In the study of managerial incentives, two questions remain unresolved due to conflicts between theory and evidence. The first is how managerial incentives are related to product market competition; the second is how they are related to risk.

It seems fair to say that most economists believe that product market competition has a positive influence on managerial incentives and hence the productive efficiency of firms. The available empirical evidence is weak, but supports this view. Efforts to pin down this idea theoretically, however, have produced mixed results. Greater competition may lead to stronger incentives for agents because principals are better informed about their agents’ actions (Oliver Hart, 1983), or because greater effort is required to avert the threat of bankruptcy (Klaus M. Schmidt, 1997). Most predictions are ambiguous, however. One reason is that if greater competition leads to lower profits, the marginal benefit of reducing costs or raising demand, and hence the value of managerial effort, may fall as well.

With the relationship between managerial incentives and risk, the problem is the other way around: a seemingly clear theoretical prediction turns out to be empirically elusive. A central prediction of principal–agent theory is that a risk-neutral principal provides weaker incentives to a risk-averse agent the noisier the measure on which the agent’s compensation is based. However, empirical evidence of an inverse relationship between risk and incentives is scarce. Evaluating two dozen studies of the relationship between performance-pay sensitivity and measures of risk in various occupations, Canice Prendergast (2002) concludes that studies finding a positive or insignificant relationship greatly outnumber those that find a negative relationship. In a survey of research on franchising, Francine Lafontaine and Slade (2000) reach a similar conclusion.

The purpose of this paper is to bring theory closer to the evidence on both issues raised. I study a model of an oligopolistic industry in which firms provide incentives to managers to reduce marginal costs. Product market competition and the provision of incentives are jointly determined as part of the industry equilibrium. A central assumption is that the market structure is endogenously determined by free entry and exit. This assumption reflects a view that, influenced by John Sutton (1991), has steadily gained popularity among students of industrial organization: while market structure is often fixed in the short run or in regulated industries, it is in the long run determined by more fundamental conditions under which firms compete.

Changes in the fundamentals of competition can take different forms, three of which are captured by parameters of the model studied here. When products become more substitutable, a market becomes more competitive in the sense that prices fall, even though with free entry some firms exit or merge. Competition can also be said to increase when the market size increases or the cost of entry falls, since both changes induce new firms to enter the market, leading to lower prices.

For a fixed number of firms, an increase in competition due to greater product substitutability affects managerial incentives in two ways.
First, there is a **business stealing effect**: with more elastic firm-level demand functions, a firm with a cost advantage can more easily attract business from its rivals. Hence, for given prices set by its rivals, greater competition increases a firm’s marginal benefit of reducing its costs. Second, there is a **scale effect**: a firm whose rivals charge lower prices loses market share and therefore has less to gain from reducing its costs. The net outcome of these two effects is in general ambiguous; in the particular model studied here, they exactly cancel each other.

With endogenous market structure, however, the predicted impact of greater product substitutability is unambiguous. Prices and profits fall, which induces some firms to exit, until the remaining firms’ profits are zero again. Each surviving firm produces a larger output and therefore has a greater incentive to reduce its costs. Thus, with more substitutable products, firms provide stronger incentives to their managers, in contrast to the ambiguous results obtained in models that assume a fixed number of firms.

Similarly, changes in market size or the cost of entry lead to stronger managerial incentives if and only if they lead to higher firm-level output in equilibrium. Thus, when market size increases, new firms enter the market, but each firm also produces more, and once again an increase in competition leads to greater incentives. In contrast, if the cost of entering the market falls, new firms enter and firm-level output falls, leading firms to provide weaker managerial incentives.

By affecting the elasticity of demand as well as output, changes in competition also lead to changes in the variance of firm’s profits, even when the variance of costs is held fixed. The variance of profits increases with product substitutability or market size, but decreases when the cost of entry falls. Thus, in each of the three changes considered, the variance of firms’ profits changes in the same direction as managerial incentives. In other words, changes in competition, for whatever reason, induce a positive correlation between firm risk and managerial incentives, without any direct link between the two.

This result illustrates that firm risk and the risk an agent is exposed to may be very different things. In the model, incentives are negatively related to the riskiness of observed costs, i.e., an agent’s measured performance, as standard theory would predict. On the other hand, incentives are positively related to firm risk because changes in competition change the value of cost reductions and the variance of firms’ profits in the same direction. It is then not surprising that many empirical studies, whose measures of risk are often measures of firm risk, find a positive relationship between incentives and risk.

