices are an equilibrium price schedule and no bundling occurs. For any $\delta_1, \delta_2$ such that $9/10 > \delta_1 > 89/110$, $1/2 > \delta_2 > 1/10$, and $\delta_1 + \delta_2 < 1$, an equilibrium price schedule exists and is given by

$$
\pi_1(t_1, p_1) = 3.9 + \delta_1 + \delta_2, \quad \pi_1(t_1, p_2) = 2, \\
\pi_1(t_1, p_3) = 2 + \delta_1, \quad \text{and} \\
\pi_2(t_2, p_1) = 3.9 + \delta_1 + \delta_2, \\
\pi_2(t_2, p_2) = 1.9 + \delta_2, \quad \text{and} \\
\pi_2(t_2, p_3) = 2.9.
$$

Firms now earn: $Z_1 = 110\delta_1$, and $Z_2 = 110\delta_2$.

Bundling occurs at all these equilibria. The constraints $\delta_1 \geq 89/110$ and $\delta_2 \geq 1/10$ guarantee that each firm earns at least as much by inducing bundling as by not, and the constraints $\delta_1 \leq 9/10$ and $\delta_2 \leq 1/10$ guarantee prices are an equilibrium. There is an equilibrium price schedule correspond-
ing to any pair of firm profits \((Z_1, Z_2)\) such that \(99 > Z_1 > 89, 11 > Z_2 > 9\).

In this example there is a Pareto dominating outcome for the firms (an equilibrium with associated payoff strictly preferred to all other equilibria) with payoffs \(Z_1 = 99\) and \(Z_2 = 11\). The price equilibrium is \(\pi_1(t_1, p_1) = 4.9, \pi_2(t_2, p_2) = 2\), and \(\pi_i(t_i, p_3) = 2.9, i = \{1, 2\}\). At this equilibrium the price for \(m_1\) is 4.9 which is strictly greater than the total cost for either firm!

For this two-firm game, multiple price equilibria with bundling exist. However, there is a single equilibrium that is strictly preferred by both firms. In general, even if there are multiple equilibrium price schedules, there may not be an equilibrium that is preferred by all firms. Changing the data of the last example slightly, so that; \(v_2(t_2, p_1) = 4.0\) demonstrates this. Now for any \(\delta_1, \delta_2\) such that \(\delta_1 > \gamma/120, \delta_2 > \gamma/120\) and \(\delta_1 + \delta_2 < \gamma/30\), an equilibrium price schedule inducing bundling exists: for both firms \(\pi_1(t_i, p_1) = 4.0, \pi_2(t_2, p_3) = 1.9 + \delta_1, \pi_i(t_i, p_3) = 2 + \delta_2\); firm profits are \(Z_1 = 110\delta_1\) and \(Z_2 = 110\delta_2\), and no equilibrium price schedule is strictly preferred by the firms to any other.

5.3. Equilibrium Price Schedules and Network Designs of Duopolists

Next, special equilibrium properties of duopoly competition are explored. Assume that the cost of bundling between paths provided by the two firms is greater than or equal to the cost of bundling between the same paths provided by a single firm. This is restrictive but not unreasonable.

**Assumption 5.1.** Suppose \((p_1, p_2)\) is a bundle connecting two nodes. Let \(p_1\) be provided by firm \(i\) and \(p_2\) be provided by firm \(j\). For all \(k \in \{i, j\}\), if \(p\) is the path using the same arcs as \((p_1, p_2)\) but provided by firm \(k\), then \(s(t, (p_1, p_2)) > e_k(t, p) - e_k(t, p_2)\).

**Theorem 5.1.** Consider a transportation market with only two firms and let Assumption 5.1 hold. Equilibrium price schedules ignoring bundling are equilibrium price schedules allowing bundling but bundling does not occur. Equilibrium price schedules allowing bundling are generally not unique and firm profits are different at different equilibrium price schedules.

The equilibrium price schedules ignoring bundling give each firm its minimum profits in equilibrium. Therefore, if each firm chooses a network design anticipating that the least favorable equilibrium price schedule will arise, then equilibrium network designs are equilibrium network designs for the original game which ignores bundling.

The proof is given in the appendix. This theorem shows that analysis of duopoly competition allowing bundling can ignore bundling if it is assumed that firms choose network designs to maximize their minimum profits. For this new game, an equilibrium price schedule always exists and an equilibrium network design does too if Assumption 4.1 holds.

These results contrast with the potential absence of an equilibrium price schedule with more than two firms. Even if an equilibrium price schedule exists, the least any firm receives in equilibrium can be less than its equilibrium payoff if bundling is ignored. Thus, analyzing price competition in networks by ignoring bundling may overestimate firm profit if bundling is possible and occurs in equilibrium.

