Measuring Operations Performance

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International Manufacturing Strategy in a Time of Great Flux

Springer

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ISSN 2363-9970 ISSN 2363-9989 (electronic)
Measuring Operations Performance
ISBN 978-3-319-25350-3 ISBN 978-3-319-25351-0 (eBook)
DOI 10.1007/978-3-319-25351-0

Library of Congress Control Number: 2016946941

Printed on acid-free paper

This Springer imprint is published by Springer Nature
The registered company is Springer International Publishing AG Switzerland

Do Improvement Programs Complement Each Other?

Phillip J. Lederer

Abstract Improvement programs of various types have been adopted by many corporations and other organizations. In some cases, multiple programs have been implemented. An important question is whether such programs are complements to each other? In other words, is the value added of a pair of programs larger than value generated by the sum of each instituted separately. This chapter studies that question for some common improvement programs. Complementarity is studied for three program types: uncertainty reduction about customers' values for service, accounting programs like ABC that eliminate biased cost estimates, and operations efforts. Three kinds of operations improvements are considered: reducing variable cost, reducing capacity cost and reducing non-value added time. Research by Milgrom and Roberts (Am Econ Rev 80(3):511–528, 1990) argue that many modern improvement programs are complementary. But in this theoretical work the conclusion is a direct result of the technical assumptions made. Specifically, their assumption of supermodularity properties directly leads to the results. Missing from this analysis, but explored here, is whether realistic, well understood cost functions lead to complementary properties. Initially we assume that cost is driven by queuing-like production technology. Because batching/lot sizing and fixed charge problems have costs like queuing, the results apply broadly. In this case, the first two programs can be either complements or substitutes. But they are *both* complementary to direct cost savings and capacity cost reduction. The situation with reduction of cost estimate bias is more complex: it is complementary to direct cost savings and the reduction in non-value added time but is a substitute to reducing capacity cost. Complementarity properties are also studied for general demand and cost functions, with sufficient conditions presented. The managerial conclusion is that care must be taken in assuming the complementarity of real programs, and that more central oversight of improvement efforts probably is warranted to better estimate the value of programs.

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L. Brennan and A. Vecchi (eds.), *International Manufacturing Strategy in a Time of Great Flux*, Measuring Operations Performance,
DOI 10.1007/978-3-319-25351-0_7

Keywords Improvement programs · Complements · Operations management · Queuing · Coordinating investments · TQM · Cost reduction · ABC accounting · Marketing research

1 Introduction

During the past two decades firms have adopted many types of functional improvement programs. To give only a few examples: operations programs such as Total Quality Management (TQM), Materials Resource Planning II (MRP-II), and Just-in-Time (JIT) have been adopted to reduce cost, increase quality and enhance customer service. In marketing, ServQual and customer satisfaction programs have been instituted. In accounting, Activity Based Costing (ABC), and other product costing programs have been implemented. The net impact on the firm of these programs is complex but can result in more desirable products, better service to customers, lower cost, better pricing, and higher margins.

Milgrom and Roberts (1990) and researchers thereafter have applied ideas of complementarity to model interactions of improvement programs. That is, the total benefit of the two activities performed together is greater than the sum of benefits when each activity is performed alone. Complementarity of activities X and Y holds if: when activity X is performed alone firm value increases by ΔV_X, and when activity Y is performed alone firm value increases by ΔV_Y, then when activity X and Y are performed together the firm value increases by ΔV_{X+Y} with the property that $\Delta V_{X+Y} \geq \Delta V_X + \Delta V_Y$.

The important managerial issue is that a firm benefits most by coordinating adoption of complementary activities because the valuation of each program separately undervalues joint adoption. On the other hand, two programs are substitutes when doing two activities together causes a gain that is less than the gain from doing each separately, thus the independently valued activities have less value than the independent valuations suggest. The upshot is that a well managed enterprise ought to engage in complementary activities and take care with substitute activities because the gain may be less than anticipated. This paper gives some direction as to which sets of improvement programs gain the most from coordinated decision making by studying the complementary/substitutability of several types of programs.

In order to guarantee complementarity, the modern literature has assumed supermodularity[1] which is mathematical condition which implies complementarity of, say, two activities. In the above cited research, the cost function is most often *assumed* to be supermodular. That is, supermodularity properties are not derived

[1]The definition of supermodularity for a real valued function $f(x, y)$ on $\mathbb{R}^2$ is: given any $x_1 \leq x_2$; $y_1 \leq y_2$ then $f(x_2, y_1) - f(x_1, y_1) \leq f(x_2, y_2) - f(x_1, y_2)$, and an identical property if x and y are switched. (This is often called the "increasing differences property").

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from fundamental principles of a general operations process. An unresearched question is whether *realistically* modeled cost and demand functions cause commonly adopted programs to be complements or substitutes. By "realistic" cost models I mean those most often recognized and used by operations management researchers, which include queuing technology and or inventory control processes. Next, I describe the types of programs I model.

Marketing research is sometimes used to better estimate parameters of consumer preferences, such as demand elasticity with respect to price and service levels. I refer to this type of program as "uncertainty reduction". The value added of such a program is better decision-making through reduced uncertainty. If uncertainty reduction is interpreted as "learning", then these results are important as they show the effect of "learning" on other improvement programs.

Aside from uncertainty reduction, I explore two other types of improvement programs. The second type studied is removing bias in cost estimates and the other is improving operations. Bias elimination programs occur in accounting systems when cost estimates are biased due to misallocation of fixed costs. For example, ABC accounting systems improve product costing by more carefully allocating fixed cost to their drivers. Kaplan and Cooper (1997) explain that many cost accounting systems over-allocate costs to high volume simple products and under-cost more complex lower-volume products. Zimmerman (1979) explains that cost allocation is a "second-best" process that helps an accountant to estimate marginal costs. The fact that the process is second-best and is often applied to all products mechanically reinforces the argument that errors in cost estimation do occur. A bias reduction program begins with the intellectual understanding that some cost allocations are biased and the direction of the bias often can be inferred ex-ante. One of the programs I study involves removing such bias.

The third type of improvement program studied is an improvement program in operations. This might focus on many aspects such as higher conformance to technical specifications, reduced lead-time, and reduced product cost. In this study the focus is on reducing direct cost, reducing non-value added lead-time and reducing capacity costs. I study several types of operations improvement programs and show that they are mutually complementary but not necessarily to other programs. This suggests that simultaneous operations improvement programs are generally more valuable than "stand-alone" economic assessments would imply.

In this analysis, common, but specific types of operations, marketing and accounting improvement programs are studied. Specificity is required because the structure of each program type affects the value function differently, and thus generates different basic sufficient conditions. Although our results are not generic for all programs, the approach to establishing sufficient conditions is generic.

I show that many improvement programs are not necessarily complementary but are instead substitutes or independently behaved. Table 1 describes the results of Sect. 2 where the demand function is assumed convex in price and the cost function concave in production quantity. The latter is an important case as our results apply to cost and service environments driven by queuing. I show that a program that adds value by reducing uncertainty about some decision-affecting-parameters can be a complement or

Table 1 Complementarity of improvement programs studied in Sect. 2

	Reduce uncertainty about value of early schedule delivery (ξ_a)	Reduce uncertainty about cost of late delivery (ξ_T)	Remove cost bias (r)	Reduce variable cost (c)	Reduce short term capacity cost (γ)	Reduce non value added time (t_a)
Reduce uncertainty about customers value of early scheduled delivery (ξ_a)		+	±	+	+	+
Reduce uncertainty about cost to the customer of late delivery(ξ_T)			–	Additive	Additive	+
Remove cost bias (r)				+	–	+
Reduce variable cost (c)					+	+
Reduce short term capacity cost (γ)						+
Reduce non value added time (t_a)						

The differing complementary properties of programs related to operations (cost related), accounting (removing cost bias) and marketing (uncertainty reduction of customer demand parameters). *Key* + complement, − substitute, ± either, *additive* independent

a substitute to other programs. Reducing uncertainty about the customers value of early delivery promises is complementary to reducing uncertainity about the value of late delivery and to cost related operations programs, but could be either complementary or a substitute for removing cost estimate bias. Reducing uncertainty about the customer's non-pecuniary cost of late delivery is a substitute for the cost bias program, a complement to reducing non-value added time but independent to other operations improvements such as reducing variable cost or short term capacity cost. A program to reduce bias in cost estimates is complementary to variable cost reduction or a non-value added time reduction program but is a substitute to reducing short-term capacity cost! Finally, programs that reduce variable cost, short-term capacity cost and non-value added time are mutual complements.

One of the contributions of this paper is studying complementarity in a queuing environment. The operations management literature has often adopted queuing as one of the key processes to model services as well as manufacturing. This is because queuing processes enable mapping production decisions into resulting service and inventory levels. Thus, how production decisions affects service levels and inventory costs can be modeled. It also is observed that other prominent operations processes share the cost behavior of queuing where total cost displays increasing returns to scale with respect to production volume. The most prominent of these are production processes with setup cost. These include the economic-order-quantity (EOQ) model and any other model with a fixed charge, such as location siting (see Francis et al. 1992). Thus, results about queuing processes apply more commonly than might first be suspected.

Section 3 explores more general cost and demand situations than are explored in Sect. 2. Here sufficient conditions that imply complementarity or substitutability of improvement programs when the demand and cost functions are non-linear are presented. Demand is assumed to be a convex decreasing function of service quality. The cost function can be convex, concave or neither as a function of production quantity. Several examples are used to demonstrate the power of the sufficient conditions are presented.

In the analysis of this paper, I ignore the significant cost of implementing an improvement program. This is because my focus is on the complementarity of improvement program *benefits*. For a specific project, the cost of implementation may be significant, but often can be assumed to be independent of the implementation of other programs. Thus, I focus on the value of an improvement program and not its value net of implementation cost. Next, I survey the literature on complementarity focused on improvement.

1.1 Literature Review

A recent overview of the use of supermodularity and complementarity in economics and game theory is found in Amir (2005). Milgrom and Roberts (1990, 1995) present models of a profit maximizing firm making operations and process choice decisions. The goal is to analyze comparative statics of the firm under optimal decisions about pricing, process choice, product innovation, lead-time to delivery, etc. Their modeling assumptions concerning supermodularity of functions leads to the result that the programs studied are complementary. Similarly, Bagwell and Ramey (1994) study the complementary relationship between processes, discount buying and falling consumer prices. In these papers, the authors seek to understand conditions that guarantee complementarity of decision variables, and monotone changes in optimal solutions with respect to exogenous variables. Also along these lines, Topkis (1995) generalizes the cited papers by Milgrom and Roberts and demonstrates necessary and sufficient assumptions to assure complementarity and monotonicity of optimal solutions. In order to get their results in all of these papers, assumptions are made that state that demand and cost functions are supermodular in decision variables. Complementarity of decision variables is shown by the property that maximization of a supermodular function with respect to some of the decision variables yields a function that is again supermodular in the remaining variables (Topkis 1998, Theorem 2.7.6). Monotonicity of optimal solutions with respect to non-decision variables is assured when the objective function is supermodular in the decision variable and the non-decision variables (Topkis 1998, Theorem 2.8.2). In short, assumptions about supermodularity, demand and cost lead directly to the results. The assumed cost functions are treated as "black boxes" meaning that the physics of the inherent processes is neither modeled nor developed. An unanswered question is if typical operations technologies (such as queuing, inventory management, fixed charge problems, etc.) lead to cost functions that are consistent with supermodularity, and when paired with a reasonable demand function, will total profit be supermodular in decision variables? Thus, a gap exists between the assumptions made in existing literature on complementarity and properties of important real operations technologies.

