UNIFORM SPATIAL PRICING

Phillip J. Lederer

William E. Simon Graduate School of Business Administration, University of Rochester, Rochester, NY 14627. E-mail: Lederer@simon.rochester.edu

ABSTRACT. Uniform spatial pricing means that a firm delivers its product to any customer at a fixed price, independent of location. Economic theory explains the use of uniform pricing by the added profit generated by absorbing freight charges of distant customers. I extend this insight by demonstrating that when demand elasticity and transportation cost are positively enough correlated, uniform pricing generates higher profits than mill pricing. I show that this result can better explain observed patterns of price policy choice by mail order and web firms. A second result is application of this idea to firms with many shipping facilities.

1. INTRODUCTION

Uniform pricing is a pricing policy that is common in many types of markets, both industrial and retail. By uniform spatial pricing (hereafter simply uniform pricing) I mean a delivered pricing policy by which a firm delivers a given product to any customer at a fixed price independent of the customer’s location. For example, in retail trade many mail-order outlets, such as L.L. Bean and Lands’ End use uniform pricing (although the price may be quantity dependent). In the quickly growing area of web commerce, many Internet retailers use uniform pricing. Surprisingly, many web retailers of furniture (such as Furniturefromhome.com) use uniform pricing apparently belying the hypothesis that uniform pricing is explained by insignificant delivery cost.

In wholesale trade, uniform pricing is used by consumer goods manufacturers for sales to supermarket chains. For example, in the United States uniform pricing is used in the consumer goods market for many cleaning and food categories: laundry cleaners, coffee, juice, shortening and oils, baking mixes, cereal, and many others. Major food processing and home products manufacturing firms all use uniform pricing for nearly all of their products. From supermarket buyers, I learned that manufacturers such as Procter & Gamble (P&G), Colgate Palmolive, Clorox, and Lever Brothers use uniform pricing almost exclusively.

Uniform pricing is also used for industrial goods. Greenhut (1981) empirically studied the use of spatial pricing policies by industrial firms in the United States, West Germany, and Japan whose transportation cost to the customer was at least 5 percent of the price. These studies found that 21 percent of United States, 27 percent of German, and 32 percent of Japanese firms used uniform prices exclusively. Norman (1981) cites a number of U.K. Price Commission studies that report uniform pricing’s use by firms producing cans, glass containers, and sugar syrup.

Despite the many examples listed above the academic literature on uniform pricing is rather small. Economic theory provides the insight that uniform pricing is a form of spatial price discrimination where freight cost of distant customers are subsidized by nearby markets. This was recognized by Hoover (1937) and Singer (1937). The idea is...
that firms can sometimes extend their market territories (compared to mill pricing\(^1\)) by absorbing freight to distant customers at the cost of higher delivered cost to nearby ones. Smithies (1941) was first to recognize that for a monopolist, the shape of customers demand function affected the attractiveness of this spatial freight absorption.

Given that a firm chooses only between mill or uniform pricing, Smithies (1941) shows that concavity (convexity) of a customer’s demand function yields higher (lower) profits for uniform pricing than mill pricing. Smithies limits the choice of pricing policy because discriminatory spatial pricing patterns are illegal under the Robinson–Patman Act (and related laws in Europe). Indeed, Scherer and Ross (1980) presents the long history of government enforcement against firms employing spatial pricing that is not directly tied to cost, particularly distribution/transport cost. But, uniform pricing can be viewed as the one type of discriminatory spatial pricing that can be used without legal jeopardy. Uniform pricing has never been legally interpreted as discriminatory, and therefore illegal. Thus, many papers in the literature including mine study the attractiveness of uniform pricing compared to mill pricing.

Another factor related to the adoption of uniform pricing is the degree of competition in spatial markets. A recent group of papers relates the degree of competition in markets to the choice of mill or uniform prices. Lederer (2011) studies competition between local retailers and distant mail order or web retailers. In this model, the local retailers are in effect using mill prices as customers provide transport themselves, but the mail-order firms who, of course, provide the delivery have a choice of mill or uniform pricing. The finding is that more competitive situations tend to cause the mail/web retailers to adopt mill prices, with less competitive situations inducing the choice of uniform prices. Closely related to this work is Zhang and Sexton (2001) which studies spatial competition in pricing between two firms offering the same good. The major insight is that as competition becomes more severe, mill pricing becomes more attractive to the duopolists than uniform pricing, and vice versa. Note that generally as competition increases, the demand elasticity each firm faces also increases. Thus, the implication is that higher demand elasticity in a spatial market tends to increase the attractiveness of uniform pricing.

The above papers assumed that all customers have the same demand function. My paper assumes that the demand functions are not spatially identical. When customers have different demand functions, then the spatial pattern of demand elasticity has a profound effect on the attractiveness of mill and uniform pricing. The spatial pattern of elasticity (and thus competition) is captured by the correlation of local demand elasticity and transportation cost. I show that over a spatial market, higher (lower) correlation of transportation cost with demand elasticity induces the use of uniform (mill) prices. This result is also shown to be independent of demand curve convexity (concavity).

As noted, the papers by Lederer (2011) and Zhang and Sexton (2001) indicate higher demand elasticity for all spatially distributed customers tends to increase the attractiveness of mill pricing. In contrast, the result of this paper shows that as demand elasticity increases with distance, uniform pricing becomes more attractive relative to mill pricing. This is a new result and leads to additional insights.

For example, consider a firm that serves a continentally sized market, where the degree of local competition varies by geography. If distant markets tend to be more price elastic, then uniform pricing is relatively more attractive. Later in this paper I argue that this elasticity pattern tends of hold for “standard” type goods (such as common consumer products such as men’s clothing) in the United States. This helps us to understand why

\(^1\)I use the term “mill” pricing rather than “f.o.b.” pricing throughout this paper. I mean the terms to be synonyms.

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many U.S. web retailers use uniform pricing. It also helps to explain why some web retailers do not. For example, web retailers such as Dinnerware Depot (fine china), Bikesmart (bicycle parts), B&H (photography, computer, and electronics), and Vegetarian Store (specialty foods), all assess distance related shipping charges, not uniform pricing. All of these products are somewhat “specialized” which I argue shows that demand elasticity may be expected to fall in distance in the United States.

These conclusions contrast with those based upon the notion that higher competition induces mill pricing. Based upon competitive effects, one would expect the opposite conclusions to the above. For example, in the very competitive men’s clothing market one would expect mill pricing to arise, while for specialized goods with less competition uniform prices would be chosen. Thus, this paper helps to explain choice of pricing policy by web retailers, when existing theory cannot easily do so.

A technical contribution is that I show that Smithies results must be qualified; a firm in a market where all customers have convex demand functions will use uniform pricing, not mill pricing, if transportation cost to distant markets is positively enough correlated with demand elasticity.

Another related contribution is the choice of pricing policy for firms with multiple factories or warehouses. This question is motivated by the observation that large consumer goods firms with several geographically distributed plants tend to use uniform pricing. This mystery is compounded by the technical result that a multifacility firm using mill pricing may earn higher profit by using mill pricing from a strict subset of facilities (compared to mill pricing from all). This adds additional profit to the mill pricing policy, and thus affects price policy choice potentially in favor of mill pricing. To help explain the use of uniform pricing I present a sufficient condition for a profit-maximizing firm to find mill pricing from a subset of facilities to be suboptimal. Then, our previous results hold. The sufficient condition is shown to hold for both linear and nonlinear demand functions.

These two results help to better understand the usage (or lack of usage) of uniform pricing in industries as diverse as mail-order retail and wholesale consumer goods. These results are analytically developed under the assumption of linear demand functions. However, I demonstrate that they apply with more general demand functions.

