Rational Sentiments and Economic Cycles*

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Abstract

We study the interaction between credit market sentiments and real economic outcomes in a fully rational environment. We capture sentiment as lenders’ choice of power of test when granting credit. The credit market exhibits the symptoms of overheating, when lenders choose lax lending standards. In these periods, a mixed quality of credit is issued at a low interest rate inducing high credit growth, high economic output and a deteriorating quality of credit applications. When the pool of applications is sufficiently low, lenders switch to tight standards implying high credit spreads, high quality and low quantity of issued credit. This leads to an improving pool of credit applications, eventually triggering a shift to lax lending standards. We characterize when the implied endogenous economic cycles feature long booms, and when the economy suffers lengthy, possibly double-dip recessions. A constrained planner often prefers a cycling economy to one with persistently high or persistently low sentiments which can be implemented by carefully chosen risk-weighted capital requirements. Our predictions match stylized facts on the co-movement of credit composition and spreads, lending standards, rebalancing and realized returns of credit portfolios, and real outcomes.

1 Introduction

A growing body of empirical evidence suggests that periods of overheating in credit markets forecasts low excess bond returns and recessions. These periods are characterized by incre-

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ased total quantity of credit, low interest rates, and, importantly, deteriorating quality of issued credit. Then, in subsequent recessions credit turns to scarce and expensive even for ex-post high-value projects. (Greenwood and Hanson, 2013; López-Salido et al., 2017)

A major conundrum for policy makers and academics alike is how economic policy should respond to periods of overheating and subsequent recessions. For this, we need to understand what triggers these periods, what determines the lengths of the phases and how various policy measures can effect their characteristics.

To answer these questions we build a rational model to analyse the two-way interaction between credit market sentiments and real economic outcomes. We capture sentiment as lenders choice of the power of the test they apply when granting credit. We show that the credit market exhibits the symptoms of overheating or high sentiment, when lenders choose lax lending standards. In these periods, a mixed quality of credit is issued at a low interest rate inducing high credit growth, high economic output and a deteriorating quality of credit applications. When the pool of applications is sufficiently low, lenders switch to tight standards implying high credit spreads, high quality and low quantity of issued credit. This leads to an improving pool of credit applications, eventually triggering a shift to lax lending standards. We characterize when the implied endogenous economic cycles feature long booms, and when the economy suffers lengthy, possibly double-dip recessions. A constrained planner often prefers a cycling economy to one with persistently high or persistently low sentiments which can be implemented by carefully chosen risk-weighted capital requirements. Our predictions match stylized facts on the co-movement of credit composition and spreads, lending standards, market fragmentation, rebalancing and realized returns of credit portfolios, and real outcomes.

In our stochastic OLG model, entrepreneurs running projects which are occasionally subject to liquidity shocks forcing them to request credit from investors for continuation. The scale of these projects is a choice and depends on the equilibrium interest rate and credit quantity. Entrepreneurs’ project has two dimensional type. A project is good or bad. An entrepreneur with a bad project ultimately defaults on its credit. A project is also opaque or transparent. For a large group of investors, none of these dimensions are

\[1\] See also Morais et al. (2019) for US and international evidence lax lending standards in booms in bank loan market, and Baron and Xiong (2017) on the negative relationship between bank’s credit expansion and banks’ equity returns. More generally, there is ample evidence for the coexistence of pro-cyclical volume and countercyclical value of investment in a wide range of contexts. For instance, Eisfeldt and Rampini (2006) demonstrates this for sales of property, plant and equipment, while Kaplan and Stromberg (2009) shows similar evidence on venture capital deals.
observable. These investors, whom we refer to as unskilled, can run imperfect tests to decide which entrepreneurs grant credit to. A **bold test** or a **cautious test** is available. A bold test passes all opaque projects and transparent good projects rejecting only transparent bad projects. A cautious test rejects all opaque projects and transparent bad projects, accepting only transparent good projects. Unskilled investors make a rational choice over the test to run based on the fundamentals of the economy. At the end of each period, entrepreneurs might exit either because of natural death or by going bankrupt when they cannot obtaining credit after a liquidity shock. In each case, they are replaced by newborns drawn from an exogenous pool of types. The resulting type distribution serves as the evolving state of the economy.

We start the analysis with the stage game for a given fraction of bad and good projects. The equilibrium in an economy with a small fraction of bad projects shows the symptoms of overheating. In this case, all unskilled investors choose the bold test, and credit to all type of entrepreneurs is issued at the same, relatively low interest rate. Because the bold tests implies false positive mistakes, a fraction of bad projects are also financed implying mixed quality of issued credit. That is, credit market conditions are lax. Because of this, entrepreneurs choose to run large projects implying high output.

When a large fraction of projects are bad, the equilibrium shows the symptoms of a low sentiment economy with a fragmented credit market. In this case, all unskilled investors choose to be cautious providing credit only to transparent good projects. For these projects, the interest rate is low, because unskilled capital is in large supply and the obtained loan quality is high due to the cautious test. Opaque good entrepreneurs instead can only obtain credit at high interest rates from the few skilled investors. Bad entrepreneurs cannot obtain credit at all. That is, credit conditions are tight reducing the total output of opaque good entrepreneurs and bad entrepreneurs. However, issued credit is high quality.

Turning to the dynamic implications, we derive the implied ergodic type distribution. Typically, our economy features deterministic cycles. These cycles are an outcome of the two-way interaction between credit sentiment and the fundamentals of the economy. In recessions investors are cautious because the fraction of opaque and bad applicants are large in the pool of credit applications. Being cautious implies tight lending standards, high interest rate and little credit for opaque projects, which stops opaque bad entrepreneurs to maintain their investments after a liquidity shock. Hence, they are replaced by newborns which improves the quality of the credit pool. At some point, the fraction opaque bad applicants is sufficiently low that investors are willing to switch to be bold. That implied
lax lending standards imply more credit. However, as a result the quality of the credit pool starts to deteriorate. Therefore, the cycle continues. We show that the model can generate a rich pattern of cyclical behaviour. Cycles might feature long booms and short recessions, or the vice-versa. They might also feature a double-dip recession. We characterize how cycles change in response to changing fundamentals. For instance, higher fixed cost (or lower opportunity cost) of lending leads to longer booms and shorter recessions.

Once we have a theory on the origin and properties of economic cycles induced by credit market sentiment, we turn to welfare and economic policy. First, we study a constrained planner’s problem where the only possible intervention is to choose which test investors should use in each aggregate state. For instance, if the planner forces investors to always use a bold test, it implies overheated credit markets in all states. As a result, bad opaque projects are not selected out and the economy converges to state with a high fraction of bad projects. In contrast, choosing a cautious test in all state pushes the economy towards a small fraction of bad projects but permanently high credits spreads and low investment in good, but opaque projects. Second, we show that often the planner prefers a cycling economy instead, where periods of overheated booms are interchanging with low sentiment recessions. The intuition is that in recessions cautious investors help to keep the fraction of bad projects in bay, which make the outcome of high investment in boom periods much more beneficial. Third, we show that such an optimally cycling economy can be achieved by a realistic macro-prudential regulatory tool: risk weighting in investors’ capital requirements.

Finally, we contrast our results to a wide range of stylized facts on market segmentation, the fluctuation of credit market sentiment, output, the heterogeneity of returns and portfolios of investors and international spillovers of monetary policy.

**Literature.** To be completed. (Among many others we will include the stream of papers on collateral cycles (Kiyotaki and Moore, 1997; Martin, 2005; Lorenzoni, 2008; Mendoza, 2010; Gorton and Ordonez, 2014; Gorton and Ordoñez, 2016; Asriyan et al., 2018; Fishman et al., 2019) with special emphasis on the last three which have an information choice angle, and on Martin (2005) which features endogenous cycles, models with similar features but exogenous information (Kurlat, 2016; Farboodi and Kondor, 2018), rational models of sentiments (Morris and Shin, 2002; Angeletos and Pavan, 2007; Angeletos and La’O, 2013), behavioural models of sentiment and credit cycles (Bordalo et al., 2018; Greenwood et al., 2019), and the stream of papers on flight-to-quality (Caballero and Krishnamurthy, 2008; Vayanos, 2004; Fishman and Parker, 2015; Bolton et al., 2016)).
2 Set Up. Rational Sentiments and Economic Cycles

In this part, first we present the set up in the stage game in each period. Then, we explain how we build on this stage game to create a dynamic economy.

2.1 Stage Game

There are entrepreneurs and investors. The horizon of the stage game is one day. Within each day, there is a morning, afternoon and night. Each agent is endowed with a unit of the good at the morning, and each of them are risk-neutral.

There is a unit mass of entrepreneurs alive each day. Each entrepreneur has a single project with a two dimensional type distribution. The project is good or bad, $\tau = g, b$, and either opaque or transparent, $\omega = 0, 1$. Sometimes we refer to the type of the project $(\tau, \omega)$ of the entrepreneur as her type. Entrepreneurs know their own type. In the stage game, we take the type distribution, as given. We denote the fraction of opaque and transparent bad entrepreneurs by $\mu_0$ and $\mu_1$, respectively. In full dynamics, the type distribution evolves endogenously and these fractions serve as state variables.

Entrepreneurs invest in the morning, raise loan in the afternoon and produce in the evening. Each entrepreneur $(\tau, \omega)$ chooses initial investment $I(\tau, \omega)$ in the morning and saves the rest of her endowment for the afternoon. Each unit of $I(\tau, \omega)$ costs one unit of consumption good. In the afternoon, each entrepreneur is hit by an idiosyncratic liquidity shock with probability $\phi$. These entrepreneurs have to inject $\xi$ consumption good for each unit of initial investment to maintain it, otherwise that unit fully depreciates. They can choose $i(\tau, \omega)$, the number of investment units to be maintained where

$$i(\tau, \omega) \leq I(\tau, \omega),$$

and abandon the rest of their initial investment. Each unit of maintained investment leads to $\rho_\tau$ goods in the evening.

When an entrepreneur is hit by a liquidity shock she might get a loan from investors collateralized by her maintained investment. Each investor can seize $\xi$ fraction of the collateral to cover the interest and principal of the loan from entrepreneurs with good projects, but cannot seize any from entrepreneurs with bad projects. Thus, if the entrepreneur is facing
interest rate, $r(\tau, \omega)$, she can choose to maintain a unit of investment by obtaining $\frac{1}{1 + r(\tau, \omega)}$ fraction of the $\xi$ cost as a loan and covering the remaining $\frac{r(\tau, \omega)}{1 + r(\tau, \omega)}$ fraction from her own, saved endowment. This gives the budget constraint\(^2\)

$$I(\tau, \omega) + \phi \xi i(\tau, \omega) \frac{r(\tau, \omega)}{1 + r(\tau, \omega)} = 1.$$ \hspace{1cm} (2.2)

We constrain the set of technological parameters throughout the paper as follows.

**Assumption 2.1**

$$\rho_g > \xi > \rho_b \hspace{1cm} (2.3)$$

$$\rho_g > 1 + \xi \phi \hspace{1cm} (2.4)$$

$$\rho_b > \frac{1}{1 - \phi}. \hspace{1cm} (2.5)$$

Assumption (2.3) ensures that maintaining bad projects is socially harmful, while maintaining good projects is beneficial. Assumption (2.4) implies that at least when facing zero interest rates, full continuation for the good entrepreneur is socially beneficial. Assumption (2.5) together with (2.3) ensure that without access to credit, all entrepreneurs invest all their endowment in the morning and fully abandon production in the case of a liquidity shock, instead of saving towards the liquidity shock on their own. As we will see, this assumption implies the existence of a maximum interest rate $\bar{r}$ at which entrepreneurs are willing to borrow.

Each investor, $h \in [0, w_0 + w_1]$, lives for one day. A small, $w_1$, mass of investors are skilled, while a large, $w_0$ mass of investors are unskilled. Skill is privately observable. All investors are born in the morning, provide loans in the afternoon, and consume and die in the evening. As an outside option, investors can always invest in a risk-free asset generating $1 + r_f$ growth return.\(^3\)

Skilled investors can observe the type of each project. Unskilled investors instead observe

\(^2\)The multiplier $\phi$ in (2.2) shows that we allow for a state-contingent saving technology at actuarially fair terms. That is, an entrepreneur saving a unit in the morning can obtain $\frac{1}{\phi}$ units when hit by liquidity shock. There is no such saving technology is available across periods in the dynamic version. If the reader prefers, can think of this saving technology as provided by a competitive fringe of risk-neutral commercial banks, living for a single period, who can accept deposits, but cannot lend missing the technology to evaluate collateral.

\(^3\)The parameter $r_f$ plays a role only when we think about implications of exogenous shocks to the relevant risk-free rate in Section 6. Until that point, normalizing $r_f$ to zero is innocuous.
only green and red signals for a sample of projects. These signals are generated by a test of their choice: they can opt for a bold test or a cautious test. For transparent projects, each test gives a green signal for every good project and a red signal for every bad project. However, none of the tests can distinguish between good and bad projects if the project is opaque. Instead, the bold test gives the green signal for any opaque project pooling them with good transparent ones, while the cautious test gives the red signal for any opaque project pooling them with bad transparent ones. The size of the sample an investor tests is limited by the investor’s unit endowment; she cannot test more applications than the quantity she could finance if all pass her test. The cost of the test on this sample is $c$, and unskilled investors run exactly one test.

The credit market opens in the afternoon after entrepreneurs realize whether hit by the liquidity shock. Each investor advertise an interest rate, $\bar{r}(h)$, at which they are willing to give loans to applications passing their tests, while each entrepreneur submits loan applications at advertised rates not larger than a chosen reservation interest rate, $\bar{r}(\tau, \omega)$. The credit market clears starting from the lowest interest rate and unskilled investors sample first. We provide further detail on the market clearing protocol and some features of the implied credit demand in Appendix A.

Throughout the analysis, we focus on the case when there are many unskilled investors, but few skilled investors in the following sense.