### I. Model

1. **Entry.**—*n* firms enter the market and position themselves symmetrically around a circle of circumference 1. The cost of entry is \( F \). Each firm consists of a risk-neutral principal and a risk-averse agent. The principal makes all entry, personnel, and pricing decisions, while the agent can influence the firm’s costs. The number of firms is determined by the assumption of free entry and exit. For simplicity, I treat \( n \) as a continuous variable, which implies that in equilibrium, each firm’s profit net of the entry cost is zero.

2. **Costs and Contracts.**—Each firm \( i \) has a constant marginal cost given by \( c_i = \bar{c} - e_i - u_i \), where \( \bar{c} \) is a constant, \( e_i \) is the effort exerted by firm \( i \)’s agent, and \( u_i \) is a random variable. I assume that \( u_i \) is normally distributed with zero mean and variance \( \sigma^2 \), and is independent of the other firms’ cost shocks. I assume that realized cost is contractible. Each principal offers her agent a total compensation of \( w_i = s_i + b_i(c_i - c) \), where \( s_i \) is a salary, \( b_i \) is a “piece rate” and \( c_i \) the observed cost reduction.

3. **Effort Choice.**—All agents simultaneously choose effort levels, which are unobservable. Each agent’s utility is given by

\[
\text{Utility} = \exp\left\{ -r [w_i - ke_i^2/2] \right\},
\]

where \( r \) is the agent’s degree of (constant absolute) risk aversion and \( ke_i^2/2 \) is his disutility of exerting effort. The agent’s wage has an expected value of \( s_i + b_i e_i \) and a variance of \( b_i^2 \sigma^2 \). Given the normal distribution of \( u_i \), maximizing utility is therefore equivalent to maximizing

\[
s_i + b_i e_i - \frac{1}{2} rb_i^2 \sigma^2 - \frac{k}{2} e_i^2.
\]

The agent accepts any contract \( (s_i, b_i) \) with an expected utility of at least his reservation utility, which I normalize to zero.
4. Price Competition.—After agents have exerted effort, each firm learns its realized cost $c_i$, which is its private information. The firms then simultaneously choose prices to maximize their expected profit.

5. Demand.—The circle is populated by a continuum of consumers with a uniform density of $m$. Each consumer buys exactly one unit of the good. If a consumer located at $x$ purchases from firm $i$ located at $z_i$, his resulting utility is $V_i(x) = y + a - p_i - t(x - z)^2$. Here, $y$ is income, $a$ is the utility of consuming the most preferred variety (namely $x$), and $t(x - z)^2$ is the disutility associated with consuming variety $i$ instead, which is quadratic in the distance between the consumer and the firm.

Three parameters affect the degree of competition among firms: the unit transport cost $t$, the market size $m$, and the cost of entry $F$. Markets with lower values of $t$ can be interpreted either literally as markets with lower transport costs, or more figuratively as markets in which goods are more substitutable. They are more competitive in the sense that prices are lower (and as I show below, welfare higher) than in markets with higher $t$. Other things equal, larger markets (higher $m$) and markets with lower entry costs $F$ can also be said to be more competitive because in both cases, the equilibrium number of firms in the market is larger and hence prices lower.

To keep the analysis tractable, I restrict the parameters of the model such that in equilibrium, every firm in effect competes only with its immediate neighbors. What must be ruled out is that one firm’s cost is so much lower than a neighbor’s cost that it can capture all of the neighbor’s consumers. Sufficient conditions for the existence of a symmetric interior equilibrium can be stated in terms of an upper bound to the equilibrium number of firms, given by $n = a m/ F$. Large random cost differences can be ignored if the variance of cost shocks $\sigma^2$ is sufficiently small.6

**ASSUMPTION 1:** $\sigma^2 < t^2/(3n^4)$.

In addition, the optimal piece rate offered by a principal must be small enough such that her agent will not cut costs to a level that allows a firm to take over a neighbor’s entire market. This is ensured by assuming that the disutility parameter $k$ is sufficiently large:

**ASSUMPTION 2:** $2kt(1 + kr \sigma^2) > \tilde{m}$.

Since marginal cost is linear in effort and the disutility quadratic, $t, m$, and $F$ can be varied only over certain intervals for Assumptions 1 and 2 to hold. The benefit of the linear-quadratic setup is that it leads to closed-form expressions for the resulting equilibrium.

II. Equilibrium

I solve for a symmetric equilibrium by backward induction. At stage 4 of the game, when the firms simultaneously choose their prices, each firm knows its own realized cost, but not the other firms’ costs. However, in equilibrium every firm knows any other firm’s expected cost, and therefore knows the other firms’ expected price.