Literature Review

Aside from the papers previously cited, there has been relatively little research on conditions to induce uniform pricing. Within this literature, Anderson and Neven (1990) show that under Cournot competition uniform pricing can arise as the result of two firms competing in a linear market. However, to get the result, an extreme amount of symmetry is required. A paper by Norman (1981) shows that a monopolist that wishes to maximize sales revenue (not profit) will prefer uniform pricing as long as demand elasticity rises with price. This result is similar in spirit to the first result found here. In a very different context than this paper, Kats and Thisse (1993) study location and price competition between two profit-maximizing firms both of which can choose either mill or uniform pricing policies. Their demand assumptions differ from my paper as they assume each consumer buys at most one unit of a good, at a maximum price as specified by its reservation price, compared to the elastic demand function found here. They also assume a uniform pricing firm has the ability to not serve individual customers. Under these assumptions, with a low reservation price, the only equilibrium is one where both firms use uniform pricing, and with a high reservation price, equilibria exist with both using mill, or both using uniform pricing. In the latter case both firms’ equilibrium profit is higher when uniform pricing is used. This is because the ability not to serve some customers allows the uniform pricing firm to avoid unprofitable markets, and results in
heightened price competition. Because demand is inelastic, the equilibrium uniform price is just the reservation price.

On the other hand, several papers focus on why firms would be expected not to use uniform pricing. Thisse and Vives (1988) show that delivered pricing always dominates uniform or mill pricing. DePalma, Pontes, and Thisse (1987) demonstrate that a Bertrand equilibrium cannot exist when two firms sell identical goods and use uniform prices. Their paper also shows that a location–price equilibrium in uniform pricing can exist when goods are differentiated.


A number of papers study a wider range of pricing methods for shipped goods, including the policy of customer self-service pickup. My paper does not consider this latter alternative because it is unobserved in the markets discussed. In some sense, these papers study pricing policies that are a mixture or a variation of mill and uniform pricing. Furlong and Slotsve (1983) allows a customer to choose between self-service transportation or a delivered uniform price. This method is known as menu pricing in the literature. Demand is initially assumed given by a linear function and prices are allowed to be high enough to reduce demand to zero. Because of the flexibility of menu pricing, menu pricing is shown to be more profitable than mill or uniform pricing. Similar assumptions are made in Basu and Mazumdar (1995). When customers are dispersed, it is shown that menu pricing generates significantly less profit than the optimal spatial discriminatory plan, but when customers are concentrated, menu pricing generates profit almost equal to optimal discriminatory spatial pricing. The analysis is reworked with a constant elasticity demand function, which is a convex function. Consistent with Smithies’ results, this case favors mill pricing, but in addition it is shown that a menu scheme generates spatial prices close to mill pricing. The authors conclude, as Smithies did, that the shape of the demand function does have an important impact on the relative profitability of pricing methods.

Basu, Ingene, and Mazumdar (2004) study the above five methods of setting delivery prices and also a N-zone pricing plan, where N uniform pricing areas are set. Profit and social welfare properties of each are computed and it is explained why some of the methods are used and others are not. It is pointed out that zonal pricing is rarely observed and that it is economically profitable only when demand is very dispersed and the cost of adding zones is small. Zonal pricing is undermined by arbitrage, but it is argued that because of fixed cost of self-service, arbitrage is actually rather rare.

Other explanations of the use of uniform pricing have been offered. These include the simplicity of the system, the prevention of spatial arbitrage, and the insignificance of shipping cost. Related, but not identical to the freight absorption argument is the finding

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2Consumer goods companies like Procter & Gamble allow customer pickup, rebating exact transportation cost to the customer’s site as “freight allowance.” As the allowance is no greater than the customer’s transport cost, there is no advantage in pickup.

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that shape of consumers’ demand function. None of these explanations in themselves seem capable of justifying its use.

For example, Norman (1981) and Cheung and Wang (2006) hypothesize that uniform pricing occurs because of simplicity in billing, that is, billing cost is low. However, this hypothesis does not explain why some firms find billing for shipping “complex” and others find individual billing simple enough to execute. For example, why does Lands’ End set uniform prices while Dinnerware Depot uses mill pricing, when the latter is a much smaller firm with (presumably) higher billing cost?

Cheung and Wang also theorize that the presence of arbitrageurs who shift goods from market to market explains the use of uniform pricing. The idea is that with mill prices, such arbitrageurs transship goods to exploit delivered price differences. Arbitrage is only possible when a firm sells to a retailer or distributor, not to the final consumer, and thus, this hypothesis cannot explain use of uniform pricing in retail markets such as mail-order. But even in wholesale markets if prices between markets differ because of differences in transportation cost, arbitrage is only profitable when arbitrageurs have strictly lower transportation costs than the selling firm. This is not likely for very high sales volume firms such as Procter & Gamble. Thus, arbitrage does not explain use of uniform pricing in the consumer products market.

Another explanation informally suggested in the business literature is the low cost of transportation. Low transport cost makes charging for shipping economically unimportant. Certainly, this might be a valid explanation for uniform pricing in a business where profit is high, and shipping cost is a small fraction of revenue. In such a case, shipping cost will not affect firm profits appreciably. But, for many uniform pricing firms, profit margins are small. One measure of profit is the operating margin (the ratio of profit before taxes to sales revenue). Amazon makes an interesting case study. Amazon reported an operating margin of 4.4 percent in its 2007 annual report. Shipping cost amounted to 8 percent of total sales. Indeed, shipping cost exceeded its shipping revenue by $434 million while Amazon’s net profit before taxes was only $476 million. The magnitude of these numbers belies the proposition that transportation costs are not significant. Amazon is well motivated to consider how shipping charges are assessed. This argument signals that Amazon’s choice of uniform pricing was a carefully chosen strategy to maximize profit.

The rest of the paper is organized as follows. Section 2 presents Smithies’ (1941) model and results that are both qualified and generalized. Section 3 studies choice of pricing policy when customers are heterogeneous. Section 4 examines how a profit-maximizing multifacility manufacturing or distribution firm prices. It is shown that Smithies’ result holds as long as the firm does price its goods out of all of its facilities. A brief case study of a multifacility firm is presented with pricing policy choice that is consistent with the theories developed here. The paper concludes with discussion and suggestions for future work.

2. DEMAND SIDE EFFECTS: SHAPE OF DEMAND FUNCTIONS—A RESULT BY SMITHIES (1941)

Smithies (1941) showed that the shape of customers’ demand functions affects the profitability of uniform and mill pricing. If a firm chooses only between mill or uniform pricing, Smithies showed that concavity (convexity) of a customer’s demand function yields
A profit-maximizing firm is assumed to be at a single fixed location and seeks to supply spatially distributed customers with a single good. The firm has a constant unit production cost which, without loss of generality, is assumed to be zero. The firm’s transportation cost to deliver a unit of good to a customer at point \( z \) is \( t(z) \).

(2) The customers are distributed in the market according to the density function \( p(z) \). Each customer has an identical linear and downsloping demand function, \( D(p) \), for the good, where \( p \) is the total delivered price to the customer.

(3) The firm has lower transportation costs than customers, so that customers prefer the firm to provide delivery as long as the gradient of delivered prices charged does not exceed the customers’ transport cost.

(4) The firm limits its choice of pricing policy to mill pricing or uniform pricing because of potential conflict with the Robinson–Patman Act. Under mill pricing, the firm will charge a price, \( p_m \); under uniform pricing strategy, the firm will charge the unit price, \( p_u \).

(5) For all demand functions and associated pricing policies, profit-maximizing prices are low enough so that all customers are served.

All of these assumptions are commonly used in spatial pricing models, except the last, which guarantees that all customers are served. This assumption is an important one because as if demand is truncated at high prices, the shape of the demand function will change. For example, with very high prices a linear demand function generates zero demand. This causes the demand function to have a convex shape. See Figure 1. As already described, Smithies’ showed, this causes the choice of pricing policies to favor mill pricing, all other things being equal. As I wish to study other effects beyond the demand function shape, I make this assumption to clarify what is going on.

With mill pricing a customer will bear the transport cost, \( t(z) \), per unit shipped. Whether the customer provides the transport itself, or the firm provides the transport and charges the customer for these charges does not affect the modeling or results. Under uniform pricing strategy, the firm will charge the same unit price, \( p \), to all customers, and the firm bears the transport cost. The firm is free to choose price policy and the price using...
that policy. The question asked is: under what conditions will uniform pricing generate higher profit than mill pricing?