**Assumption 2.2** The mass of skilled and unskilled investors, $w_1, w_0$, satisfies the following criteria.

(i) Skilled investors capital, $w_1$, is not sufficient to cover the credit demand of all opaque good entrepreneurs at any interest rate, $r(g, \omega) \leq \bar{r}$, good entrepreneurs would be willing to take.

(ii) Unskilled investors capital, $w_0$, is abundant. In particular, it is sufficiently large that it covers the credit demand of all entrepreneurs unskilled investors are willing to lend to at any interest rate $r(\tau, \omega) \leq \bar{r}$.

We look for a stage game equilibrium defined as follows.

**Definition 2.1** A decentralized stage game equilibrium is a set of the entrepreneurs’ investment plans, $I(\tau, \omega), i(\tau, \omega)$, measures of loan applications, and maximum interest rate
choices $r_{\text{max}}(\tau,\omega)$ along with investors’ advertised interest rate schedule $\bar{r}(h)$, unskilled investors’ choice of test, and with the implied equilibrium realized interest rate $r(\tau,\omega)$ and credit allocation, $\ell(\tau,\omega)$ for each entrepreneur, such that

(i) each agent’s choice is optimal given the strategy profile of all other agents;

(ii) the implied credit allocation and interest rates are consistent with the investment plans and budget constraints (2.2) for each entrepreneur.

This definition implies that we are searching for a Nash Equilibrium where each entrepreneur with the same type $(\tau,\omega)$ follow the same strategy, and all their qualifying loans are filled at the same interest rate $r(\tau,\omega)$.

2.2 Dynamic Economy

The dynamic economy consists of infinite periods or days, $t = 0, 1, ...$. The stage game describes the sequence of events within each period. Each generation of investors live for one day only and replaced by a new, identical generation next day. As we explain next, entrepreneurs live for a random period of time implying a stochastic overlapping generation model for entrepreneurs. The fraction of different type of entrepreneurs serve as state variables.

At the end of each period, a $\delta$ fraction of all the entrepreneurs die and is replaced by a randomly chosen $\delta$ mass of new entrepreneurs. Furthermore, any entrepreneur who is hit by the liquidity shock but is not able to raise financing dies and is replaced by a randomly chosen entrepreneur. For the type distribution of new entrants we assume that $\lambda$ fraction of entrants are bad, and $\frac{1}{2}$ are opaque (independent from each other). Thus $\frac{1}{2}\lambda$ fraction of new entrants have bad opaque (and bad transparent) projects, and $\frac{1}{2}(1 - \lambda)$ fraction have good opaque (and good transparent) projects.

This endogenous replacement is formally stated in the following assumption.

**Assumption 2.3** Any entrepreneur suffering a liquidity shock, but not financed by investors is replaced by a randomly chosen entrepreneur from the initial pool in the next period.

For simplicity, we assume that there is no credit history recorded for entrepreneurs. That is, investors cannot learn from the past. Also, there is no saving technology available across
periods. Therefore, entrepreneurs consume their wealth at the evening of each period, and, if survive, start the new period with their unit endowment received in the morning.

Note that we intentionally constructed the dynamic set up in a way that, apart from the law of motion for the fraction of different types of entrepreneurs, each period is independent. That is, the dynamic equilibrium can be defined as follows.

**Definition 2.2** A decentralized dynamic equilibrium, is a sequence of stage game equilibria. In each period, the stage game equilibrium is consistent with the realized type distribution \((\mu_0, \mu_1)\), while the dynamics of \((\mu_0, \mu_1)\) is consistent with Assumption 2.3.

In principle, each equilibrium objects defined for the stage game should be conditioned on the state variables. For simplicity, we will suppress this dependence whenever it does not cause any confusion.

### 3 Credit Market Equilibrium

In this part, we solve for the equilibrium of our model. As we will show, our structure allows for solving for the full equilibrium in steps. First we solve for the credit market equilibrium in the stage game. Second, we characterize the credit market dynamics. Third, in Section 4, we describe the real economy outcomes in the stage game and its dynamics.

#### 3.1 Stage Game

For a simpler and more intuitive exposition, in this part we fix the type distribution at \(\mu_0 = \mu_1 = \frac{1}{2}\) and characterize the equilibrium as a function of the fraction of bad entrepreneurs, \(\lambda\). (This would be the type distribution, if all entrepreneurs would die during the night, \(\delta = 1\).) We show in the next part, that the generalization of the results to the case of \(\mu_0 \neq \mu_1\) is straightforward.

Before we characterize the equilibrium, it is useful to define the benchmark when credit markets are not functioning, perhaps because Assumption 2.2 does not hold as the total mass of investors \(w_0 + w_1 = 0\). In this case, as assumptions (2.5)-(2.3) imply \(\xi > \frac{1}{1-\phi}\), each entrepreneur chooses to invest her full endowment in the morning and abandon all investment after a liquidity shock. We refer to this case as *autarchy*.
As we show in this section, we need only the following property of credit demand to derive the equilibrium on credit markets. In the stage game both good and bad entrepreneurs prefer to maintain all their investments after a liquidity shock if they can obtain loans at an interest rate \(r(g, \omega) < \bar{r}\) where

\[
\bar{r}(\rho_g, \xi, \phi) \equiv \frac{\rho_g - \xi}{\rho_g \xi (1 - \phi) - (\rho_g - \xi)}.
\]  

(3.1)

To see this, define \(\eta(\tau, \omega) \equiv \frac{i(\tau, \omega)}{I(\tau, \omega)}\), as the equilibrium fraction of maintained investment for a given type. Then, a binding budget constraint (2.2) gives

\[
I(\tau, \omega) = \frac{1}{1 + \phi \xi \eta(\tau, \omega) \frac{r(\tau, \omega)}{1 + r(\tau, \omega)}}.
\]

A good entrepreneur is maximizing expected consumption which is expected production minus expected repayment:

\[
\rho_g ((1 - \phi) I(g, \omega) + \phi i(g, \omega)) - (1 + r(g, \omega)) \phi \xi \frac{i(g, \omega)}{(1 + r(g, \omega))} =
\]

\[
(\rho_g (1 - \phi) + \phi (\rho_g - \xi) \eta(g, \omega)) I(\tau, \omega) =
\]

\[
\rho_g (1 - \phi) + (\rho_g - \xi) \phi \eta(g, \omega) \frac{r(g, \omega)}{1 + r(g, \omega)}.
\]  

(3.2)

It is easy to see that this is increasing in \(\eta(g, \omega)\) if and only if \(r(\tau, \omega) < \bar{r}\). Note that \(\bar{r}\) is independent of the choice of \(i(g, \omega)\) or \(I(g, \omega)\) and positive under (2.3)-(2.5).

For a bad entrepreneur expected consumption equals expected production,\n
\[
\rho_b \frac{(1 - \phi) + \phi \eta(b, \omega)}{1 + \phi \xi \eta(b, \omega) \frac{r(b, \omega)}{1 + r(b, \omega)}},
\]  

(3.3)

as she does not intend to pay back. This is also increasing in \(\eta(g, \omega)\) if \(r(g, \omega) \leq \bar{r}\).

We are ready to characterize interest rates and information choice, that is, the stage game equilibrium in credit markets. In Section 4 we proceed to the real outcomes.

We first show that the unique equilibrium which arises in our economy can be one of three distinct types, depending on the parameters. In order to do so, it is useful to define three interest rate functions.
Definition 3.1 (Interest Rates)

\[
\begin{align*}
    r_B(\lambda, c, r_f) &\equiv \frac{1}{2} \frac{\lambda}{1-\lambda} + \frac{1-\lambda}{1-\lambda} r_f + \frac{1}{1-\lambda} c \\
    r_C(\lambda, c, r_f) &\equiv r_f + \frac{2}{1-\lambda} c \\
    r_I(\lambda, c, r_f) &\equiv \frac{\lambda}{1-\lambda} + \frac{1}{1-\lambda} r_f + \frac{1+\lambda}{1-\lambda} c.
\end{align*}
\]

(3.4) \hspace{1cm} (3.5) \hspace{1cm} (3.6)

We will show in the next proposition that in each type of equilibrium, the entrepreneurs who can obtain credit face exactly one of these three interest rates or the maximum interest rate \(\bar{r}\).

Proposition 3.1 When \(\bar{r}(\rho_g, \xi, \phi) > \min(r_C(\lambda, c, r_f), r_B(\lambda, c, r_f))\) then

(i) If \(r_B(\lambda, c, r_f) < \min(\bar{r}(\rho, \xi, \phi), r_C(\lambda, c, r_f))\), there is a pooling equilibrium, where all entrepreneurs obtaining credit at interest rate \(r_B(\lambda, c, r_f)\). Each unskilled investor runs the bold test. We refer to this as the bold equilibrium.

(ii) If \(r_C(\lambda, c, r_f) < \min(r_B(\lambda, c, r_f), \bar{r}(\rho, \xi, \phi))\) and \(\bar{r}(\rho, \xi, \phi) < r_I(\lambda, c, r_f)\), there is a separating equilibrium, where opaque good entrepreneurs obtaining credit at interest rate \(\bar{r}(\rho, \xi, \phi)\), while transparent good entrepreneurs obtain credit at \(r_C(\lambda, c, r_f)\). None of the bad entrepreneurs are obtaining credit. Each unskilled investor runs the cautious test. We refer to this as the cautious equilibrium.

(iii) If \(r_C(\lambda, c, r_f) < \min(r_B(\lambda, c, r_f), \bar{r}(\rho, \xi, \phi))\) and \(\bar{r}(\rho, \xi, \phi) > r_I(\lambda, c, r_f)\) there is a semi-separating equilibrium, where opaque good and bad entrepreneurs obtain credit at interest rate \(r_I(\lambda, c, r_f)\). All good transparent entrepreneurs obtain credit at interest rate \(r_C(\lambda, c, r_f)\). Some unskilled investors run the bold test, while others run the cautious test. We refer to this as the mix equilibrium.

When \(\bar{r}(\rho_g, \xi, \phi) < \min(r_C(\lambda, c, r_f), r_B(\lambda, c, r_f))\) then the only equilibrium is autharchy.

The intuition behind this proposition relies on the following observations. The assumption that unskilled capital is in abundant supply implies a zero profit condition. In any equilibrium, interest rates received by unskilled investors running any test has to be such
that their expected return on their loan portfolio equals the risk-free rate. If their expected
return were higher, all who are not lending in equilibrium would be motivated to enter and
offer a lower interest rate. If their expected return were lower, they would not enter. In fact,
interest rates \( r_B (\lambda, c, r_f) \), \( r_C (\lambda, c, r_f) \) and \( r_I (\lambda, c, r_f) \) are the rates at which
an unskilled investor is indifferent between lending to entrepreneurs and earning the risk free rate
without running a test under different informational choices. In particular, \( r_B (\lambda, c, r_f) \)
makes a bold investor indifferent whether to lend if all entrepreneurs are expected to apply for
loans at that rate. The rate \( r_C (\lambda, c, r_f) \) makes a cautious investor indifferent whether to
lend under the same condition. While the rate \( r_I (\lambda, c, r_f) \) makes a bold investor indifferent
whether to enter if all opaque entrepreneurs, but not transparent good entrepreneurs are expected
to apply for loans at that rate. Note that any of these rates are feasible only if its smaller than
\( \bar{r} (\rho, \xi, \phi) \).

The zero profit interest rates, \( r_B, r_C \) and \( r_I \) depends on investors’ choice of test, because
of a trade-off. Using the cautious test results in a loan portfolio of high quality. As only
good entrepreneurs pass these test, they end up lending only to good entrepreneurs who pay
back. However, their rejection rate is higher than with a bold test as a cautious test fails all
the opaque good entrepreneurs. As running the test has a fixed cost, not lending to tested
applications is costly. Under the bold test, the investor’s loan portfolio is of lower quality,
but she rejects less applications. On each panel of Figure 1, we plotted the relevant zero
profit interest rates along with \( \bar{r} \), as a function of the proportion of bad projects. The two
panels correspond to a different parametrization.

Given these observations, the bold equilibrium arises when the rate at which bold
investors are willing to enter is smaller than the rate at which cautious investors are,
\( r_B (\lambda, c, r_f) < r_C (\lambda, c, r_f) \). This is the first region on each panel of Figure 1, on the left
of the (first) vertical line. In that case, cautious investors cannot compete with bold
investors implying that all active investors are bold. All good entrepreneurs choose \( r_B (\lambda, c, r_f) \)
as a reservation rate. Bad entrepreneurs choose a higher reservation rate as they do not intend
to pay back. All skilled and unskilled investors advertise the same rate \( r_B (\lambda, c, r_f) \) at which
they are willing to lend. While all investors would prefer to lend at interest rates higher than
\( r_B (\lambda, c, r_f) \), good entrepreneurs do not demand credit at higher interest rate. This deters
investors from lending. Good entrepreneurs would prefer to borrow at interest rates lower
than \( r_B (\lambda, c, r_f) \), but separation is not possible for transparent good entrepreneurs because
(i) all investors observe the same signal on all good entrepreneurs, and hence good entre-
preneurs cannot be served at different markets, and (ii) for any interest rate the demand of
bad entrepreneurs is weakly higher than that of good entrepreneurs, as their effective cost of credit is lower due to the fact that they are not paid back. Hence, bad entrepreneurs follow good ones to any market. It thus follows that all entrepreneurs that are able to issue bonds do so at the same interest rate $r_B(\lambda, c, r_f)$. Skilled and unskilled investors differ in the quality of the pool of entrepreneurs which they finance, but not in the financing terms.