As previously stated, this paper analyzes the complementarity of uncertainty reduction and other improvement programs. Several papers have been written on the interaction of risk and supermodularity. An important paper is Athey (2002) that studies the monotonicity of the solution to an expected utility problem with respect to parameters that shift the utility function and the associated probability distribution. The question asked is what restrictions are necessary so that an arbitrary parameterized objective function from a restricted class of functions and an arbitrary parameterized probability distribution from a restricted class of probability distributions yields an expected value function that is supermodular in the decision variables and parameters. This question is important because it sets conditions for optimal decisions to be monotone in the common parameters. This question is quite unlike that of this paper that studies the complementarity of decisions to adopt different improvement programs, and not how parameters affect solutions.

Particularly important is the work of Siggelkow (2002) which demonstrates the cost of misperceiving complementarity or substitutability of programs. He shows that generally incorrectly assuming complementary effects are more costly than incorrectly assuming substitution effects. Siggelkow's work underscores the value of knowing when improvement programs possess either (or neither) of these properties. My research helps to better understand when complementarity or substitutability is likely to be present through analytical modeling of demand and processes.

The rest of the paper is organized as follows. Section 2 demonstrates analysis of a queuing process in a market with constant demand elasticity. The analysis studies whether direct cost savings, uncertainty reduction, and cost bias elimination are complements or substitutes. Section 3 generalizes conditions of Sect. 2 to other cost and demand functions. Section 4 presents a summary of results and some suggestions for further research.

2 A Study in Complementarity: The Three Improvement Programs in a System with Queuing

In this section I model a profit maximizing firm and its production system. The firm uses one of the basic operations processes: queuing. What is interesting about the queuing process is that if direct cost and delay dependent costs (e.g., work-in-process inventory, delay costs to customers) are considered there is a nonlinear production relationship between inputs and output cost. I assume that the firm wishes to improve its profitability by engaging in improvement programs. Consistent with the discussion so far, the firm is uncertain about some parameters that affect customers' demand. One possible improvement program is reduction in this uncertainty, which can be interpreted as a marketing-related program. In addition to uncertainty reduction, I study two other types of improvement programs: (1) cost bias elimination (which is accounting system related), and (2) direct cost savings (which is production related). Cost bias elimination improves profit by recognizing that often firms make production decisions based upon biased cost estimates. In a real firm, many productivity improvement programs are possible, such as direct cost savings, defect elimination, yield enhancement, shortening production lead time, decreasing process variability, and shortening cycle times, etc. In modeling, I need to be specific and focus primarily on direct cost savings and, secondarily, on reduction in non-value added lead time, and capacity costs.

Consider a one-stage, one-product system that produces to customer order. For simplicity, it is assumed that the manager seeks to maximize profit and is risk neutral. The manager's decisions are: production rate, what delivery date to promise, and how much short-term capacity (such as direct labor) to employ. The time to produce an order can be reduced by adding short-term capacity to speed up the production rate, but at some expense. The promised delivery date is the date when the delivery to the customer is supposed to be made: however due to production problems the delivery date may be missed and the order may be tardy. We assume that the price that a customer is willing to pay is a function of the market's demand rate, the time to delivery and the tardiness of such the order. The firm's costs are the sum of direct cost and capacity related cost. To formally describe the problem, the following notation is used:

p	the \$ price for the product/unit (a function of sales rate)
P	the "full" price is the \$ sum of the unit sales price, p, plus the sum of the costs of delivery delay, and expected tardiness
q	the steady state sales rate in each period for the product (units/period) (decision variable)
δ_a	promised delivery date (periods in the future) (decision variable)
δ_K	the short-term capacity for cost center (rate of production/period) (decision variable)
$C(\delta_K)$	the total \$ cost of short-term capacity at cost center. We assume $C(\delta_K) = \gamma\delta_K$ (\$/rate of production in units/period)
$L = L(q, \delta_K)$	the total production lead time for production rate q per period with capacity
δ_K	Lead time is itself divided into two components: non-value added lead time, and process time (periods)
$L_N = t_o$	non-value added lead time (periods)

$$L_P = \frac{1}{\delta_K - q} = \text{process lead time (periods)} \tag{2.1}$$

T the expected tardiness of the order (expected difference between the actual delivery date and the promised delivery date (periods)

c the direct production \$ cost per unit of the product

ξ_a customers' cost per period for planned delivery wait (\$/period)

ξ_T customers' cost per period of delay beyond the promised delivery date (\$/period)

As is traditional, random variables will be indicated by a tilde above the character and mean value will be indicated by a bar over the random variable.

Customers value fast planned delivery and adherence to the schedule. We assume that customers' cost per period for a planned delivery horizon is ξ_a. We also assume that the customer's cost of delay per period in actual delivery beyond the "promised" date δ_a is ξ_T. Once delivery date has been promised, then customers find no value to delivery before that date. The cost structure is very much like that of a customer running a project. The project is complex and activities must be coordinated. Once an item (off the critical path) has been scheduled for delivery, earlier arrival is valueless. However, tardiness (late arrival) causes disruptions and additional project cost.

For analytic tractability, the customers demand function is assumed to be of a simple constant elastic form:

$$P(q) = \sqrt{\frac{\alpha}{q}}, \quad \text{thus,} \quad p + \xi_T T + \xi_a \delta_a = \sqrt{\frac{\alpha}{q}}. \tag{2.2}$$

In Sect. 3, I show that this assumption can be generalized to a broad range of demand functions.

The firm is uncertain about a customer's taste for quick delivery and tardiness, but has an unbiased estimate of cost parameters $\tilde{\xi}_a$ and $\tilde{\xi}_T$. Customers may more highly value quick delivery than reduced tardiness $(\xi_a > \xi_T)$, or may view early delivery more valuable than an earlier promised delivery date $(\xi_T > \xi_a)$. The PDF's for these two random variables are: f_a and f_T. A notation we will often use is to write the expected value of these two random variables as, respectively, $\bar{\xi}_a$ and $\bar{\xi}_T$.

Lead-time consists of two components: non-value added time and process-related time queuing time, $L = L_N + L_P$. Non-value added time captures the time in which an item is neither in production nor in queue for production. Non-value added time is an important component of total lead-time, and has been a major focus of waste reduction in total quality management.

To calculate process-related queuing time we assume an M/M/1 queuing process which implies that the arrival of orders is given by a Poisson process and the service times are exponentially distributed with rate δ_K. Thus, the actual queuing time is a function of production capacity and manager's capacity-related effort, and expected waiting time is given by the M/M/1 formula $L_P = \frac{1}{\delta_K - q}$. Using the M/M/1 assumption, process time due to congestion is stochastic with an exponential distribution and the actual completion time t has a distribution function

$$h(t\,|\,\delta_K, q) = \frac{1}{\delta_K - q} e^{-(\delta_K - q)(t - t_o)}. \tag{2.3}$$

If a is the promised delivery date, the expected tardiness has a value of

$$T(\delta_a, q, \delta_K) = \frac{1}{\delta_K - q} e^{-(\delta_K - q)(\delta_a - t_o)}. \tag{2.4}$$

The firm's objective is to choose a production rate for the product and short-term capacity to maximize its profit.

$$\operatorname*{Max}_{q,\delta_a,\delta_K} \quad \Pi = pq - cq - \gamma\delta_K \tag{2.5}$$

$$\text{subject to} \quad p = \sqrt{\frac{\alpha}{q}} - \xi_a\delta_a - \xi_T T \tag{2.6}$$

$$q \geq 0,\ \delta_a \geq 0,\ \delta_K \geq 0. \tag{2.7}$$

This problem can be written separating the inner problem that minimizes cost with respect to decision variables:

$$\operatorname*{Max}_{q} \ \operatorname*{Min}_{\delta_a,\delta_K} \ \left[\sqrt{\alpha q} - [(\xi_a\delta_a + c + \xi_T T)q + \gamma\delta_K]\right],$$

$$\text{or,} \quad \operatorname*{Max}_{q} \left[\sqrt{\alpha q} - \operatorname*{Min}_{\delta_a,\delta_K} \left[(\xi_a\delta_a + c + \xi_T \frac{1}{\delta_K - q} e^{-(\delta_K - q)(\delta_a - t_o)})q + \gamma\delta_K\right]\right] \tag{2.8}$$

subject to (2.7).

It will be useful to define the cost function

$$\begin{aligned} C(q, \delta_a, \delta_K, \xi_a, \xi_T) &= (\xi_a\delta_a + c + \xi_T T)q + \gamma\delta_K \\ &= \left(\xi_a\delta_a + c + \xi_T \frac{1}{\delta_K - q} e^{-(\delta_K - q)(\delta_a - t_o)}\right)q + \gamma\delta_K \end{aligned} \tag{2.9}$$

thus the objective can be written $\operatorname*{Max}_{q} \left[\sqrt{\alpha q} - \operatorname*{Min}_{\delta_a,\delta_K} \ C(q, \delta_a, \delta_K, \xi_a, \xi_T)\right]$.

We next derive optimal values of the decision variables.

2.1 Optimization of Decisions

Proposition 2.1 *For fixed q, the values of* δ_K*, and* δ_a*, that solve the inner problem are:*

$$\delta_a^*(q) = \begin{cases} t_o & \text{if} \quad \xi_a \geq \xi_T \\ t_o + \dfrac{t_o}{\sqrt{\dfrac{\gamma}{\xi_a\left(1-\log\left(\frac{\xi_a}{\xi_T}\right)\right)q}}} \log\left(\frac{\xi_T}{\xi_a}\right) & \text{if} \quad \xi_a < \xi_T \end{cases} \tag{2.10}$$

$$\delta_K^*(q) = \begin{cases} q + \sqrt{\frac{\xi q}{\gamma}} & \text{if} \quad \xi_a \geq \xi_T \\ q + \sqrt{\dfrac{\xi_a\left(1-\log\left(\frac{\xi_a}{\xi_T}\right)\right)q}{\gamma}} & \text{if} \quad \xi_a < \xi_T \end{cases}, \tag{2.11}$$

with associated tardiness

$$T^*(q) = \begin{cases} \frac{1}{\delta_K^* - q} = \sqrt{\frac{\gamma}{\xi_T q}} & \text{if} \quad \xi_a \geq \xi_T \\ \frac{1}{\xi_T}\sqrt{\dfrac{\gamma\xi_a}{\left(1-\log\left(\frac{\xi_a}{\xi_T}\right)\right)q}} & \text{if} \quad \xi_a < \xi_T \end{cases}. \tag{2.12}$$

Optimal decisions are derived by differentiating by the appropriate variable and solving the first-order condition (and checking the second-order conditions). These routine computations are omitted.