**PROPOSITION 1:** If firm $i$’s marginal cost is $c_i$, and it expects its rivals to set a price of $E(p)$, its profit-maximizing price and the resulting profit are

$$ p_i(c_i, E(p)) = \frac{t}{2n^2} + \frac{E(p) + c_i}{2} \quad \text{and} $$

$$ \pi_i(c_i, E(p)) = \frac{nm}{4t} \left[ E(p) - c_i + \frac{t}{n^2} \right]^2. $$

**PROOF:**

See the Appendix.

In a symmetric equilibrium, the expected price $E(p)$ must equal the expression in (2) for

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4 The transport cost thus is a measure for the “toughness of price competition,” which Sutton (1991, p. 9) defines as a function linking market structure to prices or unit margins, not to the level of prices or unit margins observed in equilibrium.

5 Total revenue cannot exceed $am$ (willingness to pay times market size), and so each firm’s expected profit in a symmetric equilibrium cannot exceed $aml/n - F$.

6 This assumption is common in both financial economics and the industrial organization literature, see, e.g., Xavier Vives (1999, Ch. 8).
a firm whose cost equals its expected value $E(c)$, which leads to

$$E(p) = E(c) + \frac{t}{n^2}. \quad (3)$$

Substitute (3) into (2) to obtain (without further proof) firm $i$’s equilibrium price and expected gross profit as a function of its realized cost:

**PROPOSITION 2:** For given costs \{\(c_1, \ldots, c_n\)\} and an expected cost of $E(c)$, there exists a unique Nash equilibrium in prices, given by

$$\pi_i(c_i, E(c)) = \frac{mt}{n} \left[ \frac{1}{n} + \frac{n}{2t}[E(c) - c_i] \right]^2. \quad (4)$$

For the stated expressions to be valid requires that $c_i - E(c) \leq 2t/n^2$. Assumption 1 ensures that realizations of $c_i$ that violate this condition can be safely ignored.\(^7\)

Firm $i$’s cost is in part determined by the effort its agent exerts. The agent maximizes (1) with respect to $e_i$ and hence chooses the effort $e_i = b_i/k$. Using (1), one can then calculate the (negative) salary the agent must be paid to obtain an expected utility of zero. At the contracting stage, each firm chooses its piece rate $b_i$ to maximize its expected profit net of the agent’s total compensation, which is given by (4) minus the agent’s wage $w_i = s_i + b_i(c - c_i)$. We then obtain:

**PROPOSITION 3:** There exists a unique symmetric Nash equilibrium in contract choices, in which each firm chooses a piece rate of

$$b = \frac{m}{n(1 + kr\sigma^2)}, \quad (5)$$

and its resulting expected profit net of the setup cost is

$$\frac{mt}{n^3} + \frac{nm\sigma^2}{4t} - \frac{m^2}{2kn^2(1 + kr\sigma^2)} - F. \quad (6)$$

**PROOF:**

See the Appendix.

Proposition 4 below establishes that (6) is decreasing in $n$. At stage 1 of the game, firms enter until (6) equals zero.

### III. Competition and Managerial Incentives

In spite of evidence to the contrary, many previous analyses predict an ambiguous relationship between competition and incentives because of two countervailing effects. When the market structure is taken as given, the same two effects reappear in the model studied here. When the market structure is treated as endogenous, the predictions are unambiguous and consistent with empirical work.

**A. Given Market Structure**

According to (5), the equilibrium piece rate $b$ is proportional to each firm’s expected demand $m/n$. Thus, for given $n$, firms in larger markets provide stronger incentives, whereas for any given market size, firms in less concentrated (and in this sense more competitive) markets provide weaker incentives (cf., also Stephen Martin, 1993).

However, the piece rate does not depend on $t$, the measure of product substitutability. This is the result of two effects which in this model exactly cancel each other. To isolate these effects, differentiate firm $i$’s profit in (2) with respect to $c_i$ to obtain firm $i$’s marginal gain of reducing its cost:

$$m \left[ c_i - E(p) \right] n^2 - t \quad (7)$$

Since $c_i - E(c) \leq 2t/n^2$, (7) is negative; i.e., a cost reduction increases firm $i$’s expected profit. Moreover, (7) is increasing in $t$ and decreasing in $E(p)$, which implies that for a given

\(^7\) More precisely, Assumption 1 can be rephrased as

$$2\sqrt{3}\sigma < \frac{2t}{n}. \quad \text{Thus, since } n \equiv \bar{n}, c_i - E(c) \text{ can exceed } 2t/n^2 \text{ only if } c_i \text{ deviates from its mean by more than } 2\sqrt{3}\sigma = 3.46\sigma, \text{ the probability of which is well below 0.1 percent. By making Assumption 1 more restrictive, any desired confidence level can be achieved. On the other hand, for all results below that rely on Assumption 1, only the much weaker condition } \sigma^2 < 4\sigma^2/n^2 \text{ is needed.}$$
number of firms, an increase in product substitutability has two effects:

1. A decrease in \( t \) increases the gain from reducing marginal cost. Intuitively, with a lower value of \( t \) demand is more elastic, making it easier for a firm to increase its demand by cutting its price. This business stealing effect implies that for given prices of its rivals, each firm has a greater incentive to lower its cost, and hence to increase its piece rate.\(^8\)

2. On the other hand, a decrease in \( t \) also leads to lower equilibrium prices, and a fall in \( E(p) \) decreases a firm’s gain from reducing its cost. Intuitively, when firm \( i \)’s rivals reduce their price, \( i \)’s optimal response is to match the price cut, but not fully [cf., (2)]. The result is a decrease in \( i \)’s demand. Since the value of reducing cost is proportional to demand, it is optimal for firm \( i \) to reduce its piece rate, which is a scale effect.\(^9\)

When the number of firms is fixed, these two effects exactly cancel each other because in the spatial framework used here, total market demand is price-inelastic, implying that each firm’s demand is always \( m/n \). If, in contrast, total demand were decreasing in price (such as in a linear demand model), then greater product substitutability would lead to lower prices and higher firm-level demand, and hence stronger incentives. Thus, for given \( n \), the net effect of tougher price competition on the piece rate \( b \) is generally ambiguous.

\(^8\) The role of product substitutability is also emphasized in recent papers by Jean-Etienne De Bettignies (2000) and Gene M. Grossman and Elhanan Helpman (2002), who study how vertical integration depends on competition. R. Preston McAfee and John McMillan (1995) and Lafontaine and Slade (2000), too, argue that optimal incentives should be increasing in the elasticity of demand, but do not study explicit oligopoly models, where the business stealing effect would be counteracted by a scale effect.

\(^9\) The scale effect described here corresponds to Benjamin E. Hermalin’s (1992) change-in-the-relative-value-of-actions effect and Schmidt’s (1997) value-of-a-cost-reduction effect. Because of Schmidt’s assumption that the agent is risk neutral but wealth constrained, the latter effect arises only when the agent’s participation constraint is not binding. Here, where the agent is risk averse, the scale effect is present even though the agent’s participation constraint is always binding.

B. Endogenous Market Structure

To see how the results change when market structure is endogenous, we first need to know how the number of firms varies with different measures of competition:

**PROPOSITION 4:** The equilibrium number of firms increases with transport costs (decreases with product substitutability), increases, but less than proportionally, with market size, and decreases with the cost of entry.

**PROOF:**

See the Appendix.

Lower transports mean that products are more substitutable and lead to lower equilibrium prices and profits. On the other hand, given fluctuations in costs translate into larger fluctuations in demand, leading to higher expected profits, as reflected in the second term in (6). This effect is common in models with stochastic costs or demand, see, e.g., Raith (1996). Intuitively, the gain in profit from a given cost advantage (relative to a firm’s rivals) exceeds the loss in profit from a cost disadvantage of the same magnitude. Assumption 1 ensures that the first effect dominates the second.

Similarly, it is straightforward that an increase in market size induces entry when costs and qualities are exogenous. As Sutton (1991) has shown, however, when investments in quality improvements or cost reductions are endogenous (as here), then increases in market size may lead to an escalation of firms’ strategic investments that dissipates any profit gains and prevents further entry, and the market may remain concentrated irrespective of market size. Assumption 2 rules out this escalation effect.

A main result of this paper states how equilibrium incentives depend on competition:

**PROPOSITION 5:** With endogenous market structure, piece rates are higher in markets with more substitutable products and in larger markets, but lower in markets with lower entry costs.

**PROOF:**

See the Appendix.

The intuition behind Proposition 5 is that the higher firm-level output, the greater is a firm’s
incentive to invest in cost reductions, and therefore the stronger incentives it will provide, cf. (5). Hence, how incentives are related to competition depends on how firm-level output varies with the degree of competition. An increase in product substitutability leads to lower prices and profits for any given number of firms, and induces some firms to exit. Each surviving firm produces a larger output and provides stronger incentives. If the density of consumers \( m \) increases, each firm’s demand increases in proportion. Higher profits induce entry, but since \( n \) increases less than proportionally with \( m \), each firm in the market still produces more than originally, and the net effect is again an increase in incentives. Finally, a decrease in the entry cost \( F \) has no direct effect on the piece rate, but leads to the entry of new firms. In equilibrium, each firm produces a smaller output, and hence provides weaker incentives to its agent.