The second part of the proposition describes the cautious equilibrium. This arises if cautious investors are willing to enter at a lower interest rate than bold investors, $r_B(\lambda, c, r_f) > r_C(\lambda, c, r_f)$, and the interest rate at which bold investors would be willing to enter a market with no transparent good entrepreneurs is not feasible, $r_I(\lambda, c, r_f) > \bar{r}(\rho, \xi, \phi)$. This is the region on the right of the (second) vertical line on each of the panels of Figure 1. In this case, all investors are cautious in equilibrium. It follows that skilled and unskilled investors finance a different set of entrepreneurs at different interest rates. In particular, transparent good projects are financed at $r_C(\lambda, c, r_f)$ by unskilled investors. In contrast, opaque good entrepreneurs can raise financing only from skilled investors at the maximum feasible interest rate. This is, so because when unskilled investors are running the cautious test, only skilled investors can distinguish opaque good entrepreneurs from opaque bad entrepreneurs. The interest rate is high, because the supply of skilled capital is less than the demand from opaque good entrepreneurs by Assumption 2.2. Still, the rate is not sufficiently high to tempt investors to run the bold test and enter to that market where transparent good entrepreneurs are not present.

The third part of the proposition shows that there might be an intermediate case which we refer to as the mix equilibrium. If the maximum feasible interest rate, $\bar{r}$, is sufficiently high, the interest rate $r_I(\lambda, c, r_f)$ becomes feasible. This is the case on the left panel of Figure 1 between the two vertical lines. In that case, an asymmetric separating equilibrium arises where opaque good entrepreneurs are financed by bold unskilled investors and skilled investors at a relatively high interest rate, $r_I(\lambda, c, r_f)$. In this equilibrium, unskilled investors also finance opaque bad entrepreneurs at that interest rate which they mistake for good entrepreneurs. Cautious unskilled investors finance all transparent good entrepreneurs at the lower interest rate $r_C(\lambda, c, r_f)$. Interest rates $r_I(\lambda, c, r_f)$ and $r_C(\lambda, c, r_f)$ are such that unskilled investors are indifferent whether to be bold, take on a worse quality credit portfolio for the higher interest rate, or be cautious and finance a high quality credit portfolio for a lower interest rate.

In anticipation of the dynamic results which generate endogenous cycles, and in line with Figure 1, it is useful to map out how the type of the equilibrium changes as the share of bad
entrepreneurs, $\lambda$, changes in the economy. The corollary follows directly from expressions (3.4)-(3.6) and Proposition 3.1.

**Corollary 3.1** Let $\lambda_B \equiv \frac{2c}{1+r_f}$ and $\lambda_C \equiv \frac{\bar{r}-r_f-c}{1+c+r_f}$.

(i) When $\min\{r_B(\lambda,c,r_f), r_C(\lambda,c,r_f)\} < \bar{r}(\rho_g, \xi, \phi)$:

(a) There is a bold equilibrium in the range $\lambda \in [0, \lambda_B]$.

(b) There is a cautious equilibrium in the range $\lambda \in [\max\{\lambda_B, \lambda_C\}, 1]$.

(c) There is a mix equilibrium in the range $\lambda \in [\lambda_B, \max\{\lambda_B, \lambda_C\}]$.

(ii) When $\min\{r_B(\lambda,c,r_f), r_C(\lambda,c,r_f)\} \geq \bar{r}(\rho_g, \xi, \phi)$ the economy is in autarchy.

We focus on case (i) of the Corollary when $\bar{r}$ is sufficiently large as case (ii) is uninteresting. Note that for the set of parameters implying $\lambda_B > \lambda_C$, the equilibrium can be only bold or cautious. Sometimes we will refer this case as a 2-stage economy. When the opposite holds it can be bold, cautious or mix. We will refer to this case as a 3-stage economy. The two panels of Figure 1 illustrates these two cases.

Note that a bold equilibrium exhibits several features of an overheated, or high sentiment credit market. Interest rates are uniformly low and, apart from transparently bad projects, all other type of projects are financed. Consequently, in this type of equilibrium the overall quality of initiated credit contracts is low with a significant share eventually defaulting. This
is in contrast with the cautious equilibrium which exhibits feature of a low sentiment credit market. Most importantly, this market is fragmented. Transparently good entrepreneurs enjoy low interest rates, and sufficient founding. However, credit for opaque good entrepreneurs is available only for very high rates, if at all. On the bright side, bad projects are not financed at all. Therefore, the total loan quantity is relatively low, but it is quality is high. This implies that the subsequent realized returns is expected to be high.

In Section 4, we characterize output and welfare in our economy. We argue that a bold equilibrium corresponds to a boom, while a cautious equilibrium corresponds to a recession. In Section 6 we contrast the qualitative features of our model with various stylized facts of financial markets and the real economy. However, first, in the next section, we explain why our equilibrium generates endogenous cycles.

### 3.2 Credit Market Dynamics and Endogenous Cycles

In this section, we construct the dynamic equilibrium using the stage game explained in section 2.1 as the building block. In the dynamic version, the fraction of different type of entrepreneurs serve as state variables. The key to the dynamics is that these states respond endogenously to credit market sentiment. In particular, the quality of the pool of credit applications deteriorates in the credit market is overheated, i.e., in a bold equilibrium, and improves when credit market sentiment is low, i.e., in a cautious equilibrium. At the same time, the changing type distribution induces rational shifts in investors information acquisition decisions implying fluctuations in sentiment. This endogenous interaction leads to deterministic economic cycles without exogenous aggregate shocks to the system. Moreover, our model is capable to generate a wide range of different cycles. In particular, in some economies long booms are alternating with short recessions, while in others we see the opposite pattern. We also show that sometimes the economy must experience a double-dip recession, before eventually recovering.

Recall that \( \mu_0 \) and \( \mu_1 \) denote the measure of bad opaque and bad transparent entrepreneurs, respectively. While the corresponding measures are equal to \( \frac{\lambda}{2} \) in the newborn pool (and in our illustration of the stage game), they evolve differently in the dynamic equilibrium. Note that we are not using a time-subscript for \( \mu_0, \mu_1 \), instead we denote next period values as \( \mu'_0, \mu'_1 \) when necessary. We conjecture and verify that in the dynamic economy, the measures of good opaque and good transparent entrepreneurs are both equal to \( \frac{1-\mu_0-\mu_1}{2} \).
As such, $\mu_0$ and $\mu_1$ are sufficient state variables to characterize each period of the dynamic economy.

Before deriving the endogenous law of motion, we show that our analysis of the stage game equilibrium is easy to generalize for the case of a state where $\mu_0$ and $\mu_1$ are different. First, we carefully replace $\lambda$ in (3.4-3.6) to redefine the zero profit interest rates as follows.

**Definition 3.2 (Dynamic Interest Rates)**

\[
\begin{align*}
 r_B(\mu_0, \mu_1, c, r_f) &\equiv \frac{\mu_0}{1 - \mu_1 - \mu_0} + \frac{1 - \mu_1}{1 - \mu_1 - \mu_0} r_f + \frac{1}{1 - \mu_1 - \mu_0} c \\
 r_C(\mu_0, \mu_1, c, r_f) &\equiv r_f + \frac{2}{1 - \mu_1 - \mu_0} c \\
 r_I(\mu_0, \mu_1, c, r_f) &\equiv \frac{2\mu_0}{1 - \mu_1 - \mu_0} + \frac{1 + \mu_0 - \mu_1}{1 - \mu_1 - \mu_0} r_f + \frac{1 + \mu_1 + \mu_0}{1 - \mu_1 - \mu_0} c.
\end{align*}
\]  

(3.7)  

(3.8)  

(3.9)

The generalized interest functions imply that given $\mu_0$ and $\mu_1$, the economy is either in the stage of a bold, cautious or mix equilibrium. In fact, the following Proposition is generalization of Proposition 3.1 and Corollary 3.1 where we have carefully replaced $\lambda$ with expressions of $\mu_0, \mu_1$.

**Proposition 3.2** Let $\bar{\mu}_0(\mu_1) \equiv \bar{\mu}_0(\mu_1) \equiv \frac{\bar{\mu}_0 - c - \mu_1(\bar{\mu}_0 + c - r_f)}{2 + c + \bar{\mu}_0 + r_f}$.

(i) When

\[
\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r}(\rho_g, \xi, \phi)
\]

(3.10)

holds:

(a) There is a bold stage if $\mu_0 \in \left[0, \frac{c}{1 + r_f}\right]$.

(b) There is a cautious stage if $\mu_0 \in \left[\max\left\{\frac{c}{1 + r_f}, \bar{\mu}_0(\mu_1)\right\}, 1\right]$.

(c) There is a mix stage if $\mu_0 \in \left[\frac{c}{1 + r_f}, \max\left\{\frac{c}{1 + r_f}, \bar{\mu}_0(\mu_1)\right\}\right]$.

(ii) When $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} > \bar{r}(\rho_g, \xi, \phi)$, then the economy is in autarchy.
In each of these stages the credit market is characterized as the corresponding equilibrium in the stage game.

Next, we turn to the characterization of the law of motion of the state variables. The critical observation is to understand how Assumption 2.3 affects the dynamics in different stages. When at least some lenders run the bold test, all three groups of opaque and transparent good projects and opaque bad projects are financed after hit by a liquidity shock. However, when lenders are all cautious opaque bad projects are not financed. This implies that in that stage entrepreneurs with opaque bad projects hit by a liquidity shock are exiting the pool of projects and are replaced by a new draw from the pool. Therefore, when lenders are cautious the fraction of opaque bad projects changes to
\[
\mu'_0 = \mu_{0C}(\delta, \lambda, \phi, \mu_0, \mu_1),
\]
while the fraction of transparent bad projects changes to
\[
\mu'_1 = \mu_{1C}(\delta, \lambda, \phi, \mu_0, \mu_1)
\]
where
\[
\mu_{0C}(\delta, \lambda, \phi, \mu_0, \mu_1) = (1 - \delta)(1 - \phi)\mu_0 + (\delta + (1 - \delta)\phi(\mu_0 + \mu_1))\frac{\lambda}{2}
\]
(3.11)
and
\[
\mu_{1C}(\delta, \lambda, \phi, \mu_0, \mu_1) = (1 - \delta)(1 - \phi)\mu_1 + (\delta + (1 - \delta)\phi(\mu_0 + \mu_1))\frac{\lambda}{2}.
\]
(3.12)

On the other hand, if at least some lenders run a bold test, the fraction of opaque bad projects changes to \(\mu'_0 = \mu_{0B}(\delta, \lambda, \phi, \mu_0, \mu_1)\), while the fraction of transparent bad projects changes to \(\mu'_1 = \mu_{1B}(\delta, \lambda, \phi, \mu_0, \mu_1)\) where
\[
\mu_{0B}(\delta, \lambda, \phi, \mu_0, \mu_1) = (1 - \delta)\mu_0 + (\delta + (1 - \delta)\mu_1\phi)\frac{\lambda}{2}
\]
(3.13)
and
\[
\mu_{1B}(\delta, \lambda, \phi, \mu_0, \mu_1) = (1 - \delta)(1 - \phi)\mu_1 + (\delta + (1 - \delta)\phi\mu_1)\frac{\lambda}{2}.
\]
(3.14)

The law of motion for the state variables is quite intuitive. For instance, consider the mass of opaque bad types \(\mu_0\). When the economy is not in a cautious stage, function \(\mu_{0B}(\delta, \lambda, \phi, \mu_0, \mu_1)\) determines this measure in the next period. From the existing entrepreneurs, fraction \((1 - \delta)\) survives. Also, out of the \(\delta\) measure exogenously replaced, a fraction \(\lambda/2\) will be of this type. Finally, \((1 - \delta)\phi\mu_1\) is the measure of the transparently bad types who are hit by a liquidity shock, survived the exogenous replacement, but not financed. Hence, they are also replaced from the original pool by Assumption 2.3. All the other cases follow a similar intuition.

The next proposition summarizes the law of motion for fraction of opaque and transparent bad entrepreneurs.

**Proposition 3.3** When \(\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r} (\rho_g, \xi, \phi)\) so that the eco-
nomics is not in autarchy, then the law of motion is described as follows.

(i) If \( \mu_0 \in \left[ 0, \max \{ \frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1) \} \right] \), then law of motion for \( \mu_0 \) and \( \mu_1 \) follows equations (3.11) and (3.12).

(ii) If \( \mu_0 \in \left[ \max \{ \frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1) \}, 1 \right] \), then law of motion for \( \mu_0 \) and \( \mu_1 \) follows equations (3.13) and (3.14).

It is easy to see that opaque good and transparent good types subject to the same law of motion both in and out of recessions. This is validates our conjecture that as long as we start the economy with an equal measure of opaque good and opaque bad types, these measures will continue to be equal to \( \frac{1-\mu_0-\mu_1}{2} \). This validates that our system can be described with two state variables only, even if we have four types.

The last step is to characterize the prevailing ergodic steady state of our economy. First, we consider the simpler case where \( \max \{ \frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1) \} = \frac{c}{1+r_f} \) along the equilibrium path. This corresponds to a two-stage economy where the investors are either all bold or all cautious. Then, we will proceed to the full characterization.

### 3.2.1 Simple Endogenous Cycles: Two-Stage Economy

In this part, we discuss the case when \( \max \{ \frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1) \} = \frac{c}{1+r_f} \) along the equilibrium path, and we characterize all the possible ergodic distributions of the economy. This is the case where the dynamic economy is characterized by a single state variable, \( \mu_0 \).

To ensure \( \frac{c}{1+r_f} > \tilde{\mu}_0(\mu_1) \) in the long run, we maintain the following assumption throughout this section.

**Assumption 3.4** For a 2-stage economy, assume

\[
\frac{\bar{r} - r_f - c}{\bar{r} + c} - \frac{2c}{1+r_f} \leq \frac{\delta \lambda}{2(\delta (1 - \phi) + \phi) - \lambda \phi (1 - \delta)}. \tag{3.15}
\]

Assumption 3.4 is a sufficient condition for investors being all bold or cautious. In order to characterize the credit cycles of the economy, we need to first define four constant levels of \( \mu_0 \). These constant are defined as: (1) \( \tilde{\mu}_{0B}(\delta, \lambda, \phi) \): value of \( \mu_0 \) in a steady state where every
lender is bold and remains bold forever, (2) \( \bar{\mu}_{0B}(\delta, \lambda, \phi) \): value of \( \mu_0 \) in a steady state where every lender is cautious and remains cautious forever. Note that these two values correspond to steady states that are not cycles. (3,4) \( \mu_{0*}^B(\delta, \lambda, \phi) \) and \( \mu_{0*}^C(\delta, \lambda, \phi) \) are the values of \( \mu \) if the economy fluctuates between two states in the long run ergodic set, one in which every one is bold (\( \mu_{0*}^B(\delta, \lambda, \phi) \)), and one in which everyone is cautious (\( \mu_{0*}^C(\delta, \lambda, \phi) \)). Note that this corresponds to a cycle of length 2. The appendix provides detail for derivation of these levels. It also shows that \( \bar{\mu}_{0B}(\delta, \lambda, \phi) > \mu_{0*}^C(\delta, \lambda, \phi) > \mu_{0*}^B(\delta, \lambda, \phi) > \bar{\mu}_{0C}(\delta, \lambda, \phi) \).