To simplify notation for later expressions, I write:

$$\lambda(\xi_a, \xi_T) = \begin{cases} \xi_T & \text{if } \xi_T \leq \xi_a \\ \xi_a\left(1 - \log\left(\frac{\xi_a}{\xi_T}\right)\right) & \text{if } \xi_T \geq \xi_a \end{cases}. \tag{2.13}$$

If $\xi_T \leq \xi_a$, the firm prefers to announce its earliest possible delivery date (t_o). Then the expected tardiness is just the expected time in system for an order. If $\xi_a \leq \xi_T$, the firm will announce a date after t_o .

Substituting the results of Proposition 2.1, the firm's objective function is in the form:

$$\Pi(q, \xi) = \underset{q}{\text{Max}} \ \sqrt{\alpha q} - \left[(\xi_a t_o + c + \gamma)q + 2\sqrt{\gamma\lambda(\xi_a, \xi_T)q}\right]. \tag{2.14}$$

We proceed, and solve explicitly for an optimal sales rate:

$$q(\xi) = \left(\frac{\frac{\sqrt{\alpha}}{2} - \sqrt{\gamma\lambda(\xi_a, \xi_T)}}{\xi_a t_o + c + \gamma} \right)^2, \tag{2.15}$$

in which case the firm's profit function is:

$$\Pi(q(\xi), \xi) = \frac{\left(\frac{\sqrt{\alpha}}{2} - \sqrt{\gamma\lambda(\xi_a, \xi_T)}\right)^2}{\xi_a t_o + c + \gamma} = (\xi_a t_o + c + \gamma) q(\xi). \tag{2.16}$$

Interestingly, profit is directly proportional to the sales rate. If $\xi_a \leq \xi_T$, the Hessian of the profit function with respect to (ξ_a, ξ_T) can be shown to be positive definite, so that this function is strictly convex in (ξ_a, ξ_T). Likewise if $\xi_a \geq \xi_T$, the same conclusion holds and I will show in the next section that information about ξ_a is valuable even though choice of delivery date is fixed at t_o.

We next explicitly compute the value of marketing research, cost estimation, and process improvement programs.

2.2 Value of an Uncertainty Reduction Program

If there is uncertainty in the value of the demand parameters $\tilde{\xi}_a$ and $\tilde{\xi}_T$ when its decisions are made, the firm's problem is:

$$\underset{q}{\text{Max}} \ \underset{\delta_a, \delta_K}{\text{Max}} \ E_f[\sqrt{\alpha q} - \tilde{\xi}_a \delta_a q - cq - \tilde{\xi}_T T q - \gamma \delta_K] \text{ subject to (2.7).}$$

Computing the expectation, the optimization problem becomes:

$$\underset{q}{\text{Max}} \ \underset{\delta_a, \delta_K}{\text{Max}} \ \sqrt{\alpha q} - \bar{\xi}_a \delta_a q - cq - \bar{\xi}_T T q - \gamma \delta_K, \text{ subject to (2.7).}$$

(Recall, a bar over a random variable indicates its mean). The firm's problem is:

$$\Pi(q, \bar{\xi}) = \underset{q}{\text{Max}} \ \sqrt{\alpha q} - \left[\left(\bar{\xi}_a t_o + c + \gamma\right) q + 2\sqrt{\lambda(\bar{\xi}_a, \bar{\xi}_T)}\sqrt{\gamma q} \right].$$

Now, the decisions the manager makes are exactly those specified in (2.10) and (2.11) but with $\bar{\xi}_a$ and $\bar{\xi}_T$ used instead of ξ_a and ξ_T. The profit functions will be:

$$\Pi(q, (\bar{\xi}_a, \bar{\xi}_T)) = \underset{q}{\text{Max}} \ \sqrt{\alpha q} - \left[\left(\bar{\xi}_a t_o + c + \gamma\right) q + 2\sqrt{\lambda(\bar{\xi}_a, \bar{\xi}_T)}\sqrt{\gamma q} \right].$$

The optimal sales rate and profit are:

$$q(\bar{\xi}_a, \bar{\xi}_T) = \left(\frac{\frac{\sqrt{\alpha}}{2} - \sqrt{\lambda(\bar{\xi}_a, \bar{\xi}_T)}}{(\bar{\xi}_a + h)t_o + c + \gamma} \right)^2 \tag{2.17}$$

$$\Pi(q(\bar{\xi}_a, \bar{\xi}_T), (\bar{\xi}_a, \bar{\xi}_T)) = \frac{\left(\frac{\sqrt{\alpha}}{2} - \sqrt{\gamma}\sqrt{\lambda(\bar{\xi}_a, \bar{\xi}_T)} \right)^2}{\bar{\xi}_a t_o + c + \gamma}. \tag{2.18}$$

The profit in absolute terms is just $q(\bar{\xi})\left(\bar{\xi}_a t_o + c + \gamma\right)$. Note that as before, profit is linear in the sales rate, and directly proportional to the unit direct cost, $\xi_a t_o + c + \gamma$.

The expected gain from instituting an *uncertainty reduction program* is

$$\begin{aligned} &E\Pi(q(\tilde{\xi}_a, \tilde{\xi}_T)), (\tilde{\xi}_a, \tilde{\xi}_T)) - \Pi(q(\bar{\xi}_a, \bar{\xi}_a), (\bar{\xi}_a, \bar{\xi}_a)) \\ &= \frac{\sqrt{\alpha\gamma}\sqrt{\lambda(\bar{\xi}_a, \bar{\xi}_T)} + \gamma\lambda(\bar{\xi}_a, \bar{\xi}_T)}{\bar{\xi}_a t_o + c + \gamma} - E\frac{\sqrt{\alpha\gamma}\sqrt{\lambda(\tilde{\xi}_a, \tilde{\xi}_T)} + \gamma\lambda(\tilde{\xi}_a, \tilde{\xi}_T)}{\tilde{\xi}_a t_o + c + \gamma} > 0. \end{aligned} \tag{2.19}$$

As in the last section, the Hessian of (2.18) is positive definite in (ξ_a, ξ_T), thus is a convex function of (ξ_a, ξ_T), which implies that (2.19) is strictly positive. But observing definition (2.13) it is seen that even if $\xi_a \geq \xi_T$, positive expected value-added occurs when uncertainty about ξ_a and ξ_T are reduced. Although the due date (δ_a) decision is not affected by reduced uncertainty about ξ_T, the optimal production rate is. We conclude that reduction in uncertainty for either, or both parameters is valuable.

2.3 Value of Bias Reduction

In this section I introduce bias into cost estimates. I assume a common accounting procedure and heuristic: the firm uses *average* operating cost to estimate a component, or the whole of *marginal* cost. We consider distortion of the non-linear cost of production $\gamma q + \sqrt{\lambda\gamma q}$ by use of its average cost. Using average cost has the effect of *reducing* marginal cost by 50 %. If the firm uses (2.9) and (2.10) to set δ_a and δ_K, then at any volume level, q':

$$\text{Average } \textit{real} \text{ operating cost} = \xi_a t_o + c + \gamma + \sqrt{\frac{\lambda\gamma}{q'}}, \tag{2.20}$$

where q' is output used to set the average cost. Now the firm's profit as a function of production (assuming the other components are estimated correctly) is:

$$\hat{\Pi}(q,\xi) = \sqrt{\alpha q} - (\xi_a t_o + c + \gamma)q - \sqrt{\lambda\gamma q} - \sqrt{\frac{\lambda\gamma}{q'}}q. \tag{2.21}$$

The derivative of profit with respect to demand is now

$$\frac{\partial\hat{\Pi}(q,\xi)}{\partial q} = \frac{\sqrt{\alpha}}{\sqrt{q}} - (\xi_a t_o + c + \gamma) - \frac{1}{2}\frac{\sqrt{\lambda\gamma}}{\sqrt{q}} - \frac{\sqrt{\lambda\gamma}}{\sqrt{q'}}.$$

When $q = q'$, we find the optimal production level is just

$$\hat{q}(\xi) = \left(\frac{\frac{\sqrt{\alpha}}{2} - \frac{3}{2}\sqrt{\lambda\gamma}}{\xi_a t_o + c + \gamma}\right)^2. \tag{2.22}$$

Using biased cost, $\hat{q}$ is the perceived "optimal q". I write $\hat{\Pi}$ and $\hat{C}$ for respectively, the profit and cost functions computed with biased cost. The biased cost function is $\hat{C}(q,\hat{\delta}^*) = (\xi_a t_o + c + \gamma)q + \frac{3}{2}\sqrt{\gamma\lambda(\xi_a,\xi_T)q}$ instead of the true cost function: $C(q,\delta^*) = (\xi_a t_o + c + \gamma)q + 2\sqrt{\gamma\lambda(\xi_a,\xi_T)q}$ (2 is the *proper* weighting on the non-linear term in q, but when misestimating, the *improper* weighting is 3/4 of the *proper* value. This tells us that one component of marginal cost is misestimated by a factor of *¾,* that is marginal cost is 25 % too low). With misestimated marginal cost, true profit is:

$$\Pi(\hat{q}(\xi),\xi) = \frac{\left(\frac{\sqrt{\alpha}}{2} - \frac{3}{4}\sqrt{\lambda\gamma}\right)\left(\frac{\sqrt{\alpha}}{2} - \frac{5}{4}\sqrt{\lambda\gamma}\right)}{\xi_a t_o + c + \gamma}. \tag{2.23}$$

It can be shown that (2.22) is strictly convex in (ξ_a, ξ_T), so that uncertainty reduction is valuable even with distorted production decisions through cost misestimate. By subtracting (2.16) from (2.23), the value of a bias elimination program is:

$$\Pi(q(\xi),\xi) - \Pi(\hat{q}(\xi),\xi) = \frac{\gamma\lambda(\xi_a,\xi_T)}{16(\xi_a t_o + c + \gamma)} \tag{2.24}$$

In this example, the bias causes one component of marginal cost to be taken at $r = 3/4$ of its real value. In general, if that component of marginal cost is distorted by a factor of r, the value added by removing bias can be shown to be

$$\Pi(q(\xi),\xi) - \Pi(\hat{q}(r,\xi),\xi) = \frac{(1-r)^2\gamma\lambda(\xi_a,\xi_T)}{(\xi_a t_o + c + \gamma)}. \tag{2.25}$$

This fact will be useful in the next section in analyzing complementarity when marginal cost is *over*estimated, that is, $r > 1$.