One answer by Smithies (1941) and Stevens and Rydell (1966) presented similar results many years later. Next, as I often refer to these results I present a lemma that is primarily a restatement of Smithies’ results. But, in this restatement I add (i) a minor generalization, (ii) a necessary qualification, and (iii) a new result. The generalization is that papers assumed a uniform density of consumers, which turns out to be unnecessary. The qualification is that in some of the cases, local prices must be restricted and cannot be so high as to reduce consumer demand to zero. The extension is found in assertion iv of Lemma 1 below, adding a new but related result.

**LEMMA 1:** Assume a monopolist producer at a fixed location on a bounded segment of the real line. The firm has constant marginal production cost and consumers are distributed in the bounded segment with density function \( \rho(z) \) having the same individual demand curve, then

i. If the demand function \( D \) is linear, then for any mill price there is a uniform price yielding the same profits if all customers are served with positive quantity of the good under the mill price.

ii. If the demand function \( D \) is (strictly) convex, then, for any uniform price there is a mill price that yields (strictly) higher profits.

iii. If the demand function \( D \) is (strictly) concave, then for any mill price for which all customers are served with a positive quantity of the good there is a uniform price that yields (strictly) higher profits.

iv. If the demand function \( D \) is linear and not all customers are served with a positive quantity of good under the profit-maximizing mill price, then uniform pricing generates strictly lower profits than mill pricing.

**Proof:** See the appendix.

The intuition behind this proposition is that with a concave demand curve, demand falls off rapidly with price. Thus, mill pricing suffers from very low demand far from the mill site, but uniform pricing does not have this defect. In contrast, uniform pricing can serve distant markets through freight absorption. Similarly, with a convex demand curve, demand at distant markets is still quite high and does not decline as much with high prices, resulting in higher profits for mill pricing compared to uniform. It is interesting to note that these results are independent of the transportation cost function. In particular, these results hold when the transportation cost function is concave, convex, or arbitrary in distance.

In the case of a linear demand curve, mill and uniform pricing yield identical profits. A subtlety described in (iv of Lemma 1) arises when mill price becomes so high that demand falls to zero. In this case, the demand curve effectively becomes convex, and thus, mill pricing dominates uniform pricing.

I next discuss alternative necessary conditions that show that even with linear demand functions, the demand curve need not be concave to induce uniform pricing.

3. DEMAND SIDE EFFECTS: CUSTOMERS HAVE DIFFERENT DEMAND FUNCTIONS

Although Lemma 1 explains how the shape of the demand function affects the relative profitability of uniform and mill prices, it requires all consumers to have the identical
demand function. In this section, the problem of choice of pricing policy when customers have different demand functions is studied.

Assume all customers are located on the unit interval and have demand functions of the form \( D(z, p) = (a(z) - b(z)p)\rho(z) \), where \( a(z) \) and \( b(z) \) can be location dependent. A technical requirement on \( a \) and \( b \) is that both are integrable functions on the unit interval.

Price elasticity for a customer with a linear demand function located at \( z \) is 
\[
\frac{1}{[a(z)/b(z)]p - 1},
\]
thus with a fixed price, demand elasticity is directly related to 
\[
\frac{b(z)}{a(z)}.
\]
We refer to \( b(z)/a(z) \) as a measure of price elasticity.

The next result explains how the distributions of \( a(z) \), \( b(z) \), \( t(z) \), and \( \rho \) affect the relative attractiveness of mill and uniform pricing. In this paper, I use the notation that an integral written without limits is interpreted as an integral over the entire market.

**PROPOSITION 1:** Suppose the firm has a fixed location in the interval with transport cost to point \( z \) of \( t(z) \). Assume that profit-maximizing uniform and mill prices both have the property that all customers are served. The firm’s profit-maximizing pricing policy is

\[
\begin{align*}
\text{uniform pricing if } & \frac{\int b(z)p(z)dz}{\int a(z)p(z)dz} < \frac{\int b(z)t(z)p(z)dz}{\int a(z)t(z)p(z)dz}, \\
\text{mill pricing if } & \frac{\int b(z)p(z)dz}{\int a(z)p(z)dz} > \frac{\int b(z)t(z)p(z)dz}{\int a(z)t(z)p(z)dz}, \text{ and}
\end{align*}
\]

uniform pricing and mill pricing yield the same profit if

\[
\frac{\int b(z)p(z)dz}{\int a(z)p(z)dz} = \frac{\int b(z)t(z)p(z)dz}{\int a(z)t(z)p(z)dz}.
\]

A proof is found in the appendix.

Proposition 1 describes how the transportation cost function and location dependent demand functions determine the profit-maximizing pricing policy. Roughly speaking (1) states that demand elasticity is positively correlated with transportation cost. (I say “roughly” as the ratio \( b/a \) is not properly the demand elasticity.) To see this we rewrite the equations in a different form. In particular Equation (1) can be written as

\[
\frac{\int b(z)p(z)dz}{\int a(z)p(z)dz} < \frac{\int b(z)t(z)p(z)dz}{\int a(z)t(z)p(z)dz}.
\]

The left hand side is a measure of the aggregate demand elasticity (elasticity of total demand) when uniform pricing is used. The right hand side is similar, except the main numerator and denominator are weighted by the transportation cost. If \( \rho(t) > 0 \), inequality (4) states that our measure of aggregate demand elasticity rises when transportation cost rises. That is, a distant market point tends to have higher price elasticity than nearby market points. In this case, mill pricing is less profitable than uniform pricing because too much demand is lost in distant markets and freight absorption is thus called for.

A slightly more precise characterization can be developed in terms of expected value of the ratio-function \( b(z)/a(z) \) given a modified distribution function \( \hat{\rho}(z) = a(z)p(z) \). Suppose real-valued functions on the market line segment \( W \) and \( V \). Write their integral on the segment as \( E_p W(u) = \int W(u)\rho(u)du \) and \( E_p V(u) = \int V(u)\rho(u)du \). The covariance of \( W \) and
V is defined as
\[
\text{Cov}_\rho [W, V] = \int (W(z) - E_\rho W)(V(z) - E_\rho V)\rho(z)dz 
\]
(in accordance with the usual probabilistic definition with measure \(\rho\)).

Equation (4) can be written as
\[
\int \frac{b(z)}{a(z)} t(z)\rho(z)dz - \int \frac{b(z)}{a(z)} \rho(z)dz \int t(z)\rho(z)dz > 0 
\]
which by our definition of covariance is
\[
\text{Cov}_\rho \left[ \frac{b(z)}{a(z)}, t(z) \right] > 0, 
\]
which states that the covariance of our measure of elasticity and transportation cost under measure \(\rho\) is positive. This entire discussion generalizes to the cases (2) and (3). Thus our main result is that as long as our measure of price elasticity is negatively (positively) correlated with transportation cost, then mill (uniform) pricing is profit maximizing.

One caution is that the covariance of price elasticity and transportation cost and the covariance of our measure of price elasticity and transportation cost do not always generate the same sign. Although the price elasticity and our measure of price elasticity are very closely related, it is possible to generate pathological cases where the covariances do not agree in sign. Thus, the analytical predictions in Proposition 1 must be respected and can lead to departures from intuition. The following example demonstrates the effect.