Given these preliminaries, the next proposition characterizes the steady state dynamic cycles of the economy.

**Proposition 3.4** Consider \( \bar{\mu}_{0B}(\delta, \lambda, \phi) > \mu_{0*}^C(\delta, \lambda, \phi) > \mu_{0*}^B(\delta, \lambda, \phi) > \bar{\mu}_{0C}(\delta, \lambda, \phi) \) as defined above, and assume assumption 3.4 holds. The ergodic set of the economy is characterized as follows.

(i) \( \frac{e}{1+r_f} > \bar{\mu}_{0B} \): the ergodic distribution is degenerate, \( \mu_0 \to \bar{\mu}_{0B} \), thus that the economy converges to a permanent bold stage.

(ii) \( \frac{e}{1+r_f} < \bar{\mu}_{0C} \): the ergodic distribution is degenerate, \( \mu_0 \to \bar{\mu}_{0C} \), implying that the economy converges to a permanent cautious stage.

(iii) \( \mu_{0*}^B < \frac{e}{1+r_f} < \mu_{0*}^C \): the ergodic distribution has a two-point support, \( \mu_{0*}^C, \mu_{0*}^B \), implying that the economy oscillates between a one-period bold stage and one-period cautious stage for ever. Thus the economy has a credit cycle of length 2.

(iv) \( \mu_{0*}^C < \frac{e}{1+r_f} < \bar{\mu}_{0B} \): the ergodic distribution has more than two points of support. The credit cycle consists of a multi-period bold stage (while \( \mu_0 \) increases), followed by a one-period cautious stage when \( \mu_0 \) declines to that of the bold stage with the lowest \( \mu_0 \).

(v) \( \bar{\mu}_{0C} < \frac{e}{1+r_f} < \mu_{0*}^B \): the ergodic distribution has more than two points of support. The credit cycle consists of a multi-period cautious stage (while \( \mu_0 \) decreases), followed by a one-period bold stage when \( \mu_0 \) rises to that of the cautious stage with the highest \( \mu_0 \).

The Proposition shows that depending on the size of \( \frac{e}{1+r_f} \), the threshold determining investors switching from bold to cautious, our dynamic system can show a large diversity of features. When this value is very large, then the economy is doomed to end in a permanent overheated bold stage: a boom. This is so, because the fraction of opaque bad investors never reaches a value where it would be wise to be cautious. Hence, unskilled investors remain to be bold.
Figure 2: The law of motions of state variables (first panel), interest rates (second panel), total gross output (third panel) in a multi-period boom and a one period recession cycle in a 2-stage economy.
As a mirror image when \( \frac{c}{1+r_f} \) is very low, we have a permanent low-sentiment cautious stage: a recession.

In contrast, as cases (iii)-(v) in Proposition 3.4 describes, when \( \frac{c}{1+r_f} \in \bar{\mu}_0C, \bar{\mu}_0B \) then our economy features endogenous, deterministic cycles of various types. We refer to this set of parameters as the cyclicality region. Within this region, when \( \frac{c}{1+r_f} \) is relatively high, the ergodic distribution features a multi-period boom and a one-period recession. In that case, a short recession is enough to improve the quality of loan applications sufficiently that investors are happy to be bold again. A further decrease of \( \frac{c}{1+r_f} \) implies a symmetric ergodic distribution, one-period booms are followed by one-period recessions. An even lower \( \frac{c}{1+r_f} \) implies multi-period recessions followed by one period booms.

Naturally, as the critical thresholds \( \bar{\mu}_0B, \mu^*_0C, \mu^*_0B, \bar{\mu}_0C \) all depend on \( \lambda, \delta \) and \( \phi \) any change in these parameters can shift the economy across these regions even if \( \frac{c}{1+r_f} \) remains constant.

The next proposition characterizes how the cyclicality region changes with these parameters.

**Proposition 3.5** The cyclicality region and its subregions change in \( \lambda, \phi \) and \( \delta \) as follows.

(i) As the probability of a liquidity shock, \( \phi \), the probability of survival, \( (1 - \delta) \), or the fraction of bad new projects diminishes, \( \lambda \), so does the cyclical region:

\[
\lim_{\phi \to 0} \bar{\mu}_0B, \mu^*_0C, \mu^*_0B, \bar{\mu}_0C = \lim_{\delta \to 1} \bar{\mu}_0B, \mu^*_0C, \mu^*_0B, \bar{\mu}_0C = \frac{\lambda}{2}, \lim_{\lambda \to 0} \bar{\mu}_0B, \mu^*_0C, \mu^*_0B, \bar{\mu}_0C = 0.
\]

(ii) The upper (lower) threshold of the cyclical region, \( \bar{\mu}_0B \) (\( \bar{\mu}_0C \)), is increasing (decreasing) in the probability of a liquidity shock \( \phi \) and in the probability of survival \( (1 - \delta) \). Therefore, the cyclicality region is expanding in the probability of a liquidity shock and in the probability of survival. The cyclicality region is shrinking in the fraction of good new projects, \( 1 - \lambda \).

(iii) Within the cyclicality region, (a) the subregions of multi-period boom and one-period boom and recession are both expanding in the fraction of good new projects and the probability of survival in the sense of

\[
\frac{\partial \bar{\mu}_0B}{\partial \mu^*_0C}, \frac{\partial \mu^*_0B}{\partial \mu^*_0B}, \frac{\partial \mu^*_0C}{\partial \bar{\mu}_0B}, \frac{\partial \mu^*_0C}{\partial \bar{\mu}_0C} > 0,
\]

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(b) the subregion of multi-period recessions is expanding in the probability of survival in the sense of \( \frac{\partial \mu_0}{ \partial (1-\delta) } > 0 \).

The top panel on Figure 2 helps to understand the technical intuition behind deterministic cycles. Given the OLG structure of entrepreneurs type distribution, under a fixed information choice of investors, the proportion of bad opaque types would converge to a single attractor. In particular, the upper dashed horizontal line denotes this attractor when investors are bold. In contrast, the lower dashed horizontal line denotes this attractor if investors are cautious. The earlier has to be higher than the latter, as there is the exit rate of opaque bad entrepreneurs is higher when investors are cautious. It is easy to see that if the threshold for investors to switch between cautious and bold lies between these two attractors, the economy must exhibit deterministic cycles of one type or another. For instance, if we start the system with a \( \mu_0 \) below the threshold, then investors are bold hence \( \mu_0 \) move towards the higher attractor. Therefore, there must be a point when \( \mu_0 \) becomes larger than the threshold triggering a switch to cautious. But then, immediately \( \mu_0 \) starts to move towards the lower attractor. Intuitively, the length of booms and recessions depend on how many steps the system needs in any of these stages to cross the threshold. The Figure depicts the case when booms are long and recessions are short.

The middle panel of Figure 2 plots the corresponding ergodic distribution of interest rates and output which helps to understand the economic intuition. The cycles are an outcome of the two-way interaction between credit sentiment and the fundamentals of the economy. In recessions investors are cautious because the fraction of opaque and bad applicants are large in the pool of credit applications. Being cautious implies tight lending standards, high interest rate and little credit for opaque projects, which stops opaque bad entrepreneurs to maintain their investments after a liquidity shock. Hence, they are replaced by newborns which improves the quality of the credit pool. At some point, the fraction opaque bad applicants is sufficiently low that investors are willing to switch to be bold. That implied lax lending standards imply more credit. However, as a result the quality of the credit pool starts to deteriorate. Therefore, the cycle continues.

3.2.2 Full Dynamics: The Three-Stage Economy and the Double-dip Recession

When \( \frac{c}{1+r_f} > \bar{\mu}_0(\mu_1) \) does not hold everywhere along the equilibrium path (in the long run), our economy features even more elaborate dynamics. In this case, we need two state variables
to characterize the dynamic economy, \((\mu_0, \mu_1)\).

Importantly, in this case the economy can cycle through all three stages, going from bold to mix and then to cautions, and then jumping back to bold stages. Such a three-stage economy is interesting because it features two different type of recessions. The mix stage has signs of low credit market sentiment as it leads to a similar fragmentation of the market as the cautious stage. However, the mixed stage corresponds to a recession which is not sufficiently deep to trigger the purifying effect of the entrepreneur pool we observed in the cautious stage. This is so, because even a small mass of bold investors is sufficient to give enough credit to bad entrepreneurs by mistake which keeps them from exiting. Therefore, in a mix stage the fraction of bad entrepreneurs keeps increasing to the point when the cautious stage is triggered with sufficiently tight credit conditions to reverse the direction of the economy.

The following assumption ensures a three-stage cycle for the dynamic economy.

**Assumption 3.5** Assume

\[
(i) \quad \frac{c}{1+r_f} < \frac{\delta \lambda (c+\phi)}{c+\phi+2} < \frac{\lambda (\delta + \phi(1-\delta))}{(1-\delta)(2-\lambda)} \phi + 2 \\
(ii) \quad \frac{c}{1+r_f} > \frac{\delta \lambda (2-\delta)(\delta + \phi(1-\delta)) (2+(2-(2-\lambda)\phi)(1-\delta))}{2(2-\delta)^2 \beta^2 - (1-\delta)^4 (2-\lambda)(1-\lambda) \phi^2 + 3\delta(2-\delta)(1-\delta)^2 (2-\lambda) \phi + 4(1-\lambda) - (2-\delta) \lambda^2 + 6(1-\lambda)) (\phi(1-\delta))^2}.
\]

Assumption 3.5 ensures that \(\mu^*_C < \frac{c}{1+r_f} < \mu^*_B(\mu_1) < \bar{\mu}_B\). This is sufficient to make sure that in the long run, the economy fluctuates in all the three stages, as formalized in the following proposition.

**Proposition 3.6** Consider \(\bar{\mu}_B(\delta, \lambda, \phi) > \mu^*_C(\delta, \lambda, \phi) > \mu^*_B(\delta, \lambda, \phi) > \mu^*_C(\delta, \lambda, \phi)\), and assume assumption 3.5 holds. Then the ergodic distribution has more than two points of support. The credit cycle consists of a multi-period bold stage (while \(\mu_0\) increases), followed by a multi-period mix stage (while \(\mu_0\) still increases), followed by a one-period cautious stage when \(\mu_0\) declines to that of the bold stage with the lowest \(\mu_0\).

In this case, Figure 3 portrays an example of the dynamic cycle of the economy. The top panel shows illustrates the dynamics of the state variables. In the beginning of each upward path, \(\mu_0\) is sufficiently small and the composition of borrowers sufficiently good, \(\mu_0 < \frac{c}{1+r_f}\). In this region (below the dotted blue line) all investors are bold. Between the dotted blue line
and the yellow curve, some investors turn cautious but some are still bold (semi-separating equilibrium). In this region, opaque bad entrepreneurs still get funded (although at worse interest rates), so $\mu_0$ is still growing. When $\mu_0$ becomes sufficiently large, i.e. it crosses the yellow threshold, then the composition of borrowers become sufficiently bad that no investor chooses to be bold. Thus all investors turn cautious and a one period crash happens.

The second panel on Figure 3 shows the interest rates in our example of a 3-stage economy. Observe that although the dynamics of the state variables are qualitatively similar to the 2-stage economy depicted in Figure 2, the implied economy is quite different. We see that already in the mix stage there is a considerable spread between the interest rates faced by opaque and transparent entrepreneurs. This spread drops to zero again, only when the economy reverses to the bold stage. Looking at the output dynamics at the bottom panel shows that output crashes twice in this economy in each cycle, creating a double dip recession. We return to this phenomenon in the Section 4.2, when we analyse the implied dynamics of the real economy.

A few more cases are worth mentioning, which arise when assumption 3.4 and 3.5 are both violated. First, if $\mu_{0B}^* < \frac{c}{1+r_f} \leq \bar{\mu}_0(\mu_1) < \mu_{0C}^*,$ then the ergodic set consists of the same two point distribution as described in Proposition 3.4. (iii), i.e. a two-stage economy with a cycle of length two which fluctuates between all lenders being bold or cautious.

Second, it is not possible to have a three-stage economy with a long downturn. In other words, whenever there is a cycle which consists of multiple consecutive cautious stages, it ends either by a single bold stage (if $\frac{c}{1+r_f} > \bar{\mu}_0(\mu_1))$, or by a single mix stage (if $\frac{c}{1+r_f} < \bar{\mu}_0(\mu_1))$. The key to this observation is that whenever $\mu_0$ falls below $\max\{\frac{c}{1+r_f}, \bar{\mu}_0(\mu_1)\}$, the dynamics is dictated by $\mu_{0B}(\delta, \lambda, \mu_0, \mu_1)$ function and is upward sloping, so it cannot enter a third (lower) stage.

Finally, note that given Proposition 3.2 the threshold $\bar{\mu}_0(\mu_1)$ depends one of the state variable $\mu_1$. This implies that our dynamic economy might even fluctuate between a two-stage and a three-stage economy. This can lead to cycles with varying length.

4 Investment and Output

In this part, we characterize entrepreneurs’ investment and output. We start with the stage game and then proceed to the dynamics. Just as before, when characterizing the stage game
Figure 3: The law of motions of state variables (first panel), interest rates (second panel), total gross output (third panel) in a multi-period boom, multi-period mix recession and a one period cautious recession cycle in a 3-stage economy.
outcomes we apply the restriction $\mu_0 = \mu_1 = \lambda$, which we relax in the discussion of dynamics.