An increase in the piece rate induces more effort and results in a lower marginal cost. Moreover, since the agent is exposed to greater income risk, the firm must increase his expected total compensation to hold expected utility constant. Hence we have:

**COROLLARY 1:** Marginal costs are lower, and average compensation for agents higher, in markets with more substitutable products and larger markets. The opposite holds for markets with lower entry costs.

The above results do not depend on the specific setup of the model: if market demand were price-elastic, lower prices due to greater competition would lead to greater total and hence firm-level output. This would reinforce the effect that each firm produces more with greater product substitutability or a larger market, and would only partially offset the effect that firms produce less when entry costs are lower.

According to Proposition 5, the relationship between competition and managerial incentives depends on what causes variations in the degree of competition, which poses a challenge to empirical tests. To my knowledge, few empirical studies of incentives account for the role of competition. Classifying semiconductor products in broad groups, Coughlan (1985) finds that more substitutable products are more likely to be sold through middlemen than in-house. Slade (1998) finds that oil companies are more likely to delegate pricing decisions to gas stations the greater the price and cross-price elasticities of demand. More indirectly related, Nickell’s (1996) cross-industry study documents a weakly negative relationship between concentration and productivity growth.

One particular challenge is that with endogenous market structure, concentration indices alone are poor measures of competition unless it is clear what causes their variation: when markets vary in size or entry costs, less concentrated markets will tend to be more competitive. In contrast, if markets vary in product substitutability or other dimensions of the toughness of competition, high levels of concentration are indicative of intense competition, not a lack of it, cf. Sutton (1991) and George Symeonidis (2000a) for evidence.\(^{10}\)

While detailed empirical studies are scarce, the predictions of Proposition 5 are consistent with long-term changes in competition and the use of incentive pay in many industries. Incentive pay has become more prevalent throughout the past decades; for evidence covering a variety of occupations, see W. Bentley MacLeod and Daniel Parent (1999). One possible explanation is the national and global integration of markets, which has been attributed to decreases in shipping, travel and communication costs, and to reductions in tariffs.\(^{11}\) All of these changes correspond to decreases in \( t \), the parameter given some emphasis in this paper. Second, deregulation often both opens a market to entry and introduces price competition that was previously absent. The model predicts that incentives and productivity are more likely to increase when deregulation leads to a more con-
centrated market with larger firms. Indeed, this has often occurred, notably in the U.S. airline industry (cf. Steven A. Morrison, 1998). Third, in growing markets firms often adopt technologies better suited to large-scale production. Such shifts often entail an increase in setup costs and a decrease in marginal costs (the exogenous part of which is \( c \) in the model). The model predicts greater concentration and stronger managerial incentives as a consequence, reinforcing the basic result that incentives are positively related to market size.

C. Competition and (X-)Efficiency

Leibenstein (1966) termed the failure of firms to minimize production costs “X-inefficiency,” and argued that lack of competition is often its cause. George J. Stigler (1976) questioned the “Xistence” of X-inefficiency, arguing that production within firms always entails costs of enforcing contracts, which profit-maximizing firms must balance against production costs. Production at a higher than technologically feasible cost should therefore not per se be associated with inefficiency.

Stigler’s view is opposite in the context of the model studied here. Lower production costs can be achieved only by incurring higher agency costs. While firms always maximize profit, cost reductions are not realized unless there is pressure to do so. Nevertheless, greater competition in any of the three senses discussed in this paper lead to an overall gain in efficiency:

PROPOSITION 6: Expected per capita welfare is higher in markets with lower transport costs (greater product substitutability), larger markets, and markets with lower entry costs.

PROOF: See the Appendix.

Proposition 6 hence confirms Leibenstein’s general claim that competition increases productive efficiency (allocative efficiency is unaffected because every consumer always purchases one unit). With greater product substitutability, however, welfare is higher only in part because firms have lower marginal costs.

The other, possibly larger, effect is that with lower margins, fewer firms are viable in the market, which saves on setup costs. Similarly, when the entry cost falls, the overall effect is an increase in welfare, but a side effect is that firms provide weaker incentives and hence have higher marginal costs.

IV. Managerial Incentives and Risk

Greater competition also leads to changes in the impact of given cost fluctuations on realized profits:

PROPOSITION 7: With endogenous market structure, the variance of firms’ gross and net profits is higher in markets with more substitutable products and in larger markets, but lower in markets with lower entry costs.

PROOF: See the Appendix.

Proposition 7 states that for a given variance of cost shocks, the variance of firms’ profits depends on the degree of competition. According to (4), any change in competition causes a change in the level of prices without affecting their variance. But with greater product substitutability or a larger market, the variance of profit increases because demand is both larger and more elastic; a decrease in the entry cost has the opposite effect. Propositions 5 and 7 immediately imply the second main result of this paper:

COROLLARY 2: Piece rates are positively correlated with the variance of firms’ profits across markets that differ in product substitutability, market size, or entry costs.