**EXAMPLE 1:** Consider the market interval \((1 + \varepsilon, 2]\), with \(\varepsilon > 0\) and small. I assume customers are uniformly distributed. Let \(a(z)/b(z) = z\) for \(z \in (1 + \varepsilon, 2]\). The firm is located at 1, and transportation cost from there to a customer at point \(z\) is just \(z\). Thus the mean transportation cost is \((0.5 + \frac{\varepsilon}{2})\). For convenience, we set the uniform price equal to 1. For the customer at \(z\), price elasticity and our measure of price elasticity are respectively: \(\frac{1}{1 - z}\) and \(\frac{1}{z}\). The covariance of price elasticity and transportation cost is then
\[
\int_{1+\varepsilon}^{2} \frac{z-(0.5+\varepsilon/2)}{z-1}dz, 
\]
but \(\lim_{\varepsilon \to 0} \int_{1+\varepsilon}^{2} \frac{z-(0.5+\varepsilon/2)}{z-1}dz = -\infty\). The covariance of our measure of price elasticity and transportation cost is
\[
\int_{1+\varepsilon}^{2} \frac{z-(0.5+\varepsilon/2)}{z}dz = 0.5 - \varepsilon - \log(\frac{2}{1+\varepsilon}) \]
which has the limit \(\lim_{\varepsilon \to 0} (0.5 - \varepsilon - \log(\frac{2}{1+\varepsilon})) = 0.5 - \log(2) > 0\). Thus the covariances do not have the same sign for sufficiently small values of \(\frac{1}{z}\). A prediction of which pricing policy is optimal must depend upon use of our measure of price elasticity.

We next demonstrate that for other demand functions a similar result holds: if price elasticity and transportation cost are sufficiently negatively correlated, then uniform pricing is profit maximizing.

**Generalization to Other Demand Functions**

Thus far, this papers shows that there are two effects that can induce uniform pricing to be profit maximizing. The first effect is due to Smithies’ result: mill pricing (uniform pricing) is favored when the demand function that all markets share is concave (convex). The other effect is that for linear demand functions, if our measure of demand elasticity is positively correlated with transportation cost then uniform pricing is profit maximizing.

Two important questions remain unanswered. First, does the latter effect apply to more general demand functions, and second, is this effect truly independent of Smithies’ result? Through an example, I now show that the answer to both questions is yes.

A general demonstration is difficult to analytically prove so my argument relies on an extreme example. I again assume a linear market, but with only two market points. I assume each market point has a different demand function. As these functions are strictly convex, Smithies’ result clearly shows that any of these functions have a propensity to
cause mill pricing to be profit maximizing. I show that even with these convex demand functions, if demand elasticity is highly enough correlated to transportation cost, then uniform pricing is profit maximizing.

EXAMPLE 2: Consider two point markets (labeled near and far) having demand functions

\[ D_{\text{near}}(p) = e^{-\varepsilon p} \quad \text{and} \quad D_{\text{far}}(p) = ge^{-\theta \varepsilon p}, \]

with parameters \( g > 0, \theta \geq 1, \varepsilon > 0 \) which represent, respectively, the relative size of the far market, the relative demand elasticity of the far market, and a parameter that sets the price elasticity in the near market. (Note that the price elasticity of the far market is \(-\theta \varepsilon p\)). Assume that the transportation cost between the markets is 1 per unit good, and that the variable production cost is zero.

The example shows that high demand elasticity in distant markets causes uniform pricing to be optimal and \( \theta \) can be independent of the concavity or convexity of demand. Figure 2 shows values of \( g \) and that induce uniform pricing to yield higher profit than mill pricing when demand elasticity parameter is \( \varepsilon = 0.3 \). As long as the demand elasticity in the far market is 20 percent or higher than that in the nearby market, uniform pricing is optimal. Unlike the linear demand case, market size does affect the optimal pricing policy. But market size may have less of an effect on optimal pricing policy than one might expect. For example Figure 2 shows that uniform pricing is optimal even if the size of the far market is 10 percent of the near market.

Figure 3 assumes the size of the far market is 60 percent of the near market \((g = 0.6)\). The figure shows that as the near market’s demand elasticity \((\varepsilon p)\) rises, the relative elasticity of the far market \((\theta \varepsilon p)\) needs to increase to assure that uniform pricing yields higher profit. This is because as demand elasticity rises, so does the demand function’s second derivative with respect to price. That is, the demand function’s convexity increases.

\[ \varepsilon \]

FIGURE 2: Uniform Pricing is Optimal when the Relative Elasticity of the far Market is Sufficiently Large. Uniform pricing is optimal when the elasticity in the far market is about 20 percent higher than that in the near market. Note that the relative size of the far market has little effect on this result. In this example, the near market demand elasticity \( \varepsilon = 0.3 \).
FIGURE 3: As Elasticity of near Market Increases, the Relative Elasticity of the far Market must Increase for Uniform Pricing to Remain Optimal. Here the Relative size of the far Market is $g = 0.6$.

shows that demand elasticity must be more highly correlated to distance (transportation cost) than in the linear demand case to overcome convexity effects.

Next, some implications of Proposition 1 are presented.

**Implications**

Given reasonable assumptions about the type of good and the distribution of population density and transportation cost, Proposition 1 permits some positive predictions. It has often been observed in the regional science literature that household income is positively correlated with population density.\(^4\) It is reasonable to assume that buyers in higher population density areas have lower search costs for standardized, everyday, low cost consumer goods\(^5\) than consumers in lower population density areas. The difference in search costs is due to high population density consumers having relatively more retail opportunities, ubiquitous advertising, and shorter travel distances from home to retail locations. All of these factors enable comparison shopping especially on price. The overall effect is that low search costs increase buying opportunities and thus increase demand elasticity. Thus, for low-cost standard consumer goods, higher population density can imply higher demand elasticity.

In contrast, search costs for specialized goods and services can be high in any market. This is because specialized products are sold in relatively few retail stores, local advertising (especially mentioning price) will be sparse, and time to find the desired good is, on average, relatively large. Income effects exaggerate these effects, as search is particularly costly for higher income populations. Simply speaking, for higher income individuals the cost of time is higher than for lower income individuals. If search cost rises with household income, demand elasticity will fall with income. That is, all things being equal, higher

\(^4\)This point was made in Jacobs (1969). Glaeser (1999) quotes Marshall on agglomeration effects. Empirical support of the relationship of urbanization and wealth is found in Glaeser and Mare (2001).

\(^5\)For example most consumer consumables such as food and clothing fall in this category.
prices can prevail in high income areas compared to lower income areas. If income is positively related to population density, it is expected that for specialized goods, demand elasticity (including price, time, and convenience costs) tends to fall with population density and result in higher prices. This distinction between demand elasticity behavior for low-cost consumer goods, and specialized goods has implications for optimal pricing policy for these two categories of goods.

For a firm attempting to sell standard consumer goods into markets where shipping distance is positively correlated with population density, my results suggest that uniform pricing policy will tend to be profit maximizing. For example, a mail-order firm selling everyday clothing, serving the entire United States and located in the Midwest, would find distance positively related to population density, and thus would tend to use uniform pricing. This is consistent with the practice of major mail-order (and internet) sellers of clothing in the United States.

In contrast, a mail-order firm in the United States selling specialized goods will tend to choose mill pricing, when shipping distance is positively correlated with population density, and thus relative demand elasticity is positively related to distance. A firm selling country wide, and wishing to compete in the distant coastal markets, will tend to observe this pattern. Mail-order catalogs for specialty goods for hard-to-find items such as: Dinnerware Depot (fine china), Bikesmart (bicycle parts), PartsAmerica (spare parts for appliances), and Vegetarian Store (specialty foods), all use mill prices.

These implications contrast with conclusions from existing theory. Based upon competitive effects, one would expect the market for standard goods to be very competitive, and thus mill prices would be observed. On the other hand, in the less competitive specialized good market, uniform pricing is expected. These conclusions are not observed. For example, in the very competitive men's clothing market, one would expect mill pricing to arise, while for specialized goods with less competition uniform prices would be chosen. Thus, my results help to explain choice of pricing policy by web retailers when existing theory cannot.

An important assumption in this section’s analysis is that all customers are being served at profit-maximizing mill and uniform prices. Strictly speaking, this restricts the results to situations where the transportation cost is sufficiently small so that positive demand can be generated at all points. As is clear from Lemma 1, point iv, truncating the demand function to assure it never becomes negative tends to favor the choice of mill pricing. This is an important limitation that introduces a conflicting effect and requires further research.

Next, multiple-location effects on pricing policies are analyzed.