4.1 Investment and Output in the Stage Game

We start with the characterization of the investment and maintenance decisions, $I(\tau, \omega), i(\tau, \omega)$. As we argued, both type of entrepreneurs prefer to maintain as much as their investment as possible as long as $r \leq \bar{r}$. Therefore, if they are unconstrained in the credit market, they choose $\eta(\tau, \omega) = 1$ implying

$$I(\tau, \omega) = i(\tau, \omega) = \frac{1 + r(\tau, \omega)}{1 + (\phi \xi + 1)r(\tau, \omega)}.$$  \hspace{1cm} (4.1)

As output of entrepreneur $(\tau, \omega)$ is

$$y(\tau, \omega) \equiv \rho_\tau ((1 - \phi) I(\tau, \omega) + \phi i(\tau, \omega)) = \rho_\tau \frac{(1 - \phi) + \phi \eta(\tau, \omega)}{1 + \phi \xi \eta(\tau, \omega)} \frac{r(\tau, \omega)}{1 + r(\tau, \omega)}.$$  \hspace{1cm} (4.2)

clearly, $\eta(\tau, \omega) = 1$ leads to the maximum expected output subject to the budget constraint (2.2) and the interest rate $r(\tau, \omega)$. This expected output is decreasing in the interest rate $r(\tau, \omega)$. We refer to (4.1) as the investment plan with unconstrained maintenance. However, in certain states maintenance is limited by the capital of some investors groups. In this case, $\eta(\tau, \omega) < 1$ and expected output is reduced by insufficient liquidity.

The next proposition states the main features of entrepreneurs investment plans in each type of equilibria. We spell out the explicit formulas in the Appendix.

**Proposition 4.1** In any equilibrium transparent bad entrepreneurs are not financed by experts hence their investment plan is $I(b, 1) = 1, i(b, 1) = 0$.

(i) In the bold equilibrium, all good entrepreneurs implement the investment plan with unconstrained maintenance (4.1) with interest rate $r_B$ while bad opaque entrepreneurs’ investment plan is limited by unskilled investors’ mistakes implying $0 < \eta(b, 0) < 1$.

(ii) In a cautious equilibrium, all transparent good entrepreneurs implement the investment plan with unconstrained maintenance (4.1) with interest rate $r_C$ while good opaque investment $i(g, 0), I(g, 0)$ is limited by the capital of skilled experts and the interest rate $\bar{r}$ implying $0 < \eta(g, 0) < 1$. Bad opaque entrepreneurs are not financed by experts hence their investment plan is $I(b, 0) = 1, i(b, 0) = 0$. 26
(iii) In a mix equilibrium, all transparent good entrepreneurs implement the investment plan with unconstrained maintenance \((4.1)\), all opaque good entrepreneurs implement the investment plan with unconstrained maintenance \(I(g,0) = i(g,0)\) pinned down by \((2.2)\) and the interest rate \(r_I\). Opaque bad investment \(i(b,0), I(b,0)\) is limited by unskilled investor’s mistake implying \(0 < \eta(b,0) < 1\).

Consider first the bold equilibrium, all entrepreneurs apply for loans at the interest rate \(r_B\). As unskilled capital is in large supply and all unskilled investors using the bold test receive a green signal for all good entrepreneurs, good entrepreneurs can obtain all the credit they are willing to absorb at interest rate \(r_B\). Transparent bad entrepreneurs cannot obtain credit as unskilled investors receive red signals for their applications. Opaque bad entrepreneurs can obtain some credit, because unskilled investors are receiving green signals for their applications and cannot distinguish them from good entrepreneurs. However, their credit and maintenance is limited by the mistakes those unskilled investors are making who decide to enter. The fact that investment is high under this case, as all good types and even some bad types can maintain some of their investment, justifies our label of a boom for this type of equilibrium.

In the cautious equilibrium, good transparent entrepreneurs obtain credit from unskilled investors using the cautious test at interest rate \(r_C\). Unskilled capital is in large supply, therefore good transparent entrepreneurs can implement \((4.1)\). Good opaque entrepreneurs instead are obtain credit from skilled investors at the maximum feasible interest rate \(\bar{r}\). As the capital of skilled investors is in short supply, their capital limits the maintenance of these entrepreneurs implying low expected output.

As investment is low under this case, because none of the bad types can maintain their investment after a liquidity shock, and even some good types face high interest rates and low credit quantities, which justifies our label of recession for this equilibrium.

Finally, the mix equilibrium is in between the other two regimes of equilibria. The masses of entering cautious and bold unskilled investors are such that the first group can satisfy the credit demand of transparent good investors at the low interest rate \(r_C\) while the second group, together with skilled capital, can satisfy the credit demand at the higher interest rate \(r_I\). Therefore, the investment plans of both of these groups are given by \((4.1)\) after applying the differing interest rates. Similarly to the bold equilibrium, the credit to opaque bad entrepreneurs is given by the share of capital of cautious unskilled investors which is allocated to those entrepreneurs loan applications by mistake.
Given the idiosyncratic nature of liquidity shocks, aggregate output of entrepreneurs is deterministic in a given state and given by

\[ Y(\lambda) \equiv \frac{1}{2} - \frac{\lambda}{2} \rho_g (y(g, 1) + y(g, 0)) + \frac{\lambda}{2} \rho_b (y(b, 1) + y(b, 0)). \]  

(4.3)

A natural observation is that aggregate output is smoothly monotonically decreasing in the fraction of bad entrepreneurs within any range of parameters where the type of the equilibrium is not changing. This is so because of two reasons. First, the average productivity of entrepreneurs is smaller when \( \lambda \) is smaller by inequality (2.3). Second, Figure 1 illustrates that within the range of a given equilibrium interest rates are (weakly) increasing in \( \lambda \). A larger proportion of bad entrepreneurs increases the equilibrium interest rates. This increases the cost of maintaining production, which decreases maintained investment and total output according to Proposition 4.1.

More interestingly, the change in total output is not smooth when the economy switches between types of equilibria. Continuous changes in \( \lambda \) can lead to discontinuous jumps in \( Y(\lambda) \). In this sense, the economy crashes around the thresholds. In the case of welfare, these jumps can even lead to a non-monotonic pattern in \( \lambda \). This is what we state in the next proposition and illustrate on Figures 4-5. In each Figure, the left panel corresponds to a 3-stage economy, while the right panel corresponds to a 2-stage economy.

**Proposition 4.2** Consider a set of parameters for which \( r_B (\lambda_B, c, r_f) < \bar{r} (\rho, \xi, \phi) \). Total output, \( Y(\lambda) \), jumps downward both at \( \lambda_B \) and \( \lambda_C \) in a 3-stage economy and at \( \lambda_B \) in a 2-stage economy.

The downward jumps in output at \( \lambda_B \) and \( \lambda_C \) is intuitive. For a marginal increase in \( \lambda \) around any of the thresholds there is at least one type of entrepreneurs who experience a discontinuous downward jump in the credit it obtains. For instance, when the equilibrium switches from mix to cautious, bad entrepreneurs stop getting credit. When the equilibrium switches from bold to mix, interest rate jumps for the opaque projects leading to a downward jump in obtained credit for all opaque groups. When the equilibrium switches to cautious from bold in the 2-stage economy, both happens at the same time. As \( \rho_g, \rho_b > 0 \), the resulting drop in maintained investment leads to an unambiguous crash in output.
Figure 4: Gross output as the fraction of bad investors, \( \lambda \), changes in a 3-stage economy (left panel) and in a 2-stage economy (right panel).

4.2 Real Economy Dynamics

Given the characterization of the ergodic distribution of the state variables, \( \mu_0 \) and \( \mu_1 \), in Section 3.2, and that of the real outcomes in the stage game, analysing the dynamics of the real economy is relatively straightforward. Just as in Section 3.2, we only have to carefully replace \( \lambda \) with \( \mu_0 \) and \( \mu_1 \) in our expressions characterizing the investment plans then apply the results in Section 3.2. We spell out the implied expression for aggregate output \( Y(\mu_0, \mu_1) \) in the Appendix.

The middle and bottom panels on Figure 2 illustrate the cyclicality of output, \( Y(\mu_0, \mu_1) \), and its co-movement with the spread between opaque and transparent rates in our 2-stage economy. Comparing with the top panel shows the co-movement with the fraction of opaque bad entrepreneurs \( \mu_0 \). In line with the corresponding Figure 4 for the stage game, it is not surprising that larger fraction of bad opaque entrepreneurs implies smaller output. Note, however, the effect of the change from an overheated credit market to a low sentiment in credit market. The top panel of Figure 2 shows that this switch occurs in periods 5, 11 and 17 in our example. While \( \mu_0 \) increases only slightly in those periods, the right panel of Figure 2 shows a sizeable drop in output. This is the result of the switch in sentiment. At that point, the deterioration of the pool in credit applications triggers investors to switch to be cautious. Therefore all bad projects lose financing, while opaque good projects are significantly squeezed. As the middle panel on Figure 2 shows, there fragmentation in the credit market opaque entrepreneurs facing a significantly larger interest rate than transparent entrepreneurs. On the bright side though, this crash has a purification effect on the economy. Entrepreneurs with a liquidity shock and a bad project are bankrupt and exit the economy. This leads to a sufficient improvement in the pool for the next period to trigger investors to switch to bold test. The credit market becomes overheated again.
It is interesting to compare Figure 2 and Figure 3 illustrating the dynamics in a 3-stage economy. The bottom panels on Figure 3 show that even if the interest rate in the mix stage and in the cautious stage is at similar levels, the output effect of switching to the cautious stage is significant. In fact, the output dynamics shows a double-dip recession. Despite the significant drop in output around period 3, the recession is not deep enough for the economy to experience the purifying effects of a cautious equilibrium. Therefore, output drops further until a second drop in output occurs in period 6. This is the period, when finally the fraction of opaque bad entrepreneurs drops, triggering a boom in period 7.

5 Welfare, Economic Cycles and Capital Regulation

In the previous sections, we demonstrated how fluctuations of sentiment and that of fundamentals feed on each other creating endogenous cycles. As we model explicitly the mechanism which turns booms to recessions and vice-versa, our framework is well suited to explore how certain economic policies could and should influence these economic cycles. We start the analysis we defining welfare in the stage game. Then, we turn to the dynamic economy and study a constrained planner’s problem where the only possible intervention is to choose which test investors should use in each aggregate state. This choice determines the dynamics of the economy through the mechanism we explored in the previous chapters. We show that the planner often prefers a cycling economy where periods of overheated booms are interchanging with low sentiment recessions, instead of choosing one where the economy is permanently in one of these states. Third, we show that such an optimally cycling economy can be achieved by a realistic macro-prudential regulatory tool: risk weighting in investors’ capital requirements.

5.1 Stage Game Welfare

A natural welfare measure is the aggregate consumption of all entrepreneurs and investors. While it is very close to $Y(\lambda)$, it is not identical, as some of the consumption originates from the risk-free technology as opposed to the output of entrepreneurs. Using expressions (3.2)-(3.3) and the fact that unskilled investors always earn the risk-free return in expectation,
while skilled ones earn the highest available return on the market, aggregate consumption is

\[ W(\lambda) \equiv \frac{1 - \lambda}{2} \left( \frac{\rho_g (1 - \phi) + (\rho_g - \xi) \phi \eta (g, 1)}{1 + \phi \xi \eta (g, 1) \frac{r(g,1)}{1+r(g,1)}} + \frac{\rho_g (1 - \phi) + (\rho_g - \xi) \phi \eta (g, 0)}{1 + \phi \xi \eta (g, 0) \frac{r(g,0)}{1+r(g,0)}} \right) + \]

\[ \frac{\lambda}{2} \left( \frac{\rho_b (1 - \phi) + \phi \eta (b, 0)}{1 + \phi \xi \eta (b, 0) \frac{r(b,0)}{1+r(b,0)}} + \frac{\rho_b (1 - \phi) + \phi \eta (b, 1)}{1 + \phi \xi \eta (b, 1) \frac{r(b,1)}{1+r(b,1)}} \right) + \]

\[ + w_0 (1 + r_f) + w_1 \left( 1 + \max_{(\tau,\omega)} r (\tau, \omega) \right). \] (5.1)

Because of analogous reasons to aggregate gross output, our welfare measure is also smoothly monotonically decreasing in the fraction of bad entrepreneurs within any range of parameters where the type of the equilibrium is not changing. Also, just as \( Y(\lambda) \) does, welfare discontinuously drops at every point where the type of equilibrium changes. This is what we state in the next proposition and illustrate on Figure 5. The left panel corresponds to a 3-stage economy, while the right panel corresponds to a 2-stage economy.

**Proposition 5.1** Consider a set of parameters for which \( r_B (\lambda_B, c, r_f) < \bar{r} (\rho, \xi, \phi) \). In a 3-stage economy, \( W(\lambda) \) jumps downward at \( \lambda_C \) and \( \lambda_B \). In a 2-stage economy, \( W(\lambda) \) jumps downward at \( \lambda_B \).

