Anomalies

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A unified theory of asset pricing anomalies based on value maximization of firms

Outline

- Anomalies
- Perspective
- Model
- Intuition
Empirical Asset Pricing

- The value anomaly

\[(\text{Market/Book equity})_t \uparrow \Rightarrow \bar{r}_{t+1} \downarrow\]  
Stronger in small firms

Fama and French (1992, 1993); Lakonishok, Shleifer, and Vishny (1994)

- The investment anomaly

\[(\text{Investment/Asset})_t \uparrow \Rightarrow \bar{r}_{t+1} \downarrow\]

Titman, Wei, and Xie (2004); Anderson and Garcia-Feijóo (2005); Polk and Sapienza (2005); Xing (2005)
Empirical Corporate Finance

- The seasoned-equity-offering anomaly

\[(\text{Seasoned equity}/\text{Asset})_t \uparrow \Rightarrow \bar{r}_{t+1} \downarrow \quad \text{Stronger in small firms}\]


- The payout anomaly

\[(\text{Payout}/\text{Asset})_t \uparrow \Rightarrow \bar{r}_{t+1} \uparrow \quad \text{Stronger in value firms}\]

Lakonishok and Vermaelen (1990); Ikenberry, Lakonishok, and Vermaelen (1995); Michaely, Thaler, and Womack (1995)
The post-earnings-announcement drift

\[(Earnings\ surprise)_{t} \uparrow \Rightarrow \bar{r}_{t+1} \uparrow\]  
Stronger in small firms

Ball and Brown (1968); Bernard and Thomas (1989, 1990)

The profitability/expected-profitability anomalies

\[Profitability_{t} \uparrow \Rightarrow \bar{r}_{t+1} \uparrow\]  
\[E_{t}[Profitability_{t+1}] \uparrow \Rightarrow \bar{r}_{t+1} \uparrow\]  
Stronger in small firms

Haugen and Baker (1996); Frankel and Lee (1998); Piotroski (2000); Cohen, Gompers, and Vuolteenaho (2002); Fama and French (2005)
Anomalies — empirical characteristic-return relations not predicted or captured by existing expected-return models

Two lines of investigation:

\[ \hat{r}_{t+1} = \mathbb{E}_t[r_{t+1}] + \epsilon_{t+1} \]

I focus on deriving and testing structural expected-return models
Why structural models? Moving beyond the Fama-French (1993) model

“[J]ust because a factor model happens to work well does not necessarily mean that we are learning anything about the economic drivers of average returns” (Barberis and Thaler (2003, p. 1091)).

“We hope future research will help us understand why the market appears to overreact in some circumstances and underreact in others” (Michaely, Thaler, and Womack (1995, p. 606), Fama (1998, p. 287)).
Model

- The basic idea: linking characteristics to expected returns analytically
  - Cochrane (1991) and Berk, Green, and Naik (1999)

- Two periods, $t$ and $t+1$

- The operating-profit function, $\Pi(K_t, X_t)$

- Firms invest in period $t$, produce in $t$ and $t+1$, liquidate at the end of $t+1$

- $K_{t+1} = (1 - \delta)K_t + I_t$, adjustment costs $(a/2)(I_t/K_t)^2 K_t$

- Exogenous $M_{t+1}$ correlates with the aggregate component of $X_{t+1}$
The market value of the firm

\[
\max_{\{I_t\}} \left\{ \Pi(K_t, X_t) - I_t - \frac{a}{2} \left[ \frac{I_t}{K_t} \right]^2 K_t + E_t \left[ M_{t+1} \left[ \Pi(K_{t+1}, X_{t+1}) + (1 - \delta)K_{t+1} \right] \right] \right\}
\]

Payout/Outside equity at period \(t\)

The first-order condition with respect to \(I_t\):

\[
1 + a \left[ \frac{I_t}{K_t} \right] = E_t \left[ M_{t+1} \left[ \frac{\Pi_1(K_{t+1}, X_{t+1}) + (1 - \delta)}{1 + a \left( \frac{I_t}{K_t} \right)} \right] \right]
\]

Marginal cost of investment at time \(t\)

\[
E_t[M_{t+1}r^{I}_{t+1}] = 1; \quad r^{I}_{t+1} \equiv \frac{\Pi_1(K_{t+1}, X_{t+1}) + (1 - \delta)}{1 + a \left( \frac{I_t}{K_t} \right)}
\]

Investment return

\(r^S_{t+1} = r^I_{t+1}\) based on Hayashi (1982)
Covariances versus Characteristics


- Surprise — characteristics can be sufficient statistics of expected returns!

\[
E_t\left[r_{t+1}^S\right] = E_t\left[r_{t+1}^I\right]
\]

Risk \quad \text{Expected returns} \quad \text{Characteristics}

\[
r_{ft} + \beta_{Mt} \lambda_{Mt} = E_t[r_{t+1}^S] = E_t[r_{t+1}^I]
\]

