Guest Lecture:

The Rational Expectations Approach to the Cross Section of Returns

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Applying rational expectations economics to the cross section of returns

- Facts
- Equilibrium cross section of returns
- The value premium
- Anomalies
- Future
Anomalies:

Observed characteristic-return relations in the cross section not predicted or captured by current asset pricing models

Rational expectations versus behavioral finance:

\[
\hat{r}_{t+1} = E_t[r_{t+1}] + \epsilon_{t+1}
\]

I apply rational expectations economics to derive and test expected-return models
Facts: Empirical Asset Pricing

- The value anomaly

\[(\text{Market/Book equity})_t \uparrow \Rightarrow \bar{r}_{t+1} \downarrow\] Stronger in small firms

Fama and French (1992, 1993); Lakonishok, Shleifer, and Vishny (1994)

- The investment anomaly

\[(\text{Investment/Asset})_t \uparrow \Rightarrow \bar{r}_{t+1} \downarrow\]

Titman, Wei, and Xie (2004); Anderson and Garcia-Feijóo (2005); Polk and Sapienza (2005); Xing (2005)
The seasoned-equity-offering underperformance anomaly

\[
(\text{Seasoned equity}/\text{Asset})_t \uparrow \Rightarrow \bar{r}_{t+1} \downarrow \quad \text{Stronger in small firms}
\]


The payout anomaly

\[
(\text{Payout}/\text{Asset})_t \uparrow \Rightarrow \bar{r}_{t+1} \uparrow \quad \text{Stronger in value firms}
\]

Lakonishok and Vermaelen (1990); Ikenberry, Lakonishok, and Vermaelen (1995); Michaely, Thaler, and Womack (1995)
Facts: Capital Markets Research in Accounting

- The post-earnings-announcement drift

  \[(\text{Earnings surprise})_t \uparrow \Rightarrow \bar{r}_{t+1} \uparrow\]  
  \[\text{Stronger in small firms}\]

  Ball and Brown (1968); Bernard and Thomas (1989, 1990)

- The profitability/expected-profitability anomalies

  \[\text{Profitability}_t \uparrow \Rightarrow \bar{r}_{t+1} \uparrow\]  
  \[\text{E}_t[\text{Profitability}_{t+1}] \uparrow \Rightarrow \bar{r}_{t+1} \uparrow\]  
  \[\text{Stronger in small firms}\]

  Haugen and Baker (1996); Frankel and Lee (1998); Piotroski (2000); Cohen, Gompers, and Vuolteenaho (2002); Fama and French (2005)
Prescott (2004, Prize lecture): “Prior to the transformation, macroeconomics was largely separate from the rest of economics. Indeed, some considered the study of macroeconomics fundamentally different and thought there was no hope of integrating macroeconomics with the rest of economics, that is, with neoclassical economics.”

Empirical finance is like macro 60 years ago:

Koopmans (1947): “Measurement without theory”

Time for the Lucas-Prescott neoclassical transformation in finance
Methodology


- Surprise — characteristics can be sufficient statistics of expected returns!

\[ r_{ft} + \beta_{Mt} \lambda_{Mt} = E_t[r_{t+1}^S] = E_t[r_{t+1}^I] \]

Consumption-based asset pricing        Investment-based asset pricing

- \( \beta_{Mt} = \frac{E_t[r_{t+1}^I] - r_{ft}}{\lambda_{Mt}} \) — relative measurement errors
Gomes, Kogan, and Zhang (2003, Journal of Political Economy)

The first DSGE model with the cross section of returns

Using the Kydland-Prescott (1982) methodology, we find:

- The cross-sectional predictability associated with size and book-to-market
- The predictability subsists after controlling for empirical estimates of beta
- Empirical success of size and book-to-market can be consistent with the CAPM
The model is not neoclassical, however

Market-clearing requires **aggregation** across the cross section of returns

We assume all firms have equal amount of growth options

This assumption allows analytical aggregation

But:

- Value and growth firms have same amount of growth options
- Growth firms are more profitable, but cannot invest more
- Growth firms pay out more, and have long cash-flow duration than value firms