The drop in aggregate consumption around the thresholds always comes from a drop in consumption for some groups originating from a drop in the loan quantity or a jump in the interest rate (or both). For instance, in the 2-stage economy as the equilibrium switches from bold to cautious, bad opaque entrepreneurs loose all their financing, while good opaque entrepreneurs experience both a jump in their interest rate from \( r_B \) to \( \bar{r} \) and a drop in the fraction they can maintain as unskilled investors stop financing them. \(^4\)

\(^4\)One might expect that there is also a positive effect as financing the continuation of bad entrepreneurs was socially negative NPV by assumption (2.1) in the bold equilibrium. That social cost raised the interest rate \( r_B \) in the bold equilibrium as unskilled investors passed on their losses to entrepreneurs in this form. In principal, some good entrepreneurs should get a lower interest rate at that switch as this cross-subsidization is not present in the cautious equilibrium. However, there is a counterweighting effect leading to \( r_B = r_C \). The fixed cost of sampling is paid by a smaller pool of borrowers in the cautious equilibrium. The effect of less cross-subsidization, but higher per loan cost of sampling exactly cancels out. In fact, investors switch at that \( \lambda_B \), because this is the point where the two effects cancel out, as we explained when we have constructed the equilibrium.
5.2 Dynamic Welfare: a Constrained Planner’s Problem

As the focus of our analysis is the relationship between the choice of investors’ test and that of fundamentals, it is instructive to study the following constrained planner’s problem. As Proposition 3.2 described, in the decentralized equilibrium investors are bold if and only if $\mu_0 \in \left[0, \frac{c}{1+rf}\right]$. Instead, consider an economy where investors are bold if and only if $\mu_0 \in [0, K]$ where the parameter $K$ is chosen by a planner, once and for all. Note, that the planner can force all investors to always be cautious by choosing $K = 0$, or to always be bold by choosing $K = 1$. We assume that this is the only intervention a planner can do. Therefore, each state $(\mu_0, \mu_1)$ together with $K$ determine whether investors are bold and cautious implying a stage game equilibrium with corresponding interest rates and investment plans. Then, laws of motion (3.11-3.14) determine the next state. We calculate average welfare across all the points in a cycling economy.

Panels (a) and (b) on Figure 6 shows the outcome of this exercise. The left panel depicts the average welfare over the ergodic steady state for different planner’s choice of $K$ (left panel). The vertical dashed lines show the regions for different cycle dynamics. If $K$ is in the first interval on the left than the economy is in a permanent cautious phase. If $K$ is in the second interval, the economy features long recessions and short booms. In the third interval there are short recessions and short booms, etc.

Note that changing $K$ does not have any effect on welfare within the permanent cautious interval and the permanent bold interval, because in each of these cases the distribution of the state variables are degenerate. The same is true within the short boom/short recession cycle in the middle region, because the state jumps between two points $\mu^*_0B$ and $\mu^*_0C$ (and the corresponding points of $\mu_1$), in this phase, hence the average welfare is constant. This is not the case in the remaining two intervals. For example, within the long-boom/short recession
interval welfare is changing with $K$. The reason is that larger $K$ within this interval makes booms longer.

It is apparent that in our example shown in Figure 6, with the planner’s optimal choice $K^*$ the economy is oscillating across short booms and short recessions. The intuition is illustrated on the right panel of 6 which shows the implied dynamics of the fraction of opaque bad entrepreneurs $\mu_0$ under $K^*$ along with the one in the decentralized economy. By choosing a $K$ for which $\mu_0^* B < K < \mu_0^* C < \frac{c}{1+r_f}$, the planner can keep the fraction of opaque bad entrepreneurs at bay. The planner uses cautious investors in the recession phase to reduce the fraction of opaque bad entrepreneurs. Then, it lets them to operate a bold test for one period to boost investment and output. With a low $\mu_0$ the overheated period is not too costly, because most investments are positive NPV. However, letting the economy to stay more in the overheated period would increase the fraction of opaque bad entrepreneurs too much.

### 5.3 Optimal Cycles and Capital Requirements

In the previous section, we have showed that a planner who can force investors to run a bold or a cautious test in a given state can improve welfare. In reality, it might not be obvious that the regulator can directly affect the lending standards of financial institutions. In this part, we study whether a common policy tool, namely, risk-weights in capital requirements, can achieve a similar effect. We show that this is indeed the case.

We model risk-weighted capital requirements as follows. Let $v_g$ be the capital a bold investor invests in loans which have produced a green signal $a$ by the bold test. Suppose that the regulator imposes a risk weight $x \geq 1$ on those investments as they are risky.\footnote{Introducing a different risk-weight for cautious investors’ loans would be straightforward. For simplicity, and because they are investing only in loans which certainly pay back, we omit that treatment.} Let $v_r$ her investment in the risk-free asset. Recall that a bold investor, after financing all the loans producing a green-light invests all of her remaining capital in risk-free assets. As this is safe investment, its risk-weight is 1. That is, the investor has to respect the capital requirement $v_g x + v_r = 1$. If $x = 1$, this reduces to the budget constraint of the investor in our unregulated economy. Furthermore, we know that in the sample of a bold investor $1 - \mu_1$ fraction of the applications produce a green signal. That is $\frac{v_g}{v_g + v_r} = (1 - \mu_1)$ by definition.
Figure 6: Mean welfare under the planner with various choices of $K$, and with various choices of risk-weights $x$ (left panels). Implied paths of bad opaque entrepreneurs $\mu_0$ under $K^*$ and in the decentralized economy, and that of welfare under all three economies (right panels). On the left panels, the vertical dashed lines from left to right are $\bar{\mu}_0C, \mu^*_0B, \mu^*_0C, \bar{\mu}_0B$ determining the regions for different cycle dynamics. On the upper right panel, the horizontal lines are the thresholds $\frac{c}{1+r_f}$ (dashed) and $K^*$ (dotted) where investors in the respective economies switch between cautious and bold tests.
Solving for \( v_g \) and \( v_r \) gives

\[
v_g = \frac{1 - \mu_1}{x(1 - \mu_1) + \mu_1}, \quad v_r = \frac{\mu_1}{x(1 - \mu_1) + \mu_1}.
\]

Now, we can rewrite the free-entry condition of unskilled investors using a bold test as

\[
\frac{1 - \mu_1}{x(1 - \mu_1) + \mu_1} (1 + r_B) \frac{(1 - \mu_1 - \mu_0)}{1 - \mu_1} + \frac{\mu_1}{x(1 - \mu_1) + \mu_1} (1 + r_f) - c = (1 + r_f).
\]

With \( x = 1 \), this equation would give back the definition of \( r_B(\mu_0, \mu_1, c, r_F) \) in (3.7). When \( x > 1 \), it defines the interest rate \( r_Bx(\mu_0, \mu_1, c, r_F, x) \) instead, at which unskilled investors are indifferent to enter and use the bold test or to stay outside. As some unskilled investors also use the bold test in the mix equilibrium, their indifference condition defines a \( r_Ix(\mu_0, \mu_1, c, r_F, x) \) instead of expression (3.9). It is easy to see that both functions shift upward with higher risk-weight. Given these expressions, we solve for the equilibrium as before. That is, all investors optimally choose their test subject to the risk-weighted capital regulation. The implied interest rates and loan quantities determine entrepreneurs investment and maintenance plans and the law of motion of state variables.

Panel (c) of Figure 6 illustrates the effect of changing risk-weight \( x \). The vertical dashed lines show the regions for different cycle dynamics just as in panel (a). As a smaller \( K \) has a similar effect to a larger \( x \), we have reversed the x-axis for easier comparison between panel (a) and (c). That is, in both panels the first interval on the left corresponds to an economy with a permanent cautious phase, and the second interval corresponds to long recessions and short booms, etc. Comparing the two panels, it is apparent that welfare jumps significantly when the intervention switches the types of the implied cycle. Note that just as in the planner’s problem, welfare is largest at a point when the economy features short booms and short recessions under capital requirements too. The main difference between the two figures are coming from the fact that within an interval with a similar cycle, a larger \( x \) reduces welfare. The reason is that it implies a larger interest rate in a boom, which reduces investment and output. This is also a reason why the optimal economy under capital regulation produces less aggregate consumption than that under the constrained planner’s solution \( K^* \).

Panel (d) of Figure 6 compares the welfare paths in the three economy. It is apparent that the optimal economy under capital regulation is in-between our baseline case and between the constrained optimum. An \( x^* > 1 \), makes the bold test costly, inducing unskilled investors
to switch to cautious testing more often. Just as in the constrained optimum, this helps the
regulator keeping the fraction of bad projects at bay, improving the outcome. However, this
is achieved at the cost of less pronounced booms, because of higher interest rates.

6 Model and Facts

Despite its simple structure, our model generates a rich set of empirical predictions. In this
part, we point out some of these and contrast it with empirical evidence, if available.

The quality-spread and the tightness of credit When mapping the model outcomes
to the data, a critical question is how to think about credit to opaque and to transparent
entrepreneurs. We argue that the earlier corresponds to risky or low quality credit, while
the latter corresponds to less risky or high quality credit. The idea is that from the point
of view of an unskilled investor the opaque group is heterogeneous, hence providing credit
to these is risky: some of these entrepreneurs would pay back, while others would not. Our
implicit assumption, therefore, is that the econometrician’s information set is close to an
unskilled investor.\footnote{We could think of an econometrician as an unskilled investor who can run both tests. In this way,
he can distinguish opaque entrepreneurs from transparent entrepreneurs and measure their credit issuance. However,
just as unskilled investors, he can still not distinguish bad entrepreneurs from good entrepreneurs
except by their ex-post default.} Therefore, we think of the quality spread, the difference between yields
of comparable BAA and AAA corporate bonds as a proxy between the interest rates faced by
these groups. Basic predictions of our model are that (1) the quality spread is countercyclical,
(2) it strongly co-moves with proxies for the tightness of credit.

Figure 7 demonstrates that these predictions are in line with the data. The solid curve
is the percentage of senior loan officers claiming to tighten C&I loan conditions in the given
quarter, while the dashed curve is Moody’s seasoned AAA-BAA spread. The shaded areas
are NBER recessions.\footnote{For a more systematic treatment see Morais et al. (2019) who finds both US and international evidence
for lax lending standards in booms in the bank loan market.}

Credit composition and realized returns There is an also a growing body of evidence
suggesting that periods of overheating in credit markets forecasts low excess bond or loan
returns. This is not a tautology, if credit market overheating is measured ex-ante by the

36
Figure 7: The percentage of senior loan officers claiming to tighten C&I loan conditions in the given quarter (Survey of Senior Loan Officers, Federal Reserve, solid curve, left axis), and the Moody’s seasoned BAA-AAA spread (FRED, St. Louis FED, dashed curve, right axis). The shaded areas are NBER recessions.

quantity or composition of credit. As an influential example, Greenwood and Hanson (2013) show that the share of bond issuance with low credit rating out of total issuance inversely predicts the excess return on these bonds. The left panel on Figure 8 is the illustration of this fact using the reproduction of Stein (2013). The blue curve is the share of low-grade bond issued in a given period as a fraction of all issuance measured on the right axis. The black curve is excess return on those low-grade bonds in the subsequent two years measured on a reverse scale on the left axis. For instance, periods when both curves are high corresponds to overheated periods with low subsequent returns. Periods where both curves are tend to correspond to recessions: low sentiment credit markets with high subsequent returns.

On the right panel of Figure 8, we plotted the model-equivalent time-series in our economy studied on Figure 4. In our model, the share of transparent credit to all credit is

$$S(\mu_0, \mu_1) \equiv \frac{C(\mu_0, \mu_1, \omega = 0)}{C(\mu_0, \mu_1, \omega = 0) + C(\mu_0, \mu_1, \omega = 1)}$$
where

\[ C(\mu_0, \mu_1, \omega = 0) \equiv \mu_0 \xi \phi \frac{i(B, 0)}{1 + r(B, 0)} + \frac{1 - \mu_0 - \mu_1}{2} \xi \phi \frac{i(G, 0)}{1 + r(G, 0)} \]

and

\[ C(\mu_0, \mu_1, \omega = 1) \equiv \frac{1 - \mu_0 - \mu_1}{2} \xi \phi \frac{i(G, 1)}{1 + r(G, 1)} \]

are the total credit provided to opaque and transparent entrepreneurs respectively, where \( i(\tau, \omega) \) and \( r(\tau, \omega) \) also depends on the state variables as we described in Sections 3.2 and 4.

The net return on opaque credit is

\[ R(\mu_0, \mu_1) \equiv \frac{1 - \mu_0 - \mu_1}{2} \xi \phi i(G, 0) \]

where the denominator is the total credit opaque projects receive in a given period, while the numerator is the total repayment on that credit in the subsequent period. Note the strong co-movement between \( S(\mu_0, \mu_1) \) and – on a reverse scale – \( R(\mu_0, \mu_1) \) both within the bold phase and across periods. To show the intuition, we calculate these expressions in the limit when skilled capital is negligible:

\[ \lim_{w_1 \to 0} S(\mu_0, \mu_1) = \frac{c - \mu_0(1 + r)}{1 + \mu_0 - \mu_1}, \quad \lim_{w_1 \to 0} R(\mu_0, \mu_1) = \frac{1 + \mu_0 - \mu_1}{2(1 - \mu_1)} \]
in the bold stage and

$$\lim_{w_1 \to 0} S(\mu_0, \mu_1) = 0, \lim_{w_1 \to 0} R(\mu_0, \mu_1) = \bar{r} - r_f$$

in the cautious stage. It is apparent that the (inverse) co-movement within the bold stage is driven by both of our state variables. The co-movement across the bold and cautious stages are driven by the fact that in the cautious stage, $S(\mu_0, \mu_1)$ is converges to its minimal value, while $R(\mu_0, \mu_1)$ converges to its maximal value for $w_1 \to 0$.

The inverse relationship between credit expansion and subsequent returns is remarkably widespread across various financial markets. For instance, Baron and Xiong (2017) documents the negative relationship between bank’s credit expansion and banks’ equity returns, Kaplan and Stromberg (2009) finds a similar inverse relationship between venture capitalists aggregate flow to new investments and their subsequent returns. A related early work is Eisfeldt and Rampini (2006), who shows that volume of transactions are procyclical while return on transactions is counter-cyclical in the sales of property, plant and equipment.  

**Market fragmentation and heterogeneous portfolio rebalancing** We argue that by the unique structure of our stage game, our model provides crucial insights on the mechanism of market fragmentation in recessions. Recall that as a bold stage turns to a cautious stage skilled and unskilled investors rebalance their portfolio very differently. Skilled investors rebalance from low-yield transparent entrepreneurs to high-yield opaque entrepreneurs, while unskilled investors do the opposite. This implies that fundamentally similar good entrepreneurs suddenly face very different experiences. Transparent good entrepreneurs face abundant credit supply, while opaque good entrepreneurs are squeezed, even if in the bold stage they faced the same market conditions. This market fragmentation and the implied heterogeneity of the effect of a recession is a unique feature of our model.