4. MULTIPLE LOCATION EFFECTS: PRICING POLICIES FOR A FIRM WITH MANY FACILITIES

Firms that serve a large geographical area and offer products with high transportation cost often have multiple plants and warehouses from which to ship to customers. Firms like Procter & Gamble, Unilever, etc. have multiple plants producing the same product and several facilities from which they ship to customers. The main question asked and answered in this section is, “do Smithies’ results hold when a firm has multiple facilities?” The answer is: not necessarily because the presence of multiple shipping facilities raises the possibility that a mill pricing firm could set mill prices from a subset of facilities, and thereby increase its profit. But I show that in reasonable situations firms find mill pricing from a subset does not raise profit.

---

6Population density in the United States is highest on the two coasts.
Assume that a firm has \( N = \{1, 2, \ldots, n\} \) facilities located at fixed locations, \( \{x_1, x_2, \ldots, x_n\} \), within the interval \([0,1]\). I allow the firm to set mill prices from a subset of the \( n \) facilities in order to increase its profit. The firm chooses \( m \) (\( m \leq n, n \geq 2 \)) facilities from which mill price is computed. I refer to these \( m \) as “mill sites.” The set of mill sites is denoted \( M \). The location of the facilities is fixed.

My nomenclature, “mill site,” is related to the term “basing point” in the delivered pricing literature. A basing point is a market point from which mill prices are set. The interesting cases of the basing point literature are those where a profit-maximizing firm sets mill prices from a point that is not its own shipping point\(^7\). In this paper, firms must set mill prices from a subset of its shipping points. Thus our pricing regime is related but distinct from the usual term “basing point pricing.” What is different is that mill sites are always shipping points, but not all shipping point are mill sites. Because I assume that discriminatory pricing is not permitted, using nonshipping sites as mill pricing points is not allowed because billed transport cost from there will always differ from actual transport cost. Further, allowing nonshipping points to be mill sites permits any point in the market to be a mill pricing site. This causes the pricing pattern to be just basing point pricing which is a form of discriminatory pricing.

A customer is quoted a delivered price from the “mill site” with the least delivered price. However, I require that the \( m \) “mill sites” from which delivered prices are quoted have the same mill price. The mill prices are assumed the same to prevent antitrust arguments based upon spatial price discrimination, or customer complaints of unfair dealing.

I assume a general distribution of customers on the interval, \( \rho(z) \). The transport cost for a unit shipment from facility \( i \) to a customer at \( z \) is \( t_i(x_i, z) \). At any point \( z \) on the interval, I denote the lowest transport cost from the firm’s facilities to market point \( z \) by \( t^*(z) = \min_{i=1,\ldots,n}[t_i(x_i, z)] \). The transportation cost from the closest mill site to a market point \( z \) is \( t_{M}(z) = \min_{i \in M}[t_i(x_i, z)] \). A customer at \( z \) will place an order at the least delivered mill price offered. Thus the facility minimizing \( \min_{i \in M}[t_i(x_i, z)] \) will establish the delivered mill price and the good will be shipped to \( z \) from the facility minimizing \( \min_{i=1,\ldots,n}[t_i(x_i, z)] \). Let \( S \) denote the entire market and let \( S_M \) denote the set of market points that are not actually supplied by the closest mill site in set \( M \) but are instead served by a closer shipping facility that is not a mill site.

I first show that if all facilities are used as mill sites, Smithies’ results hold.

**PROPOSITION 2:** If the assumptions of Proposition 1 hold but with multiple shipping locations with mill pricing from each, then Lemma 1 holds.

**Proof:** See the appendix.

In brief this result holds because the sum of linear functions is linear and similarly, the sum of concave (convex) functions is concave (convex).

However, I next show it is possible that mill pricing from a subset of facilities can raise a firm’s profit above that of uniform pricing. Initially I assume that all customers have the identical linear demand function: \( D(p) = a - bp \), with \( a, b \) constant for all customers on the linear market interval \( S = [0,1] \).

The next proposition gives an answer to the question of which pricing policy maximizes firm profit when multiple shipping points exist.

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\(^7\)For example in the case of “Pittsburgh plus” pricing, steel firms set delivered prices as if Pittsburgh was the shipping point, even though these firms neither produced nor shipped from Pittsburgh.

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PROPOSITION 3: Suppose that the firm with \( n \) shipping points has chosen \( m < n \) facilities to act as "mill sites." All customers have the identical linear demand function and buy goods at the lowest delivered price offered. Also assume that at the profit-maximizing mill price, all customers are served. Then, mill pricing from the subset \( M \) dominates uniform pricing (and mill pricing from all facilities) if and only if

\[
\int \left[ t_M(y) - \frac{\int t_M(z)p(z)dz}{\int p(z)dz} \right] \left[ (t_M(y) - t'(y)) - \frac{\int (t_M(z) - t'(z))p(z)dz}{\int p(z)dz} \right] p(y)dy < 0.
\]

It is understood that all integrals without limits are those over the entire interval, \( S \). Further, the profit-maximizing uniform and profit-maximizing mill prices are respectively:

\[
p^*_U = \frac{\int (a - bt'(z))p(z)dz}{2b\int p(z)dz},
\]

\[
p^*_M = \frac{\int (a - 2bt'(z) + bt_M(z))p(z)dz}{2b\int p(z)dz}.
\]

The difference in profits when optimal prices are used is

\[
\Pi_U - \Pi_M = b \int \left[ t_M(y) - \frac{\int t_M(z)p(z)dz}{\int p(z)dz} \right] \left[ (t_M(y) - t'(y)) - \frac{\int (t_M(z) - t'(z))p(z)dz}{\int p(z)dz} \right] p(y)dy.
\]

**Proof:** See the appendix

An initial observation is that if every facility is a mill site, then (9) will be identically zero as \((t_1(y) - t'(y)) = 0\) for all \( y \). Thus when all facilities are used as mill sites, then mill and uniform pricing result in equal profit which is consistent with Proposition 2. Mill pricing is only strictly superior to uniform pricing when mill pricing from a subset of facilities is the profit-maximizing configuration.

It is useful to interpret condition (6) in terms of the way that the functions \( t_M(z) \) and \((t_M(z) - t'(z))\) covary on the set \( S \). The first bracketed term in (6) is the transport cost used to set delivered mill price to market point \( z \) less the average of this cost for the entire market. The second bracketed term is the extra unit revenue generated at \( z \) by using mill sites \( M \) instead of all \( n \) facilities as mill sites, less the average of this difference for the

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FIGURE 4: Layout of Example 4: Location of the Two Facilities, \( x_1 \) and \( x_2 \) on the Market [0,1] and the Transportation Cost for Shipments From Them.

equation: \[
\text{expression}
\]

Expression (9) will always have the same sign as (10) as they differ only by a constant multiplicative factor of \( \frac{1}{\int \rho(z)dz} \).

Thus, Equation (6) states that when the covariance of \( t_M(z) \) and \( (t_M(z) - t'(z)) \) is negative, mill pricing yields higher profit than uniform pricing. Condition (10) holds if \( (t_M(z) - t'(z)) \) tends to be above its average value on the entire space \( S \) when \( t_M(z) \) is below its average value on \( S \). Recall that \( (t_M(z) - t'(z)) \) is nonzero only in set \( \bar{S}_M \).

I next present an example where mill pricing from a subset of facilities yields higher profit than mill pricing from all.

EXAMPLE 3: Assume a market interval [0,1] distributed with constant density \( \frac{\rho(z)}{\int \rho(z)dz} \) by consumers with an identical demand function. Assume \( N = \{1,2\} \). Let a shipping facility be located at \( x_1 = 0.8 \) and also at \( x_2 = 0.9 \). See Figure 4. Let transportation cost be linear in distance at a rate of $1 per unit distance.