**Counterfactual economic mechanism underlying the simulated value premium**
The Value Premium

- Relax the equal-growth assumption, but sacrifices general equilibrium

Contribution

- A neoclassical explanation of the value premium:
  - Asymmetry causes countercyclical risk of value-minus-growth strategy
  - Countercyclical price of risk interacts with and propagates the effect
- Endogenous cross section of returns with aggregate uncertainty
Adjustment costs increase risk in a production economy. Jermann (1998):

- Capital adjustment helps the firms smooth cash flow streams
- Adjustment costs are the offsetting force of changing capital

In the data and in the model, high book-to-market (value) signals sustained low profitability, and low book-to-market (growth) signals sustained high profitability
Asymmetry:

- **Adjustment cost**

- **VALUE**

- **GROWTH**

- **Investment-to-capital**

- **Time-varying risk** — value is riskier than growth in bad times, and growth is riskier than value in good times, but to a somewhat lesser extent
Propagation effects of countercyclical price of risk

Strengthen the time-varying risk pattern

A high average value premium can coexist with a low average risk difference

\[
\text{Value Premium} = \text{Risk Difference} \times \text{Price of Risk}
\]

<table>
<thead>
<tr>
<th></th>
<th>Risk Difference</th>
<th>Price of Risk</th>
<th>Value Premium</th>
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<tbody>
<tr>
<td>good times</td>
<td>slightly negative</td>
<td>low</td>
<td>slightly negative</td>
</tr>
<tr>
<td>bad times</td>
<td>high</td>
<td>high</td>
<td>very high</td>
</tr>
<tr>
<td>on average</td>
<td>low</td>
<td>average</td>
<td>high</td>
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The Model

- The Hopenhayn (1992) IE model augmented with aggregate uncertainty
  - See a recent GE extension by Gala (2005)

- Production function:

  \[ y_{jt} = e^{(x_t + z_{jt})k_{jt}^\alpha} \]

  where \( x_t \): aggregate productivity; \( z_{jt} \): idiosyncratic productivity

- Industry demand:

  \[ e^{p_t} = Y_t^{-\eta} \]

  where \( 0 < \eta < 1 \) is the inverse price elasticity of demand.
Exogenous stochastic discount factor:

\[ \log M_{t+1} = \log \beta + \gamma (x_t - x_{t+1}) \]

\[ \gamma = \gamma_0 + \gamma_1 (x_t - \bar{x}) \quad \gamma_1 < 0 \]

Reflects the focus on the firm side, not consumer side

Suppose a representative agent with isoelastic utility:

\[ \log M_{t+1} = \log \beta + \sigma (c_t - c_{t+1}) \]

\[ c_t \approx a + bx_t \]

\[ \Rightarrow \quad \gamma \approx \sigma b \]

Time-varying risk aversion generates countercyclical market price of risk
Value-Maximization of firms:

\[
v(k_t, z_t; x_t, p_t) = \max_{\{i_t\}} \left\{ \begin{array}{c}
\text{Current Period Dividend} \\
\text{Expected Continuation Value}
\end{array} \right.
\]

\[
e^{x_t + z_t + p_t k_t^\alpha - f - i_t - c(i_t, k_t)} + M_{t+1} v(k_{t+1}, z_{t+1}; x_{t+1}, p_{t+1})
\]

subject to : \quad k_{t+1} = i_t + (1 - \delta) k_t

The adjustment-cost function is asymmetric and quadratic:

\[
c(i_t, k_t) = \frac{\theta_t}{2} \left( \frac{i_t}{k_t} \right)^2
\]

where \(\theta^- \geq \theta^+ > 0\) and \(\theta_t = \theta^+ \cdot \chi_{\{i_t \geq 0\}} + \theta^- \cdot \chi_{\{i_t < 0\}}\)
Aggregation