2.4 The Value of Direct Cost Savings

In this section, I explicitly compute the value of reduction in variable production cost (c). Suppose that all parameters are known with certainty. If the linear cost can be reduced to zero ($c = 0$), the optimal production and profit become

$$q(\xi, 0) = \left(\frac{\frac{\sqrt{\alpha}}{2} - \sqrt{\gamma\lambda}}{\xi_a t_o + \gamma}\right)^2, \text{ and} \tag{2.26}$$

$$\Pi(q(\xi, 0), \xi) = \frac{\left(\frac{\sqrt{\alpha}}{2} - \sqrt{\gamma\lambda}\right)^2}{\xi_a t_o + \gamma} = (\xi_a t_o + \gamma) q(\xi, 0). \tag{2.27}$$

Profit increases by

$$\Pi(q(\xi, 0), \xi) - \Pi(q(\xi, c), \xi) = \Pi(q(\xi, c), \xi)\left(\frac{(\xi_a t_o + c + \gamma)}{\xi_a t_o + \gamma} - 1\right) \tag{2.28}$$

which is clearly positive.

2.5 Complementary/Substitute Improvement Programs

By explicit computation, the complementarity or substitutability between the three improvements is demonstrated. I show that complementarity or substitutability cannot be universally assumed for all parameter values. Thus, care must be taken in assuming complementarity of improvement programs.

Suppose the firm is using an *average costing system* and has uncertainty about demand parameters that cannot be resolved before production begins.

If the firm engages in a cost bias elimination program, the gain has been shown via (2.24) to be $\frac{\gamma\lambda(\bar{\xi}_a, \bar{\xi}_T)}{16(\bar{\xi}_a t_o + c + \gamma)}$. If the firm also implements an uncertainty reduction program, the additional benefit is now, $E_{\tilde{\xi}} \frac{\gamma\lambda(\tilde{\xi}_a, \tilde{\xi}_T)}{16(\tilde{\xi}_a t_o + c + \gamma)} - \frac{\gamma\lambda(\bar{\xi}_a, \bar{\xi}_T)}{16(\bar{\xi}_a t_o + c + \gamma)}$.

If uncertainty in ξ_T *but not* ξ_a has been eliminated, the value added is $E_{\tilde{\xi}_T} \frac{\gamma\lambda(\bar{\xi}_a, \tilde{\xi}_T)}{16(\bar{\xi}_a t_o + c + \gamma)} - \frac{\gamma\lambda(\bar{\xi}_a, \bar{\xi}_T)}{16(\bar{\xi}_a t_o + c + \gamma)}$. This is negative as the value added by cost bias elimination, (2.24), is concave in ξ_T : $\frac{d}{d\xi_T^2}\left[\frac{\gamma\lambda(\xi_a, \xi_T)}{16(\xi_a t_o + c + \gamma)}\right] = -\frac{\gamma}{16(c + \gamma + t_o\xi_a)\xi_a^2} < 0.$

Thus, cost bias reduction is a *substitute* for uncertainty reduction in ξ_T. This surprising result contrasts with the complementarity properties of uncertainty reduction in ξ_a which we show next.

If uncertainty in ξ_a *but not* ξ_T has been eliminated complementarity properties are ascertained by convexity properties of the value added and there are two cases. This is a bit complicated because, as remarked earlier, if $\bar{\xi}_T \leq \xi_a$, then the promised delivery date will be t_o. If $\tilde{\xi}_T \leq \tilde{\xi}_a$ holds with probability 1, the expected value added is convex independent of the other parameters, thus cost bias elimination and uncertainty reduction are complements.

If $\tilde{\xi}_T \geq \tilde{\xi}_a$ holds with probability 1, the expected value added is neither concave nor convex: $\frac{d^2}{d\xi_a^2}\left[\frac{\gamma\lambda(\xi_a,\bar{\xi}_T)}{16(\bar{\xi}_a t_o + c + \gamma)}\right] = \frac{-\gamma(c+\gamma)^2 - t_o^2\xi_a^2 + 2(c+\gamma)t_o\lambda(\xi_a,\bar{\xi}_T)}{16\xi_a(c+\gamma+t_o\xi_a)^3}$. Unfortunately, the sign is ambiguous. However some sufficient conditions for convexity can be derived. There are two sub-cases related to the relative ratio of $\tilde{\xi}_a$ and $\bar{\xi}_T$. If $\bar{\xi}_T/\tilde{\xi}_a > \mathrm{e}^{\frac{1}{2}\left(-2+\frac{c+\gamma}{t_o\tilde{\xi}_a}+\frac{t_o\tilde{\xi}_a}{c+\gamma}\right)}$ with probability 1, then the second-order derivative is negative and *substitutability* is implied; if $\bar{\xi}_T/\tilde{\xi}_a < \mathrm{e}^{\frac{1}{2}\left(-2+\frac{c+\gamma}{t_o\tilde{\xi}_a}+\frac{t_o\tilde{\xi}_a}{c+\gamma}\right)}$ with probability 1, then the *second-order derivative is positive*, and *complementarity* is implied. Thus, the complementarity of bias elimination and uncertainty reduction cannot be taken as a given, and must be considered carefully.

Do these results depend on the fact that our model assumes that one component of marginal cost is underestimated? What happens when that component is *over*estimated? By observing the general case for marginal cost bias as expressed in (2.25), it can be seen that the above conclusions about complementarity hold when bias results in *over* or *under*estimates of marginal cost.

Next, I show that direct cost savings and uncertainty reduction are complements. When cost c is reduced to zero but there is uncertainty about parameters, then the expected value added by direct cost savings is $\Pi(q(\bar{\xi},c),\bar{\xi})\left(\frac{\bar{\xi}_a t_o + c + \gamma}{\bar{\xi}_a t_o + \gamma} - 1\right)$. If uncertainty about parameters has been resolved, then the value added by direct cost savings is $E_{\tilde{\xi}}\left[\Pi(q(\tilde{\xi},c),\tilde{\xi})\left(\frac{\tilde{\xi}_a t_o + c + \gamma}{\tilde{\xi}_a t_o + \gamma} - 1\right)\right]$. Note that as Π and $\left(\frac{\xi_a t_o + c + \gamma}{\xi_a t_o + \gamma} - 1\right)$ are both positive, convex in ξ_a and monotone decreasing in ξ_a, it follows that the product of Π and $\left(\frac{\xi_a t_o + c + \gamma}{\xi_a t_o + \gamma} - 1\right)$ is also convex in ξ_a. Thus, $E_{\tilde{\xi}}\left[\Pi(q(\tilde{\xi},c),\tilde{\xi})\left(\frac{\tilde{\xi}_a t_o + c + \gamma}{\tilde{\xi}_a t_o + \gamma} - 1\right)\right] > \Pi(q(\bar{\xi},c),\bar{\xi})\left(\frac{\bar{\xi}_a t_o + c + \gamma}{\bar{\xi}_a t_o + \gamma} - 1\right)$: direct cost savings is unambiguously complementary to uncertainty reduction in ξ_a. By an identical argument, reducing the value of γ or t_o are seen to be complements to bias elimination.

Because the second term in the product is not a function of ξ_T, uncertainty reduction in ξ_T does not increase or decrease the value added by direct cost savings: these programs have *additive values*. Again, the same conclusion can be made about γ or t_o: reducing these has no effect on the value added by reducing uncertainty in ξ_T.

The relationship between bias reduction and direct cost savings is easily inferred from (2.23). The increase in value due to bias reduction is complementary to c and t_o, but is a substitute to reduction in γ.

Is uncertainty reduction in ξ_T complementary to uncertainty reduction in ξ_a? The answer is yes. This is shown via the following proposition:

Proposition 2.2 *Consider two random variables,* $\tilde{\xi}_a$ *and* $\tilde{\xi}_T$ *with respective means* $\bar{\xi}_a$ *and* $\bar{\xi}_T$*. Also assume three continuous functions defined on a compact set* G *denoted* $F_a(x)$, $F_T(x)$ *and* $J(x)$ *with* $x \in G$*. Now define* $x(\xi_a, \xi_T) = \underset{x}{ArgMax}[J(x) - \xi_a F_a(x) - \xi_T F_T(x)]$ *then*

$$\begin{aligned}
&E_{\tilde{\xi}_a \tilde{\xi}_T}\left[J(x(\tilde{\xi}_a, \tilde{\xi}_T)) - \tilde{\xi}_a F_a(x(\tilde{\xi}_a, \tilde{\xi}_T)) - \tilde{\xi}_T F_T(x(\tilde{\xi}_a, \tilde{\xi}_T))\right] \\
&- E_{\tilde{\xi}_T}\left[J(x(\bar{\xi}_a, \tilde{\xi}_T)) - \bar{\xi}_a F_a(x(\bar{\xi}_a, \tilde{\xi}_T)) - \tilde{\xi}_T F_T(x(\bar{\xi}_a, \tilde{\xi}_T))\right] \\
&\quad \geq E_{\tilde{\xi}_a}\left[J(x(\tilde{\xi}_a, \bar{\xi}_T)) - \tilde{\xi}_a F_a(x(\tilde{\xi}_a, \bar{\xi}_T)) - \bar{\xi}_T F_T(x(\tilde{\xi}_a, \bar{\xi}_T))\right] \\
&- \left[J(x(\bar{\xi}_a, \bar{\xi}_T)) - \bar{\xi}_a F_a(x(\bar{\xi}_a, \bar{\xi}_T)) + \bar{\xi} F_T(x(\bar{\xi}_a, \bar{\xi}))\right]
\end{aligned}$$

.