6. Conclusions

Although our model is simple, a number of important economic results are presented. This paper shows that a profit maximizing firm's network design should not be chosen to minimize its costs but, surprisingly, the industry's total cost (both firms' and customers') under the restrictive assumption that congestion costs are not important. Under this assumption, the equilibrium network designs chosen in real world markets are those that minimize total industry costs. This shows that competition in networks can lead to socially efficient network configurations. Because equilibrium network designs minimize total industry cost, the configuration of equilibrium networks are sensitive to the relative magnitudes of firm and customer costs. To properly predict industry structure, the relative magnitudes of the cost components must be estimated. A test of the model is to see whether observed network structures minimize industry costs: for example, are hub and spokes really the cost minimizing configurations for passenger airlines? Further, we have gained some
in sight why prices are unstable in some competitive, network-based transportation markets: when customers have the option to bundle, equilibrium prices may not exist or, if they do exist, may not be unique. This result can be shown to hold in markets where demand is price elastic and costs are not linear in volume.

APPENDIX

Proof of Theorem 5.1

Denote the firms I = {1, 2} and their network designs t. Consider any market, say m = (n₁, n₂). For this market, let firm 1’s (firm 2’s) least cost path be p₁(p₂). Without loss of generality assume v₁(t, p₁) + e₁(t, p₁) < v₂(t₂, p₂) + e₂(t, p₂).

Let the set of arcs on path p₁ be denoted (n₁₁, n₁₂, n₁₃, ..., n₁b, n₁m) and those on p₂ by (n₂₁, n₂₂, n₂₃, ..., n₂c, n₂m), where n₁₁ = n₂₁ = n₁ and n₁m = n₂m = n₂. We write

S₁ = \{k|v₁(t, (n₁k, n₁k+1)) + e₁(t, (n₁k, n₁k+1))
\leq v₂(t, (n₁k, n₁k+1)) + e₂(t, (n₁k, n₁k+1)),

k = 1, ..., b - 1\},

\bar{S}_1 = \{k|v₁(t, (n₁k, n₁k+1)) + e₁(t, (n₁k, n₁k+1))
> v₂(t, (n₁k, n₁k+1)) + e₂(t, (n₁k, n₁k+1)),

k = 1, ..., b - 1\},

S₂ = \{k|v₂(t, (n₂k, n₂k+1)) + e₂(t, (n₂k, n₂k+1))
\leq v₁(t, (n₂k, n₂k+1)) + e₁(t, (n₂k, n₂k+1)),

k = 1, ..., c - 1\},

\bar{S}_2 = \{k|v₂(t, (n₂k, n₂k+1)) + e₂(t, (n₂k, n₂k+1))
> v₁(t, (n₂k, n₂k+1)) + e₁(t, (n₂k, n₂k+1)),

k = 1, ..., c - 1\},

S₁(\bar{S}_2) contains the indices of the arcs of p₁(p₂) on which firm 1 (firm 2) has the total cost advantage. \bar{S}_1(\bar{S}_2) are indices of the arcs of p₁(p₂) on which firm 2 (firm 1) has the strict total cost advantage.

Consider the bundle c’ made up of arcs where firm 1 provides arcs with indices in S₁ and firm 2 provides arcs with indices in \bar{S}_1 and let (t, c) be the bundling cost for this bundle. If firm 1 prices at v₁(t₁, p₁) + e₁(t₁, p₁) on p₁ and prices at v₂(t₂, (n₁k, n₁k+1)) + e₂(t, (n₁k, n₁k+1)) - e₁(t, (n₁k, n₁k+1)) on each arc of S₁, then to induce demand for bundle c, firm 2 must price at a total of v₂(t₂, p₂) + e₂(t, p₂) - \Sigma_{k \in S₁}[v₂(t₂, (n₁k, n₁k+1)) + e₂(t, (n₁k, n₁k+1))] - s(t, c) on arcs in S₁. Firm 2 will have a contribution per unit of traffic m of at most v₂(t₂, p₂) + e₂(t, p₂) - \Sigma_{k \in S₁}[v₂(t₂, (n₁k, n₁k+1)) + e₂(t, (n₁k, n₁k+1))] - s(t, c). The total of the two summations is the total cost for firm 2 of path p₁ less the customers’ cost of bundling between the elements of S₁ and \bar{S}_1 when offered by firm 2 alone. By Assumption 5.1, the customers’ cost of bundling between elements S₁ and \bar{S}_1 is less than s(t, c). The contribution per unit of traffic m served is non positive since the total cost of path p₁ is less than or equal to the total cost of path of p₂. Therefore, firm 2 does not have an incentive to induce demand for this bundle when firm 1 sets equilibrium price schedules that ignore bundling. Firm 2’s best response to firm 1’s prices is to choose prices for p₂ that are equilibrium price schedules that ignore bundling.