The law of motion of the cross-sectional distribution of firms, $\mu_t$, is:

$$\mu_{t+1}(\Theta; x_{t+1}) = T(\Theta, (k_t, z_t); x_t)\mu_t(k_t, z_t; x_t)$$

where

$$T(\Theta, (k_t, z_t); x_t) \equiv \iint \chi\left\{(k_{t+1}, z_{t+1}) \in \Theta\right\} Q_z(dz_{t+1}|z_t)Q_x(dx_{t+1}|x_t)$$

Industry output:

$$Y_t \equiv \iint y(k_t, z_t; x_t) \mu_t(dk, dz; x_t)$$
Equilibrium

A recursive competitive equilibrium is composed of: (i) a law of motion for the cross-sectional distribution of firms; (ii) a value function and a set of investment decision rules; and (iii) an output price, such that:

- Optimality conditions hold.
- Consistency conditions hold.
- Market clears
Solution

- The endogenous state variable, $p_t$, depends upon the firm distribution, $\mu_t$

- Use approximate aggregation: Krusell and Smith (1998):

$$p_{t+1} = a_1 + a_2 p_t + a_3 (x_t - \bar{x})$$

- Solve the firms’ problem by the value function iteration method.

- Simulate the economy with 5,000 firms for 12,000 monthly periods.

- Use the data in the stationary region to update the coefficients $a_1, a_2,$ and $a_3$.

- Check convergence; if yes, go to next step; otherwise go back to step 2.

- Check goodness-of-fit; if yes, done; otherwise try a different specification.
Anomalies

Zhang (2006, Manuscript)

A neoclassical theory of asset pricing anomalies with rational expectations

Contribution

Unify many anomalies using one single, analytical framework

Propose a theoretically-motivated, easy-to-implement asset pricing test
The Model

- The basic idea: linking characteristics to expected returns analytically
  - Cochrane (1991) and Berk, Green, and Naik (1999)

- Two periods, $t$ and $t+1$

- The operating-profit function, $\Pi(K_t, X_t)$

- Firms invest in period $t$, produce in $t$ and $t+1$, liquidate at the end of $t+1$

- $K_{t+1} = (1 - \delta)K_t + I_t$, adjustment costs $(a/2)(I_t/K_t)^2 K_t$

- Exogenous $M_{t+1}$ correlates with the aggregate component of $X_{t+1}$
The market value of the firm

\[
\max_{I_t} \left\{ \Pi(K_t, X_t) - I_t - \frac{a}{2} \left[ \frac{I_t}{K_t} \right]^2 K_t + \mathbb{E}_t \left[ M_{t+1} \left( \Pi(K_{t+1}, X_{t+1}) + (1 - \delta)K_{t+1} \right) \right] \right\} \\
\text{Payout/Outside equity at period } t
\]

The first-order condition with respect to \( I_t \):

\[
1 + a \left[ \frac{I_t}{K_t} \right] = \mathbb{E}_t \left[ M_{t+1} \left( \frac{\Pi_1(K_{t+1}, X_{t+1}) + (1 - \delta)}{1 + a (I_t/K_t)} \right) \right]
\]

Marginal cost of investment at time \( t \)

\[
\mathbb{E}_t[M_{t+1} r^I_{t+1}] = 1;
\]

Investment return

\[
r^S_{t+1} = r^I_{t+1} \text{ based on Hayashi (1982)}
\]
The investment anomaly:

- Low investment-to-asset firms
- High investment-to-asset firms

\[ E_t[r_{t+1}] \]

Cochrane (1991)
Brealey, Myers, and Allen (2006, Chapter 6)
The value anomaly:

\[
1 + a \left( \frac{I_t}{K_t} \right) = \sqrt[\alpha]{q_t} = \sqrt[\beta]{Q_t}
\]

Marginal cost of investment

Marginal benefit of investment

Market-to-book

\[E_t[r_{t+1}]\]

Growth firms invest more than value firms

Fama and French (1995, Table I)
The seasoned-equity-offering anomaly:

\[
\text{Outside equity + operating profits} = \text{Investment + adjustment costs}
\]

- The sources of funds
- The uses of funds

\[
\mathbb{E}_t[r_{t+1}]
\]

Issuing firms invest more than nonissuers

Loughran and Ritter (1997, Figure 1)

Matching nonissuers

Issuing firms

\[
\frac{I_t}{K_t}
\]
The payout anomaly:

\[
\text{Operating profits} = \text{Payout + investment + adjustment costs}
\]

The sources of funds

The uses of funds

High payout firms invest less

High payout firms

Low payout firms

\[E_t[r_{t+1}]\]
Second-order effects:

The value anomaly is stronger in small firms; the payout anomaly is stronger in value firms; the SEO anomaly is stronger in small firms.