Corollary 2 helps to explain the apparent contradiction between theory and evidence discussed in the introduction. Many tests of the standard principal-agent model rest on the premise that variations in firm risk correspond to variations in the riskiness of agents’ measured performance, but are unrelated to the value of effort. As argued here, precisely the opposite may be the case and thus lead to the opposite prediction.
Notice that the principal–agent model used here is entirely standard: according to (5), the piece rate is inversely related to both the actual cost risk $\sigma^2$ and the agent’s risk aversion $r$. What drives Corollary 2 is that since agents are paid for observed cost reductions, greater competition affects firm risk without affecting the riskiness of measured agent performance. Greater competition changes the marginal value of an agent’s effort in the same direction as it changes firm risk (Propositions 5 and 7), and therefore incentives and risk are positively related without any direct causal link between them.

Corollary 2 rests on the assumption that costs are contractible. Suppose instead that each agent’s compensation is based on realized profit. If all firms’ profits are contractible, then optimal relative performance evaluation leads to the same result as above, since the firms’ costs can be calculated from their profits [by inverting the system (A8)]. In contrast, if an agent’s compensation is based on his own firm’s but not other firms’ profits, the relationship between firm risk and incentives is ambiguous, since greater competition makes cost reductions more valuable, but also exposes agents to more risk.

Testing Corollary 2 requires an environment in which variations in competition are observable. Executive compensation and franchising, for example, seem suitable areas in which to test the theory; agricultural contracting, on the other hand, perhaps less so, since agricultural markets are always quite competitive and the crop risk is largely exogenous.\textsuperscript{12}

For tests of a negative relationship between risk and incentives predicted by standard models, Corollary 2 has two implications. First, the degree of competition between firms is an important determinant of the value of managerial effort. The problem of how to account for differences in the value of managerial effort has always plagued empirical research, see Lafontaine and Slade (2000) for approaches taken in the literature. Studies that account for product market effects on the value of effort have to my knowledge not yet been conducted.

Second, it is important to distinguish between exogenous shocks and the variability of firm performance. While some studies measure cost or demand uncertainty using, say, input prices or exchange rates, most studies employ measures of firm risk such as the variability of sales (e.g., Robert E. Martin, 1988), firm survival rates (e.g., Lafontaine, 1992), or the variance of stock returns (e.g., Rajesh K. Aggarwal and Andrew A. Samwick, 1999). The problem with these measures is that their variation may be caused by differences in competition, not only differences in exogenous risk. It therefore seems desirable to obtain data on the measures on which agents’ compensation is actually based, or at least to obtain measures of risk that can be treated as exogenous.

Lafontaine and Slade (2000) and Prendergast (2002) offer alternative explanations for a positive relationship between risk and incentives. Lafontaine and Slade argue that if agents are given greater incentives (for whatever reason), their actions may induce a greater variability of profits. Thus, an observed positive correlation between risk and incentives may be the result of a reverse causation. In Prendergast’s model, an increase in the riskiness of available projects directly increases the value of delegating project choice (and providing incentives to make the right choice) to an agent who is better informed about the projects than the principal. Here, in contrast, both firm risk and the value of effort are endogenously determined by the degree of competition.

V. Conclusion

This paper has studied how managerial incentives in an oligopolistic industry depend on the degree of product market competition. With greater competition due to increased product substitutability or a larger market, firms provide stronger incentives to their managers to reduce costs, even though profits become more vola-

\textsuperscript{12} I would like to thank an anonymous referee for pointing this out. Douglas W. Allen and Dean Lueck (1995) document that most studies of agricultural contracting find a positive relationship between crop risk and the use of cash-renting contracts as opposed to crop-sharing. C. H. Hanumatha Rao (1971) attributes this relationship to increases in the importance of a farmer’s “entrepreneurship” that higher crop risk brings about.
tile. The key to this result is the assumption of free entry, under which both greater product substitutability or a larger market lead to an increase in firm-level output, and therefore an increase in both the value of cost reductions and the volatility of profits.

These results shed light on two unresolved issues in the study of managerial incentives. First, while most economists appear to believe that competition positively affects incentives, previous theoretical research has suggested a much more ambiguous relationship. Second, most empirical work finds a positive or insignificant relationship between risk and incentives, which is at odds with a basic result of the theory.