The question is will profit increase if only one site is used as mill site? Choose \( M = \{1\} \). Then the region served by shipping facility 2 is the interval [0.85,1]. This is also the set where delivered mill prices will rise when a change occurs from two mill sites to one; \( \bar{S}_M = [0.85,1] \). On the whole market, the average transportation cost from the single mill site is \( \frac{\int t_1(y)\rho(y)dy}{\rho(z)dz} = 0.34 \). As constructed: the distance from any point \( x \in \bar{S}_M \) to the mill site \( x_1 \) is less than the average distance from the mill site to all points in [0,1]. That is if \( z \in \bar{S}_M \) then, \( t_1(x_1,z) - \frac{\int t_1(y)\rho(y)dy}{\rho(z)dz} < 0 \). On the other hand, the average value of \( (t_1(y) - t'(y)) \) on \( \bar{S}_M \) is 0.0125. For all points \( y \in [0.8551251 < y < 1] \subset \bar{S}_M \), we find

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that $t_1(y) - t'(y) > 0.0125$. In a segment with very small measure, $[0.85, 0.8551251]$, the sign reverses: $t_1(y) - t'(y) < 0.0125$. In the expression (6) the first term is negative and the second is almost everywhere positive. Condition (6) holds: it is profit maximizing for the firm to use a single mill site at facility 1. Under our assumptions, mill pricing from just $x_1$ yields higher profit than uniform pricing or mill pricing from both shipping points.

Whether or not condition (6) holds depends on facility locations. It is rational for a firm to choose its pricing policy and locations to maximize profit. Next, I present an example where facility locations are profit maximizing and the profit is maximized by mill pricing from all facilities, and as implied by Proposition 3, uniform pricing.

EXAMPLE 4: Consider the interval $[0,1]$ populated uniformly by consumers possessing an arbitrary identical linear demand function. Let three facilities exist: $N = \{1,2,3\}$ with locations at $z_1 = 1/6$, $z_2 = 1/2$, and $z_3 = 5/6$.

Let $p'_{M}$ maximize uniform profit and for any set $M$, set mill price to $p'_{M} = p'_{U} - \frac{\int_{t(\cdot)}(y) - t(y)dy}{\int_{\rho}(\cdot)dy}$. I assume that with these mill prices, all customers in the linear market are served with positive quantities. Under the specified uniform price, these symmetric locations serve segments in a cost minimizing fashion, and thus, maximize uniform profit. Profit is jointly maximized in location and uniform price. By Proposition 2, if $M = N$ then mill profit is maximized by these locations as well.

However, will mill profit rise if one or more facilities are dropped as mill sites? Suppose $M = \{1\}$, then Cov$[t(t) - t'(y)] = 41/648 > 0$, if $M = \{1, 2\}$, then Cov$[t(t) - t'(y)] = 1/48 > 0$, and if $M = \{1, 2, 3\}$or $M = \{2\}$ then Cov$[t(t) - t'(y)] = 0$. Thus, profit cannot increase by adopting mill pricing from a subset of locations by changing locations. Profit is maximized by using all facilities as mill sites or using uniform price.

Does profit-maximizing location always result in identical profit for mill and uniform pricing? The next example shows that the answer is no. This implies that even with a linear demand function, profit maximization in location can result in mill pricing dominating uniform pricing.

EXAMPLE 5: Consider a market comprising the interval $[0,L]$ made up of two segments $[0,R]$ and $[R,L]$ (See Figure 5.) I write $Q = L - R$. Customer density is set as follows: There is a customer market of point mass $M_1$ at $z = 0$, and a customer market of point mass $M_2$ at $z = R$. On the segment $[R,L]$ customers are continuously and uniformly distributed with density $\rho > 0$. Let $N = \{1, 2\}$. We assume that (i) $M_2 > \rho Q$, (ii) $\frac{\rho Q^2}{3} < M_1 R$, and (iii) $R < \frac{\rho Q^2}{2M_1 + \rho Q}$.

Assume uniform pricing. Then total transportation cost is proportional to

$$M_1 \min[z_1, z_2] + M_2 \min[|R - z_1|, |R - z_2|] + \rho \int_{R}^{L} \min[|x - z_1|, |x - z_2|]dx.$$  

FIGURE 5: Layout of Example 5: Location of Two Point Masses of Customers at the Origin and $R$. There is a continuum of customers from $R$ to $L$. Given a two-facility firm and under stated conditions (including transportation cost is a function of Euclidean distance), the profit-maximizing locations are at $z = 0$ and $z = R$. The associated profit-maximizing subset of mill sites is a single mill site located at $z = L$. In this example, profit is maximized by using mill pricing from a subset of facilities and the locations are set to maximize profit.

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The total transportation cost location-minimum must have both shipping points in the set \(\{0, [R,L]\}\). To see why, if \(z_1 = 0\) then by (i) cost minimizing \(z_2\) must be \(z_2 = R\). If both locations are in the closed interval \([R,L]\) and (i) holds then the cost minimizing locations must be \(z_1 = R\) and \(z_2 = R + \frac{2}{3}Q\). Condition (ii) guarantees that this latter pair of locations generates strictly greater total cost than \(z_1 = 0\) and \(z_2 = R\).

Now I show that (6) holds at these locations with \(M = \{2\}\). An easy computation yields
\[
\int \frac{t_M'(z)p(z)dz}{p(z)dz} = \frac{\nu_2 + M_1 R}{M_1 + M_2 + pQ}.
\]
Therefore, \(\int [t_M(y) - \int \frac{t_M'(z)p(z)dz}{p(z)dz}] [t_M(y) - t'(y)]p(y)dy < 0\) if \(t_M(0) - \int \frac{t_M'(z)p(z)dz}{p(z)dz} = R - \frac{\nu_2 + M_1 R}{M_1 + M_2 + pQ} < 0\). Solution for \(R\) satisfying \(R - \frac{\nu_2 + M_1 R}{M_1 + M_2 + pQ} < 0\) yields condition (iii). Note conditions (i), (ii), and (iii) are not mutually exclusive so solutions exist: for fixed \(M_1\), so long as \(M_2\) is taken to be sufficiently large, a \(R\) satisfying (iii) exists. Thus profit-maximizing location and (6) can both hold. In other words, under profit-maximizing facility location, it is possible for mill pricing from a subset to dominate uniform pricing.

Thus, Smithies’ results do not completely generalize to multiple shipping facilities even when the demand function is linear. On the face of this result, analysis of the relative profitability of uniform and mill pricing in the general situation of nonlinear demand functions appears to be difficult. However, I next develop a sufficient condition for the nonlinear case for mill pricing from a subset to be suboptimal.

Suppose customers have identical but nonlinear demand functions. As before I will write
\[
\text{Cov}[d(p + t_M), t_M - t'] = \int d(p + t_M)(t_M - t')\rho(z)dz - \int d(p + t_M)\rho(z)dz \int (t_M - t')\rho(z)dz.
\]

For economy, I have suppressed the dependence of \(t_M(z)\) and \(t'(z)\) on \(z\).

**PROPOSITION 4:** Let the set \(M \subset N\) be used as mill sites. When using mill sites \(M\), let \(p_M^*\) be the profit-maximizing mill price. If
\[
\text{Cov}[d(p_M^* + t_M), (t_M - t')] < 0,
\]
then firm profit is higher when using all \(N\) facilities as mill sites compared to using \(M\) facilities as mill sites. When (11) holds for all sets \(M \subset N\), mill pricing from a subset of facilities is suboptimal. In this case Smithies’ results hold.

Before presenting the proof, please note that although the proposition calls for computation of optimal prices for different subsets of mill sites, in many cases the assumption that demand falls with price is sufficient to determine the sign of covariance (11). Thus, this result may be easier to use than appears at first read. This section will end with an example of such use.