The investment-return relation is also convex, $\frac{\partial^2 E_t[r_{t+1}]}{\partial (I_t/K_t)^2} > 0$.

By the chain rule, the convexity leads to the second-order effects.

The investment-value-SEO-payout anomalies are basically the same phenomenon.
Fama (1998): the “granddaddy” of anomalies

Earnings + depreciation = Operating cash flow

The rate of depreciation
\[ \frac{N_{t+1}}{K_{t+1}} + \delta = \frac{\Pi_{t+1}}{K_{t+1}} \]

Marginal product of capital
\[ = \frac{\Pi_1(K_{t+1}, X_{t+1})}{Q_t} \]

Expected marginal product of capital
\[ E_t[r_{t+1}] = \frac{E_t[\Pi_1(K_{t+1}, X_{t+1})]}{1 + a(I_t/K_t)} + 1 - \delta \]

Expected profitability
\[ = \frac{E_t[N_{t+1}/K_{t+1}]}{Q_t} + 1 \]

The expected-profitability anomaly, the loading \(1/Q_t\) decreases in size, \(P_t\)
Post-Earnings-Announcement Drift

Profitability is highly persistent, Fama and French (1995, 2000, 2004)

\[
\frac{N_{t+1}/K_{t+1}}{N_t/K_t} = \frac{\bar{n}(1 - \rho_n) + \rho_n(N_t/K_t)}{1} + \frac{\bar{n}(1 - \rho_n)}{1} + \frac{\rho_n}{1} + \frac{\epsilon_{t+1}}{1}
\]

Profitability↑ ⇒ Expected profitability↑ ⇒ Expected return↑

\[
E_t[r_{t+1}] = \frac{1}{Q_t} [\rho_n(N_t/K_t) + \bar{n}(1 - \rho_n) + 1]
\]

Earnings surprise↑ ⇒ Profitability↑ ⇒ Expected return↑

\[
E_t[r_{t+1}] = \frac{1}{Q_t} [\bar{n}(1 - \rho_n)(1 + \rho_n) + \rho_n^2(N_{t-1}/K_{t-1}) + \rho_n \epsilon_t + 1]
\]
The theory implies a new, structural expected-return model

\[ E \left[ r_{t+1}^S - \frac{\Pi_1(K_{t+1}, X_{t+1}) + (1 - \delta)}{1 + a(I_t/K_t)} \right] \otimes \mathbb{Z}_t = 0 \]

- Add multi-period, operating costs, and leverage
- See Whited and Zhang (2006, Testing the \( q \)-theory of anomalies)
Summary

- A neoclassical theory of asset pricing anomalies with rational expectations:

\[ \mathbb{E}_t[r_{t+1}] = \mathbb{E}_t[\Pi_1(K_{t+1}, X_{t+1})] + 1 - \delta \]

\[ \frac{1 + a (I_t/K_t)}{1 + a (I_t/K_t)} \]

The earnings-return relation via the “cash-flow” channel

The investment-value-SEO-payout anomalies via the “discount-rate” channel

- A theoretically motivated, easy-to-implement expected-return model
The Future

- The neoclassical transformation of the cross section of returns has only started
- Most of the qualitative analysis needs to be quantified
- Model other corporate decisions such as mergers and acquisitions; debt and corporate savings, research and development; corporate governance; and accruals
- Applying dynamic, quantitative tools in asset pricing and corporate finance
- Corporate finance is even more abysmal than asset pricing
- The neoclassical transformation: Wave of the future!