The results of this paper seem quite robust. They rely on the central assumption that an increase in output raises the value to each firm of gaining a cost or quality advantage over its rivals. This assumption is neither specific to the circle model chosen here, nor to the assumption that effort is directed at reducing costs. It is also inconsequential whether firms must induce managers to exert effort, as assumed here, or to use private information optimally. For related results in different settings see De Bettignies (2000), Symeonidis (2000b), and Prendergast (2002).

The results demonstrate the importance of studying firms’ provision of incentives within an explicit model of market interaction. First, a variety of changes in a market may be described as an “increase in competition,” but how incentives are affected depends on the nature of the change. Second, greater competition changes the marginal value of investments that allow a firm to steal business from others, and hence affects the provision of incentives. Third, market structure typically adjusts to changes in the nature of competition, which leads to fundamentally different predictions about incentives compared to models in which market structure is taken as given. Of course, market structure may sometimes fail to adjust, for instance because of regulatory restrictions, or because of integer constraints that are relevant when entry costs are large. In practice, however, entry, exit, and mergers are pervasive features of most markets, which suggests that market structure should be treated as endogenous.

**APPENDIX**

**PROOF OF PROPOSITION 1:**

A consumer located between firms $i$ and $i + 1$ at distance $x$ from firm $i$ purchases from $i$ if $p_i + tx^2 \leq p_{i+1} + t(1/n - x)^2$, where $1/n$ is the distance between the two firms. Straightforward calculations lead to firm $i$’s total demand

\[
q_i = m \left\{ \frac{1}{n} + \frac{n}{2t} \left( (p_{i+1} - p_i) + (p_{i-1} - p_i) \right) \right\}.
\]

If the expected price of firm $i$’s rivals is $E(p)$, then $i$’s expected gross profit is

\[
\pi_i = (p_i - c_i)E(q_i)
\]

\[
= (p_i - c_i)m \left\{ \frac{1}{n} + \frac{n}{t} \left[ E(p) - p_i \right] \right\}
\]

and maximization with respect to $p_i$ leads to (2).

**PROOF OF PROPOSITION 3:**

Using (1) and $e_i = b_i/k$, firm $i$ must pay its agent a (negative) salary of

\[
s_i = -\frac{1}{2k} (1 - k\rho^2)b_i^2
\]

for the agent to obtain an expected utility of zero. Firm $i$’s expected profit net of the agent’s compensation is given by (4) minus $\bar{w}_i = s_i + b_i(\bar{c} - c_i)$. Using (A3), $\bar{c} - c_i = e_i + u_i$, and $e_i = b_i/k$, this expected net profit is

\[
\Pi_i = \frac{mt}{n} \left[ \frac{1}{n} + \frac{E(\bar{c}) - \bar{c} + \frac{b_i}{k} + u_i}{2t} \right]^2
\]

\[
+ \frac{1}{2k} (1 - k\rho^2)b_i^2 - b_i \left( \frac{b_i}{k} + u_i \right).
\]

The firm’s expected profit before realization of $c_i$ is the expected value of (A4) over $u_i$:
The derivative of the resulting expression with respect to $mt$ is

$$\frac{mt}{n} \left[ \left( \frac{E(c) - \tilde{c} + \frac{b_i}{k}}{2t} \right)^2 + \frac{n^2 \sigma^2}{4t^2} \right] - \frac{1}{2k} (1 + kr^2) b_i^2.$$ 

Under Assumption 2, this expression is strictly concave in $b_i$. For given expectations $E(c)$ about the other firms’ costs, firm $i$ maximizes (A5) with respect to $b_i$, which leads to

$$b_i = km \left( \frac{2t}{n} - n[\tilde{c} - E(c)] \right) \frac{2t}{2ki(1 + kr^2) - nm}.$$ 

In a symmetric equilibrium, all firms choose the same piece rate $b$, and each agent chooses the same effort $e$. It follows that $E(c) = \tilde{c} - e = \tilde{c} - b/k$. Substituting this expression into the right-hand side of (A6) and solving for $\tilde{b}_i$ then leads to (5) and then (6).

PROOF OF PROPOSITION 4:

The derivative of (6) with respect to $n$ can be written as

$$\frac{m^2}{n^3} \left( \frac{m}{k(1 + kr^2) - \frac{2t}{n}} + m \left( \frac{\sigma^2}{4t^2} - \frac{t}{n^4} \right) \right),$$

where both terms are negative, the first because of Assumption 2, and the second because of Assumption 1. Hence profit is decreasing in $n$. Moreover, (6) is decreasing in $F$, increasing in $m$, and, given Assumption 1, increasing in $t$. Hence $n$ must be decreasing in $t$ and $F$ and increasing in $m$. To see that $n$ increases less than proportionally with $m$, it suffices to show that (6) falls when $m$ and $n$ both increase by the same factor $\lambda > 1$. Substitute $\lambda m$ for $m$ and $\lambda n$ for $n$ in (6). The derivative of the resulting expression with respect to $\lambda$ at $\lambda = 1$ equals $-\frac{m}{2n^2 t} (4t^2 - n^4 \sigma^2) < 0$, which establishes the result.