Consider the bundle c’ made up of arcs where firm 2 provides service on arcs with indices in \bar{S}_2 and firm 1 provides arcs with indices in S₂ and let s(t, c’) be the bundling cost for this bundle. If firm 2 prices at v₁(t₁, (n₂k, n₂k+1)) + e₁(t, (n₂k, n₂k+1)) - e₂(t, (n₂k, n₂k+1)) on each arc in S₁ (these are the equilibrium prices that ignore bundling), firm 1 must price at a total of v₁(t₁, p₁) + e₁(t, (n₂k, n₂k+1)) - \Sigma_{k \in S₁}[v₁(t₁, (n₂k, n₂k+1)) + e₁(t, (n₂k, n₂k+1))] - s(t, c’) or less on arcs in S₂ to induce demand for c’. Firm 1 earns contribution to profit per unit of m transported of at most v₂(t₂, p₂) + e₂(t, p₂) - \Sigma_{k \in S₂}[v₁(t₁, (n₂k, n₂k+1)) + e₁(t, (n₂k, n₂k+1))] - s(t, c’). But firm 1 can earn a profit per unit of m transported of v₂(t₂, p₂) + e₂(t, p₂) - v₁(t₁, p₁) - e₁(t, p₁) by using its equilibrium prices that ignore bundling. The difference in these unit profits is v₁(t₁, p₁) - e₁(t, p₁) - \Sigma_{k \in S₁}[v₂(t₂, (n₁k, n₁k+1)) + e₂(t, (n₁k, n₁k+1))] - \Sigma_{k \in S₂}[v₁(t₁, (n₂k, n₂k+1)) + e₁(t, (n₂k, n₂k+1))] - s(t, c’), which is non positive, because p₁ is firm 1’s least cost path for m, and Assumption 5.1 holds. Therefore firm 1 has no incentive to induce demand for c’. For m: firm 1’s optimal response to firm 2’s use of equilibrium price schedules that ignore bundling is to set the equilibrium price schedules that ignore bundling on p₁. This argument can be generalized to any other bundle for m.

These arguments may be repeated for every market in M. It can be concluded that equilibrium price schedule prices ignoring bundling are equilibrium prices allowing bundling. Using these prices, no bundling occurs, since neither firm has an incentive to set prices that induce bundling.

Denote equilibrium price schedules that ignore bundling by (π₁, π₂). Other equilibrium price schedules allowing bundling may exist, call one (π₁, π₂). By definition of an equilibrium price sched-
ule, total profit for firm 1 is higher than its profit if it used its equilibrium price schedule that ignores bundling, \( \pi_1^* \); \( Z_1^*(t_1, \pi_1^*, t_2, \pi_2) > Z_1^*(t_1, \pi_1, t_2, \pi_2) \). Pick any \( m \). If firm 1 has the total cost advantage for \( m \), the proof has shown that if firm 1 uses \( \pi_1^* \), then \( \pi_2^* \) cannot induce bundling on \( m \) unless firm 2 prices below variable cost. By assumption, firm 2 does not price below variable cost. Therefore firm 1 will serve \( m \) if prices \( (\pi_1^*, \pi_2^*) \) are used, and earn the same contribution on \( m \) as if \( (\pi_1^*, \pi_2^*) \) were used.

For any \( m \), if firm 2 has the cost advantage for \( m \), bundling may occur when firms use prices \( (\pi_1^*, \pi_2) \). Therefore, firm 1 may earn positive contribution on \( m \). Summing the contributions for all markets \( m \) we may conclude: \( Z_1^*(t_1, \pi_1^*, t_2, \pi_2) \geq Z_1^*(t_1, \pi_1^* t_2, \pi_2) \). Recalling that \( Z_1^*(t_1, \pi_1^*, t_2, \pi_2) \) \( \geq Z_1^*(t_1, \pi_1^* t_2, \pi_2) \) implies \( Z_1^*(t_1, \pi_1^*, t_2, \pi_2) \geq Z_1^*(t_1, \pi_1^*, t_2, \pi_2) \). We see that \( (\pi_1^*, \pi_2^*) \) yields the lowest profits for all equilibrium prices.

If firms choose network designs anticipating that the equilibrium price schedule least favorable to them will arise, a new game is defined. For this game, the firms' strategies are network designs and the firms' payoffs are just the payoffs for the original game where bundling is ignored. The new game has equilibrium network designs that are the same as the equilibrium network designs for the original game that ignored bundling.

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