Proof Let $H(x(\xi_a, \xi_T)) = [J(x(\xi_a, \xi_T)) - \xi_a F_a(x(\xi_a, \xi_T)) - \xi_T F_T(x(\xi_a, \xi_T))]$. H is a convex function of $(\xi_a, \xi_T) \in G$. This implies by Jensen's inequality

$$\begin{aligned}
&E_{\tilde{\xi}_a \tilde{\xi}_T}\left[J(x(\tilde{\xi}_a, \tilde{\xi}_T)) - \tilde{\xi}_a F_a(x(\tilde{\xi}_a, \tilde{\xi}_T)) - \tilde{\xi}_T F_T(x(\tilde{\xi}_a, \tilde{\xi}_T))\right] \\
&\geq E_{\tilde{\xi}_a}\left[J(x(\tilde{\xi}_a, \bar{\xi}_T)) - \tilde{\xi}_a F_a(x(\tilde{\xi}_a, \bar{\xi}_T)) - \bar{\xi}_T F_T(x(\tilde{\xi}_a, \bar{\xi}_T))\right].
\end{aligned}$$

and,

$$\begin{aligned}
&E_{\tilde{\xi}_T}\left[J(x(\bar{\xi}_a, \tilde{\xi}_T)) - \bar{\xi}_a F_a(x(\bar{\xi}_a, \tilde{\xi}_T)) - \tilde{\xi}_T F_T(x(\bar{\xi}_a, \tilde{\xi}_T))\right] \geq J(\bar{\xi}_a, \bar{\xi}_T) \\
&\quad - \bar{\xi}_a F_a(x(\bar{\xi}_a, \bar{\xi}_T)) - \bar{\xi}_T F_T(x(\bar{\xi}_a, \bar{\xi})).
\end{aligned}$$

Thus the conclusion is justified. QED

Consider the original profit function defined by (2.8). Letting $x = (\delta_a, \delta_K)$ puts (2.8) in the same general form as H. Then the conclusion of the proposition holds for the profit function. Thus, uncertainty reduction in one parameter is complementary to uncertainty reduction in another.

2.6 Complementarity of Other Operations Improvement Programs

Other observations are possible from this model by exploiting different, important parameters. The formula for optimal profit, (2.15), can be used to study complementarity of direct cost savings with two other operations improvement actions. These two actions are: reducing non-value added lead-time (t_o) and reducing the cost per unit of capacity (γ). Note that these are the only other operations-related parameters in the model aside from c. Computing the appropriate cross partials:

$$\frac{\partial^2\Pi}{\partial c\partial\gamma} = \frac{\sqrt{\alpha\gamma} + \sqrt{\lambda/\gamma}(-2\gamma + (c+\gamma+\xi_a t_o)^2}{2\sqrt{\gamma}(c+\gamma+\xi_a t_o)} > 0 \text{ if } c > 0,$$

$$\frac{\partial^2\Pi}{\partial c\partial t_o} = 2\xi_a \frac{\left(\frac{\sqrt{\alpha}}{2} - \sqrt{\gamma\lambda}\right)^2}{(c+\gamma+\xi_a t_o)} > 0,$$

$$\frac{\partial^2\Pi}{\partial\gamma\partial t_o} = \lambda\xi_a^2 \frac{\left(\frac{\sqrt{\alpha}}{2} - \sqrt{\gamma\lambda}\right)}{\sqrt{\gamma\lambda}(c+\gamma+\xi_a t_o)^2} + 2\xi \frac{\left(\frac{\sqrt{\alpha}}{2} - \sqrt{\gamma\lambda}\right)^2}{(c+\gamma+\xi_a t_o)^2} > 0.$$

Thus, these three improvement programs are complementary. Further, on the lattice defined on the positive orthant of three-space for (c, γ, t_o), the profit function is supermodular. It is significant to note that all three operations improvements are complementary to each other.

To summarize, in this section in the context of a queuing process, I have shown that although uncertainty reduction programs are not supermodular, it may be possible to make definitive statements of complementarity or substitutability. Direct cost reduction is complementary to uncertainty reduction. When average costing causes marginal cost to be biased, I have shown that cost reduction and bias elimination are complements. However, ambiguity enters in other cases. Depending on parameter values, cost bias elimination and uncertainty reduction may be complements or substitutes. Care must be taken not to make a priori assumptions that all improvement programs are complements.

The next section generalizes these results by presenting sufficient conditions for these three programs to be complements and substitutes. The sufficient conditions are presented in the context of more general demand and cost functions.

3 General Results

Complementarity of the three programs under more general demand and cost functions are analyzed in this section. Sufficient conditions for complementarity are developed for any demand and cost functions.

Consider a market with an inverse demand function, $P = P(q)$, where P is the *full price* per unit. Full price is the sum of the price charged plus service quality associated costs borne by the customer. Customers are sensitive to characteristics of service quality, such as product quality, lead time, reliability of delivery, etc. Without loss of generality, we assume a single attribute for customer service quality, s, that is weighted by a positive constant ξ. The service level is a negative attribute (higher s implies longer wait, more unreliability, poorer service, etc.). If the service level is s and the customer's sensitivity to inconvenience is parameter ξ, the *real* price paid by the customer is $p = P(q) - \vartheta(\xi)s$. In this section function $\vartheta()$ is not necessarily linear in ξ. Instead we assume the function is concave in ξ.

Making concave $\vartheta(\xi)$ assures that the expected value of additional information about ξ is non-negative. However, if $\vartheta(\xi)$ is *convex*, then the value of information has an expected value that is *non-positive*. Thus, this concavity condition is necessary for information to add value.

Given a scalar decision parameter δ and production volume q, the service level is given by the function $s(q,\delta)$. The cost of producing output q with quality $s(q,\delta)$ is $\mathcal{C}(q,\delta)$. Firm profit is

$$\Pi(q,\delta,\xi) = (P(q) - \vartheta(\xi)s(q,\delta))q - \mathcal{C}(q,\delta). \tag{3.1}$$

Here we interpret parameter ξ as affecting demand through "*Full Price*" but a slight re-interpretation of the model could have it affect the cost function. To do this, the cost $s(q,\delta)q$ would be part of the cost function, not the *Full Price*. As in Sect. 2, with a certain value of ξ, the profit maximizing choice of δ is denoted $\delta^*(q,\xi)$.

The following three assumptions insure that a local optimal production rate is a continuous function of other model parameters..

Assumptions 3.1

(a) $P(q)$ is twice continuously differentiable, down-sloping for all $q \geq 0$.

(b) $\vartheta(\xi)s(q,\delta)q + \mathcal{C}(q,\delta)$ is four-times continuously differentiable in δ and is concave in δ for all δ within the set $(q,\delta) \in \Gamma \subset \mathbb{R}^2_+$ where Γ is a closed convex set.

(c) $(q,\delta^*(q,\xi)) = (q, \underset{\{\delta|(q,\delta)\in\Gamma\}}{\text{ArgMin}}[\vartheta(\xi)s(q,\delta) + C(q,\delta)])$ is in the interior of Γ for all q. Further $\frac{d^2[\vartheta(\xi)s(q,\delta)q + \mathcal{C}(q,\delta)]}{d\delta^2}\Big|_{(q,\delta)=(q,\delta^*(q,\xi))} < 0$

(d) Let $q^* = \underset{q\geq 0}{\text{ArgMax}}\, P(q) - \vartheta(\xi)s(q,\delta^*(q,\xi))q - \mathcal{C}(q,\delta^*(q,\xi))$ then, $(q^*,\delta^*(q,\xi)) \in \text{int}\,\Gamma$ and $\frac{d^2[P(q) - \vartheta(\xi)s(q,\delta^*(q,\xi))q - \mathcal{C}(q,\delta^*(q,\xi))]}{dq^2}\Big|_{(q,\delta)=(q*,\delta^*(q,\xi))} < 0$

These assumptions generalize requirements of Sect. 2. and admit a far wider set of demand and cost functions than was previously assumed. Assumptions about continuous differentiability are not very restrictive. The differentiability assumptions are made to assure continuity of higher order derivatives derived by the inverse function theorem. If a function is not sufficiently differentiable, it may be replaced by an analytic function that is arbitrarily close to the original. With this approximation, the tools of the following pages can be used to predict complementarity properties.

Assumption 3.1.a is a standard requirement for a demand function. Assumption 3.1.b states that the optimum capacity choice will be in a closed convex set, which implies that the K-T conditions are sufficient to guarantee optimality in capacity choice. Assumption 3.1.c strengthens the latter in that given a value of q the optimum capacity choice will be global maximum within the set Γ so first order conditions are sufficient to guarantee an optimum capacity choice. Further, it guarantees that this optimum is locally continuous in other model parameters, by the inverse function theorem. Assumption 3.1.d states that the optimum choice of capacity and production rate when restricted to set Γ is in the interior of Γ. By the inverse function theorem, this assumption implies that this optimum is a continuous function of other model parameters.

These assumptions guarantee that within a set Γ, the optimum capacity choice-production volume is a continuous function of other parameters such as a, b, ξ, or K within some neighborhood of the original parameter values. Although Assumptions 3.1 only generate local properties, in the examples we present, they are in fact global properties. Finally, note that there are no formal assumptions of complementarity between functions.

For the reminder of the paper, when optimal $\delta^*(q, \xi)$ is used, we will drop the optimal decision δ^* from the notation as we assume optimal decision have been made. We write:

$$\begin{aligned} s(q,\xi) &= s(q,\delta^*(q,\xi),\xi) \\ \mathcal{C}(q,\xi) &= \mathcal{C}(q,\delta^*(q,\xi)), \\ \Pi(q,\xi) &= \Pi(q,\delta^*(q,\xi),\xi) \text{ and} \\ \Pi(\xi) &= \operatorname*{Max}_q \Pi(q,\delta^*(q,\xi),\xi). \end{aligned}$$

The next section defines the improvement programs.

3.1 Uncertainty Reduction, Cost Bias Elimination and Direct Cost Reduction Programs

As in Sect. 2, three improvement programs are studied. First, a program to reduce uncertainty about parameter ξ is shown. As before, uncertainty reduction adds profit, because $\vartheta(\xi)$ is concave in ξ.

Proposition 3.2 *The value of uncertainty reduction about* ξ *is positive:* $E_\xi \Pi(q, \tilde{\xi}) \geq \Pi(q, \bar{\xi})$.

Proof Suppose ξ is uncertain. Then $\Pi(q,\xi,\delta) = (P(q) - \vartheta(\xi)s(q,\delta))q - \mathcal{C}(q,\delta)$ is a convex function of ξ, and thus, when optimizing with respect to δ will be a convex function of ξ. Thus, the function $\Pi(q,\xi) = P(q) - \vartheta(\xi)s(q,\delta^*(q,\xi))q - \mathcal{C}(q,\delta^*(q,\xi))$ is convex in ξ. QED

The second improvement program is one where a manager misestimates a parameter used to set the service or the production level, and the improvement program corrects bias in parameter estimates so that better decisions can be made. An assumption of the form of the cost function and the nature of the bias must be assumed. The cost function of Sect. 2 is generalized as partitioned between components linear in q and non-linear in q. That is, we rewrite $\xi s(q,\xi)q + \mathcal{C}(q,\xi)$ as the sum of all its linear terms $(\gamma + c + \xi)q$ plus its nonlinear terms in q, $C(q,\xi)$.

$$(\gamma + c + \xi)q + C(q,\xi) \triangleq \xi s(q,\xi)q + \mathcal{C}(q,\xi). \tag{3.2}$$

I will assume that a manager uses inaccurate non-linear product cost $rC(q,\xi)$ with $r > 0$, when choosing "optimal" q. This cost-form is generalized at the end of this section.