This type of market fragmentation was especially salient in context of the Euro zone crisis. Indeed, Ivashina et al. (2015) and Gallagher et al. (2018) find that in 2011 a group of US money market funds stopped lending only to European banks but not to other banks.

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\(^8\)Interestingly, López-Salido et al. (2017) finds that the predictable component of the Greenwood-Hanson measure of realized returns predicts economic activity. This is in line with the deterministic nature of our economy. In our deterministic model, both the future realized return on opaque credit, $R(\mu_0, \mu_1)$, and economic activity, $Y(\mu_0, \mu_1)$ are fully predictable. Also, low $R(\mu_0, \mu_1)$ trivially forecasts subsequent recessions once the forecasting horizon is adjusted to the length of our cycles we characterize in Proposition 3.4.
which had similar fundamentals. In particular, Gallagher et al. (2018) finds that when these money market funds stopped financing entrepreneurs in a European country, they did so irrespective of an entrepreneur’s implied risk of default. These predictions are consistent with our mechanism when considering these funds as low skilled investors. Moreover, Ivashina et al. (2015) also find evidence that this process led to a significant disruption in the syndicated loan market, a possible channel for the real effects that our model predicts.  

International spillovers of US monetary policy  The applicability of our model goes beyond the a developed economy as the US. For instance, consider our set up as a description of an emerging market economy where the banking sector is sensitive to the international cost of capital. For simplicity, think of the risk-free return in our model as the return on US treasuries set by the FED. Just as in our model, this rate is effectively exogenous for the emerging market banking sector, but clearly influences their activity. Note from Corollary 3.1 and Proposition 3.4 that a small increase in the risk-free rate can have a significant adverse effect both in the short-term or in the longer term. In fact, Corollary 3.1 shows that a small increase in the risk-free rate can push a bold, overheated economy to a low-sentiment, recessionary one. Proposition 3.4 also shows that if the rate change is permanent, it can change the nature of cycles in the economy. In general, higher opportunity cost of lending leads to longer recessions and shorter booms. This is in line with several emerging market officials including Raghu Rajan’s warning, who was the Chair of India’s central bank at the time, around 2013-14 that emerging markets are expected to adversely affected when US starts to raise rates.

7 Conclusion

We present a model of rational sentiments and economic cycles. We capture sentiment by letting investors to choose the power of their test they use to grant credit. Their choice implies that whether lending standards are tight or lax. Tight credit standards let low quality entrepreneurs to exit at a larger rate implying improving quality of the borrowing pool, that is, changing fundamentals. This change then influences investors lending standards in the future. We show that the two-way interaction between sentiment and fundamentals

\[ \text{See also our companion paper Farboodi and Kondor (2018) providing a substantially richer picture on market fragmentation on the expense of treating sentiment switches as exogenous.} \]

\[ \text{See Financial Times, September 3, 2013 and January 31, 2014.} \]
generates endogenous economic cycles. We show that the planner, with carefully chosen capital requirements for investors, prefers to change the cyclicality of the economy, using recessions to keep the borrower pool quality at bay. We argue that the predictions of the model matches a large list of stylized facts related to credit cycles.

References


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### Appendix: Market Clearing Protocol and Credit Demand

Recall that each type of entrepreneur \((\tau, \omega)\) chooses an initial investment \(I(\tau, \omega)\) and saves the rest of her endowment towards the state when she is hit by a liquidity shock. In this case, she might choose to raise funds in the credit market.

We assume that the size of loan applications are standardized to correspond to an infinitesimal unit, \(dl\), of potentially maintained investment, which can serve as collateral for the loan. Therefore, if that application is financed at interest rate \(r(\tau, \omega)\), the entrepreneur obtains \(\xi \frac{1}{1+r(\tau, \omega)} dl\) funds as this is the amount that unit of maintained investment can collateralize. If the application is accepted, the entrepreneur has to post the collateral and match the loan with \(\xi \frac{r(\tau, \omega)}{1+r(\tau, \omega)} dl\) of her own savings, to have the necessary amount to finance the maintenance of the unit. The maintained unit enters a public registry, hence a maintained unit will never be financed by more than one investor (even if different maintained units might be financed by different investors). If the investor were to accept the loan application, but the entrepreneur cannot present the collateral, the loan is not granted.

We picture the market clearing protocol as follows. Each unskilled investor indexed by \(h \in H_0 \equiv [0, w_0]\) and skilled investor indexed by \(h \in H_1 \equiv [w_0, w_0 + w_1]\) chooses a test and advertises an interest rate \(\tilde{r}(h)\), at which she is willing to finance loan applications which gets to her sample and gets a green light by the test. Simultaneously, each entrepreneur chooses a maximum interest rate \(r_{\text{max}}(\tau, \omega)\) at which she is willing to take out a loan. If
max(τ,ω) r^{max}(τ,ω) \leq \min_h \tilde{r}(h), then no applications are financed. Otherwise, we start from the smallest advertised \tilde{r}(h). First, all entrepreneurs with r^{max}(τ,ω) not smaller than this \tilde{r}(h) submit a measure \sigma(r,τ,ω) loan applications at that interest rate. We assume that there is an exogenous capacity limit \bar{L}, therefore \sigma(r,τ,ω) \leq \bar{L}. It is intuitive and convenient to assume that \bar{L} happens to be max(τ,ω) I(τ,ω), the maximum amount of loan applications any entrepreneur would possibly want to submit. Given all the submitted applications at the smallest advertised \tilde{r}(h), the subset of investors advertising this rate take samples. Unskilled investors sample first, by assumption. They are ordered randomly and allocated a sample of the size corresponding to their unit endowment as explain in the main text. They grant the applications which produce a green signal and reject the rest. Rejected applications are excluded from any further sampling. Once unskilled investors advertising the given rate are allocated their samples, skilled investors take a sample of good firms only. If there are no more applications or no investors left at the given interest rate, we proceed to the next smallest advertised interest rate and repeat the process. Once all applications are exhausted or there are no remaining advertised interest rates, the process stops. Note, that \tilde{\ell}(r,τ,ω) the measure of applications granted at a given interest rate in equilibrium might be smaller than \sigma(r,τ,ω), the measure of submitted applications. Also, in principal a type (τ,ω) might have some of its applications approved at different interest rates. However, as we noted in the main text, we are searching for an equilibrium where all approved applications of a given type are at a single interest rate r(τ,ω). That is, defining the total measure of approved applications, \ell(τ,ω) \equiv \int_0^{\max \tilde{r}(h)} \tilde{\ell}(r,τ,ω) dr, we have \tilde{\ell}(r,τ,ω) = 0 if r \neq r(τ,ω) and \ell(r,τ,ω) = \ell(τ,ω) if r = r(τ,ω).

Following Kurlat (2016), we make a robustness assumption implying tie-breaking rules and avoiding multiple equilibria as we explain below.

**Assumption A.1** Any equilibrium has to be robust to a small perturbation of investors strategy. In particular, suppose that with a small probability, a randomly chosen small measure of applications submitted at an advertised interest rate are granted regardless of the type of the entrepreneur who submitted it and the loan is issued without presenting collateral. We require that the equilibrium strategy of each entrepreneur is the limit of equilibrium strategies as this small probability goes to zero.

The main consequence of assumption A.1 is that all entrepreneurs who do not expect to be able to maintain all their initial investment in a given equilibrium will strictly prefer to submit \sigma(r,τ,ω) = \bar{L} for all r \leq r^{max}(τ,ω). For instance, transparent bad types will submit \bar{L} applications at a large set of interest rates, even if they do not get any loans allocated in any equilibrium. It is so, because if, by the perturbation, they happen to be granted an extra unit of loan, they can use that unit to maintain more of their initial investment which is always preferable to liquidation. The role of the choice \bar{L} = max(τ,ω) I(τ,ω) is that those types who can maintain all their initial investment also choose \sigma(r,τ,ω) = \bar{L} for all r \leq r^{max}(τ,ω). That is, each types who choose to submit loan applications at a given interest rate submits the same amount. This feature has the convenient consequence that
the application pool at any given interest rate is independent of cross-sectional distribution of choices \( I(\tau, \omega) \), therefore, we can solve the credit market equilibrium independently of choices \( I(\tau, \omega) \). This simplifies the analysis considerably.

We derive the implied choices of \( r^{\text{max}}(\tau, \omega) \) and \( \bar{r}(h) \) in Section 3.

## B Appendix: Proofs

### Proofs of Propositions 3.1 and 3.2 and Corollary 3.1

We prove Proposition 3.1 in slightly more general form where \( \mu_1 \neq \mu_0 \neq \frac{1}{2} \). Then, we can directly use the outcome to prove Proposition 3.2. Substituting back \( \mu_1 = \mu_0 = \frac{1}{2} \) gives back the original form of the expressions in Proposition 3.1. The main steps of the proof are explained in the text. Here, we just have to specify the details.

First, we show that if all entrepreneurs submit a demand of \( \bar{L} \) to an advertised rate \( r^p_B \) than bold, unskilled investors are indifferent to stay out or enter. The superscript refers to the fact that it is a pooled market where all entrepreneurs submit. In fact, \( r^p_B \) is defined by

\[
(1 - \mu_1 - \mu_0) (1 + r^p_B) + \mu_1 (1 + r_f) - c = 1 + r_f \quad (A.1)
\]

Note that \( (1 - \mu_1 - \mu_0) + \mu_0 \) is the probability of ending up financing a good project with a bold test \( \Pr(\text{green signal}) \times \Pr(\text{good project}|\text{green signal}) \), while \( \mu_1 \) is the probability that an entrepreneur in the sample will not pass the bold test, hence the investor invests in the risk-free asset instead. Therefore, the left hand side is the expected utility of running the bold test on a proportional sample of applications. Note that we are using the assumption that unskilled investors sample first.

Similarly, a cautious investor is indifferent to enter to a pooled market at interest rate \( r^p_C \), which is defined as:

\[
\frac{(1 - \mu_1 - \mu_0)}{2} (1 + r^p_C) + \left( \frac{1 - \mu_1 - \mu_0}{2} + (\mu_1 + \mu_0) \right) (1 + r_f) - c = 1 + r_f \quad (A.2)
\]

We claim that iff \( r^p_B \leq r^p_C \) holds, \( r^p_B \) supports a bold equilibrium where the entering mass of unskilled investors is determined by the following market clearing condition. Given the fraction of bold investors’ capital financing good projects, together with the capital of skilled investors (which all finance good projects) all good projects, opaque or transparent, have all their credit demand satisfied. (This market clearing condition is spelled out below as (A.11).) Then, following the intuition in the text, it is easy to check that no one has a profitable deviation: skilled or unskilled investors do not want to change their interest rate from \( r^p_B \), and none of the entrepreneurs want to demand less than \( \bar{L} \) at that rate. While,
if the condition above did not hold, investors would be motivated to choose to be cautious advertising a rate \( \bar{r} \in (r_C^p, r_B^p) \).

Now consider a cautious equilibrium where all unskilled are cautious and advertise \( r_C^s \). This implies that opaque good projects can be financed only by skilled investors. As skilled capital is scarce, they will advertise the maximum feasible rate \( \bar{r} \). As unskilled capital is abundant, therefore \( r_C^s \) has to make cautious unskilled indifferent whether to enter. As all entrepreneurs demand credit at all advertised rate which is lower than their reservation rate, the pool of applicants in that low interest rate post is identical to the one in the pooled equilibrium at \( r_P^p \). That is, \( r_C^s \) solves (A.2) and \( r_C^s = r_C^p = r_C \) holds. If an unskilled investors is to deviate to a bold test, she has two options. She can advertise an interest rate \( \tilde{r} \leq r_C^s \) attracting the pool of all type of entrepreneurs or it can advertise a high rate \( \tilde{r} \in (r_C^s, \bar{r}] \) attracting all, but the transparent good ones. The earlier is a profitable deviation if and only if \( r_B^p \leq r_C^s \) where \( r_B^p \) solves (A.1). That is, a necessary condition for a cautious equilibrium is \( r_B^p = r_B^p > r_C \). The latter option is a profitable deviation if and only if \( r_I \leq \bar{r} \) where \( r_I \) is determined by the indifference condition

\[
\frac{(1-\mu_1-\mu_0)}{2} (1 + r_I) + \frac{\mu_1}{2} (1 + r_f) - c = (1 + r_f).
\]

Note that \( r_I > r_B \) because it refers to an adversely selected pool of applicants. Checking that neither skilled investors nor any type of entrepreneurs want to deviate from the assigned strategies concludes the construction of the cautious equilibrium.

Finally, if \( r_I < \tilde{r} \) and \( r_B > r_C \), then there is a mix equilibrium. In this case, skilled investors cannot offer \( \bar{r} \) as they would be undercut by bold unskilled ones. Instead, skilled and bold unskilled investors advertise \( r_I \). This high interest rate post is cleared similarly to the one at the bold equilibrium: the fraction of entering bold unskilled investors have to be sufficient to satisfy, together with skilled investors, all the credit demand of good opaque projects. (This market clearing condition is spelled out below as (A.13).) At the same time, a group of unskilled investors choose to be cautious and advertise \( r_C \) to serve good transparent projects. Note that the two groups of unskilled investors make the same expected profit of \( 1 + r_f \) by the definition of \( r_I \) and \( r_C \). Again, we can check that none of the agents prefer to deviate from the assigned strategies. Given that the conditions for each type of equilibria are mutually exclusive, we have uniqueness.

Corollary 3.1 is a direct consequence of expressing the equilibrium criteria in terms of \( \lambda \). For Proposition 3.2, we observe that the static reasoning can be applied in each period of the dynamic set up, and express the equilibrium criteria in terms of \( \mu_0 \).
Proofs of Proposition 3.4 and Proposition 3.5

Since the switch between the two regimes is only a function of \( \mu_0 \) in a two-stage economy, it is sufficient to compare \( \mu_0 \) across different regimes. In other words, what determines whether the economy is in a boom or a recession is measure of bad entrepreneurs who are opaque.