PROOF OF PROPOSITION 5:

According to (5), $b$ does not directly depend on $t$ or $F$ but is decreasing in $n$. It then follows from Proposition 4 that $b$ is decreasing in $t$ and increasing in $F$ in the free-entry equilibrium. Moreover, since $b$ is proportional to $m/n$, and $n$ grows less than proportionally with $m$, it follows that $b$ is increasing in $m$.

PROOF OF PROPOSITION 6:

In equilibrium, total profits are zero, and agents attain zero expected utility. Expected total welfare therefore equals the expected consumer surplus $m[a - E(p)] - 2nt \int_{(2n^2)}^{(2n^2)} x^2 dx = m[a - E(p)] - \frac{1}{12}(2n^2)$. Using (3) and $E(c) = \tilde{c} - b/k$, expected per capita welfare therefore is

$$a - \tilde{c} + b \frac{t}{k} = t \frac{a}{n^2} - \frac{t}{12mn^2}.$$ 

First, (A7) must increase with $m$ because both $n$ and $b$ are increasing in $m$. Next, substitute (5) into (A7). Under Assumption 2, the derivative with respect to $n$ is positive, which implies that (A7) is decreasing in $F$ since $n$ is. Finally, a sufficient condition for (A7) to be decreasing in $t$ is that $nt/\lambda n$ is increasing in $t$ when $n$ varies with $t$, or equivalently, that (6) falls when $t$ and $n^2$ both increase by the same factor $\lambda > 1$. Substitute $\lambda m$ for $m$ and $\lambda n$ for $n$ in (6). Then the derivative of the resulting expression with respect to $\lambda$ at $\lambda = 1$ equals $[\lambda (A7)] - F < 0$, which establishes the result.

PROOF OF PROPOSITION 7:

Using (A1), (2), and (3), firm $i$’s realized gross profit $(p_i - c_i)q_i$ can be written as

$$m \left( \frac{t}{n^2} + \frac{E(c) - c_i}{2} \right) \times \left( \frac{1}{n} + \frac{n c_{i+1} + c_{i-1}}{4t} - \frac{nc_i}{2t} \right).$$

In equilibrium, we have $E(c) = \tilde{c} - b/k$ and $c_i = \tilde{c} - b/k - u_i$, and (A8) equals

$$m \left( \frac{t}{n^2} + \frac{u_i}{2} \right) \left( \frac{1}{n} - \frac{n u_{i+1} + u_{i-1}}{4t} + \frac{nu_i}{2t} \right),$$

which has an expected value of $m(t/n^2 + \sigma^2)/(4t)$. The variance of firm $i$’s gross profit is
\[ E[(\pi_i - E(\pi))^2]. \]

Using \( E(u_i) = E(u_i u_j) = 0 \) and \( E(u_i^2) = \sigma^2 \), as well as \( E(u_i^3) = 0 \), \( E(u_i^4) = 3\sigma^4 \) and \( E(u_i^2 u_j^2) = \sigma^4 \), we obtain

\[
(A10) \quad \text{Var}(\pi_i) = \frac{m^2 \sigma^2}{32n^2t^3} (5n^4\sigma^2 + 36t^2).
\]

This variance is decreasing in \( t \). Moreover, the derivative of (A10) with respect to \( n \) has the same sign as \( 5n^4\sigma^2 - 36t^2 \), which under Assumption 1 is negative. Since \( n \) is increasing in \( t \), the variance of profits is decreasing in \( t \) both for given and for endogenous \( n \). The variance of profit net of the agent’s compensation can be calculated in a similar way by subtracting \( s_i + b_j(c_i - c_j) \) from (A8) and using (5). The resulting variance is (A10) minus

\[
(A11) \quad \frac{m^2 \sigma^2(1 + 2kr\sigma^2)}{n^2(1 + kr\sigma^2)^2},
\]

which is independent of \( t \) and increasing in \( n \). Therefore, the variance of a firm’s net profit, too, is decreasing in \( t \) for both given and for endogenous \( n \).

The variance of gross and of net profit also increases with \( m \), given that \( n \) increases less than proportionally with \( m \), cf., (A10) and (A11). Finally, a decrease in \( F \) leads to a larger number of firms and hence to a decrease in the variance of profit, according to the above argument that (A10) is decreasing in \( n \).

REFERENCES


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