Distorted cost causes a suboptimal production decision to be made, and it is that distortion which reduces real profit. If the firm uses an inaccurate cost function, the firm's profit is *ex-ante forecasted* to be

$$\Pi(q,\xi,r) = P(q)q - (\gamma + c + \xi)q - rC(q,\xi), \tag{3.3}$$

where $\Pi(q,\xi,r)$ is the perceived, but inaccurate profit. The optimum q under biased cost yields

$$q(\xi,r) = \operatorname{Arg}\operatorname*{Max}_q \left(P(q) - \gamma + c + \xi\right)q - rC(q,\xi) \tag{3.4}$$

with real profits

$$\Pi(\xi,r) = P(q(\xi,r))q(\xi,r) - (\gamma + c + \xi)q(\xi,r) - C(q(\xi,r),\xi). \tag{3.5}$$

The value added by eliminating cost estimating bias is equal to $\Pi(\xi) - \Pi(\xi,r)$.

The third improvement program reduces cost. It is a reduction in unit cost c.

The next three sections study the complementarity of these programs.

3.2 Complementarity of Uncertainty Reduction and Cost Bias Elimination Programs

We now can state sufficient conditions for the uncertainty reduction and cost bias elimination improvement programs to be complements or substitutes.

Proposition 3.3

(i) *A cost bias elimination program and uncertainty reduction program are complements if* $\Pi(\xi) - \Pi(\xi, r)$ *is convex in* ξ.
(ii) *A cost bias elimination program and uncertainty reduction program are substitutes if* $\Pi(\xi) - \Pi(\xi, r)$ *is concave in* ξ.

Proof Case i: If $\Pi(\xi) - \Pi(\xi, r)$ is convex in ξ, then $E_{\tilde{\xi}}\Pi(\xi) - E_{\tilde{\xi}}\Pi(\xi, r) \geq \Pi(\bar{\xi}, r) - \Pi(\bar{\xi})$. This implies the result. Case ii is similarly proved. QED

Discussion: One way to establish the concavity/convexity of $\Pi(\xi) - \Pi(\xi, r)$ is to study the function $\frac{d\Pi^{k+1}(\xi, q(\xi, r))}{d\xi^k dr}$. Information about the sign of this derivative when $r = 1$ will establish local convexity/concavity properties near $r = 1$. The definition of $q(\xi, r)$ requires that $\frac{d\Pi(\xi, q)}{dq}|_{q=q(1,\xi)} = 0$, for all ξ. Thus,

$$\frac{d\Pi(\xi, q(\xi, r))}{dr}|_{r=1} = \frac{d\Pi(\xi, q)}{dq}|_{q=q(\xi,1)} \frac{dq(\xi, r)}{dr}|_{r=1} = 0 \text{ for all } \xi. \tag{3.6}$$

That is, no matter how ξ may change, $\frac{d\Pi(\xi, q(\xi, r))}{dr}|_{q=q(\xi,1)} = 0$. This implies by differentiation by ξ that $\frac{d\Pi^2(\xi, q(\xi, r))}{d\xi dr}|_{r=1} \triangleq 0$ for all ξ, and in general, $\frac{d\Pi^{k+1}(\xi, q(\xi, r))}{d\xi^k dr}|_{r=1} \triangleq 0$ for any ξ and any $k \geq 1$. We might conclude that $\frac{d\Pi^3(\xi, q(\xi, r))}{drd\xi^2}|_{r=1}$ cannot be used to predict local complementarity properties based upon profit derivatives when $r = 1$.

However this assertion is false: local behavior of a properly defined function at $r = 1$ is sufficient to determine local complementarity. This is shown next. To do so I define the mixed derivative of profit ignoring the non-linear in q cost function *C:*

$$\textit{Condition A}: \ g(r, \xi) = \frac{d^3(P(q(\xi, r) - (a_1\gamma + a_2c + a_3\xi))q(\xi, r))}{drd\xi^2} > 0.$$

$$\textit{Condition B}: \ g(r, \xi) = \frac{d^3(P(q(\xi, r) - (a_1\gamma + a_2c + a_3\xi))q(\xi, r))}{drd\xi^2} < 0$$

Proposition 3.4 *Suppose Condition A holds in an open neighborhood of* ξ *for* $r = 1$. *Then there exists a neighborhood of* $r = 1$, *with* $\underline{r} < 1 < \bar{r}$, *where uncertainty reduction and cost bias elimination are complements for any distribution function on* ξ *with support within the open neighborhood. Likewise, if Condition B holds in an open neighborhood of* ξ *for* $r = 1$ *then there exists a neighborhood of* $r = 1$, *with* $\underline{r} < 0 < \bar{r}$ *where uncertainty reduction and bias elimination are substitutes for any distribution function on* ξ *with support within a neighborhood of* ξ.

A proof is found in the "Appendix".

Although it is true that $\frac{d^3\Pi(q,\xi,r)}{drd\xi^2}|_{r=1,q=q(\xi,1)}=0$, complementarity can be characterized in the neighborhood of $r = 1$ *by* studying the mixed-partial derivative of revenue minus undistorted cost. I will say that "*Condition A holds for interval* $(\underline{R},\bar{R})$," with $0\leq\underline{R}<1<\bar{R}$ if Condition A holds for all $r\in(\underline{R},\bar{R})$ on this interval. Given an interval $[\underline{\xi},\bar{\xi}]$ with $\underline{\xi}>0$, I will say that "*Condition A holds on* $[\underline{\xi},\bar{\xi}]$ *for* $(\underline{R},\bar{R})$," if Condition A holds for all $\xi\in[\underline{\xi},\bar{\xi}]$ and $r\in(\underline{R},\bar{R})$. All of these definitions can be restated in terms of equivalent relations for Condition B.

Next global conditions for complementarity of uncertainty reduction and cost bias elimination are presented:

Corollary 3.1 *If Condition A holds on* $[\underline{\xi},\bar{\xi}]$ *for* $(\underline{R},\bar{R})$ *then for any* $\xi\in[\underline{\xi},\bar{\xi}]$, $\Pi(q(s,\xi),\xi)-\Pi(q(1,\xi),\xi)$, *is convex for all* $s\in(\underline{R},\bar{R})$.

If Condition B holds on $[\underline{\xi},\bar{\xi}]$ *for* $(\underline{R},\bar{R})$ *then for any* $\xi\in[\underline{\xi},\bar{\xi}]$, $\Pi(q(s,\xi),\xi)-\Pi(q(1,\xi),\xi)$, *is concave for all* $s\in(\underline{R},\bar{R})$.

If the hypotheses of the corollary hold, then, Proposition 3.2 predicts complementarity or substitutability of uncertainty reduction and bias elimination.

The above results hold when cost bias is of a more general form. The proof of Proposition 3.3 shows the validity of the following formulation of cost bias.

Lemma 3.1 *If total cost is* $C_1(q,\xi)+C_2(q,\xi)\triangleq\xi s(q,\xi)q+\mathcal{C}(q,\xi)$ *and cost bias is of the form:* $C_1(q,\xi)+rC_2(q,\xi)$, *then the preceding Proposition 3.2, and Corollary 3.1 hold with sign*$[\frac{d^3(P(q(r,\xi))-(a_1\gamma+a_2c+a_3\xi))q(r,\xi)}{drd\xi^2}|_{r=1,q=q(\xi,1)}]$ *replaced by sign*$[\frac{d^3P(q(r,\xi)q(r,\xi)-C_1(q,\xi))}{drd\xi^2}|_{r=1,q=q(\xi,1)}]$.

I close this subsection with two examples that demonstrate the usefulness of Conditions A and B.

Example 3.1 Consider the model of Sect. 2. Recall that $\Pi(q,\xi)=\sqrt{\alpha q}-\left\{(\xi_a t_o+c+\gamma)q+2\sqrt{\gamma\lambda(\xi_a,\xi_T)q}\right\}$ therefore "misestimated" profit is $\Pi^r(r,q,\xi)=\underset{q}{\mathrm{Max}}\ \sqrt{\alpha q}-(\xi_a t_o+c+\gamma)q+2r\sqrt{\gamma\lambda(\xi_a,\xi_T)q}$. By optimizing this later equation with respect to q, one obtains the general solution: $q(r,\xi)=\left(\frac{\frac{\sqrt{\alpha}}{2}-r\sqrt{\gamma\lambda(\xi_a,\xi_T)}}{(\xi_a t_o+c+\gamma)}\right)^2$.

If we study uncertainty in parameter ξ_T, then we are interested in properties of $\frac{d^3(P(q(r,\xi_T),\xi_T)-(c+\gamma+\xi_a t_0))q(r,\xi_T)}{d\xi_T^2dr}=\frac{2r\xi_a}{(c+\gamma+t_o\xi_a)\xi_T^2}>0$. Thus, Condition A holds and uncertainty reduction and bias elimination programs are substitutes. Note that this condition holds for all ξ and all *r:* so we know this is a global property for all distribution functions and any degree of bias.

If we study uncertainty in parameter ξ_a, then we are interested in properties of $\frac{d^3(P(q(r,\xi_T),\xi_T)-(c+\gamma+\xi_a t_0))q(r,\xi_T)}{d\xi_a^2dr}=\frac{2\gamma(r-1)^2\left((c+\gamma)(c+\gamma+2_o\xi_a)-t_o^2\xi_a^2\right)}{(c+\gamma+t_o\xi_a)^3\xi_a}$. The sign of this expression is independent of r and depends on the sign of $(c+\gamma)(c+\gamma+2_o\xi_a) - t_o^2\xi_a^2$. Complementarity properties are the same if $r < 1$, or $r > 1$: if bias is positive or negative!