**Step 1.** The first step is to find the single point steady states in the dynamic model, i.e. the measures that correspond to being in a cautious market, and end up in the same cautious market, and similarly for bold market. One can use the system of equations (3.13-3.12) to get these fixed points.

\[
\tilde{\mu}_0B(\delta, \lambda, \phi) = \frac{\lambda((1 - \delta)\phi + \delta)}{2((1 - \delta)\phi + \delta) - (1 - \delta)\phi \lambda} \quad (A.3)
\]

\[
\tilde{\mu}_1B(\delta, \lambda, \phi) = \frac{\delta \lambda}{2((1 - \delta)\phi + \delta) - (1 - \delta)\phi \lambda} \quad (A.4)
\]

\[
\tilde{\mu}_0C(\delta, \lambda, \phi) = \frac{\delta \lambda}{2((1 - \delta)(1 - \lambda)\phi + \delta)} \quad (A.5)
\]

\[
\tilde{\mu}_1C(\delta, \lambda, \phi) = \frac{\delta \lambda}{2((1 - \delta)(1 - \lambda)\phi + \delta)} \quad (A.6)
\]

Note that \( \tilde{\mu}_{0C} = \tilde{\mu}_{1C} \), and \( \tilde{\mu}_0B > \tilde{\mu}_{0C} \). Since \( \bar{\mu}_0u_0 \) is exogenous to these steady states (using \( c \) and \( r_t \), we will focus on the case where \( \bar{\mu}_0 \) is in the middle, i.e. \( \tilde{\mu}_0B > \bar{\mu}_0u_0 > \tilde{\mu}_{0C} \).

**Step 2.** Two point oscillating distribution. Using the law of motions (3.13-3.12) we get the two point oscillating distribution by conjecturing that if the lower of those two points, \( \mu^*_B \) corresponds to a boom, then the implied next period value is \( \mu^*_C \) and it corresponds to a recession. Two two points has to be such that given the recession, the implied next period value is \( \mu^*_B \) again. The implied points are

\[
\mu^*_0B(\delta, \lambda, \phi) = \frac{\delta \lambda((1 - \delta)\phi + \delta)((2 - \delta)(1 - \lambda)\phi - (2 - \delta)(1 - \lambda)\phi)}{2(2 - \delta)^2\delta^2 + (1 - \delta)^2\phi^2((1 - \lambda) - (2 - \delta)(\lambda^2 + 6(1 - \lambda)))) + (1 - \delta)^2(2 - \delta)(1 - \lambda)(-\phi^2) + 3(2 - \delta)\delta(1 - \delta)^2(2 - \lambda)\phi} \quad (A.7)
\]

\[
\mu^*_1B(\delta, \lambda, \phi) = \frac{\delta \lambda((2 - \delta)(1 - \lambda)\phi - (2 - \delta)(1 - \lambda)\phi)}{2(2 - \delta)^2\delta^2 + (1 - \delta)^2\phi^2((1 - \lambda) - (2 - \delta)(\lambda^2 + 6(1 - \lambda)))) + (1 - \delta)^2(2 - \delta)(1 - \lambda)(-\phi^2) + 3(2 - \delta)\delta(1 - \delta)^2(2 - \lambda)\phi} \quad (A.8)
\]

\[
\mu^*_0C(\delta, \lambda, \phi) = \frac{\delta \lambda((1 - \delta)\phi + \delta)((2 - \delta)(1 - \lambda)\phi - (2 - \delta)(1 - \lambda)\phi)}{2(2 - \delta)^2\delta^2 + (1 - \delta)^2\phi^2((1 - \lambda) - (2 - \delta)(\lambda^2 + 6(1 - \lambda)))) + (1 - \delta)^2(2 - \delta)(1 - \lambda)(-\phi^2) + 3(2 - \delta)\delta(1 - \delta)^2(2 - \lambda)\phi} \quad (A.9)
\]

\[
\mu^*_1C(\delta, \lambda, \phi) = \frac{\delta \lambda((1 - \delta)\phi + \delta)((1 - \delta)\phi - (2 - \delta)(1 - \lambda)) + 2(2 - \delta)\delta}{2(2 - \delta)^2\delta^2 + (1 - \delta)^2\phi^2((1 - \lambda) - (2 - \delta)(\lambda^2 + 6(1 - \lambda)))) + (1 - \delta)^2(2 - \delta)(1 - \lambda)(-\phi^2) + 3(2 - \delta)\delta(1 - \delta)^2(2 - \lambda)\phi} \quad (A.10)
\]

It is clear that \( \mu^*_0C > \mu^*_0B \). Thus, the statement follows.
Each statement of Proposition 3.5 comes by direct differentiation using the analytical forms of $\bar{\mu}_0B, \mu_0^*C, \mu_0^*B, \bar{\mu}_0C$.

**Proof of Proposition 4.1**

Recall that Assumption 2.1 and the definition (3.1) imply that as long as a good entrepreneur is facing an interest rate $r \leq \bar{r}$, she prefers to submit maximum credit demand $\bar{L}$. The equilibrium credit she obtains determines maintained capital $i$. Then, constraint (2.2) determines optimal scale $I$. Observe also that given that bad entrepreneurs do not pay back, for any interest rate for which good entrepreneurs submit maximum demand, they also want to do so. Hence, equilibrium strategies and constraint (2.2) determine $i$ and $I$ along the same lines for bad entrepreneurs as well. In particular, we argue that the following expressions describe $I(\tau, \omega), i(\tau, \omega)$ in the various cases:

(i) In the bold equilibrium,

\[
I(g, \omega) = i(g, \omega) = \frac{1 + r_B}{1 + (\phi \xi + 1) r_B} \\
i(b, 0) = \frac{(1 + r_B)}{1 + (1 + \phi \xi) r_B} - \frac{1 + r_B}{(1 - \mu_0 - \mu_1) \phi \xi}w_1 \\
I(b, 0) = 1 - \frac{\phi \xi}{1 + (1 + \phi \xi) r_B} + \frac{w_1 r_B}{(1 - \mu_0 - \mu_1)} \\
I(b, 1) = 1, \ i(b, 1) = 0.
\]

(ii) In a cautious equilibrium,

\[
i(g, 1) = I(g, 1) = \frac{1 + r_C}{1 + r_C (1 + \phi \xi)} \\
i(g, 0) = 2 \frac{(1 + \bar{r}) w_1}{(1 - \mu_0 - \mu_1) \phi \xi} \\
I(g, 0) = 1 - \bar{r} \frac{w_1}{(1 - \mu_0 - \mu_1)} \\
I(b, \omega) = 1, \ i(b, \omega) = 0.
\]
(iii) In a mix equilibrium,

\[
\begin{align*}
I(g, 1) &= i(g, 1) = \frac{1 + r_C}{1 + (\phi \xi + 1) r_C} \\
i(g, 0) &= I(g, 0) = \frac{1 + r_I}{1 + r_I (1 + \phi \xi)} \\
i(b, 0) &= (1 + r_I) \left( \frac{1}{1 + r_I (1 + \phi \xi)} - \frac{w_1}{\phi \xi} \frac{2}{1 - \mu_1 - \mu_0} \right) \frac{\mu_1 + \mu_0}{2 \mu_0} \\
I(b, 0) &= 1 - \phi \xi i(b, 0) \frac{r_I}{1 + r_I} \\
I(b, 1) &= 1, \quad i(b, 1) = 0.
\end{align*}
\]

We start with the pooling equilibrium where the market clearing condition is

\[w_1 + (1 - b_P) w_0 (1 - \mu_0 - \mu_1) = \phi (1 - \mu_0 - \mu_1) \xi \frac{1}{1 + r_B} \frac{1}{1 + \phi \xi \frac{r_B}{r_B + 1} + \mu_0} \quad \text{(A.11)}\]

determining the fraction of entering unskilled capital, \(b_P\). Note that \(b_P \in (0, 1)\) as long as \(w_0\) is sufficiently large, in line with Assumption 2.2. To calculate opaque bad entrepreneurs credit, note that by definition

\[
\phi \mu_0 \frac{1}{1 + r_B} \xi i(b, 0) = (1 - b_P) w_0 \mu_0.
\]

As

\[
(1 - b_P) w_0 = \left( \phi \xi \frac{1}{1 + \phi \xi \frac{r_B}{r_B + 1} + \mu_0} - \frac{1}{1 - \mu_0 - \mu_1} w_1 \right)
\]

we have

\[
\phi \frac{1}{1 + r_B} \xi i(B, 0) = \left( \phi \xi \frac{1}{1 + \phi \xi \frac{r_B}{r_B + 1} + \mu_0} - \frac{1}{(1 - \mu_0 - \mu_1) w_1} \right)
\]

\[
i(b, 0) = \frac{(1 + r_B)}{1 + (1 + \phi \xi) r_B} - \frac{1 + r_B}{(1 - \mu_0 - \mu_1) \phi \xi} w_1.
\]

Also, by budget constraint (2.2),

\[
I(b, 0) = 1 - \phi \xi i(b, 0) \frac{r_B}{1 + r_B} = \frac{r_B}{1 + r_B} \left(1 - \phi \xi \frac{r_B}{1 + (1 + \phi \xi) r_B} \right) + \frac{w_1 r_B}{(1 - \mu_0 - \mu_1)}.
\]

Now we turn to the cautious equilibrium. Note that bad entrepreneurs are not financed by
cautious investors hence their investment plan is

\[ I(b, \omega) = 1, \ i(b, \omega) = 0. \]

Turning to transparent good entrepreneurs, they are fully financed at the \( r_C \) market, leading to

\[ i(g, 1) = I(g, 1) = \frac{1 + r_C}{1 + r_C (1 + \phi \xi)} \] (A.12)

At \( \bar{r} \), opaque good entrepreneurs are indifferent whether to borrow at all by definition. The loan, and consequently the maintained investment for opaque good entrepreneurs, \( i(g, 0) \) is limited by the capital of skilled experts and determined by

\[ (1 - \mu_0 - \mu_1) \frac{1}{2} \phi i(g, 0) \xi \frac{1}{1 + \bar{r}} = w_1 \]

or

\[ i(g, 0) = \frac{(1 + \bar{r}) w_1}{\xi \phi (1 - \mu_0 - \mu_1)} \]

The budget constraint (2.2) and (3.1) give

\[ I(g, 0) = 1 - \frac{\bar{r}}{1 + \bar{r}} \phi \xi i(g, 0) \]

and Assumption 2.2 ensures that \( I(g, 0, D) \geq i(g, 0, D) \).

Finally, for the mix equilibrium note that the total capital which goes to opaque bad entrepreneurs is \( b_I w_0 \frac{\mu_1 + \mu_0}{1 + \mu_1 + \mu_0} \), where \( b_I \) is the fraction of unskilled entering the high interest rate market. The fraction of bad and opaque investors is \( \mu_0 \) implying that

\[ \mu_0 \phi \xi i(b, 0) \frac{1}{1 + r_I} = b_I w_0 \frac{\mu_1 + \mu_0}{1 + \mu_1 + \mu_0} \]

\[ i(b, 0) = (1 + r_I) \frac{b_I w_0}{\phi \xi} \frac{\mu_1 + \mu_0}{1 + \mu_1 + \mu_0} \frac{1}{1 + \mu_1 + \mu_0} \]

\[ I(b, 0) = 1 - \phi \xi i(b, 0) \frac{r_I}{1 + r_I} \]

transparent bad entrepreneurs cannot get credit implying

\[ I(b, 1) = 1, \ i(b, 1) = 0. \]

Opaque good entrepreneurs should be fully financed at \( r_I \) implying

\[ i(g, 0) = I(g, 0) = \frac{1 + r_I}{1 + r_I (1 + \phi \xi)}. \]
At the same time, market clearing determines $b_I$ by

$$\frac{(1 - \mu_1 - \mu_0)}{2} \phi_0 \xi i(g, 0) \frac{1}{1 + r_I} = b_I w_0 \frac{(1 - \mu_1 - \mu_0)}{1 + (\mu_1 + \mu_0)} + w_1$$  \hfill (A.13)

Substituting back $b_I$ gives

$$i(b, 0) = (1 + r_I) \left( \frac{1}{1 + r_I (1 + \phi_0)} - \frac{w_1}{\phi_0} \frac{2}{1 - \mu_1 - \mu_0} \right) \frac{\mu_1 + \mu_0}{2\mu_0}.$$

Just as in the cautious equilibrium, transparent good entrepreneurs follow (A.12) by the same argument. Now market clearing on the low interest rate market determines the fraction of unskilled entering this market, $b_C$:

$$\frac{1 - \mu_1 - \mu_0}{2} \phi_0 \xi \frac{1}{1 + r_C (1 + \phi_0)} = b_C w_0 \frac{1 - \mu_1 - \mu_0}{2}.$$

Note that from the requirement of max($b_C, b_p, b_I$) < 1 one could express the second part of Assumption 2.2 in terms of the primitives.

**Proof of Propositions 4.2 and 5.1**

Proposition 4.2 follows from Assumption 2.1 as described in the text.

Welfare in the dynamic economy is

$$W(\mu_0, \mu_1) \equiv \frac{1 - \mu_0 - \mu_1}{2} \left( \rho_g (1 - \phi) + (\rho_g - \xi) \phi \eta(g, 1) \frac{r(g, 1)}{1 + r(g, 1)} + \rho_g (1 - \phi) + (\rho_g - \xi) \phi \eta(g, 0) \frac{r(g, 0)}{1 + r(g, 0)} \right) +$$

$$\mu_0 \rho_b \frac{(1 - \phi) + \phi \eta(b, 0)}{1 + \phi \xi \eta(b, 0) \frac{r(b, 0)}{1 + r(b, 0)}} + (1 - \phi) + \phi \eta(b, 1) \frac{r(b, 1)}{1 + \phi \xi \eta(b, 1) \frac{r(b, 1)}{1 + r(b, 1)}} +$$

$$w_0 (1 + r_f) + w_1 \left( 1 + \max r(\tau, \omega) \right).$$  \hfill (A.14)

To be completed.