Consumption-based asset pricing \quad Investment-based asset pricing

- \[\beta_{Mt} = \frac{E_t[r_{t+1}^I] - r_{ft}}{\lambda_{Mt}}\] — relative measurement errors
The investment anomaly

Cochrane (1991)
Brealey, Myers, and Allen (2006, Chapter 6)
The value anomaly

\[
1 + a \left( \frac{I_t}{K_t} \right) = \frac{q_t}{Q_t} = \text{Marginal benefit of investment}
\]

\[
\text{Marginal cost of investment}
\]

Growth firms invest more than value firms

Fama and French (1995, Table I)
The seasoned-equity-offering anomaly

\[
\text{Outside equity + operating profits} = \text{Investment + adjustment costs}
\]

\[
\text{The sources of funds} = \text{The uses of funds}
\]

Issuing firms invest more than nonissuers

Loughran and Ritter (1997, Figure 1)
The payout anomaly

\[ \text{Operating profits} = \text{Payout} + \text{investment} + \text{adjustment costs} \]

The sources of funds = The uses of funds

\[ \mathbb{E}_t[r_{t+1}] \]

High payout firms invest less

High payout firms

Low payout firms

\[ \frac{I_t}{K_t} \]
Convexity

- Second-order effects:

  The value anomaly is stronger in small firms; the payout anomaly is stronger in value firms; the SEO anomaly is stronger in small firms

- The investment-return relation is also convex, \( \frac{\partial^2 E_t[r_{t+1}]}{\partial (I_t/K_t)^2} > 0 \)

- By the chain rule, the convexity leads to the second-order effects.

- The investment-value-SEO-payout anomalies are basically the same phenomenon
Fama (1998): the “granddaddy” of anomalies

Earnings + depreciation = Operating cash flow

\[
\frac{N_{t+1}}{K_{t+1}} + \delta = \frac{\Pi_{t+1}}{K_{t+1}}
\]

Profitability
Average product of capital

Margin product of capital

Expected marginal product of capital

Expected profitability

\[
E_t[r_{t+1}] = \frac{E_t[\Pi_1(K_{t+1}, X_{t+1})]}{1 + a(I_t/K_t)} + 1 - \delta = \frac{E_t[N_{t+1}/K_{t+1}]}{Q_t} + 1
\]

The expected-profitability anomaly, the loading \(1/Q_t\) decreases in size, \(P_t\)!
Post-Earnings-Announcement Drift

- Profitability is highly persistent, Fama and French (1995, 2000, 2004)

\[
\frac{N_{t+1}}{K_{t+1}} = \hat{n}(1 - \rho_n) + \rho_n(N_t/K_t) + \epsilon_{n, t+1}
\]

- Profitability\textsuperscript{↑} ⇒ Expected profitability\textsuperscript{↑} ⇒ Expected return\textsuperscript{↑}

\[
E_t[r_{t+1}] = \frac{1}{Q_t} [\rho_n(N_t/K_t) + \hat{n}(1 - \rho_n) + 1]
\]

- Earnings surprise\textsuperscript{↑} ⇒ Profitability\textsuperscript{↑} ⇒ Expected return\textsuperscript{↑}

\[
E_t[r_{t+1}] = \frac{1}{Q_t} [\hat{n}(1 - \rho_n)(1 + \rho_n) + \rho_n^2(N_{t-1}/K_{t-1}) + \rho_n\epsilon_{t} + 1]
\]
Tests

- Whited and Zhang (2005): Are these mechanisms \textbf{quantitatively} important?

- Estimate the expected-return model from the $Q$-theory through GMM:

\[
E \left[ \left( r_{t+1}^S - \frac{\Pi_1(K_{t+1}, X_{t+1}) + (1 - \delta) 1 + a(I_t/K_t)}{\text{the investment return}} \right) \otimes Z_t \right] = 0
\]

- Add multi-period, operating costs, and leverage
Testing portfolios: Fama-French 25, 10 investment-to-asset, 10 SUE, 9 size-SUE

Testing procedure:

- Estimate the parameters in the operating-profit and adjustment-cost functions
- Construct the investment returns using the parameter estimates
- Report the alphas and their associated $t$-statistics
The benchmark model, unconditional moments