**Proof:** The difference in profit with mill sites \(M\) and using mill sites \(N\) is
\[
[p_M^* \int_{S - \overline{S_M}} (d(p_M^* + t_M))\rho(z)dz + \int_{\overline{S_M}} (p_M^* + t_M - t')d(p_M^* + t_M)\rho(z)dz] - p_M^* \int_{S} d(p_M^* + t')\rho(z)dz.
\]

We first note that \(\overline{S_M}\) is independent of \(p\). The first term in brackets is the total profit when only \(M\) facilities are used as mill sites, and the second is the profit when all \(N\) facilities are mill sites with mill price \(p_M^*\). Within the bracketed term, the first term is the profit on the set where profit does not change in switching from \(N\) mill sites to \(M\) mill sites. The second term is the profit where profit changes. The difference reduces to
integration on points on $S_M$ and can be written as

$$\int_S (p_{M}^{*} + t_M - t')d(p^{M} + t_M)p(z)dz - p_{M}^{*} \int_{S_M} d(p_{M}^{*} + t')p(z)dz$$

$$= p_{M}^{*} \int_{S_M} (d(p_{M}^{*} + t_M) - d(p_{M}^{*} + t'))p(z)dz + \int_{S_M} (t_M - t')d(p_{M}^{*} + t_M)p(z)dz. $$

When $N$–$M$ facilities are dropped as mill sites, two changes occur: first demand falls in regions where prices have risen (the first term) and second, extra revenue is earned in these regions (second term). Therefore, if

$$p_{M} \int_{S_M} (d(p_{M}^{*} + t_M) - d(p_{M}^{*} + t'))p(z)dz + \int_{S_M} (t_M - t')d(p_{M}^{*} + t_M)p(z)dz < 0,$$

then profit increases when using all locations $N$ as mill sites with mill price $p_{M}^{*}$ compared to using $M$ as mill sites and mill price $p_{M}$. Condition (12) is equivalent to

$$p \frac{\int_{S_M} (d(p_{M}^{*} + t_M) - d(p_{M}^{*} + t'))p(z)dz}{\int_{S_M} d(p_{M}^{*} + t_M)(t_M - t')p(z)dz} < -1.$$

It will be useful to measure total market demand elasticity when switching between $M$ and $N$ mill sites at mill price $p_{M}^{*}$. We define elasticity the usual way: the ratio of the percentage change in total density-weighted demand to the percentage change in density-weighted price. Elasticity, $\varepsilon$ is expressed in absolute value terms

$$\varepsilon(p) = -\frac{p \int_{S_M} (d(p + t_M) - d(p + t'))p(z)dz}{\int_{S_M} d(p + t_M)p(z)dz \int_{S_M} (t_M - t')p(z)dz}.$$

Note that

$$p \frac{\int_{S_M} (d(p + t_M) - d(p + t'))p(z)dz}{\int_{S_M} d(p + t_M)(t_M - t')p(z)dz} = -\varepsilon(p) \int_{S_M} d(p + t_M)p(z)dz \int_{S_M} (t_M - t')p(z)dz.$$

We assume that $0 < \varepsilon < 1$. If $\varepsilon(p_{M}^{*}) > 1$, as $N$–$M$ mill sites are dropped, the percentage decline in demand will be greater than the percentage increase in price, resulting in (12) holding.

The statement that

$$\int_{S_M} d(p_{M}^{*} + t_M)(t_M - t')p(z)dz - \varepsilon(p_{M}^{*}) \int_{S_M} d(p_{M}^{*} + t_M)p(z)dz \int_{S_M} (t_M - t')p(z)dz < 0,$$

is now seen to imply that (12) holds. If this condition holds for all $M < N$ then using all shipping points as mill sites is profit maximizing. Further, Smithies’ results will hold and can predict if the customers use mill or uniform pricing.

To conclude this section, I explain why the results of this section are important. I have shown that pricing from a subset of facilities adds another analytic factor that affects the choice of mill or uniform pricing. Even with concave demand functions whose elasticity is positively correlated with distance, the presence of multiple shipping points can cause mill pricing to be the optimal choice. Thus, the results of this section create new considerations that add to the economic understanding of choice between mill and uniform pricing beyond demand curvature and elasticity.
**A Case Study and Observations**

Theory developed in this section can help explain observed price policy choice of multiplant firms. As a case study consider Procter & Gamble (P&G) which uses only a few plants to serve its U.S. markets. As described at website, www.pg.com/jobs/jobs_us/pdf/P&G_US_Plants.pdf, P&G’s plants for Folger’s coffee are located at Kansas City, New Orleans, and Sherman Texas; for detergent products such as Cheer, plants are in New Orleans, Albany Georgia, Cincinnati, Ohio, and St Louis, Missouri; for Bounty towels plants are located in Albany Georgia, Methoopany Pennsylvania and Oxnard, California. It is striking that P&G uses uniform pricing for all these goods. Figure 6 presents the case of Folger’s Coffee. The map found there indicates Folgers’ plant locations, and the partition of the U.S. market into three transportation cost minimizing regions.

In order to determine an “optimal” pricing policy two conditions must be met. First, it must be optimal for all three facilities to be used as mill sites compared to using a subset. Second, uniform pricing must be more profitable than mill from these three locations.

The results of this section provide a weak test to determine if use of all three facilities as mill pricing points is optimal. The bottom panel of Figure 6 explores the covariance of $t_M - t'$ and $t_M$ along a line between California and Florida through the Sherman Texas plant. It shows that along this linear market if the Sherman plant is not used as a mill site, then Cov($t_M - t'$, $t_M$) is positive. This exercise can be repeated along other linear markets with other omitted mill sites with similar conclusions. Considering Proposition 4, condition (11) would seem to hold. Under mill pricing, the distant coastal markets will generate lower per customer demand than nearby markets (demand falls with $t_M$ reversing the covariance’s sign): that is, demand per customer will be below average in coastal markets (where $t_M - t'$ is highest). Thus the covariance of (11) is expected to be negative. The fact that (11) will be negative is consistent with observed uniform pricing policy. Note also, that application of the general result Proposition 4 may not require computation of equilibrium prices to establish sufficient conditions.

Supposing that mill pricing from all facilities optimal, choice of pricing policy is further guided by the results of Section 3. Even when the demand function is not linear, if demand elasticity is strongly correlated with transportation cost, then uniform pricing will be optimal. For the case of Folgers Coffee, distant, large, and price elastic markets on both coasts are served. As argued in Section 3, demand elasticity would be strongly correlated with population density (and in this case distance) for a product like coffee. Thus, price elasticity is highly correlated with distance, and it is not surprising that uniform pricing is used by P&G for pricing from these three facilities.

5. CONCLUSIONS AND FURTHER WORK

This paper presents two new results, one demand side and the other related to multiple facility locations that help understand the choice of uniform pricing. I show that if demand elasticity is highly enough correlated with transportation cost, uniform pricing is profit maximizing compared to mill pricing. This result holds for convex as well as convex demand functions. For a multifacility firm, I show that use of a subset of potential mill sites as mill pricing points can induce mill pricing to be profit maximizing.

These results give some insight into the use (or lack of use) of uniform pricing in markets like mail-order retailing, web retailing, and consumer goods. I have argued the demand elasticity of consumer goods such as standard clothing items tends to fall with distance when population falls with distance. A firm that sells these items in the U.S. market will find price elasticity to increase with distance. This explains the use of uniform pricing by firms like Lands’ End. Understanding the number and location of consumer
FIGURE 6: Procter & Gamble Plants for Folgers Coffee are in New Orleans, Kansas City and Sherman, Texas, and are Indicated by Three Large Dark Circles on the Map. The map shows the transportation minimizing regions served by each plant. In the upper panel, the lighter lines are the isodistant lines between plants. To see that it is most profitable to use all three plants as mill sites, I have drawn a linear market through Sherman Texas from California to Florida. The bottom panel shows the transportation cost profiles along this line for the three plant locations. The panel shows that along this linear market, if the Sherman Texas plant is not a mill site, then $t_M - t'$ is positively correlated to $t_M$, as $t_M - t'$ is largest when $t_M$ is above its average value, and this implies that profit will decline if this plant is not used as a mill site. Similar heuristic analysis can be performed by removing a different plant as a mill site and considering other linear markets spanning the continent. A similar conclusion arises in each case.

goods plants, I have shown a similar result for manufacturers of consumer goods such as coffee, detergent, and paper towels. In contrast, a similar argument shows that for goods with high search costs, a mail-order or web retailer will find demand elasticity to fall with distance. This explains the use of uniform pricing for web/mail-order firms dealing in specialized goods such as Dinnerware Depot, Bikesmart, B&H, and Vegetarian Store. Thus, a mail-order or web retailer must choose its pricing policy based upon the spatial pattern of demand elasticity for the product category sold.