Example 3.2 In this example I assume demand function in a different form than before, and a cost function that is convex. Let $P(q) = a - bq$ be the inverse demand function, where q is the production rate. Let the sum of linear and non linear production cost and capacity cost be $cq + q^2 + \delta\gamma$ where δ is the capacity decision and γ is the unit cost of capacity, If the customer's delay is $\frac{1}{\delta - q}$ then the cost of delay is: $\xi s(\delta, q) = \frac{\xi}{\delta - q}$. The cost function is thus $Cost = \left(c + \frac{\xi}{\delta - q}\right)q + q^2 + \gamma q$ and the profit function is $\pi(\delta, q) = \left(a - bq - c - \frac{\xi}{\delta - q}\right)q - q^2 - \gamma q$, where q is the production rate decision and δ is the capacity decision. First, I check the Assumptions 3.1: (a) holds as inverse demand is twice continuously differentiable in q; (b) The cost function is concave in δ for all $q \geq 0$ as the second derivative of cost with respect to δ is $= \frac{2\gamma^{3/2}}{\sqrt{q\xi}} > 0$. The optimal solution is $\delta^*(q, \xi) = \frac{\sqrt{\gamma}q + \sqrt{q\xi}}{\sqrt{\gamma}}$; (c) Using the optimal choice of δ, profit is a concave function of q if $q \geq \left(\frac{1}{2^2}\frac{\sqrt{\gamma\xi}}{1+b}\right)^{2/3}$. The profit function is $\pi(\delta^*, q) = (a + q - bq + \gamma)q - 2\sqrt{\gamma q\xi}$, with properties $\pi(0, \delta^*) = 0$, $\frac{d^2\pi(\delta^*, 0)}{dq^2} > 0$, $\lim_{q\to\infty}\pi(q, \delta^*) = -\infty$, $\lim_{q\to\infty}\pi\frac{d^2\pi(q,\delta^*)}{dq^2} < 0$, and with 3 (complex and real) roots to the equation $\frac{d\pi(\delta^*, q)}{dq} = 0$. The profit function can change direction at most twice when $q \geq 0$ (3 changes are impossible given the last listed properties and four changes leads to 4 real roots to the first derivative of profit on $q \geq 0$). This shows that the optimal value of q *is* either $q = 0$ *or* $q > 0$. Defining $\Gamma = \{(\delta, q) \in \mathbb{R}_+ \times [\left(\frac{1}{2^2}\frac{\sqrt{\gamma\xi}}{1+b}\right)^{2/3}, \infty]\}$, the condition $\frac{d\pi(q,\delta^*)}{dq}\Big|_{q=\left(\frac{1}{2^2}\frac{\sqrt{\gamma\xi}}{1+b}\right)^{2/3}} > 0$ can be guaranteed by examining the left hand side which is a polynomial in ξ in the form $-(e(1+b))^3\gamma + f\left((a-\gamma) - g\gamma^{1/3}\right)^3\xi$, with e, f and $g > 0$. It is seen that for fixed value of ξ, with sufficiently small values of b, and γ, or sufficiently large values of a, the inequality will hold. When it does, the global maximum production rate is in the interior of Γ and satisfied the first order equations, and assumption 3.1d holds. Thus, all assumptions hold.

Computing the "modified" profit function with non-linear cost terms dropped and the optimum production rate for cost function π^r generates the function $\pi^m(\delta^*, q(r)) = (a - b^m q - c - \gamma)q(r)$. The function $\frac{d^3\pi^m}{dr d\xi^2}\big|_{r=1,q^r}$ is shown in Fig. 1 and has a positive value in the region shown. Thus bias and uncertainty reduction are complements.

I next study the *direct cost savings program* and its complementarity to uncertainty reduction.

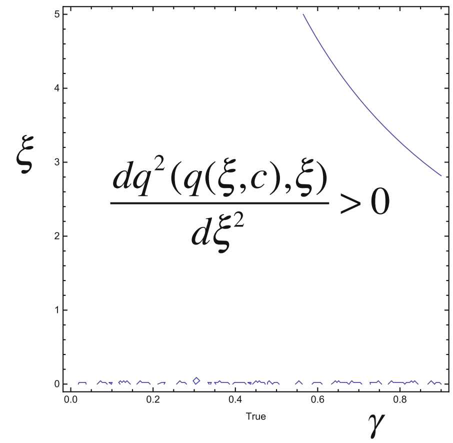

Fig. 1 For Example 3.1, the derivative $\frac{d^3\Pi(q,\xi,r)}{drd\xi^2}|_{r=1,q=q(\xi,1)}$ is positive, and the objective function is non-negative for all values of γ and ξ within the contour. For that region, uncertainty reduction and cost bias elimination are complements. Here $a = 10$, $b = 3$, $c = 0$

3.3 Complementarity of Direct Cost Savings and Uncertainty Reduction Programs

We write firm profit with a production rate q, parameter ξ, and unit production cost c as $\Pi(q, \xi, c)$. Let optimal production levels be $q(\xi, c)$. Production rates chosen using biased cost is denoted $q(\xi, c, r)$. First, a general condition for complementarity of cost savings and uncertainty reduction is presented:

Proposition 3.5

(i) *A direct cost savings and a uncertainty reduction program are complements if for any* c, c', $\Pi(\hat{q}(\xi, c, r), \xi, c) - \Pi(\hat{q}(\xi, c', r), \xi, c')$, *is concave in* ξ.
(ii) *A direct cost savings and a uncertainty reduction program are complements if for any* c, c', $\Pi(\hat{q}(\xi, c, r), \xi, c) - \Pi(\hat{q}(\xi, c', r), \xi, c')$, *is convex in* ξ.

Proof Case i: If $\Pi(\hat{q}(\xi, c, r), \xi, c) - \Pi(\hat{q}(\xi, c', r), \xi, c')$ is concave in ξ, then $E_{\tilde{\xi}}\Pi(\hat{q}(\xi, c, r), \xi, c) - \Pi(\hat{q}(\bar{\xi}, c', r), \bar{\xi}, c')$. This implies the result. Case ii is similarly proved. QED

If convexity or concavity of the above functions cannot be established, the following is an easy to compute sufficient condition for local complementarity or substitutability of cost saving and uncertainty reduction.

Proposition 3.6 *Assume the profit function has the form:*

$$\Pi(q, \xi, c) = P(q)q - cq - C(q, \xi).$$

Suppose that $q(\xi, c)$ *corresponds to the optimal production level. Then if* $\frac{d^2q(\xi,r),\xi,c)}{d\xi^2} > 0$, *direct cost savings is complementary to uncertainty reduction for all*

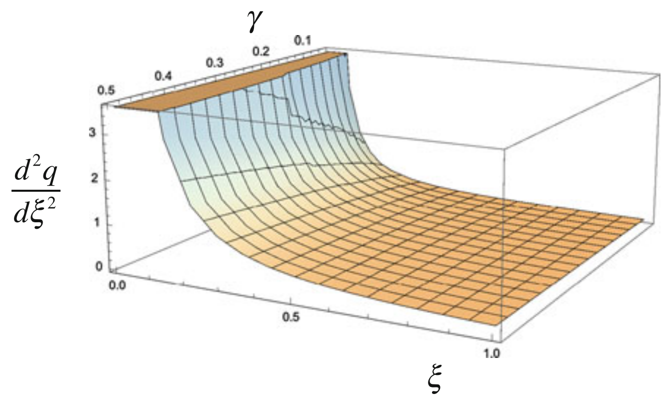

Fig. 2 For Example 3.2, the value of $\frac{dq^2(q(\xi,c),\xi)}{d\xi^2}$ is positive for the indicated values of ξ and γ. This implies that direct cost savings and uncertainty reduction are complements in this region. Here $a = 10$, $b = 3$, $c = 0$

c' in an open neighborhood of c. Alternatively, if $\frac{d^2q(q(\xi,r),\xi,c)}{d\xi^2} < 0$, direct cost savings is a substitute to uncertainty reduction for all c' in an open neighborhood of c.

Proof Differentiate the profit function with respect to *c:* $\frac{d\Pi(q(\xi,r),\xi,c)}{dc} = -q + \frac{dq}{dc}\frac{d\Pi(q(\xi,r),\xi,c)}{dq} = -q.$

It is clear that $\frac{d^3\Pi(q(\xi,r),\xi,c)}{d\xi^2 dc} = -\frac{d^2q(\xi,r),\xi,c)}{d\xi^2}$. For complementarity to hold, as *c* *increases* the curvature of profit must decrease; similarly, as *c decreases* the curvature of profit must increase. Increasing curvature increases the value added by uncertainty reduction. QED.

Example 3.1 continued When computing: $\frac{d^2q(\xi,r),\xi,c)}{d\xi_a^2}$ and $\frac{d^2q(\xi,r),\xi,c)}{d\xi_T^2}$ yield algebraic expressions that are always positive if and only if profit is positive.

Example 3.2 continued In Fig. 2, $\frac{d^2q(\xi,r),\xi,c)}{d\xi_a^2}$ is plotted as a function of γ, and ξ. The figure shows that this expression is uniformly positive, so that we know direct cost saving is complementary to uncertainty reduction for this model.

3.4 Complementarity of Bias Elimination and Direct Cost Savings Programs

For completeness, I next study the complementarity of direct cost savings and cost bias elimination. Because of the difficulty to establish global requirements, the sufficient conditions I develop are local tests. Of course, if these local tests hold on the entire domain, the results are global.

Lemma 3.2

i. *If $\frac{d^2C(q)}{dq^2}|_{q=q(c,r)} < 0$ and $\frac{d^3\Pi(q,c,r)}{dq^3}|_{q=q(c,r)} > 0$, then cost reduction and bias reduction programs are complements in a neighborhood of r.*

ii. *If* $\frac{d^2C(q)}{dq^2}|_{q=q(c,r)} > 0$ *and* $\frac{d^3\Pi(q,c,r)}{dq^3}|_{q=q(c,r)} < 0$, *then cost reduction and bias reduction programs are substitutes programs in a neighborhood of r.*

Proof See the "Appendix". It is notable that in this result the derivatives do not include differentiation with respect to *r*. This is quite unlike typical supermodularity tests involving mixed partials of variables associated with the programs in question. The next example demonstrates the proposition.

Example 3.1 Given profit and cost functions from Sect. 2:

$$\frac{d^2C(q)}{dq^2} = \frac{d^2}{dq^2}\left[\left(\bar{\xi}_a t_o + c + \gamma\right)q + 2\sqrt{\lambda(\bar{\xi}_a, \bar{\xi}_T)}\sqrt{\gamma q}\right] = -\frac{1}{2}\frac{\sqrt{\gamma\lambda(\xi_a, \xi_T)}}{q^{3/2}} < 0$$

$$\frac{d^3\Pi(q,c,r)}{dq^3} = \frac{d^3}{dq^3}\left[\sqrt{\alpha q} - \left[\left(\bar{\xi}_a t_o + c + \gamma\right)q + 2\sqrt{\lambda(\bar{\xi}_a, \bar{\xi}_T)}\sqrt{\gamma q}\right]\right]$$

$$= \frac{3}{8}\frac{\sqrt{\alpha} - 2\sqrt{\gamma\lambda(\xi_a, \xi_T)}}{q^{5/2}} > 0$$

and both expressions hold for any q. Thus the conclusion is that bias elimination and direct cost savings are complements. As this holds for any *q*, it is a global property.

Example 3.2 continued Discussion earlier in this section showed that the cost function is concave in *q*. Computing: $\frac{d^3\Pi(q,c,r)}{dq^3} = \frac{3r}{4}\sqrt{\frac{\gamma\xi}{q^5}} > 0$, so we see that bias elimination and direct cost savings are also complements.

4 Conclusion

Milgrom and Roberts (1990) show that supermodularity assumptions imply the complementarity of improvement programs, But do realistic demand function and operations processes display the necessary properties for programs to be complementary? In this paper I show that complementarity does not hold for common programs, and that some kinds of programs are substitutes while simultaneously others are complements.