<table>
<thead>
<tr>
<th></th>
<th>Fama-French 25</th>
<th>SUE</th>
<th>Size-SUE</th>
<th>Investment</th>
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<tr>
<td>( c )</td>
<td>0.184</td>
<td>0.389</td>
<td>0.230</td>
<td>0.241</td>
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<td>(0.08)</td>
<td>(0.19)</td>
<td>(0.07)</td>
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<td>( \kappa )</td>
<td>0.101</td>
<td>0.301</td>
<td>0.143</td>
<td>0.103</td>
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<tr>
<td></td>
<td>(0.07)</td>
<td>(0.12)</td>
<td>(0.09)</td>
<td>(0.08)</td>
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<td>( a_2 )</td>
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<td>(0.59)</td>
<td>(2.09)</td>
<td>(2.15)</td>
<td>(4.18)</td>
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<tr>
<td>( a_3 )</td>
<td>3.705</td>
<td>5.157</td>
<td>11.530</td>
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<tr>
<td></td>
<td>(1.20)</td>
<td>(3.99)</td>
<td>(5.78)</td>
<td>(5.28)</td>
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<td>a.a.p.e.</td>
<td>0.061</td>
<td>0.181</td>
<td>0.145</td>
<td>0.045</td>
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<tr>
<td>( J_T )</td>
<td>22.46</td>
<td>11.91</td>
<td>5.014</td>
<td>7.395</td>
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<td>d.f.</td>
<td>21</td>
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<tr>
<td>( p )-value</td>
<td>(0.37)</td>
<td>(0.06)</td>
<td>(0.41)</td>
<td>(0.29)</td>
</tr>
</tbody>
</table>
Fama-French 25 portfolios, the benchmark model, unconditional estimation

|        | Small | 2    | 3    | 4    | Big  |        | Small | 2    | 3    | 4    | Big  |
|--------|-------|------|------|------|------|--------|-------|------|------|------|------|------|
| Average stock returns |       |      |      |      |      | Average investment returns |       |      |      |      |      |
| Low    | 1.05  | 0.85 | 0.86 | 0.93 | 0.87 | 1.01  | 0.86  | 0.91 | 1.00 | 0.81 |      |
| 2      | 1.44  | 1.29 | 1.19 | 1.10 | 0.91 | 1.34  | 1.28  | 1.16 | 1.22 | 1.01 |      |
| 3      | 1.63  | 1.36 | 1.22 | 1.25 | 1.07 | 1.65  | 1.37  | 1.27 | 1.22 | 1.01 |      |
| 4      | 1.75  | 1.35 | 1.38 | 1.37 | 1.06 | 1.74  | 1.53  | 1.32 | 1.24 | 1.03 |      |
| High   | 2.00  | 1.47 | 1.45 | 1.32 | 1.16 | 1.88  | 1.57  | 1.44 | 1.28 | 1.29 |      |
| \( \alpha \) |       |      |      |      |      | \( t_{\alpha} \) |       |      |      |      |      |
| Low    | 0.04  | -0.01| -0.04| -0.07| 0.06 | 0.37  | -0.17 | -0.51| -0.80| 0.64 |      |
| 2      | 0.11  | 0.01 | 0.04 | -0.10| -0.05| 1.13  | 0.18  | 0.53 | -1.23| -0.74|      |
| 3      | -0.02 | -0.01| -0.04| 0.03 | 0.07 | -0.26 | -0.11 | -0.54| 0.39 | 0.75 |      |
| 4      | 0.01  | -0.18| 0.07 | 0.14 | 0.03 | 0.10  | -1.66 | 0.76 | 1.47 | 0.36 |      |
| High   | 0.12  | -0.09| 0.02 | 0.04 | -0.13| 1.04  | -0.83 | 0.17 | 0.30 | -0.71|      |
The benchmark model, unconditional, Fama-French 25 portfolios
One-way sorted investment-to-asset deciles

<table>
<thead>
<tr>
<th></th>
<th>Low</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>1.80</td>
<td>1.57</td>
<td>1.58</td>
<td>1.43</td>
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<td>$\alpha$</td>
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<td>$-0.15$</td>
<td>1.09</td>
<td>$-0.80$</td>
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The benchmark model, unconditional, ten investment-to-asset deciles
### One-way sorted portfolios based on Standardized Unexpected Earnings (SUE)

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### Two-way sorted portfolios based on size and SUE

<table>
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<th>Small</th>
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<td>$0.86$</td>
<td>$-1.27$</td>
<td>$1.04$</td>
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The benchmark model, unconditional, ten SUE deciles
The benchmark model, unconditional, nine size-SUE portfolios
Contribution

- A unified theory of asset pricing anomalies based on value maximization of firms

\[ E_t[r_{t+1}] = \frac{E_t[\Pi_1(K_{t+1}, X_{t+1})] + 1 - \delta}{1 + a(I_t/K_t)} \]

- The earnings-return relation via the “cash-flow” channel

- The investment-value-SEO-payout anomalies via the “discount-rate” channel

- A theoretically motivated, easy-to-implement expected-return model

- From preliminary evidence, the mechanisms seem quantitatively important
Future

- Investigate the quantitative importance of the anomaly-mechanisms
- Extend the model to incorporate other corporate policies, for example,
  - Mergers and acquisitions
  - Research and development
  - Corporate governance
  - Accruals