The results also generates testable hypotheses about the use of uniform pricing by multiple site firms such as Procter & Gamble and other consumer goods manufacturers.
These firms have multiple facilities producing the same products. Computation of the covariance function of Section 4 is predicted to show that dropping mill sites will not increase overall profit merely by switching to mill pricing from a subset of facilities.

This paper can be extended in many directions. A major research direction is that of empirical testing of various theories of uniform pricing. Particularly interesting is determining if the effect of demand elasticity for the overall spatial market has a dominating effect over the effect of demand elasticity changing with distance or vice versa. That is, is it low demand elasticity that motivates the choice of uniform pricing, or is it increasing demand elasticity with distance the dominant factor in the selection of uniform prices?

Another direction is that results about multiple shipping facilities has mostly assumed exogenous, fixed facility locations. The results of Section 4 could be reworked assuming endogenous choice of facility locations.

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APPENDIX

Proof of Lemma 1:

Assume a compact market region $S$ with customer density $\rho(z)$. 

(i) Suppose the firm charges a uniform price, $p_u$. Let the uniform demand curve be $D(p) = a - bp$. I will show that there is a mill price, $p_m$, yielding the same profit. Denote the average transport cost for the uniform firm as 

$$
\bar{t} = \frac{\int_S t(z)\rho(z)dz}{\int_S \rho(z)dz}.
$$

Define the mill price as $p_m = p_u - \bar{t}$. Under mill pricing the profit for the firm is 

$$
\Pi_m = p_m \int_S (D(p_m + t(z))\rho(z)dz.
$$

The firm’s profit under uniform pricing is 

$$
\Pi_u = (p_u - \bar{t})D(p_u) \int_S \rho(z)dz
$$

or $\Pi_u = p_m D(p_m + \bar{t}) \int_S \rho(z)dz$.

To compare the profits from the two pricing strategies consider

$$
\Pi_m - \Pi_u = p_m \left[ \int_S D(p_m + t(z))\rho(z)dz - D(p_m + \bar{t}) \int_S \rho(z)dz \right]
$$

$$
= p_m \int_S \rho(z)dz \left[ \int_S \frac{D(p_m + t(z))}{\int_S \rho(z)dz} \rho(z)dz - D(p_m + \bar{t}) \right] .
$$

If $D$ is linear (which implies that market prices are always below $\frac{a}{b}$), the difference is zero.

(ii) Using the same argument as in (i), if $D$ is convex, then by Jensen’s inequality, $\Pi_m - \Pi_u \geq 0$. Thus, the construction of $pm$ shows the result that higher profit comes with mill pricing.

(iii) If $D$ is concave, then consider a mill price, $pm$, for which all customers are served. I show that there is a uniform price with higher profit. Define $p_u = p_m + \bar{t}$, where $\bar{t}$ is as above. Then following the argument and recognizing Jensen’s inequality, I get $\Pi_m - \Pi_u \leq 0$. For any uniform price, subtract the average transport cost, and the ensuing mill price will generate higher profit than uniform.

(iv) If demand falls to zero at some market points under profit-maximizing mill prices, the market’s demand function is strictly convex. By (ii) mill pricing yields higher profits than uniform pricing.

QED
Proof of Proposition 1: If all customers are served, at mill price \( p_m \), the firm’s profit function is

\[
P_m(p_m) = \int_S p_m(a(z) - b(z)(p_m + t(z)))\rho(z)dz.
\]

By differentiation and first-order conditions, the profit-maximizing mill price is

\[
p_m^* = \frac{\int_S (a(z) - b(z)t(z))\rho(z)dz}{2\int_S b(z)\rho(z)dz}.
\]

Under uniform pricing with uniform price \( p_u \) the firm’s profit is

\[
P_u(p_u) = \int_S (p_u - t(z))(a(z) - b(z)p_u)\rho(z)dz.
\]

By differentiation and the first-order conditions, the profit-maximizing uniform price is

\[
p_u^* = \frac{\int_S (a(z) + b(z)t(z))\rho(z)dz}{2\int_S b(z)\rho(z)dz}.
\]

By substituting (A2) into (A1); and (A4) into (A3),

\[
\Pi_u(p_u^*) - \Pi_m(p_m^*) = \int_S a(z)\rho(z)dz \int_S b(z)t(z)dz - \int_S a(z)t(z)\rho(z)dz \int_S b(z)\rho(z)dz.
\]

Thus (A5) is positive if and only if (2) holds. The other cases follow similarly. \( \text{QED} \)

Proof of Proposition 2: The argument follows that of the proof of Lemma 1 with a few changes. Suppose the firm charges a uniform price, \( p_u \). I will show that there is a mill price, \( p_m \), yielding the same profit. Denote the average transport cost for the uniform firm as \( t'(z) = \frac{\int_S t'(z)\rho(z)dz}{\int_S \rho(z)dz} \). Note that \( t'(z) = \frac{\int_S t_M(z)\rho(z)dz}{\int_S \rho(z)dz} \) as \( t_M(z) = t'(z) \) everywhere. Define the mill price as \( p_m = p_u - t' \). Under mill pricing the profit for the firm is,

\[
\Pi_m = \int_S (p_m + t_M(z) - t'(z))(D(p_m + t_M(z)))\rho(z)dz = p_m \int_S (D(p_m + t_M(z))\rho(z)dz.
\]

The firm’s profit under uniform pricing is

\[
\Pi_u = (p_u - t')D(p_u) \int_S (p_u - t')D(p_u)\rho(z)dz, \text{ or } \Pi_u = p_m D(p_m + t') \int_S \rho(z)dz.
\]

To compare the profits from the two pricing strategies consider

\[
\Pi_m - \Pi_u = p_m \left[ \int_S D(p_m + t'(z))\rho(z)dz - D(p_m + t') \int_S \rho(z)dz \right]
\]

\[
= p_m \int_S \rho(z)dz \left[ \int_S \frac{D(p_m + t'(z))}{\int_S \rho(z)dz} \rho(z)dz - D(p_m + t') \right].
\]

So (i) if \( D \) is linear then the difference will be zero; (ii) and (iii) follow by Jensen’s inequality when \( D \) is respectively convex and concave. Part (iv) follows from the convexity of the truncated \( D \) function. \( \text{QED} \)
Proof of Proposition 3: Under mill pricing with mill price \( p_m \) the firm’s profit function is

\[(A6) \hspace{1cm} \Pi_m(p_m) = \int_S (p_m - t'(z) + t_M(z))(a - b(p_m + t_M(z)))\rho(z)dz.\]

Under uniform pricing with uniform price \( p_u \) the firm’s profit is

\[(A7) \hspace{1cm} \Pi_u(p_u) = \int_S (p_u - t'(z))(a - bp_u)\rho(z)dz.\]

By differentiating (A6) with respect to \( p_m \), I see that

\[(A8) \hspace{1cm} p_m^* = \frac{\int (a + bt'(z) - 2bt_M(z))\rho(z)dz}{2b\int \rho(z)dz}.\]

and likewise for (A7) yields

\[(A9) \hspace{1cm} p_u^* = \frac{a}{2b} + \frac{\int t'(z)\rho(z)dz}{2\int \rho(z)dz}.\]

Substituting (A8) into (A6) and substituting (A9) into (A7) yields the profit functions for both cases. The difference between optimal uniform and mill pricing is

\[(A10) \hspace{1cm} \Pi_u^* - \Pi_m^* = \frac{b}{\int \rho(z)dz} \int_S (t_M(z) - t'(z)) \left( t_M(z) - \frac{\int t_M(y)\rho(y)dy}{\int \rho(z)dz} \right) \rho(z)dz.\]

Thus parts i, ii, iii, and iv of Proposition 3 have been shown to be true. QED