In an extended analysis, I have demonstrated these effects on a commonly assumed operations process, namely queuing. This is important because other prominent operations technologies share the cost characteristics of queuing: in the simplest models total cost is proportional to the square root of production volume. For example, the simplest versions of the economic-order-quantity (EOQ) model and many fixed-charge models have this characteristic. Thus, the results of Sect. 2 apply more commonly than suspected. In Sect. 3, I developed sufficient conditions for complementarity both locally and globally for more general demand and cost functions.

My results reinforce the value for decentralized organizations of coordinating adoption of programs across functional boundaries. As recognized by Milgrom and Roberts (1990), when complementarities exist, it is desirable for a central authority to coordinate adoption of programs even when programs are focused on decentralized subunits because more value may be created than each subunit realizes. As modeled in this paper, improvement programs might be proposed independently by the marketing, accounting, and operations organizations. However, in the examples of this paper, substitutability of programs is shown to arise. Thus, central coordination may be necessary because independently proposed projects may *not* be as valuable as "locally" estimated ones. For both reasons, positive and negative, adding or subtracting value, centralized review of projects helps maximize firm value.

Finally, empirical study of these issues is a very important research question. One way to test premises of complementarity is to study how firms organize the evaluation of proposed improvement projects. If complementary or substitute effects hold, it would be expected that firms involve players from other functions. One could ask if business functions outside of the proposer's are involved in the adoption decision of improvement projects, and what types of projects merit this scrutiny. Another way to determine if a type of improvement program is complementary to other programs is to ask if the firm's policies make it easier to adopt this program than other types of programs. For example, are cost reduction projects (which we show tend to be complementary to other programs) evaluated at a lower discount rate than other types of proposals.

Appendix

Proof of Proposition 3.4 Define $\Pi^r(q,\xi,r) = P(q)q - (a_1\gamma + a_2c + a_3\xi)q - rC(q,\xi)$, which we distinguish from $\Pi(q,\xi) = P(q)q - (a_1\gamma + a_2c + a_3\xi)q - C(q,\xi)$.

Let $q(\xi,r)$ be defined as the solution to the first-order equation,

$$\frac{d\Pi^r(q(\xi,r),\xi,r)}{dq} = \frac{dP(q)q}{dq}|_{q=q(r,\xi)} - (a_1\gamma + a_2c + a_3\xi) - r\frac{dC(q,\xi)}{dq}|_{q=q(r,\xi)} = 0. \tag{A.1}$$

This implies $\frac{dC(q,\xi)}{dq}|_{q=q(r,\xi)} = \frac{1}{r}\left(\frac{dP(q)q}{dq}|_{q=q(r,\xi)} - (a_1\gamma + a_2c + a_3\xi)\right)$. It follows that we can express the derivative of actual profit with respect to q as:

$$\frac{d\Pi(q,\xi)}{dq}|_{q=q(r,\xi)} = \left(\frac{dP(q)q}{dq}|_{q=q(r,\xi)} - (a_1\gamma + a_2c + a_3\xi)\right)\left(1-\frac{1}{r}\right). \quad (A.2)$$

The derivative of (actual) profit with respect to r under production decision $q(\xi, r)$ is

$$\frac{d\Pi(q(\xi,r),\xi)}{dr}|_{q=q(r,\xi)} = \frac{d\Pi(q,\xi)}{dq}\frac{dq(\xi,r)}{dr}|_{q=q(r,\xi)}.$$

Substituting (A.2) into this last equation yields:

$$\frac{d\Pi(q(\xi,r),\xi)}{dr}|_{q=q(r,\xi)} = \left(\frac{dP(q)q}{dq}|_{q=q(r,\xi)} - a_1\gamma + a_2c + a_3\xi\right)\frac{dq(r,\xi)}{dr}\left(1-\frac{1}{r}\right)|_{q=q(r,\xi)}. \quad (A.3)$$

My goal is to determine the sign of $\frac{d\Pi^3(q(\xi,r),\xi)}{drd\xi^2}|_{q=q(r,\xi)}$ near $r = 1$. To do this I need to show that $\frac{dq(r,\xi)}{dr}$ is C^2 in an open neighborhood of $(1, \xi)$. Differentiating (A.1) with respect to r, $\frac{d\Pi^{r^2}(q(\xi,r),\xi,r)}{drdq} = -\frac{\partial C(q(\xi,r),\xi)}{\partial\xi\partial q} + \frac{\partial\Pi^{r^2}(q(\xi,r),\xi)}{\partial q^2}\frac{dq(\xi,r)}{dr} = 0$.

If $\frac{\partial\Pi^{r^2}(q(\xi,r),\xi)}{\partial q^2}|_{q=q(r=1,\xi)} \neq 0$ the implicit function theorem states that $\frac{dq(\xi,r)}{dr} = \frac{\frac{\partial C(q(\xi,r),\xi)}{\partial q}}{\frac{\partial\Pi^{r^2}(q(\xi,r),\xi)}{\partial q^2}}$. Additionally, this derivative will be twice continuously differentiable in a neighborhood of $r = 1$.

Differentiating (A.3) with respect to ξ twice:

$$\frac{d^2\Pi(q,\xi,r)}{drd\xi}|_{q=q(r,\xi)} = \left(\begin{array}{c}\frac{d^2P(q)q}{dq^2}\frac{dq(r,\xi)}{d\xi}\frac{dq(r,\xi)}{dr}\\ +\left(\frac{dP(q)q}{dq} - (a_1\gamma + a_2c + a_3\xi)\right)\frac{d^2q(r,\xi)}{drd\xi} - a_3\frac{dq(r,\xi)}{dr}\end{array}\right)|_{q=q(r,\xi)}\left(1-\frac{1}{r}\right)$$

$$\frac{d^3\Pi(q,\xi,r)}{drd\xi^2}|_{q=q(\xi,r)} = \left(\begin{array}{c}\left(\frac{d^3P(q)q}{dq^3}\right)\left(\frac{dq(\xi,r)}{d\xi}\right)^2\frac{dq(\xi,r)}{dr} + \left(\frac{d^2P(q)q}{dq^2}\right)\frac{d^2q(\xi,r)}{d\xi^2}\frac{dq(\xi,r)}{dr}\\ +2\left(\frac{d^2P(q)q}{dq^2}\frac{dq(\xi,r)}{d\xi} - a_3\right)\frac{d^2q(\xi,r)}{drd\xi}\\ +\left(\frac{dP(q)q}{dq} - (a_1\gamma + a_2c + a_3\xi)\right)\frac{d^3q(\xi,r)}{dr\xi^2}\end{array}\right)|_{q=q(\xi,r)}\left(1-\frac{1}{r}\right). \quad (A.4)$$

Similar to the above argument that used the implicit function theorem, $\frac{dq(\xi,r)}{d\xi}, \frac{d^2q(\xi,r)}{d\xi dr}, \frac{d^3q(\xi,r)}{d\xi^2dr}, \frac{d^3q(\xi,r)}{d\xi dr^2}, \frac{d^3q(\xi,r)}{d\xi^3}$, can be derived and found to be continuously differentiable in (ξ, r) within a neighborhood of $r = 1$. All the terms in the large brackets within (A.4) are continuously differentiable in (ξ, r) within a neighborhood of $r = 1$. Thus, the sign of $\frac{d^3P(q(\xi,r)-(a_1\gamma+a_2c+a_3\xi))q(\xi,r)}{drd\xi^2}$ will determine if concavity with respect to ξ increases with r or decreases in r. I note that if the bias was underweighting cost, that is $r < 1$, *or* bias was over weighting cost, that is, $r > 1$, concavity with respect to ξ changes in the same direction.

Proof of Lemma 3.2 The profit function is written as $\Pi^r(q,c,r) = P(q)q - cq - rC(q,\xi)$.

q(c,r) maximizes Π^r, and can be found by solving $\frac{d\Pi^r(q(c,r),c,r)}{dq} = 0$. Differentiating with respect to cost yields $\frac{d}{dc}\frac{d\Pi(q(c,r),c,r)}{dq} = \frac{d^2\Pi(q(c,r),c,r)}{dq^2}\frac{dq(c,r)}{dc} - 1$. Differentiating with respect to r again yields:

$$\frac{d}{dr}\left[\frac{d^2\Pi(q(c,r),c,r)}{dq^2}\frac{dq(c,r)}{dc} - 1\right] = \frac{d^3\Pi(q(c,r),c,r)}{dq^3}\frac{dq(c,r)}{dc}\frac{dq(c,r)}{dr} - \frac{d^2C(q(c,r))}{dq^2}\frac{dq(c,r)}{dc} + \frac{d^2\Pi(q(c,r),c,r)}{dq^2}\frac{d^2q(c,r)}{dcdr}.$$

Thus, $\frac{d^2q(c,r)}{dcdr} = \frac{\frac{dq(c,r)}{dc}\left[\frac{d^2C(q(c,r))}{dq^2}\frac{dq(c,r)}{dc} - \frac{d^3\Pi(q(c,r),c,r)}{dq^3}\frac{dq(c,r)}{dr}\right]}{\frac{d^2\Pi(q(c,r),c,r)}{dq^2}}$. If the conditions of part i) hold, then $\frac{d^2q(c,r)}{dcdr} > 0$. Now consider $\Pi(q,c) = P(q)q - cq - C(q,\xi)$.

$$\frac{\partial^2\Pi(c,r)}{\partial c\partial r} = \frac{\partial}{\partial c}\left[\frac{d[P(q)q - cq - C(q)]}{dq}\Big|_{q=q(c,r)}\right] + \frac{d[P(q)q - cq - C(q)]}{dq}\frac{\partial^2 q(c,r)}{\partial c\partial r}.$$

The first term is zero. If I assume r > 1, then *q(c,r)* is less than *q(c,1) and* $\frac{d[P(q)q-cq-C(q)]}{dq}\Big|_{q=q(c,r)} > 0$. I conclude that $\frac{\partial^2\Pi(c,r)}{\partial c\partial r} > 0$ which implies that c and r are complements. Removing estimating bias (*decreasing* r from above $r = 1$) and *reducing* cost (reducing c) causes the cross partials of Π to be positive. If instead, $r < 1$, then $\frac{d[P(q)q-cq-C(q)]}{dq}\Big|_{q=q(c,r)} < 0$ and thus $\frac{\partial^2\Pi(c,r)}{\partial c\partial r} < 0$. Increasing r and decreasing c causes the cross partials of Π again to be positive: direct cost savings and bias elimination are complements. If the conditions of part ii) hold, then $\frac{d^2q(c,r)}{dcdr} < 0$. Following the logic of the above proof, if $r > 1$, reducing r and decreasing c results in the cross partials of to be negative. Similarly, if $r < 1$, then increasing r and decreasing c causes the cross partials of Π to be negative. In short, under conditions in part ii), bias elimination and direct cost savings are substitutes. *QED*

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